BITS F464 - Semester 1 - MACHINE LEARNING

PROJECT - MACHINE LEARNING FOR SUSTAINABLE DEVELOPMENT GOALS (SDGs)

Team number: 43

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Id number of all students in the team: 2021A7PS0182H, 2021A7PS0127H, 2021A7PS0097H, 2021A7PS2175H, 2021A3PS2937H

Please refer to the email providing the assignment of project and follow the instructions provided in the project brief.

In [ ]:
```
import numpy as np
import pandas as pd
import seaborn as sns
import matplotlib.pyplot as plt
import plotly.graph_objects as go
from sklearn.metrics import classification_report
import random
from collections import Counter
```

1. Preprocessing of Dataset

In [ ]:
```
#importing dataset
df = pd.read_csv('Carbon Emissions.csv')
df.head()
```

Out[ ]:

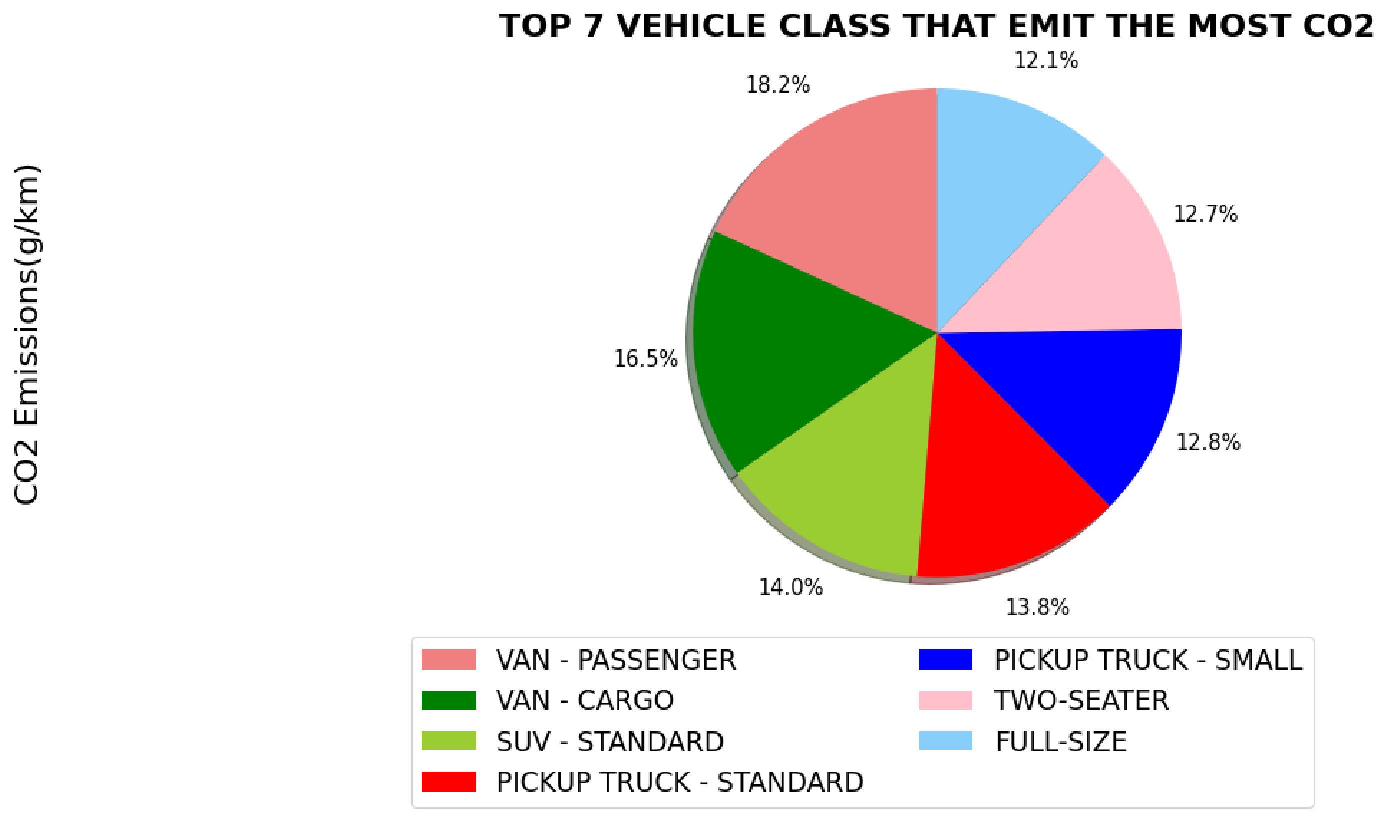
	Make	Model	Vehicle Class	Engine Size(L)	Cylinders	Transmission	Fuel Type	Fuel Consumption City (L/100 km)	Fuel Consumption Hwy (L/100 km)	Fuel Consumption Comb (L/100 km)	Fuel Consumption Comb (mpg)	Emissions
0	ACURA	ILX	COMPACT	2.0	4	AS5	Z	9.9	6.7	8.5	33	
1	ACURA	ILX	COMPACT	2.4	4	M6	Z	11.2	7.7	9.6	29	
2	ACURA	ILX HYBRID	COMPACT	1.5	4	AV7	Z	6.0	5.8	5.9	48	
3	ACURA	MDX 4WD	SUV - SMALL	3.5	6	AS6	Z	12.7	9.1	11.1	25	
4	ACURA	RDX AWD	SUV - SMALL	3.5	6	AS6	Z	12.1	8.7	10.6	27	

In [ ]:
```
numerical_columns = df.select_dtypes(include=[int, float])
plt.figure(figsize=(10,8))
sns.set_context('paper', font_scale=1.4)
sns.heatmap(numerical_columns.corr(), annot=True, cmap='Accent')
```

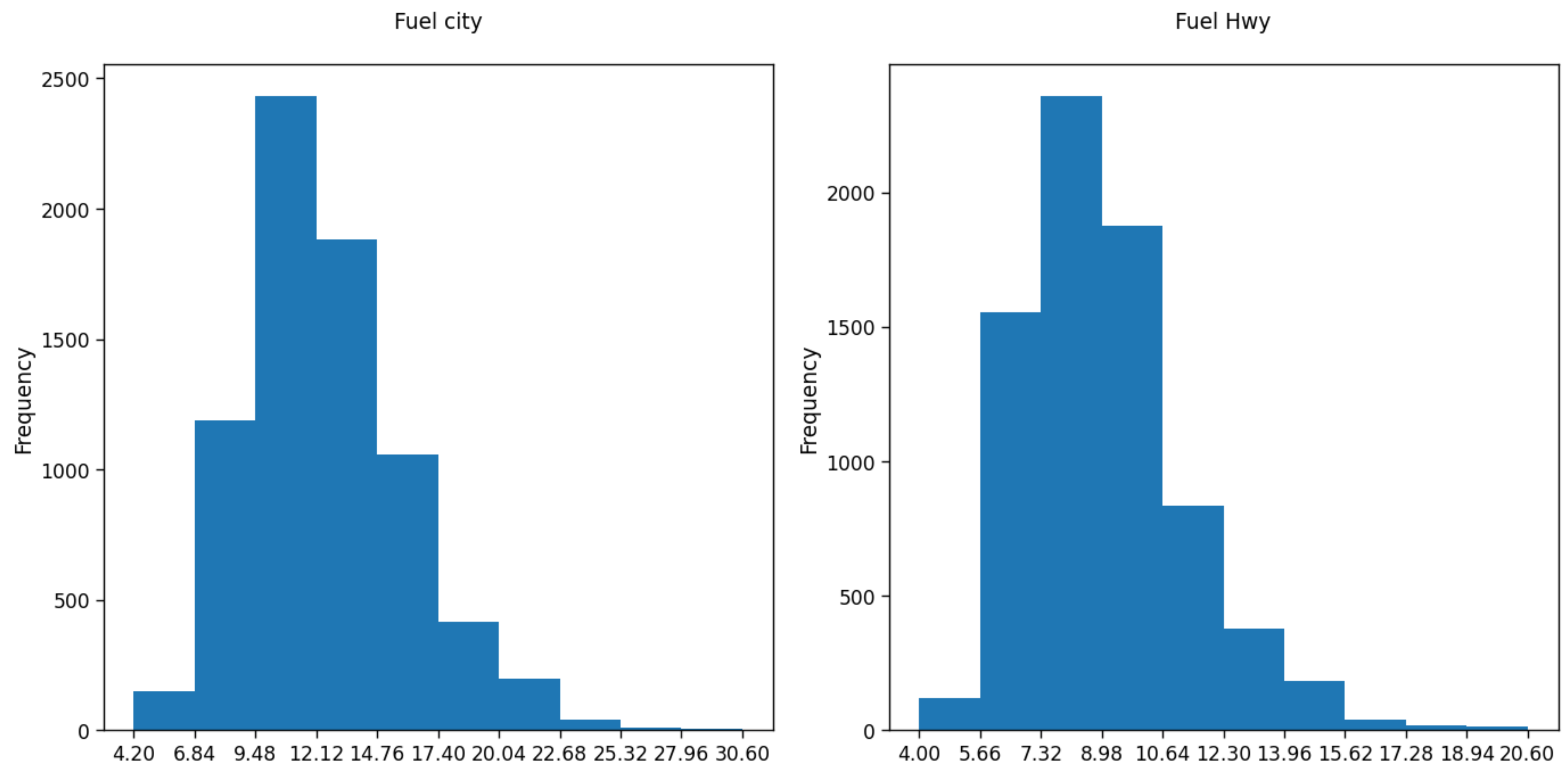
Out[ ]: <Axes: >

In [ ]:

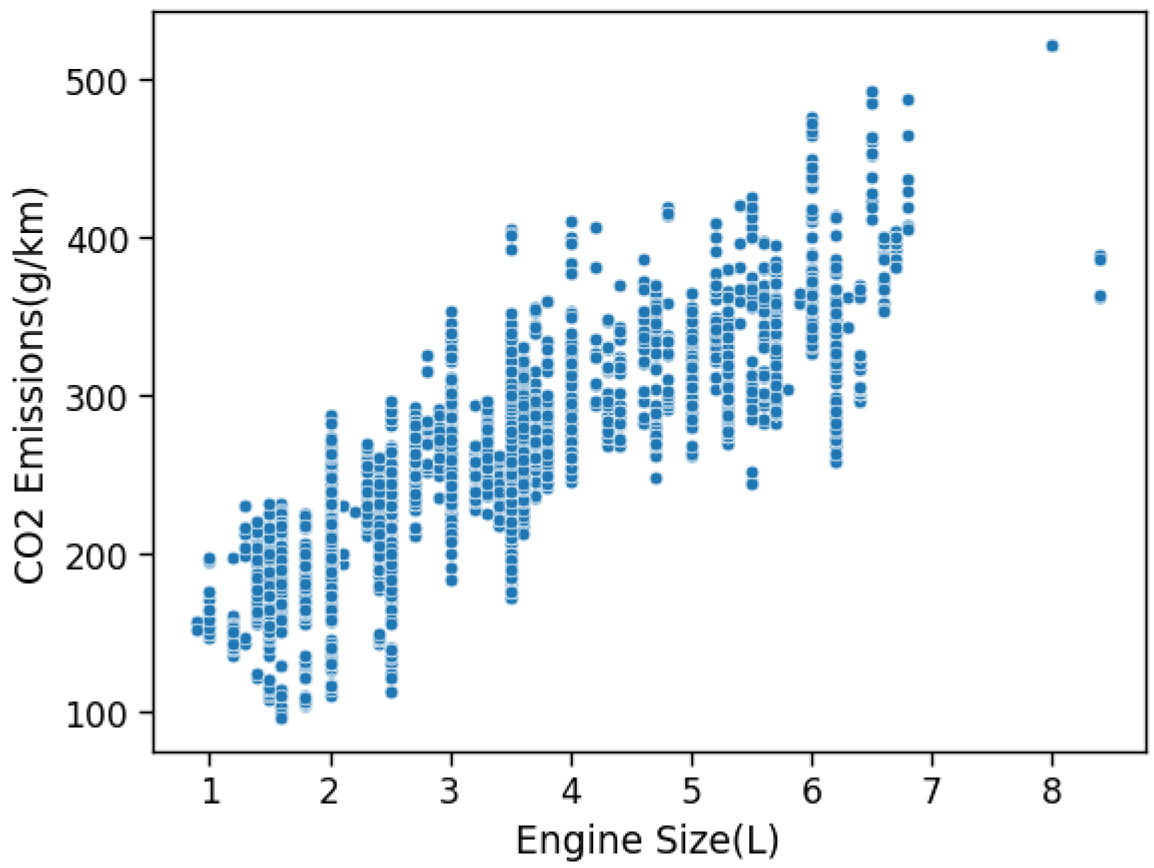
```
df1 = df.groupby(["Vehicle Class"])[["CO2 Emissions(g/km)"]].mean().sort_values("CO2 Emissions(g/km)", ascending = False)
df1["CO2 Emissions(g/km)"].head(7).plot(kind = "pie",
                                       figsize = (13, 6),
                                       colors = ["lightcoral", "green", "yellowgreen", "red", "blue", "pink", "lightskyblue"],
                                       shadow = True, explode = [0, 0, 0, 0, 0, 0, 0],
                                       autopct = "%.1f%%", textprops = {"fontsize":10.5} ,
                                       labels = None, startangle = 90,
                                       pctdistance = 1.2)
plt.title("TOP 7 VEHICLE CLASS THAT EMIT THE MOST CO2", fontsize = "large", fontweight = "bold", y = 1.03)
plt.legend(df1.index, ncol = 2, loc = "best", bbox_to_anchor=(0.72, -0.04), fontsize = 'medium')
plt.ylabel("CO2 Emissions(g/km)", fontsize = "large")

plt.axis("equal")
plt.tight_layout()
plt.show()
```

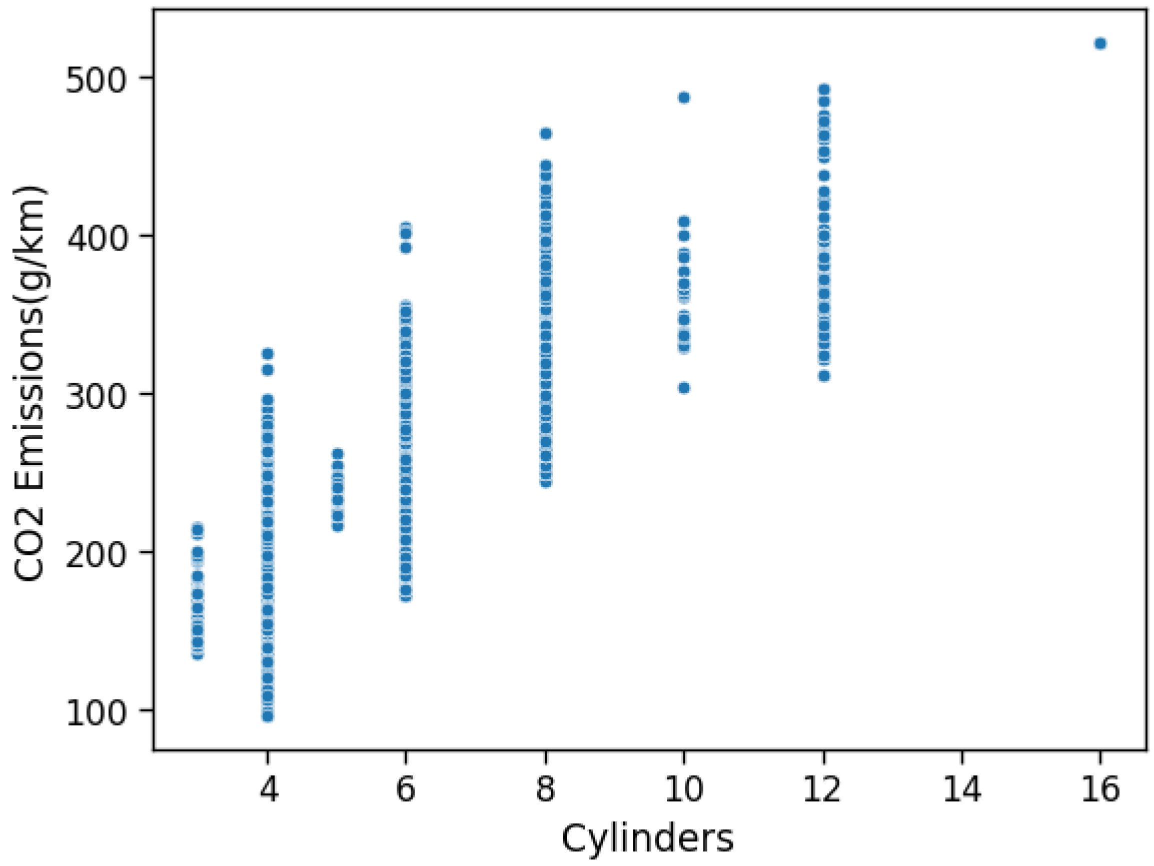


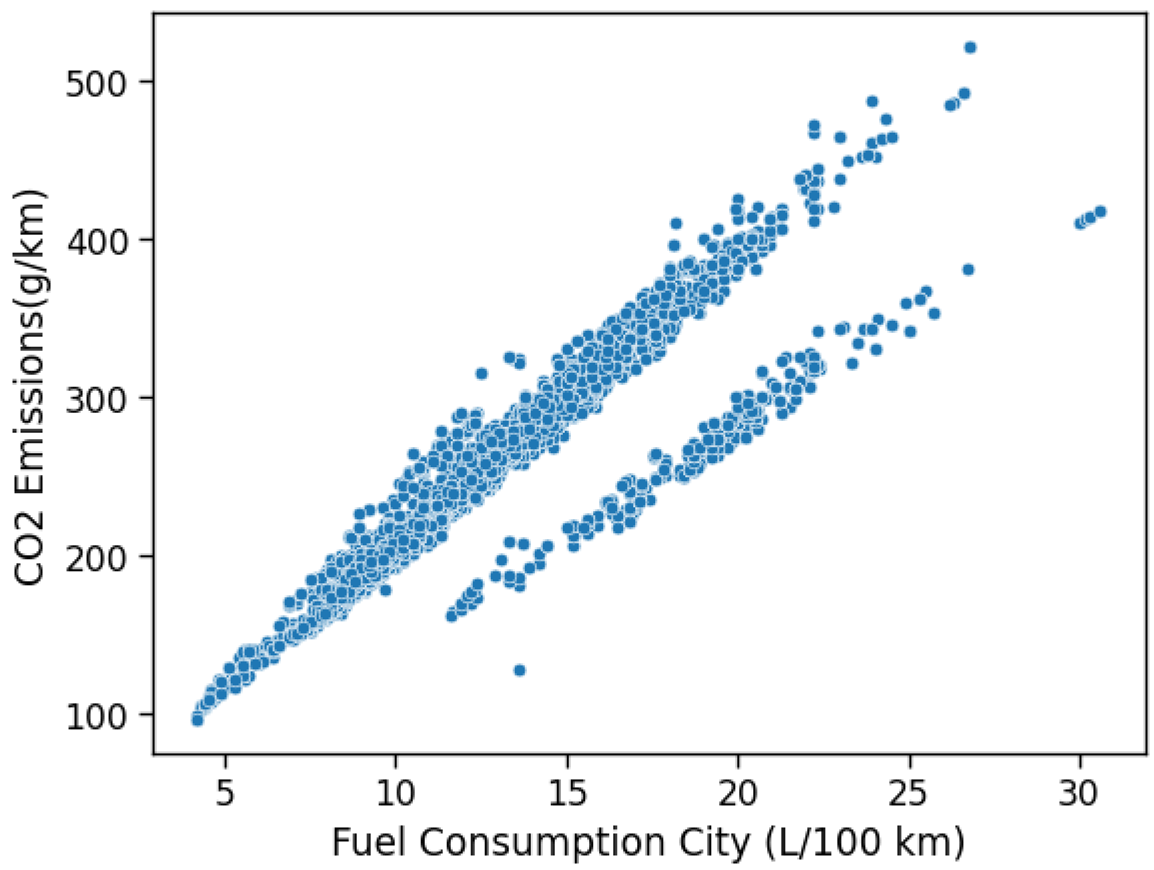
In [ ]:
```
count, bins_edge1 = np.histogram(df["Fuel Consumption City (L/100 km)"])
count, bins_edge2 = np.histogram(df["Fuel Consumption Hwy (L/100 km)"])

fig = plt.figure(figsize = (14,7))
ax1 = fig.add_subplot(1, 2, 1)
ax2 = fig.add_subplot(1, 2, 2)

df["Fuel Consumption City (L/100 km)"].plot(kind = "hist", xticks = bins_edge1, ax = ax1)
ax1.set_title("Fuel city", y = 1.04)

df["Fuel Consumption Hwy (L/100 km)"].plot(kind = "hist", xticks = bins_edge2 ,ax = ax2)
ax2.set_title("Fuel Hwy", y = 1.04)

plt.tight_layout()
plt.show()
```

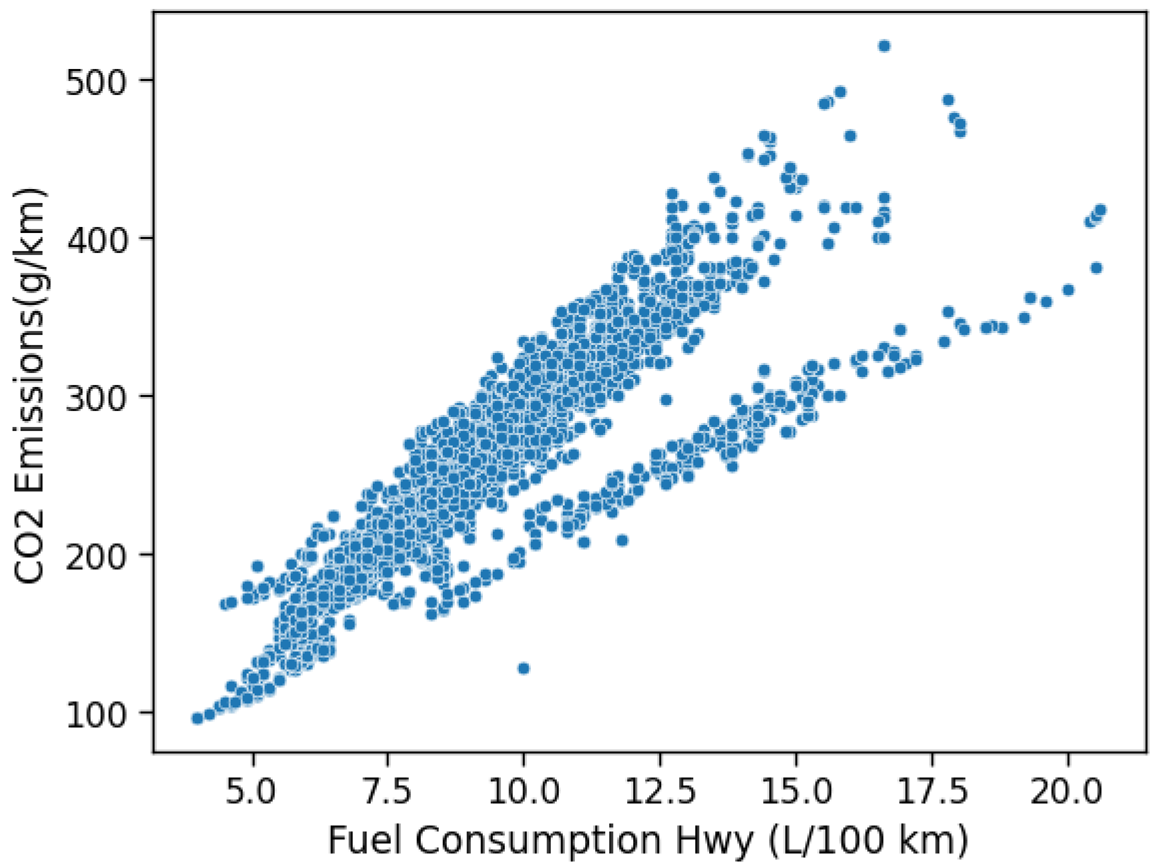


In [ ]:
```
for feature in df.select_dtypes('number').columns:
  sns.scatterplot(x=feature, y="CO2 Emissions(g/km)", data=df);
  plt.show()
```

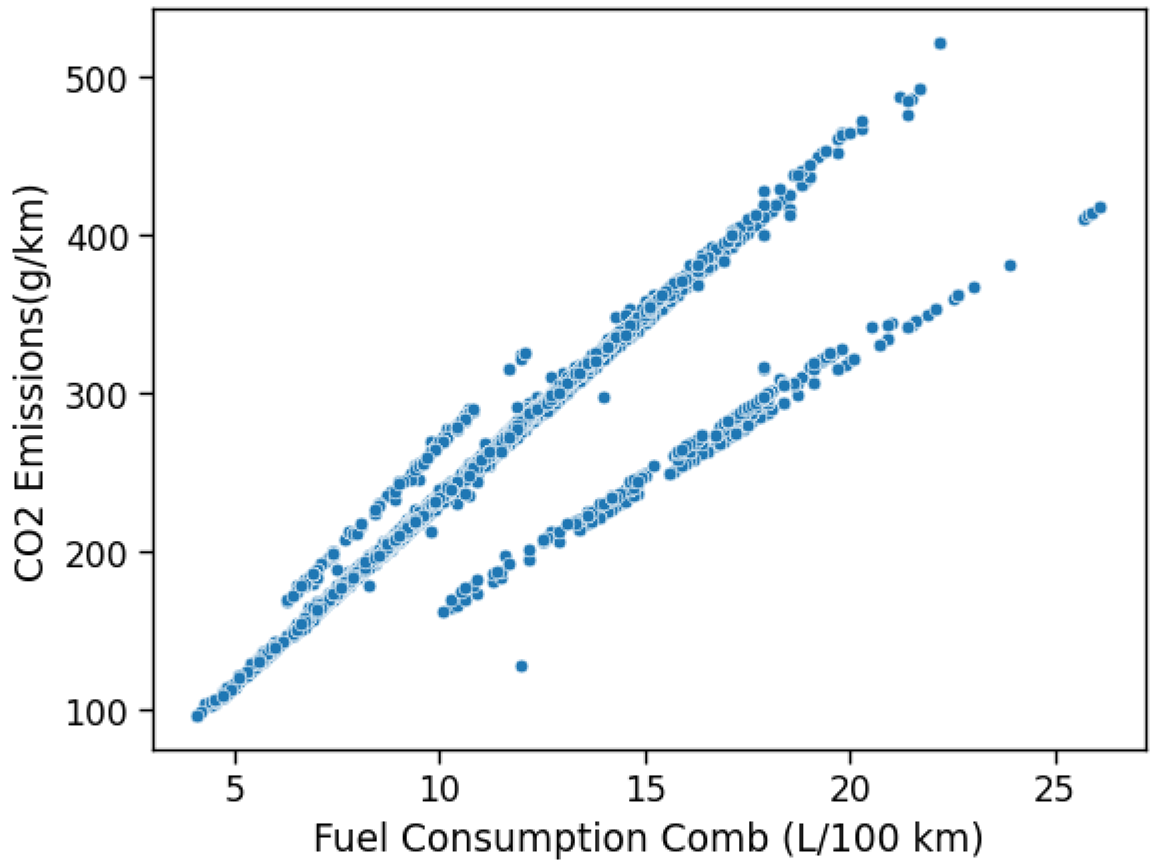

CO2 Emissions(g/km)
100
200
300
400
500
1
2
3
4
5
6
7
8
Engine Size(L)


CO2 Emissions(g/km)
100
200
300
400
500
4
6
8
10
12
14
16
Cylinders

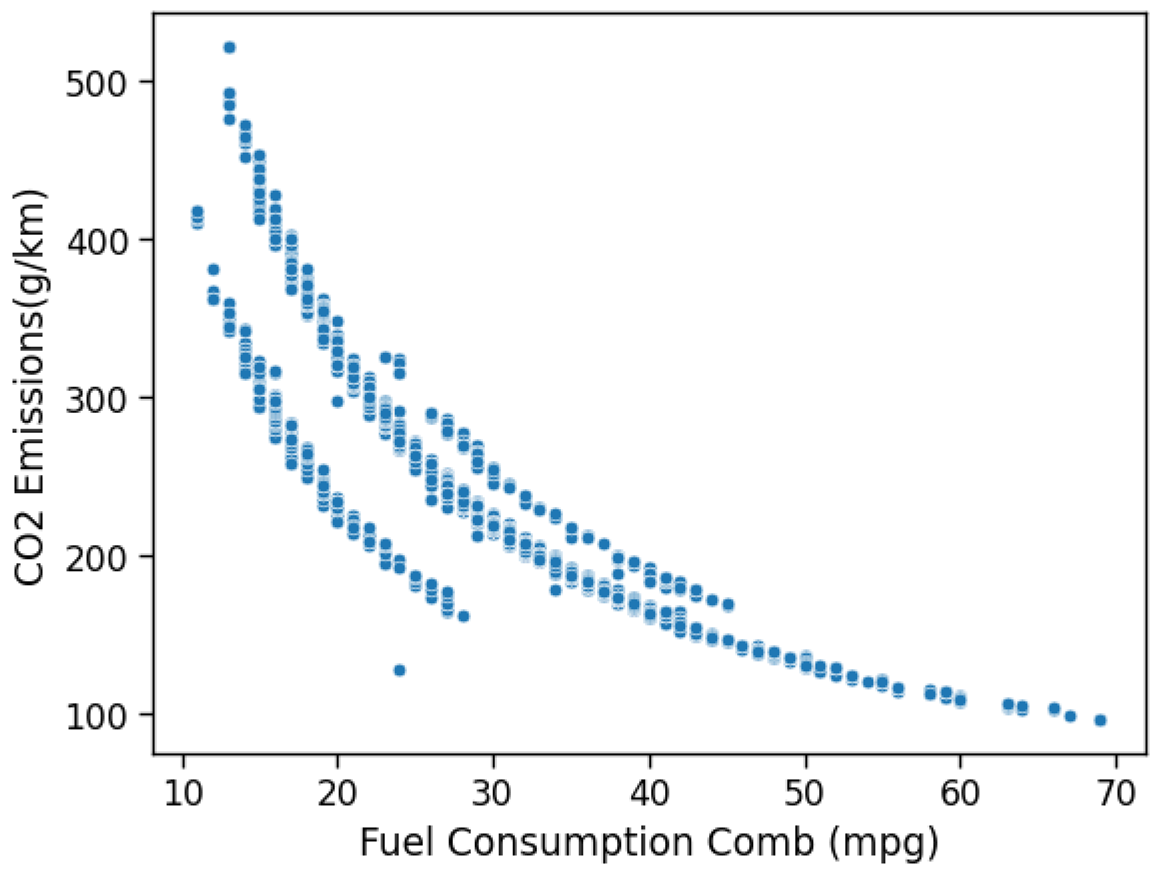

CO2 Emissions(g/km)
100
200
300
400
500
5
10
15
20
25
30
Fuel Consumption City (L/100 km)



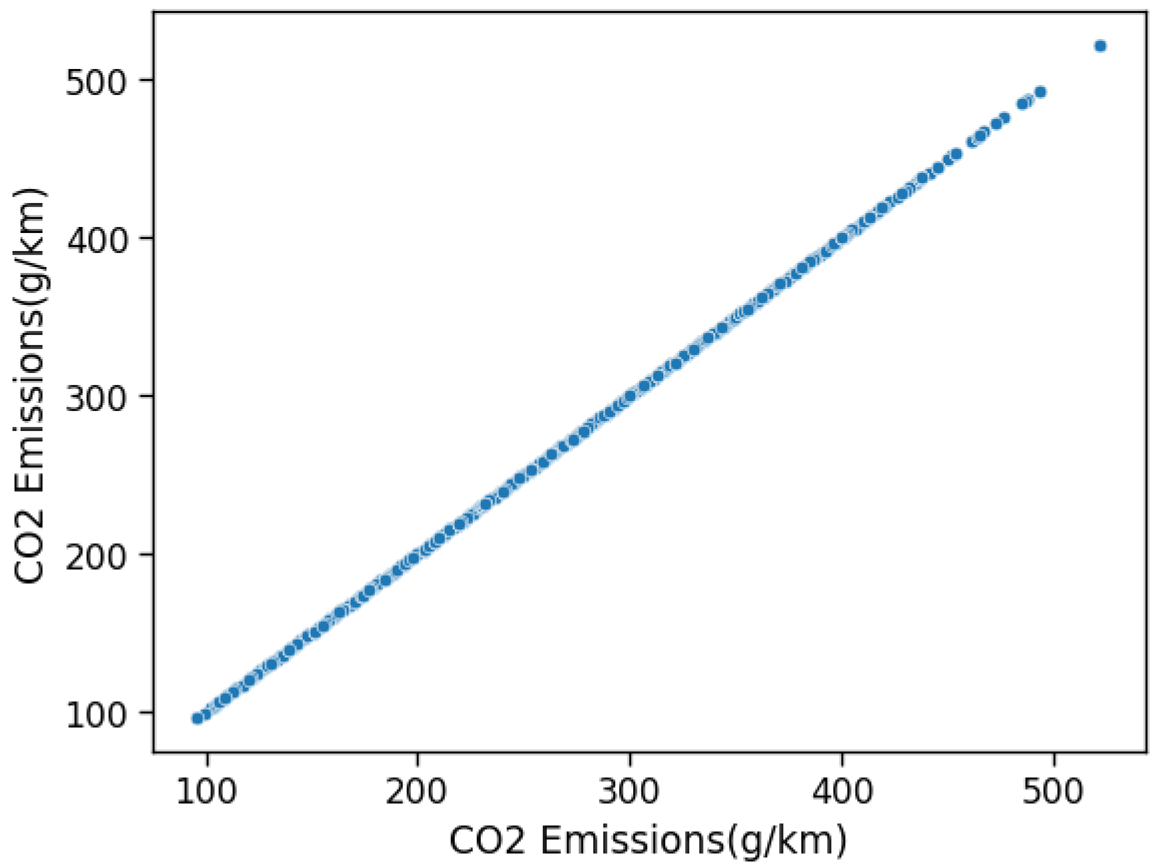
500
400
300
200
100
CO2 Emissions(g/km)
5.0
7.5
10.0
12.5
15.0
17.5
20.0
Fuel Consumption Hwy (L/100 km)



500
400
300
200
100
CO2 Emissions(g/km)
5
10
15
20
25
Fuel Consumption Comb (L/100 km)



500
400
300
200
100
CO2 Emissions(g/km)
10
20
30
40
50
60
70
Fuel Consumption Comb (mpg)



In [ ]:
```
gear_type = {'A','AS','AM','AV','M'}
gear_count = {'3','4','5','6','7','8','9','10'}
transmission = set()
for i in gear_type:
  for j in gear_count:
    transmission.add(i+j)
for u in df['Transmission']:
  transmission.add(u)
```

In [ ]:
```
gear_dict = {}
for  u in transmission:
  gear_dict[u] = 0
print(gear_dict)
gear_dict_list = []
for i in range(0,len(df.index)):
  gear_dict_list.append(gear_dict)
len(gear_dict_list)
```

```
{'AS3': 0, 'AV7': 0, 'A9': 0, 'M4': 0, 'AV6': 0, 'AV5': 0, 'AM8': 0, 'A4': 0, 'A3': 0, 'AM9': 0, 'M5': 0, 'A7': 0, 'M8': 0, 'M
6': 0, 'A5': 0, 'AS5': 0, 'AV': 0, 'AM7': 0, 'A6': 0, 'AV10': 0, 'AV3': 0, 'AS8': 0, 'AS6': 0, 'A10': 0, 'AV9': 0, 'AM5': 0, 'A
S4': 0, 'A8': 0, 'AM10': 0, 'AS9': 0, 'AM3': 0, 'M9': 0, 'AM6': 0, 'AM4': 0, 'AS10': 0, 'AS7': 0, 'AV8': 0, 'M10': 0, 'AV4': 0,
'M3': 0, 'M7': 0}
```

Out[ ]: 7385

In [ ]:
```
gearDF = pd.DataFrame.from_dict(gear_dict_list)
```

In [ ]:
```
df = pd.concat([df,gearDF ], axis=1)
df.head()
```

Out[ ]:

	Make	Model	Vehicle Class	Engine Size(L)	Cylinders	Transmission	Fuel Type	Fuel Consumption City (L/100 km)	Fuel Consumption Hwy (L/100 km)	Fuel Consumption Comb (L/100 km)	...	M9	AM6	AM4	A!
0	ACURA	ILX	COMPACT	2.0	4	AS5	Z	9.9	6.7	8.5	...	0	0	0	
1	ACURA	ILX	COMPACT	2.4	4	M6	Z	11.2	7.7	9.6	...	0	0	0	
2	ACURA	ILX HYBRID	COMPACT	1.5	4	AV7	Z	6.0	5.8	5.9	...	0	0	0	
3	ACURA	MDX 4WD	SUV - SMALL	3.5	6	AS6	Z	12.7	9.1	11.1	...	0	0	0	
4	ACURA	RDX AWD	SUV - SMALL	3.5	6	AS6	Z	12.1	8.7	10.6	...	0	0	0	

5 rows × 53 columns

In [ ]:
```
for index,row in df.iterrows():
  df.at[index,row['Transmission']]=1
```

In [ ]:
```
df_no_fuel = df.copy()
```

In [ ]:
```
df_no_fuel = df_no_fuel.drop(['Fuel Consumption City (L/100 km)'],axis=1)
df_no_fuel = df_no_fuel.drop(['Fuel Consumption Hwy (L/100 km)'],axis=1)
```

```
df_no_fuel = df_no_fuel.drop(['Fuel Consumption Comb (L/100 km)'],axis=1)
df_no_fuel = df_no_fuel.drop(['Fuel Consumption Comb (mpg)'],axis=1)
```

In [ ]:
```
df_cat = df[['Make','Vehicle Class','Fuel Type','Transmission']]
```

In [ ]:
```
df_cat = pd.get_dummies(df_cat, columns = ['Make', 'Vehicle Class','Fuel Type'])
df_cat.head()
```

Out[ ]:

	Transmission	Make_ACURA	Make_ALFA ROMEO	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUGATTI	Make_BUICK	Make_
0	AS5	1	0	0	0	0	0	0	0	
1	M6	1	0	0	0	0	0	0	0	
2	AV7	1	0	0	0	0	0	0	0	
3	AS6	1	0	0	0	0	0	0	0	
4	AS6	1	0	0	0	0	0	0	0	

5 rows × 64 columns

In [ ]:
```
df_cat = pd.concat([df_cat,gearDF ], axis=1)
df_cat.head()
```

Out[ ]:

	Transmission	Make_ACURA	Make_ALFA ROMEO	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUGATTI	Make_BUICK	Make_
0	AS5	1	0	0	0	0	0	0	0	
1	M6	1	0	0	0	0	0	0	0	
2	AV7	1	0	0	0	0	0	0	0	
3	AS6	1	0	0	0	0	0	0	0	
4	AS6	1	0	0	0	0	0	0	0	

5 rows × 105 columns

In [ ]:
```
for index,row in df_cat.iterrows():
  df_cat.at[index,row['Transmission']]=1
```

In [ ]:
```
df_cat = df_cat.drop(['Transmission'],axis=1)
```

In [ ]:
```
df = df.drop(['Transmission'],axis=1)
```

In [ ]:
```
def normalize(df,column):
    df[column] = (df[column]-df[column].mean())/df[column].std()
    return df
```

In [ ]:
```
car_models = df['Model'].to_numpy()
df = df.drop(['Model'],axis=1)
```

In [ ]:
```
df = normalize(df,'Cylinders')
df = normalize(df,'Fuel Consumption City (L/100 km)')
df = normalize(df,'Fuel Consumption Hwy (L/100 km)')
df = normalize(df,'Fuel Consumption Comb (L/100 km)')
df = normalize(df,'Fuel Consumption Comb (mpg)')
df = normalize(df,'Engine Size(L)')
df = normalize(df,'CO2 Emissions(g/km)')
```

In [ ]:
```
def percentileDivision(df):

  first_quartile = df['CO2 Emissions(g/km)'].quantile(0.25)
  third_quartile = df['CO2 Emissions(g/km)'].quantile(0.75)


  def classify_emissions(emission):
      if emission < first_quartile:
          return 1
      elif first_quartile <= emission <= third_quartile:
          return 2
      else:
          return 3


  df['Target'] = df['CO2 Emissions(g/km)'].apply(classify_emissions)
  return df
```

In [ ]:
```
df = percentileDivision(df)
```

In [ ]:
```
df.drop('CO2 Emissions(g/km)', axis = 1)
```

Out[ ]:

	Make	Vehicle Class	Engine Size(L)	Cylinders	Fuel Type	Fuel Consumption City (L/100 km)	Fuel Consumption Hwy (L/100 km)	Fuel Consumption Comb (L/100 km)	Fuel Consumption Comb (mpg)	AS3	...	AM6	AM4	AS
0	ACURA	COMPACT	-0.856663	-0.883348	Z	-0.758950	-1.052709	-0.855684	0.763059	0	...	0	0	
1	ACURA	COMPACT	-0.561279	-0.883348	Z	-0.387551	-0.603161	-0.475391	0.209952	0	...	0	0	
2	ACURA	COMPACT	-1.225893	-0.883348	Z	-1.873149	-1.457303	-1.754558	2.837208	0	...	0	0	
3	ACURA	SUV - SMALL	0.251026	0.210561	Z	0.040987	0.026206	0.043191	-0.343155	0	...	0	0	
4	ACURA	SUV - SMALL	0.251026	0.210561	Z	-0.130428	-0.153613	-0.129670	-0.066601	0	...	0	0	
...	...	...	...	...	...	...	...	...	...	...	...	...	...	
7380	VOLVO	SUV - SMALL	-0.856663	-0.883348	Z	-0.530397	-0.603161	-0.544535	0.348229	0	...	0	0	
7381	VOLVO	SUV - SMALL	-0.856663	-0.883348	Z	-0.387551	-0.333433	-0.371675	0.209952	0	...	0	0	
7382	VOLVO	SUV - SMALL	-0.856663	-0.883348	Z	-0.244705	-0.198568	-0.233386	-0.066601	0	...	0	0	
7383	VOLVO	SUV - STANDARD	-0.856663	-0.883348	Z	-0.387551	-0.333433	-0.371675	0.209952	0	...	0	0	
7384	VOLVO	SUV - STANDARD	-0.856663	-0.883348	Z	-0.101859	-0.153613	-0.095098	-0.204878	0	...	0	0	

7385 rows × 51 columns

In [ ]:
```
df_num = df[['Cylinders','Fuel Consumption City (L/100 km)','Fuel Consumption Hwy (L/100 km)','Fuel Consumption Comb (L/100 km
```

In [ ]:
```
def chiSquareTest(df_cat,df):
  from scipy.stats import chi2_contingency

  p_values = {}

  target_col = df['Target']
  significance_level = 0.0005
  for col in df_cat.columns:
      contingency_table = pd.crosstab(df_cat[col], target_col)
      chi2, p, _, _ = chi2_contingency(contingency_table)
      p_values[col] = p

  selected_features = [feature for feature, p_val in p_values.items() if p_val < significance_level]

  len(selected_features)
  return selected_features,target_col
```

In [ ]:
```
selected_features,target = chiSquareTest(df_cat,df)
df_cat = df_cat[selected_features]
df_cat.head()
```

Out[ ]:

	Make_ACURA	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUICK	Make_CADILLAC	Make_CHEVROLET	Make_CHRYS
0	1	0	0	0	0	0	0	0	
1	1	0	0	0	0	0	0	0	
2	1	0	0	0	0	0	0	0	
3	1	0	0	0	0	0	0	0	
4	1	0	0	0	0	0	0	0	

5 rows × 74 columns

In [ ]:
```
df = pd.concat([df_cat,df_num], axis=1)
```

In [ ]:
```
def PCA(df):

  centered_data = df - df.mean()

  covariance_matrix = centered_data.cov()

  eigenvalues, eigenvectors = np.linalg.eig(covariance_matrix)

  total_variance = sum(eigenvalues)
  explained_variance = [(i / total_variance) * 100 for i in sorted(eigenvalues, reverse=True)]
  cumulative_variance = np.cumsum(explained_variance)

  n_components = np.argmax(cumulative_variance >= 95) + 1

  sorted_eigenvalues = eigenvalues.argsort()[::-1]
  eigenvectors = eigenvectors[:, sorted_eigenvalues]
  selected_eigenvectors = eigenvectors[:, :n_components]

  transformed_data = np.dot(centered_data, selected_eigenvectors)

  print("Original shape:", centered_data.shape)
  print("Transformed shape:", transformed_data.shape)
  return transformed_data
```

In [ ]:
```
transformed_data_fuel = PCA(df)
```

```
Original shape: (7385, 80)
Transformed shape: (7385, 35)
```

In [ ]:
```
df_cat_no_fuel = df_no_fuel[['Make','Vehicle Class','Fuel Type','Transmission']]
```

In [ ]:
```
df_cat_no_fuel = pd.get_dummies(df_cat_no_fuel, columns = ['Make', 'Vehicle Class','Fuel Type'])
df_cat_no_fuel.head()
```

Out[ ]:

	Transmission	Make_ACURA	Make_ALFA ROMEO	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUGATTI	Make_BUICK	Make_
0	AS5	1	0	0	0	0	0	0	0	
1	M6	1	0	0	0	0	0	0	0	
2	AV7	1	0	0	0	0	0	0	0	
3	AS6	1	0	0	0	0	0	0	0	
4	AS6	1	0	0	0	0	0	0	0	

5 rows × 64 columns

In [ ]:
```
df_cat_no_fuel = pd.concat([df_cat_no_fuel,gearDF ], axis=1)
df_cat_no_fuel.head()
```

Out[ ]:

	Transmission	Make_ACURA	Make_ALFA ROMEO	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUGATTI	Make_BUICK	Make_
0	AS5	1	0	0	0	0	0	0	0	
1	M6	1	0	0	0	0	0	0	0	
2	AV7	1	0	0	0	0	0	0	0	
3	AS6	1	0	0	0	0	0	0	0	
4	AS6	1	0	0	0	0	0	0	0	

5 rows × 105 columns

In [ ]:
```
for index,row in df_cat_no_fuel.iterrows():
  df_cat_no_fuel.at[index,row['Transmission']]=1
```

In [ ]:
```
df_cat_no_fuel = df_cat_no_fuel.drop(['Transmission'],axis=1)
```

In [ ]:
```
df_no_fuel = df_no_fuel.drop(['Transmission'],axis=1)
```

In [ ]:
```
df_no_fuel = df_no_fuel.drop(['Model'],axis=1)
```

In [ ]:
```
df_no_fuel = normalize(df_no_fuel,'Cylinders')
df_no_fuel = normalize(df_no_fuel,'Engine Size(L)')
df_no_fuel = normalize(df_no_fuel,'CO2 Emissions(g/km)')
```

In [ ]:
```
df_no_fuel = percentileDivision(df_no_fuel)
```

In [ ]:
```
df_no_fuel = df_no_fuel.drop('CO2 Emissions(g/km)', axis = 1)
```

In [ ]:
```
df_num_no_fuel = df_no_fuel[['Cylinders','Engine Size(L)']]
```

In [ ]:
```
selected_features_no_fuel,_ = chiSquareTest(df_cat_no_fuel,df_no_fuel)
df_cat_no_fuel = df_cat_no_fuel[selected_features_no_fuel]
df_cat_no_fuel.head()
```

Out[ ]:

	Make_ACURA	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUICK	Make_CADILLAC	Make_CHEVROLET	Make_CHRYS
0	1	0	0	0	0	0	0	0	
1	1	0	0	0	0	0	0	0	
2	1	0	0	0	0	0	0	0	
3	1	0	0	0	0	0	0	0	
4	1	0	0	0	0	0	0	0	

5 rows × 74 columns

In [ ]:
```
df_no_fuel = pd.concat([df_cat_no_fuel,df_num_no_fuel], axis=1)
```

In [ ]:
```
df_no_fuel.head()
```

Out[ ]:

	Make_ACURA	Make_ASTON MARTIN	Make_AUDI	Make_BENTLEY	Make_BMW	Make_BUICK	Make_CADILLAC	Make_CHEVROLET	Make_CHRYS
0	1	0	0	0	0	0	0	0	
1	1	0	0	0	0	0	0	0	
2	1	0	0	0	0	0	0	0	
3	1	0	0	0	0	0	0	0	
4	1	0	0	0	0	0	0	0	

5 rows × 76 columns

In [ ]:
```
transformed_data_no_fuel = PCA(df_no_fuel)
```

```
Original shape: (7385, 76)
Transformed shape: (7385, 43)
```

In [ ]:
```
target_arr = target.to_numpy()
```

```
target_arr = target_arr.reshape((target_arr.shape[0],1))
```

In [ ]:
```
transformed_data_fuel = np.concatenate((transformed_data_fuel, target_arr), axis = 1)
transformed_data_no_fuel = np.concatenate((transformed_data_no_fuel, target_arr), axis = 1)
```

In [ ]:
```
print(transformed_data_fuel.shape,transformed_data_no_fuel.shape)
```

```
(7385, 36) (7385, 44)
```

In [ ]:
```
split = int(0.75*transformed_data_fuel.shape[0])
training_data_fuel, test_data_fuel = transformed_data_fuel[:split,:], transformed_data_fuel[split:,:]
car_models = car_models[split:]
print(training_data_fuel.shape,test_data_fuel.shape)
```

```
(5538, 36) (1847, 36)
```

In [ ]:
```
car_models.shape
```

Out[ ]:
```
(1847,)
```

In [ ]:
```
split = int(0.75*transformed_data_fuel.shape[0])
training_data_no_fuel, test_data_no_fuel = transformed_data_no_fuel[:split,:], transformed_data_no_fuel[split:,:]
print(training_data_no_fuel.shape,test_data_no_fuel.shape)
```

```
(5538, 44) (1847, 44)
```

In [ ]:
```
df_only_fuel = df[['Fuel Consumption City (L/100 km)','Fuel Consumption Hwy (L/100 km)','Fuel Consumption Comb (L/100 km)','Fu
only_fuel_data = df_only_fuel.to_numpy()
split = int(0.75*only_fuel_data.shape[0])
training_data_only_fuel, test_data_only_fuel = only_fuel_data[:split,:], only_fuel_data[split:,:]
print(training_data_no_fuel.shape,test_data_no_fuel.shape)
```

```
(5538, 44) (1847, 44)
```

2. Artificial Neural Networks (Multi-layered Perceptron)

A **Multi-Layer Perceptron (MLP)** is a type of artificial neural network that consists of multiple layers of interconnected nodes, also known as neurons or perceptrons. It is a feedforward neural network, meaning that information flows through the network in one direction—from the input layer through the hidden layers to the output layer.

Structure of an MLP:

- **Input Layer**: The first layer of the network, where each

node represents an input feature.

- **Hidden Layers**: Layers between the input and output layers. Each node in a hidden layer receives input from the nodes in the previous layer and passes its output to the nodes in the next layer.
- **Output Layer**: The final layer that produces the network's output.

1. **Input Layer:** x_i
2. **Hidden Layers:**

   $z_j^l = \sum_{k=1}^{n^{l-1}} w_{jk}^l a_k^{l-1} + b_j^l$

   $a_j^l = \sigma(z_j^l)$

   - n^{l-1} : is the number of neurons in the previous layer.
   - w_{jk}^l : is the weight between kth neuron in layer l - 1 and the jth neuron in layer l
   - a_k^{l-1} : output of kth neuron in layer l - 1
   - b_j^l : bias term for jth neuron in layer l
3. **Activation Function:**

   $a_j^l = \sigma(z_j^l)$
4. **Backward Pass (Backpropagation):**
   - The network is trained by adjusting the weights and biases based on the error between the predicted output and the actual target. The backpropagation algorithm is used to update the weights and biases.
   - The error E for a given example is computed using a loss function (e.g., mean squared error or cross-entropy)
   - In our model we have used cross entropy error, which is also known as log loss error since its a classification problem.

     $E = -\sum_{i=1}^{n_{\text{output}}} (y_i \log(\hat{y}_i) + (1 - y_i) \log(1 - \hat{y}_i))$

- Modified Error Term:

$$\delta_j^L = \frac{\partial E}{\partial z_j^L} = \hat{y}_j - y_j$$

```
* Where :
  * ${n_{\text{output}}}$ is the number of neurons in output layer
  * $y_i$ is the actual output
  * $\hat{y_i}$ is the predicted output
```

- Weight Update Rule:
  - $\Delta w_{jk}^L = -\eta \delta_j^L a_k^{L-1}$
  - $\Delta b_j^L = -\eta \delta_j^L$

  Here: η is the leanring rate.

- **Output Layer:**
  - $t_i = \sum_{j=1}^{n^L} w_{ij}^L a_j^L + b_i^L$
  - $\hat{y}_i = \sigma(t_i)$
- This process is repeated for each layer, propagating the error backward through the network. The weights and biases are updated iteratively until the network converges to a satisfactory solution.

In [ ]:
```python
import numpy as np
from sklearn.metrics import accuracy_score, precision_score, recall_score, f1_score, confusion_matrix
class MLP:
    def __init__(self, input_size, hidden_size, output_size):
        self.weights_hidden = np.random.randn(input_size, hidden_size)
        self.bias_hidden = np.zeros((1, hidden_size))
        self.weights_output = np.random.randn(hidden_size, output_size)
        self.bias_output = np.zeros((1, output_size))

    def sigmoid(self, x):
        return 1 / (1 + np.exp(-x))

    def softmax(self, x):
        exp_x = np.exp(x - np.max(x, axis=1, keepdims=True))
        return exp_x / np.sum(exp_x, axis=1, keepdims=True)

    def forward(self, X):

        hidden_input = np.dot(X, self.weights_hidden) + self.bias_hidden
        hidden_output = self.sigmoid(hidden_input)

        output_input = np.dot(hidden_output, self.weights_output) + self.bias_output
        output_prob = self.softmax(output_input)

        return hidden_output, output_prob

    def compute_loss(self, y_true, y_pred):
        epsilon = 1e-15  # Small constant to avoid log(0)
        y_pred = np.clip(y_pred, epsilon, 1 - epsilon)
        loss = -np.sum(y_true * np.log(y_pred)) / len(y_true)
        return loss

    def backward(self, X, y_true, learning_rate):
        hidden_output, output_prob = self.forward(X)

        error_output = output_prob - y_true
        grad_weights_output = np.dot(hidden_output.T, error_output)
        grad_bias_output = np.sum(error_output, axis=0, keepdims=True)

        error_hidden = np.dot(error_output, self.weights_output.T) * hidden_output * (1 - hidden_output)
        grad_weights_hidden = np.dot(X.T, error_hidden)
        grad_bias_hidden = np.sum(error_hidden, axis=0, keepdims=True)

        self.weights_hidden -= learning_rate * grad_weights_hidden
        self.bias_hidden -= learning_rate * grad_bias_hidden
        self.weights_output -= learning_rate * grad_weights_output
        self.bias_output -= learning_rate * grad_bias_output

    def accuracy(self, y_true, y_pred):
        correct_predictions = np.sum(y_true == y_pred)
        total_samples = len(y_true)
        accuracy = correct_predictions / total_samples
        return accuracy

    def train(self, X, y, epochs, batch_size, learning_rate):
        y_one_hot = np.eye(len(np.unique(y)))[y]

        for epoch in range(epochs):
```

```
            indices = np.arange(len(X))
            np.random.shuffle(indices)
            X_shuffled = X[indices]
            y_shuffled = y_one_hot[indices]

            for i in range(0, len(X), batch_size):
                X_batch = X_shuffled[i:i + batch_size]
                y_batch = y_shuffled[i:i + batch_size]

                self.backward(X_batch, y_batch, learning_rate)

            _, output_prob = self.forward(X)
            loss = self.compute_loss(y_one_hot, output_prob)
            predictions = np.argmax(output_prob, axis=1)
            acc_train = self.accuracy(y, predictions)
            print(f"Epoch {epoch + 1}/{epochs}, Loss: {loss}, Training Accuracy: {acc_train}")

    def predict(self, X):
        _, output_prob = self.forward(X)
        return np.argmax(output_prob, axis=1)
```

In [ ]:
```
accuracy_arr_w_f = []
accuracy_arr_n_f = []
accuracy_arr_o_f = []
prediction_fuel_mlp = []
prediction_no_fuel_mlp = []
prediction_only_fuel_mlp = []
size_arr = [1, 2, 5, 10]
```

In [ ]:
```
# with fuel


X_train = training_data_fuel[:, :-1]
y_train = training_data_fuel[:,-1].astype(int) - 1
X_test = test_data_fuel[:, :-1]
y_test = test_data_fuel[:,-1].astype(int) - 1

for size in size_arr:
    input_size = X_train.shape[1]
    hidden_size = size
    output_size = 3  # Three classes
    mlp1 = MLP(input_size, hidden_size, output_size)


    mlp1.train(X_train, y_train, epochs=100, batch_size=32, learning_rate=0.01)


    y_pred = mlp1.predict(X_test)


    accuracy = accuracy_score(y_test, y_pred)
    print(f"Accuracy: {accuracy}")

    accuracy_arr_w_f.append(accuracy)
    prediction_fuel_mlp.append(y_pred)
    precision = precision_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"Precision: {precision}")


    recall = recall_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"Recall: {recall}")


    f1 = f1_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"F1 Score: {f1}")


    conf_matrix = confusion_matrix(y_test, y_pred)
    print("Confusion Matrix:")
    print(conf_matrix)

report_fuel = classification_report(y_test, prediction_fuel_mlp[len(prediction_fuel_mlp)-1])
```

In [ ]:
```
# without fuel


X_train = training_data_no_fuel[:, :-1]
y_train = training_data_no_fuel[:,-1].astype(int) - 1
X_test = test_data_no_fuel[:, :-1]
y_test = test_data_no_fuel[:,-1].astype(int) - 1

for size in size_arr:

    input_size = X_train.shape[1]
    hidden_size = size
    output_size = 3
```

```python
    mlp2 = MLP(input_size, hidden_size, output_size)

    mlp2.train(X_train, y_train, epochs=100, batch_size=32, learning_rate=0.01)


    y_pred = mlp2.predict(X_test)

    accuracy = accuracy_score(y_test, y_pred)
    print(f"Accuracy: {accuracy}")

    accuracy_arr_n_f.append(accuracy)
    prediction_no_fuel_mlp.append(y_pred)
    precision = precision_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"Precision: {precision}")


    recall = recall_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"Recall: {recall}")


    f1 = f1_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"F1 Score: {f1}")


    conf_matrix = confusion_matrix(y_test, y_pred)
    print("Confusion Matrix:")
    print(conf_matrix)

report_no_fuel = classification_report(y_test, prediction_no_fuel_mlp[len(prediction_no_fuel_mlp)-1])
```

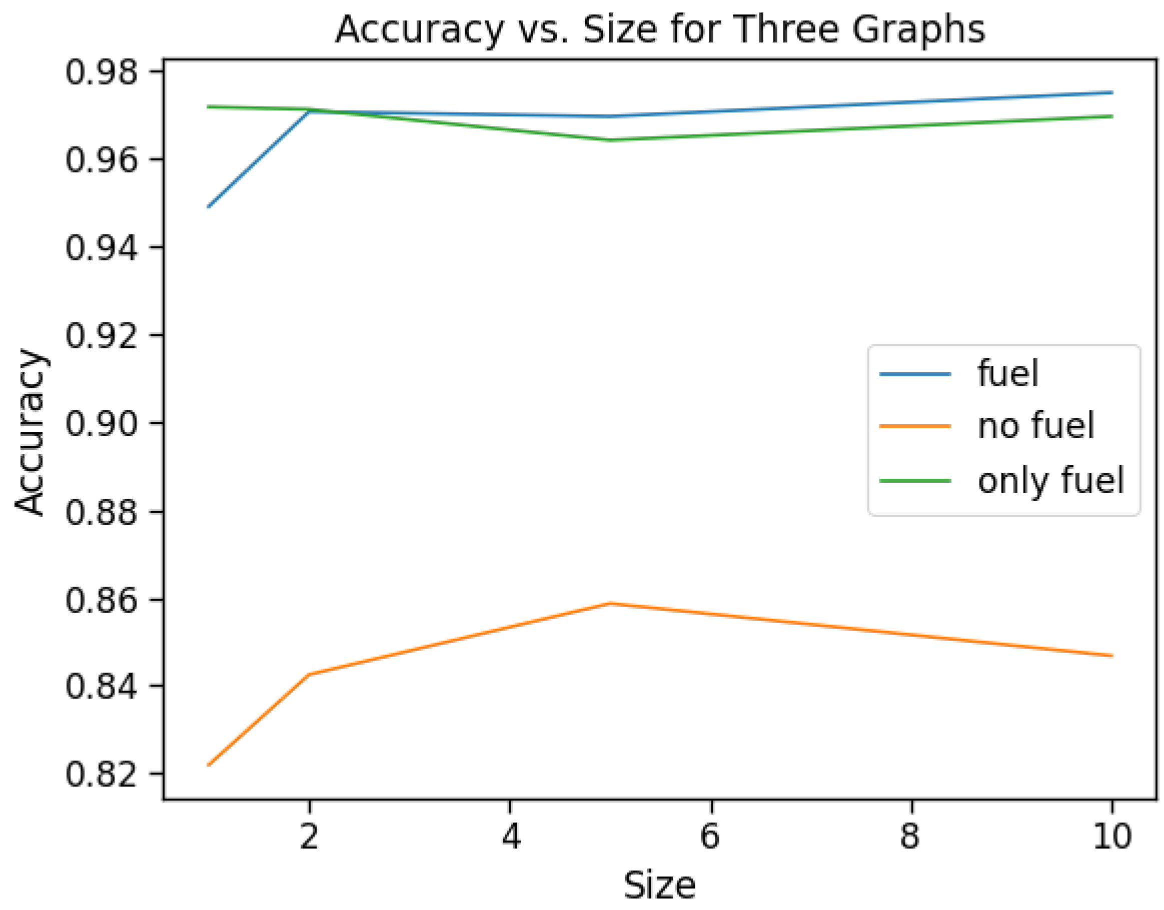
In [ ]:

```python
# only fuel


only_fuel_data_copy = np.concatenate((only_fuel_data, target_arr), axis = 1)
split = int(0.75*only_fuel_data_copy.shape[0])
training_data_only_fuel_copy, test_data_only_fuel_copy = only_fuel_data_copy[:split,:], only_fuel_data_copy[split:,:]
print(training_data_no_fuel.shape,test_data_no_fuel.shape)
X_train = training_data_only_fuel_copy[:, :-1]
y_train = training_data_only_fuel_copy[:,-1].astype(int) - 1
X_test = test_data_only_fuel_copy[:, :-1]
y_test = test_data_only_fuel_copy[:,-1].astype(int) - 1


for size in size_arr:
    input_size = X_train.shape[1]
    hidden_size = size
    output_size = 3
    mlp3 = MLP(input_size, hidden_size, output_size)


    mlp3.train(X_train, y_train, epochs=100, batch_size=32, learning_rate=0.01)


    y_pred = mlp3.predict(X_test)


    accuracy = accuracy_score(y_test, y_pred)
    print(f"Accuracy: {accuracy}")

    accuracy_arr_o_f.append(accuracy)
    prediction_only_fuel_mlp.append(y_pred)

    precision = precision_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"Precision: {precision}")


    recall = recall_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"Recall: {recall}")


    f1 = f1_score(y_test, y_pred, average='weighted', zero_division=1)
    print(f"F1 Score: {f1}")


    conf_matrix = confusion_matrix(y_test, y_pred)
    print("Confusion Matrix:")
    print(conf_matrix)

report_only_fuel = classification_report(y_test, prediction_only_fuel_mlp[len(prediction_only_fuel_mlp)-1])
```

In [ ]:

```python
plt.plot(size_arr, accuracy_arr_w_f , label='fuel')
plt.plot(size_arr, accuracy_arr_n_f , label='no fuel')
plt.plot(size_arr, accuracy_arr_o_f , label='only fuel')

plt.xlabel('Size')
plt.ylabel('Accuracy')
plt.title('Accuracy vs. Size for Three Graphs')
```

```
plt.legend()

plt.show()
```



In [ ]:
```
print(report_fuel)
```

```
              precision    recall  f1-score   support

           0       0.99      0.98      0.98       425
           1       0.96      0.99      0.98       926
           2       1.00      0.94      0.97       496

    accuracy                           0.98      1847
   macro avg       0.98      0.97      0.98      1847
weighted avg       0.98      0.98      0.98      1847
```

In [ ]:
```
print(report_no_fuel)
```

```
              precision    recall  f1-score   support

           0       0.83      0.86      0.85       425
           1       0.82      0.90      0.85       926
           2       0.94      0.74      0.83       496

    accuracy                           0.85      1847
   macro avg       0.86      0.83      0.84      1847
weighted avg       0.85      0.85      0.85      1847
```

In [ ]:
```
print(report_only_fuel)
```

```
              precision    recall  f1-score   support

           0       0.97      1.00      0.98       425
           1       0.98      0.96      0.97       926
           2       0.95      0.96      0.96       496

    accuracy                           0.97      1847
   macro avg       0.97      0.97      0.97      1847
weighted avg       0.97      0.97      0.97      1847
```

Insights:

About fuel:

1. Since there is a significant drop in accuracy when we remove fuel consumption from the set of features, we can say that fuel consumption is a necessary set of features to get an idea about CO_2 emmissions which is correctly predicted by the correlation matrix we made while pre processing.
2. The high accuracy in the features when only fuel is there, suggests that fuel alone can also be enough while predicting a range or CO_2 emmissions which again suggests a high linear correlation between fuel and CO_2 emmissions.

About model performance:

1. Impact of hyperparameters:

2. Number of Hidden Layers and Neurons:

```
  * Impact: The architecture of the MLP, including the number of hidden layers and neurons in each layer,
affects the model's capacity to learn complex patterns. Too few neurons or layers may result in
underfitting, while too many may lead to overfitting.

  * Here we used one layer since the boundaries are linear and can be easily separated with one layer, on
increasing layers from one to sixteen, accuracy increases significantly

2.  Batch Size:

  * Impact: The batch size determines the number of samples used in each iteration of gradient descent. A
smaller batch size can provide regularization effects and might help the model generalize better, while
larger batch sizes can lead to faster convergence.

  * Here we have taken a batch size of eight.

3. Number of Epochs:

  * Impact: The number of epochs is the number of times the model sees the entire training dataset. Too few
epochs may result in underfitting, while too many may lead to overfitting.

  * Here we have taken a hundred epochs.
```

4. Learning Rate:

- Impact: The learning rate controls the size of the step taken during the optimization process. A too high learning rate might cause the model to converge too quickly or even overshoot the optimal weights, leading to poor convergence. On the other hand, a too low learning rate can result in slow convergence or getting stuck in local minima.
- Here a moderately high learning rate of 0.01 works well.

3. Naive Bayes Classifier

The Naive Bayes classifier is a probabilistic machine learning algorithm based on Bayes' theorem, which describes the probability of an event based on prior knowledge of conditions that might be related to the event. Despite its simplicity, Naive Bayes is effective and widely used for various classification tasks, especially in natural language processing and spam filtering.

The "naive" in Naive Bayes comes from the assumption of feature independence. It assumes that the presence or absence of a particular feature is unrelated to the presence or absence of any other feature given the class label. While this assumption might not hold in all cases, Naive Bayes often performs surprisingly well and is computationally efficient.

Naive Bayes classifiers are generally more suitable for discrete and independent data and so a common choice is to use a Gaussian Naive Bayes classifier. In Gaussian Naive Bayes, it is assumed that the continuous features are normally distributed. This allows you to estimate the parameters of the normal distribution (mean and variance) for each class, and then use these parameters to make predictions based on the probability density function of the normal distribution.

Algorithm Overview

Steps involved in the implementation of Naive bayes

1. **Normalising the features in the dataset.**
2. **Finding the class probablities for all the 3 classes:** Naive bayes follows the bayes theorem of conditional probability which requires class probabilities.
3. **Calculating mean for all the features in the dataset.**
4. **Calculating the covariance matrix**
5. **Calculating the prior class conditional probabilties**
6. **Making a prediction: Finding the posterior class probalities using the bayes theorem:**By furnishing the prior class conditional probabilties and the class probablities we can predict the class the data point belongs to.

In [ ]:
```
X_train_fuel = training_data_fuel[:,:training_data_fuel.shape[1]-1]
y_train_fuel = training_data_fuel[:,training_data_fuel.shape[1]-1]
X_test_fuel = test_data_fuel[:,:test_data_fuel.shape[1]-1]
y_test_fuel = test_data_fuel[:,test_data_fuel.shape[1]-1]
```

In [ ]:
```
X_train_no_fuel = training_data_no_fuel[:,:training_data_no_fuel.shape[1]-1]
y_train_no_fuel = training_data_no_fuel[:,training_data_no_fuel.shape[1]-1]
X_test_no_fuel = test_data_no_fuel[:,:test_data_no_fuel.shape[1]-1]
y_test_no_fuel = test_data_no_fuel[:,test_data_no_fuel.shape[1]-1]
```

In [ ]:
```
class GaussianNB():
    def fit(self, X, y):
        self.classes = np.unique(y)
        self.mean = {}
        self.variance = {}
        self.prior = {}
        for c in self.classes:
            X_c = X[y == c]
            self.mean[str(c)] = np.mean(X_c, axis=0)
            self.variance[str(c)] = np.var(X_c, axis=0)
            self.prior[str(c)] = X_c.shape[0] / X.shape[0]

    def predict(self, X):
        probabilities = np.zeros((X.shape[0], len(self.classes)))
        for c in self.classes:
            prior = self.prior[str(c)]
            mean = self.mean[str(c)]
            variance = self.variance[str(c)]
            probabilities[:, int(c)-1] = np.log(prior) + \
                                  np.sum(np.log(1 / np.sqrt(2 * np.pi * variance)) - \
                                         (X - mean)**2 / (2 * variance), axis=1)
        return self.classes[np.argmax(probabilities, axis=1)]
```

In [ ]:
```
gnb = GaussianNB()
gnb.fit(X_train_fuel,y_train_fuel)
prediction_fuel_naive_bayes = gnb.predict(X_test_fuel)
```

In [ ]:
```
gnb = GaussianNB()
gnb.fit(X_train_no_fuel,y_train_no_fuel)
prediction_no_fuel_naive_bayes = gnb.predict(X_test_no_fuel)
```

In [ ]:
```
gnb = GaussianNB()
gnb.fit(training_data_only_fuel,y_train_no_fuel)
prediction_only_fuel_naive_bayes = gnb.predict(test_data_only_fuel)
```

In [ ]:
```
def accuracy(predictions,y_test):
    acc = 0
    for i,val in enumerate(predictions):
        if(val==y_test[i]):
            acc = acc+1
    acc = acc/len(predictions)
    return acc
```

In [ ]:
```
print("Accuracy for All Features: ",str(np.round(accuracy(prediction_fuel_naive_bayes,y_test_fuel),5)))
print("Accuracy Excluding Fuel Consumption: ",str(np.round(accuracy(prediction_no_fuel_naive_bayes,y_test_fuel),5)))
print("Accuracy for only fuel consumption: ",str(np.round(accuracy(prediction_only_fuel_naive_bayes,y_test_fuel),5)))
```

```
Accuracy for All Features:  0.84028
Accuracy Excluding Fuel Consumption:  0.70655
Accuracy for only fuel consumption:  0.90579
```

In [ ]:
```
report_fuel = classification_report(y_test_fuel, prediction_fuel_naive_bayes)
report_no_fuel = classification_report(y_test_fuel, prediction_no_fuel_naive_bayes)
report_only_fuel = classification_report(y_test_fuel, prediction_only_fuel_naive_bayes)
```

In [ ]:
```
print(report_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.84      0.83      0.83       425
         2.0       0.84      0.84      0.84       926
         3.0       0.84      0.84      0.84       496

    accuracy                           0.84      1847
   macro avg       0.84      0.84      0.84      1847
weighted avg       0.84      0.84      0.84      1847
```

In [ ]:
```
print(report_no_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.64      0.80      0.71       425
         2.0       0.78      0.59      0.67       926
         3.0       0.68      0.84      0.75       496

    accuracy                           0.71      1847
   macro avg       0.70      0.74      0.71      1847
weighted avg       0.72      0.71      0.70      1847
```

In [ ]:
```
print(report_only_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.87      1.00      0.93       425
         2.0       0.91      0.91      0.91       926
         3.0       0.95      0.82      0.88       496

    accuracy                           0.91      1847
   macro avg       0.91      0.91      0.91      1847
weighted avg       0.91      0.91      0.90      1847
```

4. Random Forest Classifier

Decision Trees

Decision Trees are binary decision trees which make a single decision on each internal node and split the node into right and left child nodes. This split depends on the information gained by the split. The node is split on the basis of which decision will give maximum information gain. Once we reach a node where no information is gained, or the number of datapoints is insufficient to determine the split to a reasonable significance or if the tree has reached maximum permissable depth, we denote that node as a leaf node. This node holds the predicted value of the data point which will take the path to the leaf while traversing the tree.Usually this predicted value is just the class label for the class with highest frequency present at this node.

Random Forests and Ensemble Learning

Random forest is a ensemble learning algorithm that uses multiple decision trees to solve classification or regression problems.

Ensemble learning is process of training multiple models over the same dataset and averaging the results from them to get better results than stand alone models.

Ensemble learing has 3 main methods:
1. Bagging 2. Boosting 3. Stacking

Random forests are similar to bagging emsemble models but not quite same. In bagging, multiple decision trees are created using a bootstraped sample but all features are considered for each decision tree.Whereas in Random Forests, only a random subset of features with size < total feature size.

The advantage of random forest lies in the injected randomness in feature selection and bootstraping.
Bootstraping:The process of drawing a bootstrap sample.
Bootstrap sample: Drawing a sample D from the dataset of size n *with replacement* such that $|D| < n$
The randomness in feature selection helps to deal with the strength of each tree vs the strength of entire forest. If the number of features selected is large (all of them in case of bagging) then each tree will be a good classifier but may overfit in overall ensemble. Whereas a subset of features will make every tree a weak classifier but the forest overall will be a good classfier.

Random forests are also immune to overfitting by the Strong Law of Large Numbers.
The Strong Law of Large Numbers:
If $\bar{X}$ is the sample mean of population with mean μ then as the sample size increases the sample mean converges to the population mean.

$$\lim_{n \to \infty} \bar{X}_n = \mu$$

So as the number of trees in the random forest keep increasing the sample mean i.e the predictions of all the trees together will be more accurate and less sensitive to noise in the data.

Entropy and Information Gain

A number of different metrics can be used to calculate the information gain. Here we use entropy as it gives larger range of values.

Entropy: It is the average level of uncertainty to a variables possible outcomes. In information theory, entropy is given by

$$H(X) = -\sum_x p(x) log_2(p(x))$$
$$\text{where } x \text{ is a random variable and } p(x) \text{ is its probability mass fucntion}$$

We can call log(1/p) information because if all events happen with probability p , it means that there are 1/p events. To tell which event have happened, we need to use log(1/p) bits.(Shannon's entropy)

Information Gain : The expected reduction in entropy caused by a split in the node.Higher information gain indicates that a particular attribute is more effective in reducing uncertainty and, therefore, more relevant for classifying the data.

$$IG(D) = H(D_{parent}) - \frac{N_{left}}{N_{total}} H(D_{left}) - \frac{N_{right}}{N_{total}} H(D_{right})$$
$$\text{here } D_{left}, D_{right}, D_{left}, D_{right} \text{ are the datasets in left,right and parent nodes.}$$

So we take a greedy approach to finding the best split. We check for all possible splits for attributes and take the best split based on the highest information gain.

The Random Forest algorithm has been made into a class called RandomForestClassifier to make have error free and easy run for multiple parameters. Given below are snippets from the functions that are implemented within that class.

Calculate Entropy:

```
def entropy(p1,p2,p3):
    e1 = p1
    e2 = p2
    e3 = p3

    if(p1!=0):
        e1 = p1*np.log2(p1)
    if(p2!=0):
        e2 = p2*np.log2(p2)
    if(p3!=0):
        e3 = p3*np.log2(p3)
    return -(e1+e2+e3)
```

Calculate Information Gain:

```
def information_gain(left_child,right_child):
    parent = left_child+right_child

    p_parent_1 = parent.count(1) / len(parent) if len(parent) > 0 else 0
    p_parent_2 = parent.count(2) / len(parent) if len(parent) > 0 else 0
    p_parent_3 = parent.count(3) / len(parent) if len(parent) > 0 else 0

    p_left_child_1 = left_child.count(1) / len(left_child) if len(left_child) > 0 else 0
    p_left_child_2 = left_child.count(2) / len(left_child) if len(left_child) > 0 else 0
    p_left_child_3 = left_child.count(3) / len(left_child) if len(left_child) > 0 else 0

    p_right_child_1 = right_child.count(1) / len(right_child) if len(right_child) > 0 else 0
    p_right_child_2 = right_child.count(2) / len(right_child) if len(right_child) > 0 else 0
    p_right_child_3 = right_child.count(3) / len(right_child) if len(right_child) > 0 else 0

    e_parent = entropy(p_parent_3,p_parent_1,p_parent_2)
    e_left_child = entropy(p_left_child_3,p_left_child_1,p_left_child_2)
    e_right_child = entropy(p_right_child_3,p_right_child_1,p_right_child_2)

    IG = e_parent - len(left_child) / len(parent) * e_left_child - len(right_child) / len(parent) *
e_right_child

    return IG
```

Random Forest Algorithm

Let N the number of trees in the random forest

$A.$ for $i = 1$ to N

1. Draw a bootstraped sample from dataset
2. To grow a decision tree T_i perform following 3 steps given that node has > 1 class
   and the maximum permisable depth is not reached and the size of node dataset is enough to perform split
   $i.$ sample $m = log_2(total\ features) + 1$
   $ii.$ Calculate information gain for all possible values using bootstraped data and m features and find best split
   $iii.$ Split into left and right child and repeat (2.) on them
3. Save root of tree T_i

$B.$ return ensemble of trees $\{T\}^B$

Random Forest Algorithm:

```
def randomForest(X_train,y_train,max_features,max_depth,sample_threshold,tree_count):
    forest = []
    for i in range(tree_count): #Step A.
        X_bootstrap,y_bootstrap = draw_bootstrap_sample(X_train,y_train) #Step A.1
        tree = construct_decision_tree(X_bootstrap,y_bootstrap,max_features,max_depth,sample_threshold)
#Step A.2
        forest.append(tree) #Step A.3
    return forest  #Step B
```

Draw Bootstrapped Sample:

```
def draw_bootstrap_sample(X_train,y_train):
    #drawing 66% of the sample
    bootstrap_indices = list(np.random.choice(range(len(X_train)), int((2/3)*len(X_train)), replace =
True))
    X_bootstrap = []
    y_bootstrap = []
    for i in bootstrap_indices:
        X_bootstrap.append(X_train[i])
        y_bootstrap.append(y_train[i])
    return np.array(X_bootstrap),np.array(y_bootstrap)
```

Construct Decision Tree:

```
def construct_decision_tree(X,y,max_features,max_depth,sample_threshold):
    root = find_best_split(X,y,max_features) #find split for root according to highest information gain
    split_node(root,max_features,sample_threshold,max_depth,1)
    return root
```

Split Nodes:

```
def split_node(node, max_features, sample_threshold, max_depth, depth):
    left_child = node['left']
    right_child = node['right']
    del(node['left'])
    del(node['right'])

    #if there is no best split possible
    if(len(right_child['y'])==0 or len(left_child['y'])==0):
        empty_child = {'y': right_child['y'] + left_child['y']}
        node['left_child'] = make_terminal(empty_child)
        node['right_child'] = make_terminal(empty_child)
        return

    #if max depth is reached
    if depth >= max_depth:
        node['left_child'] = make_terminal(left_child)
        node['right_child'] = make_terminal(right_child)
        return node

    #if number of observations (in any one child) to split node is less than threshold
    if(len(left_child['y'])<sample_threshold):
        node['left_child'] = make_terminal(left_child)
        node['right_child'] = make_terminal(left_child)
    else:
        #if none of the terminal conditions hold then child the node furthur
        node['left_child'] = find_best_split(left_child['X'],left_child['y'],max_features) #Step A.2.i and
Step A.2.ii
        split_node(node['left_child'],max_features,sample_threshold,max_depth,depth+1) #Step A.2.iii

    #if number of observations to child node is less than threshold
    if(len(right_child['y'])<sample_threshold):
        node['left_child'] = make_terminal(right_child)
        node['right_child'] = make_terminal(right_child)
```

```
    else:
        #if none of the terminal conditions hold then split the node furthur
        node['right_child'] = find_best_split(right_child['X'],right_child['y'],max_features)  #Step A.2.i
and Step A.2.ii
        split_node(node['right_child'],max_features,sample_threshold,max_depth,depth+1) #step A.2.iii
```

Find Best Split:

```
def find_best_split(X_boot,y_boot,m):
    feature_ls = []
    num_features = len(X_boot[0])

    #sample m features
    while len(feature_ls) <= m:
        feature_idx = random.sample(range(num_features), 1)
        if feature_idx not in feature_ls:
            feature_ls.extend(feature_idx)

    best_info_gain = float('-inf')
    node = None

    #for each feature selected
    for index in feature_ls:

        #splitting continuous data
        #for each value in dataset choose it as split point and calculate information gain by splitting
dataset about that point
        for split_pt in X_boot[:,index]:
            left_child = {'X':[],'y':[]}
            right_child = {'X':[],'y':[]}

            for i,val in enumerate(X_boot[:,index]):
                if(val<=split_pt):
                    left_child['X'].append(X_boot[i])
                    left_child['y'].append(y_boot[i])
                else:
                    right_child['X'].append(X_boot[i])
                    right_child['y'].append(y_boot[i])

            #calculate information gain
            IG = information_gain(left_child['y'],right_child['y'])

            #each time we get a larger infomation gain we update the left and right child nodes and
splitting condition.
            if(IG>best_info_gain):
                best_info_gain = IG
                left_child['X'] = np.array(left_child['X'])
                right_child['X'] = np.array(right_child['X'])
                node = {'left':left_child,
                        'right':right_child,
                        'split_point':split_pt,
                        'feature_index':index,
                        'info_gain' : IG
                        }
    return node
```

Create Terminal Node:

```
def make_terminal(node):
    #return class with highest frequency.
    pred = max(node['y_bootstrap'],key = node['y_bootstrap'].count)
    return pred
```

Predict class on each Decision Tree:

```
#this is traversal of binary search tree
def predictDT(tree,X_test):
    index = tree['feature_index'] #get index of feature used to split

    #search based on splitting value of the feature
    if(X_test[index] <= tree['split_point']):
        #if its internal node call function recursively
        if(type(tree['left_child'])==dict):
            return predictDT(tree['left_child'],X_test)
        else:
            #return value(class prediction) of leaf node
            return tree['left_child']
    else:
        if(type(tree['right_child'])==dict):
```

```
        #if its internal node call function recursively
            return predictDT(tree['right_child'],X_test)
        else:
        #return value(class prediction) of leaf node
            return tree['right_child']
```

Ensemble Prediction:

```
def predict_randomForest(forest,X_test):
    predictions = []
    for i in range(len(X_test)):
        tree_pred = []

        #for each datapoint predct using all trees in forest
        for tree in forest:
            tree_pred.append(predictDT(tree,X_test[i]))

        #take class with highest frequency of predictions
        predictions.append(max(tree_pred, key= tree_pred.count))
    return predictions
```

In [ ]:
```
X_train_fuel = training_data_fuel[:,:training_data_fuel.shape[1]-1]
y_train_fuel = training_data_fuel[:,training_data_fuel.shape[1]-1]
```

In [ ]:
```
X_test_fuel = test_data_fuel[:,:test_data_fuel.shape[1]-1]
y_test_fuel = test_data_fuel[:,test_data_fuel.shape[1]-1]
```

In [ ]:
```
X_train_no_fuel = training_data_no_fuel[:,:training_data_no_fuel.shape[1]-1]
y_train_no_fuel = training_data_no_fuel[:,training_data_no_fuel.shape[1]-1]
```

In [ ]:
```
X_test_no_fuel = test_data_no_fuel[:,:test_data_no_fuel.shape[1]-1]
y_test_no_fuel = test_data_no_fuel[:,test_data_no_fuel.shape[1]-1]
```

In [ ]:
```
class RandomForestClassifier:
    def __init__(self,N_estimators,max_features,max_depth,sample_threshold):
        self.N_estimators = N_estimators
        self.max_features = max_features
        self.max_depth = max_depth
        self.sample_threshold = sample_threshold

    def entropy(self,p1,p2,p3):
        e1 = p1
        e2 = p2
        e3 = p3

        if(p1!=0):
            e1 = p1*np.log2(p1)
        if(p2!=0):
            e2 = p2*np.log2(p2)
        if(p3!=0):
            e3 = p3*np.log2(p3)
        return -(e1+e2+e3)

    def information_gain(self,left_child,right_child):
        parent = left_child+right_child

        p_parent_1 = parent.count(1) / len(parent) if len(parent) > 0 else 0
        p_parent_2 = parent.count(2) / len(parent) if len(parent) > 0 else 0
        p_parent_3 = parent.count(3) / len(parent) if len(parent) > 0 else 0

        p_left_child_1 = left_child.count(1) / len(left_child) if len(left_child) > 0 else 0
        p_left_child_2 = left_child.count(2) / len(left_child) if len(left_child) > 0 else 0
        p_left_child_3 = left_child.count(3) / len(left_child) if len(left_child) > 0 else 0

        p_right_child_1 = right_child.count(1) / len(right_child) if len(right_child) > 0 else 0
        p_right_child_2 = right_child.count(2) / len(right_child) if len(right_child) > 0 else 0
        p_right_child_3 = right_child.count(3) / len(right_child) if len(right_child) > 0 else 0

        e_parent = self.entropy(p_parent_3,p_parent_1,p_parent_2)
        e_left_child = self.entropy(p_left_child_3,p_left_child_1,p_left_child_2)
        e_right_child = self.entropy(p_right_child_3,p_right_child_1,p_right_child_2)

        IG = e_parent - len(left_child) / len(parent) * e_left_child - len(right_child) / len(parent) * e_right_child

        return IG

    def draw_bootstrap_sample(self,X_train,y_train):
        bootstrap_indices = list(np.random.choice(range(len(X_train)), int((2/3)*len(X_train)), replace = True))
        X_bootstrap = []
        y_bootstrap = []
        for i in bootstrap_indices:
            X_bootstrap.append(X_train[i])
            y_bootstrap.append(y_train[i])
```

```
        return np.array(X_bootstrap),np.array(y_bootstrap)

    def find_split(self,X_boot,y_boot):
        feature_ls = list()
        num_features = len(X_boot[0])
        while len(feature_ls) <= self.max_features:
            feature_idx = random.sample(range(num_features), 1)
            if feature_idx not in feature_ls:
                feature_ls.extend(feature_idx)

        best_info_gain = float('-inf')
        node = None
        for index in feature_ls:
            for split_pt in X_boot[:,index]:
                left_child = {'X_bootstrap':[],'y_bootstrap':[]}
                right_child = {'X_bootstrap':[],'y_bootstrap':[]}

                for i,val in enumerate(X_boot[:,index]):
                    if(val<=split_pt):
                        left_child['X_bootstrap'].append(X_boot[i])
                        left_child['y_bootstrap'].append(y_boot[i])
                    else:
                        right_child['X_bootstrap'].append(X_boot[i])
                        right_child['y_bootstrap'].append(y_boot[i])

                IG = self.information_gain(left_child['y_bootstrap'],right_child['y_bootstrap'])
                if(IG>best_info_gain):
                    best_info_gain = IG
                    left_child['X_bootstrap'] = np.array(left_child['X_bootstrap'])
                    right_child['X_bootstrap'] = np.array(right_child['X_bootstrap'])
                    node = {'left_child':left_child,
                            'right_child':right_child,
                            'split_point':split_pt,
                            'feature_index':index,
                            'info_gain' : IG
                            }
        return node

    def make_terminal(self,node):
        pred = max(node['y_bootstrap'],key = node['y_bootstrap'].count)
        return pred

    def split_node(self,node, depth):
        left_child = node['left_child']
        right_child = node['right_child']
        del(node['left_child'])
        del(node['right_child'])

        #if there is no best split possible
        if(len(right_child['y_bootstrap'])==0 or len(left_child['y_bootstrap'])==0):
            empty_child = {'y_bootstrap': right_child['y_bootstrap'] + left_child['y_bootstrap']}
            node['left_split'] = self.make_terminal(empty_child)
            node['right_split'] = self.make_terminal(empty_child)
            return

        #if max depth is reached
        if depth >= self.max_depth:
            node['left_split'] = self.make_terminal(left_child)
            node['right_split'] = self.make_terminal(right_child)
            return node

        #if number of observations (in any one child) to split node is less than threshold
        if(len(left_child['y_bootstrap'])<self.sample_threshold):
            node['left_split'] = self.make_terminal(left_child)
            node['right_split'] = self.make_terminal(left_child)
        else:
            #if none of the terminal conditions hold then split the node furthur
            node['left_split'] = self.find_split(left_child['X_bootstrap'],left_child['y_bootstrap'])
            self.split_node(node['left_split'],depth+1)

        #if number of observations to split node is less than threshold
        if(len(right_child['y_bootstrap'])<self.sample_threshold):
            node['left_split'] = self.make_terminal(right_child)
            node['right_split'] = self.make_terminal(right_child)
        else:
            #if none of the terminal conditions hold then split the node furthur
            node['right_split'] = self.find_split(right_child['X_bootstrap'],right_child['y_bootstrap'])
            self.split_node(node['right_split'],depth+1)

    def construct_decision_tree(self,X_bootstrap,y_bootstrap):
        root = self.find_split(X_bootstrap,y_bootstrap)
        self.split_node(root,1)
        return root

    def fit(self,X_train,y_train):
        forest = list()
        for i in range(self.N_estimators):
```

```
            X_bootstrap,y_bootstrap = self.draw_bootstrap_sample(X_train,y_train)
            tree = self.construct_decision_tree(X_bootstrap,y_bootstrap)
            forest.append(tree)
        self.forest = forest
        return forest

    def predictDT(self,tree,X_test):
        index = tree['feature_index']
        if(X_test[index] <= tree['split_point']):
            if(type(tree['left_split'])==dict):
                return self.predictDT(tree['left_split'],X_test)
            else:
                return tree['left_split']
        else:
            if(type(tree['right_split'])==dict):
                return self.predictDT(tree['right_split'],X_test)
            else:
                return tree['right_split']

    def predict(self,X_test):
        predictions = []
        for i in range(len(X_test)):
            tree_pred = []
            for tree in self.forest:
                tree_pred.append(self.predictDT(tree,X_test[i]))
            predictions.append(max(tree_pred, key= tree_pred.count))
        return predictions
```

In [ ]:
```
def accuracy(predictions,y_test):
    acc = 0
    for i,val in enumerate(predictions):
        if(val==y_test[i]):
            acc = acc+1
    acc = acc/len(predictions)
    print("Testing accuracy: {}".format(np.round(acc,6)))
    return acc
```

In [ ]:
```
def precision(y_pred,y_test):
    count_1 = 0
    count_2 = 0
    count_3 = 0
    for i in y_pred:
        if i==1 :
            count_1 = count_1 + 1
        elif i==2:
            count_2 = count_2 +1
        elif i==3:
            count_3 = count_3 +1
    correct_1 = 0
    correct_2 = 0
    correct_3 =0

    for i,val in enumerate(y_pred):
        if y_test[i] == y_pred[i]:
            if val==1 :
                correct_1 = correct_1 + 1
            elif val==2:
                correct_2 = correct_2 +1
            elif val==3:
                correct_3 = correct_3 +1
    return correct_1/count_1, correct_2/count_2,correct_3/count_3
```

In [ ]:
```
def weighted_avg(arr_fuel,arr_no_fuel,arr_only_fuel):
    wt_avg_fuel = 0
    wt_avg_no_fuel = 0
    wt_avg_only_fuel = 0
    for i in range(len(n_estimators)):
        wt_avg_fuel = wt_avg_fuel + n_estimators[i]*arr_fuel[i]
        wt_avg_no_fuel = wt_avg_no_fuel + n_estimators[i]*arr_no_fuel[i]
        wt_avg_only_fuel = wt_avg_only_fuel + n_estimators[i]*arr_only_fuel[i]
    wt_avg_fuel = wt_avg_fuel/sum(n_estimators)
    wt_avg_no_fuel = wt_avg_no_fuel/sum(n_estimators)
    wt_avg_only_fuel = wt_avg_only_fuel/sum(n_estimators)
    print("Weigthed Average Comparison: ")
    print("With fuel comsumption: "+str(np.round(wt_avg_fuel,6)))
    print("Without fuel comsumption: "+str(np.round(wt_avg_no_fuel,6)))
    print("Only fuel consumption: "+str(wt_avg_only_fuel))
```

In [ ]:
```
n_estimators = [5,10,20,30,50]

predictions_fuel = []
predictions_no_fuel = []
predictions_only_fuel = []
accuracy_arr_fuel = []
accuracy_arr_no_fuel = []
accuracy_only_fuel = []
```

In [ ]:
```
max_features_fuel = int(np.log2(X_train_fuel.shape[1])) + 1
```

In [ ]:
```
for n in n_estimators:
    randomForestClassifier = RandomForestClassifier(n,max_features_fuel,10,10)
    forest = randomForestClassifier.fit(X_train_fuel[:1000],y_train_fuel[:1000])
    predictions = randomForestClassifier.predict(X_test_fuel)
    accuracy_arr_fuel.append(accuracy(predictions,y_test_fuel))
    predictions_fuel.append(predictions)
```

```
Testing accuracy: 0.65
Testing accuracy: 0.71
Testing accuracy: 0.73
Testing accuracy: 0.9
Testing accuracy: 0.89
```

In [ ]:
```
max_features_no_fuel = int(np.log2(X_train_no_fuel.shape[1])) + 1
```

In [ ]:
```
for n in n_estimators:
    randomForestClassifier = RandomForestClassifier(n,max_features_no_fuel,10,10)
    forest = randomForestClassifier.fit(X_train_no_fuel[:1000],y_train_no_fuel[:1000])
    predictions = randomForestClassifier.predict(X_test_no_fuel)
    accuracy_arr_no_fuel.append(accuracy(predictions,y_test_no_fuel))
    predictions_no_fuel.append(predictions)
```

```
Testing accuracy: 0.62
Testing accuracy: 0.64
Testing accuracy: 0.74
Testing accuracy: 0.73
Testing accuracy: 0.76
```

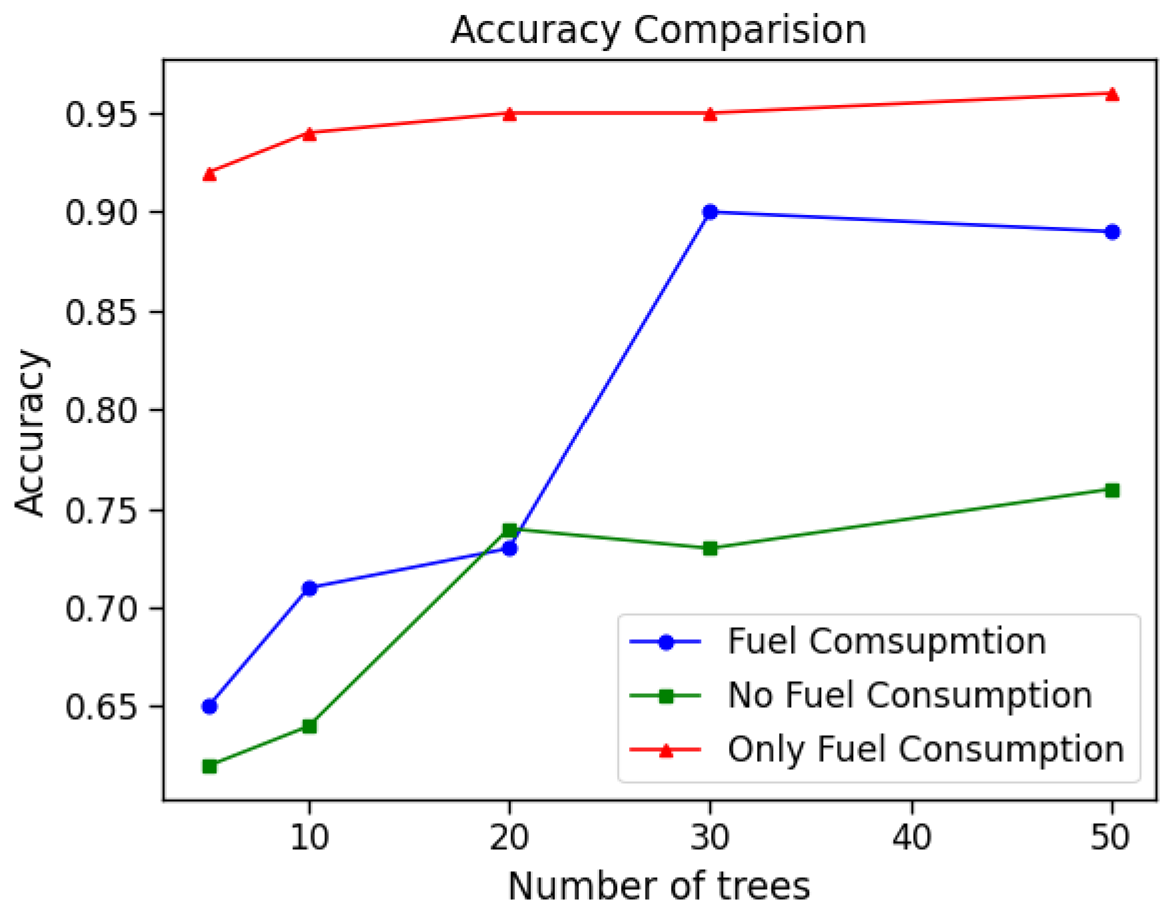
In [ ]:
```
max_features_only_fuel = int(np.log2(training_data_only_fuel.shape[1])) + 1
```

In [ ]:
```
for n in n_estimators:
    randomForestClassifier = RandomForestClassifier(n,max_features_only_fuel,10,10)
    forest = randomForestClassifier.fit(training_data_only_fuel[:1000],y_train_fuel[:1000])
    predictions = randomForestClassifier.predict(test_data_only_fuel)
    accuracy_only_fuel.append(accuracy(predictions,y_test_fuel))
    predictions_only_fuel.append(predictions)
```

```
Testing accuracy: 0.92
Testing accuracy: 0.94
Testing accuracy: 0.95
Testing accuracy: 0.95
Testing accuracy: 0.96
```

In [ ]:
```
def plot3lines(x1,y1,x2,y2,x3,y3,label):
    plt.plot(x1, y1, label='Fuel Comsupmtion', color='blue', marker='o')

    plt.plot(x2,y2, label='No Fuel Consumption', color='green', marker='s')

    plt.plot(x3,y3, label='Only Fuel Consumption', color='red', marker='^')

    plt.xlabel('Number of trees')
    plt.ylabel(label)
    plt.title(label+' Comparision')
    plt.legend()
    plt.show()
```

In [ ]:
```
plot3lines(n_estimators,accuracy_arr_fuel,n_estimators,accuracy_arr_no_fuel,n_estimators,accuracy_only_fuel,"Accuracy")
```

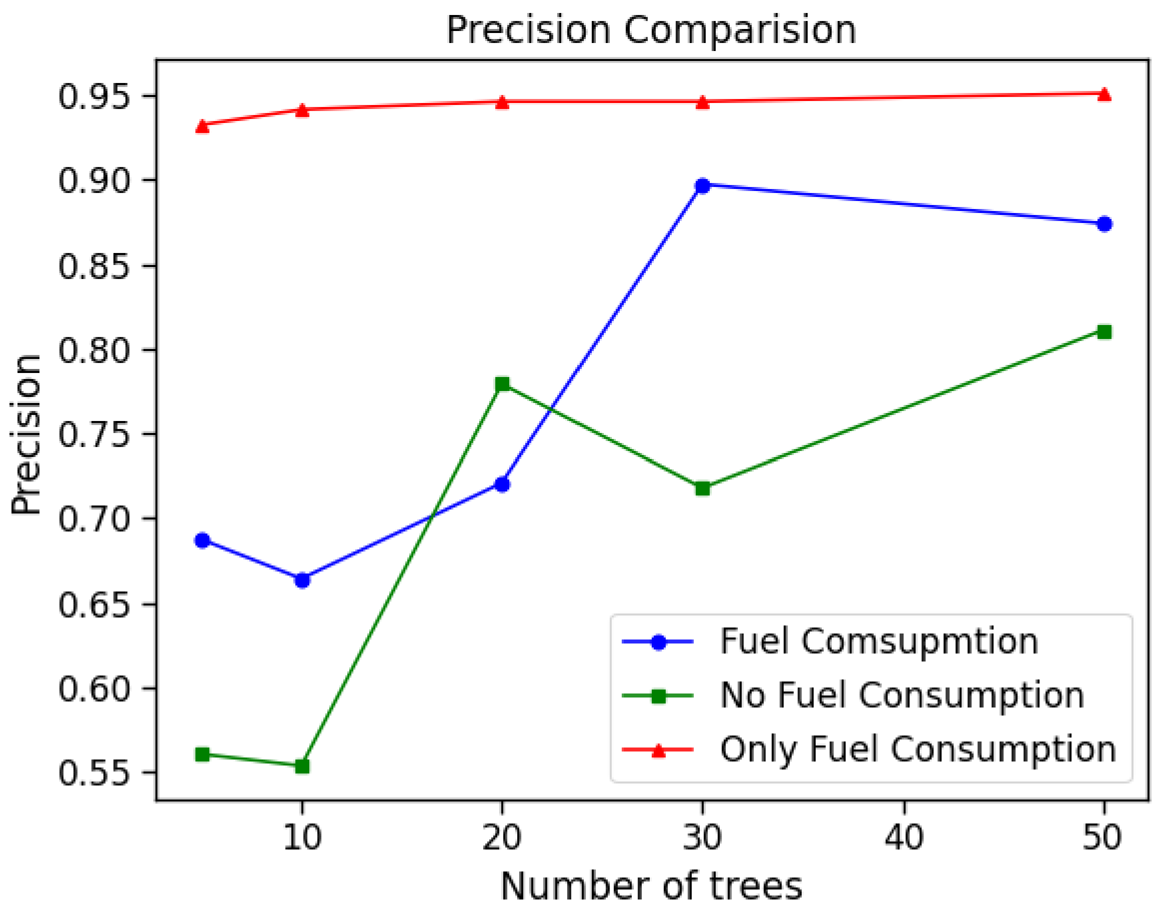


In [ ]:
```
weighted_avg(accuracy_arr_fuel,accuracy_arr_no_fuel,accuracy_only_fuel)
```

```
Weigthed Average Comparison:
With fuel comsumption: 0.838696
Without fuel comsumption: 0.732174
Only fuel consumption: 0.9521739130434783
```

In [ ]:
```
pres_fuel_1 = []
pres_fuel_2 = []
pres_fuel_3 = []
pres_no_fuel_1 = []
pres_no_fuel_2 = []
pres_no_fuel_3 = []
pres_only_fuel_1 = []
pres_only_fuel_2 = []
pres_only_fuel_3 = []

for i in predictions_fuel:
    _1,_2,_3 = precision(i,y_test_fuel)
    pres_fuel_1.append(_1)
    pres_fuel_2.append(_2)
    pres_fuel_3.append(_3)

for i in predictions_no_fuel:
    _1,_2,_3 = precision(i,y_test_no_fuel)
    pres_no_fuel_1.append(_1)
    pres_no_fuel_2.append(_2)
    pres_no_fuel_3.append(_3)

for i in predictions_only_fuel:
    _1,_2,_3 = precision(i,y_test_no_fuel)
    pres_only_fuel_1.append(_1)
    pres_only_fuel_2.append(_2)
    pres_only_fuel_3.append(_3)
```

In [ ]:
```
average_precision_fuel = []
average_precision_no_fuel = []
average_precision_only_fuel = []
for i in range(len(n_estimators)):
    average_precision_fuel.append((pres_fuel_1[i]+pres_fuel_2[i]+pres_fuel_3[i])/3)
    average_precision_no_fuel.append((pres_no_fuel_1[i]+pres_no_fuel_2[i]+pres_no_fuel_3[i])/3)
    average_precision_only_fuel.append((pres_only_fuel_1[i]+pres_only_fuel_2[i]+pres_only_fuel_3[i])/3)

plot3lines(n_estimators,average_precision_fuel,n_estimators,average_precision_no_fuel,n_estimators,average_precision_only_fuel
```



In [ ]:
```
weighted_avg(average_precision_fuel,average_precision_no_fuel,average_precision_only_fuel)
```

```
Weigthed Average Comparison:
With fuel comsumption: 0.827197
Without fuel comsumption: 0.748007
Only fuel consumption: 0.9472667979221687
```

In [ ]:
```
report_fuel = classification_report(y_test_fuel[:100], predictions_fuel[len(predictions_fuel)-1])
report_no_fuel = classification_report(y_test_fuel[:100], predictions_no_fuel[len(predictions_no_fuel)-1])
report_only_fuel = classification_report(y_test_fuel[:100], predictions_only_fuel[len(predictions_only_fuel)-1])
```

In [ ]:
```
print(report_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.88      0.78      0.82         9
         2.0       0.95      0.87      0.91        62
         3.0       0.80      0.97      0.88        29

    accuracy                           0.89       100
   macro avg       0.87      0.87      0.87       100
weighted avg       0.90      0.89      0.89       100
```

In [ ]:
```
print(report_no_fuel)
```

```
              precision    recall  f1-score   support

         1.0       1.00      0.67      0.80         9
         2.0       0.78      0.85      0.82        62
         3.0       0.65      0.59      0.62        29

    accuracy                           0.76       100
   macro avg       0.81      0.70      0.74       100
weighted avg       0.76      0.76      0.76       100
```

In [ ]:
```
print(report_only_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.90      1.00      0.95         9
         2.0       0.95      0.98      0.97        62
         3.0       1.00      0.90      0.95        29

    accuracy                           0.96       100
   macro avg       0.95      0.96      0.95       100
weighted avg       0.96      0.96      0.96       100
```

In [ ]:
```
prediction_fuel_random_forest = predictions_fuel.copy()
prediction_no_fuel_random_forest = predictions_no_fuel.copy()
prediction_only_fuel_random_forest = predictions_only_fuel.copy()
```

Insights:

For each dataset, as the number of trees increases the prediction accuracy either considerably increases or remains almost the same. This is consistent with our expectations from this algorithm it uses the law of large numbers.

Precision is defined as the ratio of number of points classified correctly as class j and the number of points predicted as class j. The precision of all models also increases with number of trees or remains almost the same which indicates powerful classification capabilities.

We can also see that for 1 tree in the forest, the accuracy is low. This shows that for a random forest, each tree is not a good classifier but the overall forest classifies with high accuracy.

The accuracy and precision for dataset that contains only fuel consumption far exceeds the dataset with all features and dataset with fuel consumption exculded. This shows that fuel consumption of all types together can easily predict the range of CO_2 emmisions thus showing that the other features are not as important as fuel consumption while checking for emmissions for vehicles. This is also complemented by the fact that the accuracy for classifying data without using fuel consumption gives lower accuracy on average from the other two.

5.Kernel K-Nearest Neighbours Classifier (Research Literature)

The Kernel K-Nearest Neighbors (Kernel KNN) algorithm is a sophisticated extension of the KNN method used for solving classification problems. It operates on the principle of leveraging similarity measures between new and existing test instances. This approach predicts the target value (label) by evaluating the proximity of the test instance to its neighbors in the training dataset, utilizing a kernel function to assess similarity and classify it into the appropriate category.

The principle of this algorithm is to identify clusters in the training data and classify the test data point in the cluster closest to it by assigning the test instance to a specific cluster based on its proximity or similarity, as determined by the chosen kernel function, facilitating accurate classification within the dataset's defined clusters.

In Kernel K-Nearest Neighbors (Kernel KNN), the use of a kernel allows for more flexible and powerful handling of the similarity measure between data points. Kernel KNN uses a kernel function to measure similarity by implicitly mapping the data into a higher-dimensional space. The kernel function enables us to handle non-linear relationships between data points by mapping them into a higher-dimensional space which is helpful in capturing complex patterns that might not be effectively captured in the original feature space. Other advantage is that it transform data in such a way that previously non-separable classes might become more separable in the new space, potentially enhancing classification accuracy

Algorithm Overview

Given a dataset with labeled instances, the KNN algorithm follows a straightforward process:

1. **Choose a value for `k`:** Select the number of neighbors (`k`) that the algorithm will consider when making predictions for a new, unseen data point.
2. **Calculate distances:** For a given test instance, calculate the distance to all instances in the training dataset. Common distance metrics include Euclidean distance, Manhattan distance, or others, depending on the problem.
3. **Calculate Gaussian Kernel:** For a given test instance, compute the similarity or weight between two data points based on their Euclidean distance. We prioritize points in close proximity by giving them more significance, while reducing the importance attributed to points that are situated farther apart.
4. **Identify nearest neighbors:** Identify the `k` instances in the training dataset with the smallest distances to the test instance. These instances become the "neighbors" that influence the prediction.
5. **Make a prediction:** For classification tasks, assign the class label that is most frequent among the `k` nearest neighbors. For regression tasks, predict the average value of the target variable for the `k` nearest neighbors.

Euclidean Distance

This is the cartesian distance between the two points which are in the plane/hyperplane. Given two points

$$A(a_1, a_2, a_3, \ldots) \quad \text{and} \quad B(b_1, b_2, b_3, \ldots)$$

in an n-dimensional space, the Euclidean distance between them is given by:

$$\text{Euclidean Distance} = \sqrt{\sum_{i=1}^{n} (a_i - b_i)^2}$$

For example, in a 3-dimensional space, the Euclidean distance:

$$= \sqrt{\sum_{i=1}^{n}(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2}$$

Gaussian Kernel

The Gaussian Kernel function computes the similarity or weight between two points, x and y, based on their Euclidean distance.

The formula for the Gaussian Kernel is expressed as:

$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right)$$

x and y represent the data points for which the similarity is being calculated.

$\|x - y\|$ denotes the Euclidean distance between x and y.

σ is a parameter known as the bandwidth that determines the width or scale of the kernel function.

Hyperparameter Tuning

The choice of `k` is a crucial hyperparameter in KNN. A small `k` may lead to overfitting, where the model is too sensitive to noise, while a large `k` may result in underfitting, where the model oversimplifies the underlying patterns.

Pros and Cons

Pros:

- **Simplicity:** Kernel KNN is easy to understand and implement.
- **No Training Phase:** Kernel KNN doesn't require an explicit training phase, making it suitable for dynamic datasets.
- **Versatility:**It can be applied to a wide range of problem types.

Cons:

- **Computationally Expensive:** Especially for large datasets, as it involves calculating distances for each test instance.
- **Sensitivity to Noise:** Kernel KNN is sensitive to irrelevant features and noisy data.

Let 's define a function to calculate the Euclidian distance between a test point and a training point using the square root of the sum of squared differences.

In [ ]:
```
def euclidean_distance(point1, point2):
    return np.sqrt(np.sum((point1 - point2)**2))
```

Let's define a function for Gaussian Kernel computation, determining the similarity between a test point and a training point. This function employs the Gaussian distribution to measure similarity, emphasizing proximity by assigning higher similarity values to nearby points.

In [ ]:
```
def gaussian_kernel(distance, bandwidth=1.0):
    return np.exp(-0.5 * ((distance / bandwidth) ** 2))
```

The ***kernel_knn_predict*** function performs the prediction using Kernel KNN for a single test point. It computes the Gaussian Kernel distances between the test point and all training points, identifies the k nearest neighbors based on the kernel distances, tallies the occurrences of each class, and makes a prediction by selecting the class with the highest count among the neighbors.

In [ ]:
```
def kernel_knn_predict(train_data, train_target, test_point, k, bandwidth=1.0):
    distances = []
    for i, train_instance in enumerate(train_data):
        dist = euclidean_distance(test_point, train_instance)
        distances.append((dist, train_target[i]))
    distances.sort(key=lambda x: x[0])
    neighbors = distances[:k]
    weights = [gaussian_kernel(neighbor[0], bandwidth=bandwidth) for neighbor in distances[:k]]
    weighted_votes = {}
    for i, (dist, label) in enumerate(distances[:k]):
        if label not in weighted_votes:
            weighted_votes[label] = 0
```

```
        weighted_votes[label] += weights[i]
    predicted_class = max(weighted_votes, key=weighted_votes.get)

    return predicted_class
```

In [ ]:
```
def kernel_knn_predict_batch(train_data, train_target, test_data, k, bandwidth=1.0):
    predictions = []
    for test_point in test_data:
        prediction = kernel_knn_predict(train_data, train_target, test_point, k, bandwidth=bandwidth)
        predictions.append(prediction)

    return np.array(predictions)
```

In [ ]:
```
X_train_fuel = training_data_fuel[:,:training_data_fuel.shape[1]-1]
y_train_fuel = training_data_fuel[:,training_data_fuel.shape[1]-1]
```

In [ ]:
```
X_test_fuel = test_data_fuel[:,:test_data_fuel.shape[1]-1]
y_test_fuel = test_data_fuel[:,test_data_fuel.shape[1]-1]
```

In [ ]:
```
X_train_no_fuel = training_data_no_fuel[:,:training_data_no_fuel.shape[1]-1]
y_train_no_fuel = training_data_no_fuel[:,training_data_no_fuel.shape[1]-1]
```

In [ ]:
```
X_test_no_fuel = test_data_no_fuel[:,:test_data_no_fuel.shape[1]-1]
y_test_no_fuel = test_data_no_fuel[:,test_data_no_fuel.shape[1]-1]
```

In [ ]:
```
only_fuel_data_copy = only_fuel_data.copy()
split = int(0.75*only_fuel_data_copy.shape[0])
training_data_only_fuel_copy, test_data_only_fuel_copy = only_fuel_data_copy[:split,:], only_fuel_data_copy[split:,:]
```

In [ ]:
```
predictions_fuel = []
predictions_no_fuel = []
predictions_only_fuel = []
k_values = [1,2,5,10]
accuracy_fuel = []
accuracy_no_fuel = []
accuracy_only_fuel = []
```

In [ ]:
```
for k in k_values:
  predictions = kernel_knn_predict_batch(X_train_fuel[:3000], y_train_fuel[:3000], X_test_fuel[:1000], k)
  accuracy_fuel.append(accuracy(predictions,y_test_fuel))
  predictions_fuel.append(predictions)
```

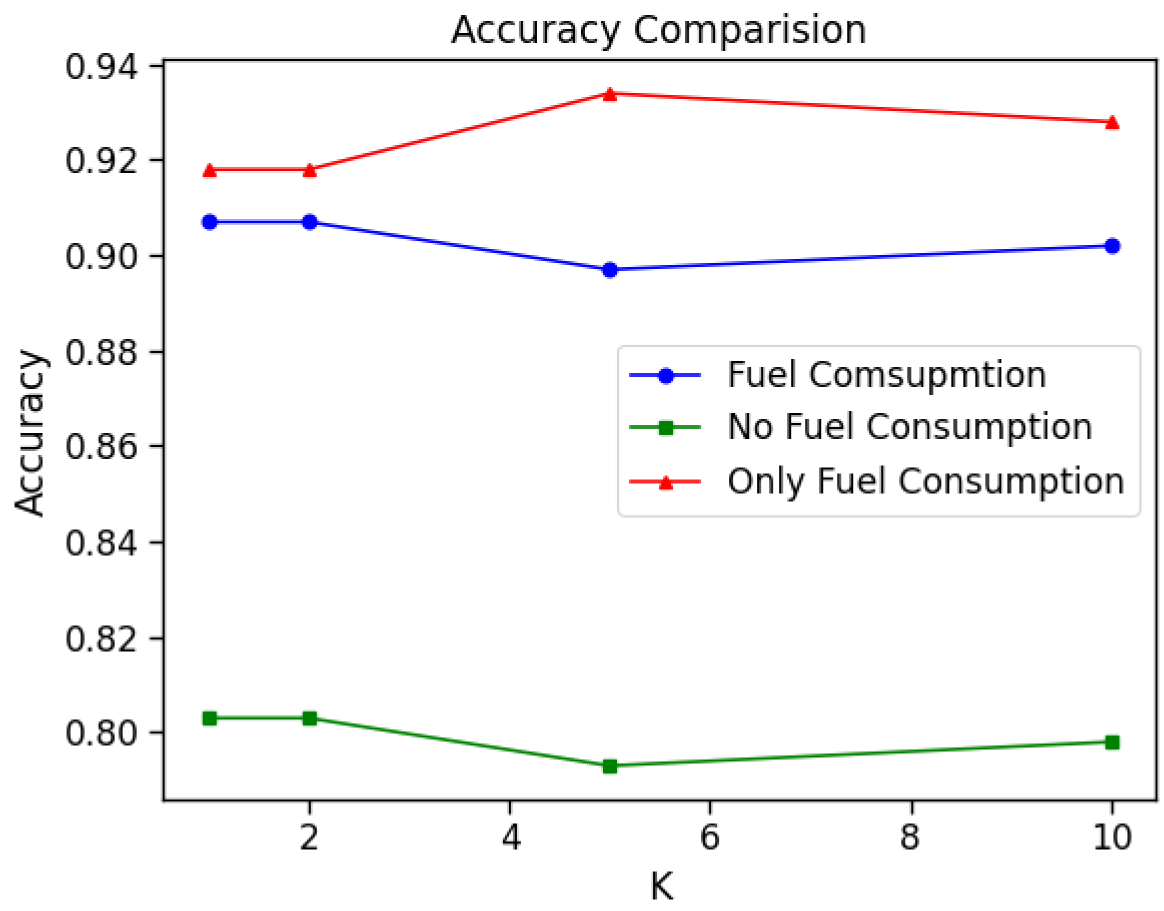
In [ ]:
```
for k in k_values:
  predictions = kernel_knn_predict_batch(X_train_no_fuel[:3000], y_train_no_fuel[:3000], X_test_no_fuel[:1000], k)
  accuracy_no_fuel.append(accuracy(predictions,y_test_no_fuel))
  predictions_no_fuel.append(predictions)
```

In [ ]:
```
for k in k_values:
  predictions =  kernel_knn_predict_batch(training_data_only_fuel_copy[:3000], y_train_fuel[:3000], test_data_only_fuel_copy[:
  accuracy_only_fuel.append(accuracy(predictions,y_test_fuel))
  predictions_only_fuel.append(predictions)
```

In [ ]:
```
def accuracy(predictions,y_test):
    acc = 0
    for i,val in enumerate(predictions):
        if(val==y_test[i]):
            acc = acc+1
    acc = acc/len(predictions)
    return acc
```

In [ ]:
```
def plot3lines(x1,y1,x2,y2,x3,y3,label):
    plt.plot(x1, y1, label='Fuel Comsupmtion', color='blue', marker='o')

    plt.plot(x2,y2, label='No Fuel Consumption', color='green', marker='s')

    plt.plot(x3,y3, label='Only Fuel Consumption', color='red', marker='^')

    plt.xlabel('K')
    plt.ylabel(label)
    plt.title(label+' Comparision')
    plt.legend()
    plt.show()
```

In [ ]:
```
plot3lines(k_values,accuracy_fuel,k_values,accuracy_no_fuel,k_values,accuracy_only_fuel,"Accuracy")
```



In [ ]:
```
report_fuel = classification_report(y_test_fuel[:1000], predictions_fuel[len(predictions_fuel)-1])
report_no_fuel = classification_report(y_test_fuel[:1000], predictions_no_fuel[len(predictions_no_fuel)-1])
report_only_fuel = classification_report(y_test_fuel[:1000], predictions_only_fuel[len(predictions_only_fuel)-1])
```

In [ ]:
```
print(report_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.86      0.93      0.90       230
         2.0       0.90      0.91      0.90       508
         3.0       0.95      0.86      0.90       262

    accuracy                           0.90      1000
   macro avg       0.90      0.90      0.90      1000
weighted avg       0.90      0.90      0.90      1000
```

In [ ]:
```
print(report_no_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.74      0.82      0.78       230
         2.0       0.80      0.81      0.80       508
         3.0       0.87      0.76      0.81       262

    accuracy                           0.80      1000
   macro avg       0.80      0.80      0.80      1000
weighted avg       0.80      0.80      0.80      1000
```

In [ ]:
```
print(report_only_fuel)
```

```
              precision    recall  f1-score   support

         1.0       0.92      1.00      0.96       230
         2.0       0.92      0.94      0.93       508
         3.0       0.94      0.85      0.90       262

    accuracy                           0.93      1000
   macro avg       0.93      0.93      0.93      1000
weighted avg       0.93      0.93      0.93      1000
```

In [ ]:
```
prediction_only_fuel_knn = predictions_only_fuel.copy()
prediction_fuel_knn = predictions_fuel.copy()
prediction_no_fuel_knn = predictions_no_fuel.copy()
```

6. Insights: Best Car Models

In [ ]:
```
def getBestCars(cars,y_pred):
  best_cars = set()
  for i,val in enumerate(y_pred):
    if(val==1):
      best_cars.add(cars[i])
  return best_cars
```

In [ ]:
```
best_cars_mlp_fuel = getBestCars(car_models,prediction_fuel_mlp[len(prediction_fuel_mlp)-1])
best_cars_naive_bayes_fuel = getBestCars(car_models,prediction_fuel_naive_bayes)
best_cars_knn_fuel = getBestCars(car_models,prediction_fuel_knn[len(prediction_fuel_knn)-1])
best_cars_random_forest_fuel = getBestCars(car_models,prediction_fuel_random_forest[len(prediction_fuel_random_forest)-1])
all_cars_fuel = set.union(best_cars_mlp_fuel,best_cars_naive_bayes_fuel,best_cars_knn_fuel,best_cars_random_forest_fuel)
print("Best Cars predicted using all features: ")
for  u in all_cars_fuel:
  print(u)
```

Best Cars predicted using all features:
Civic Coupe
Boxster S
Beetle Dune Convertible
John Cooper Works Convertible
A 220 4MATIC
Colorado
Cherokee
NX 300 AWD F SPORT
E-PACE P250
911 Turbo
Nautilus
AMG GT 53 4MATIC+ Coupe
Odyssey
Range Rover 3.0
G70
Wrangler JL Unlimited 4X4
Sierra WT
Discovery Sport P250
Rogue
John Cooper Works Clubman ALL4
Q70 AWD
RC F
Explorer FFV AWD
911 Targa 4
TTS Coupe
SL 550
Beetle Convertible
AMG S 63 4MATIC+
Enclave AWD
Sonata Hybrid SE
CLS 450 4MATIC
AMG E 53 4MATIC+ Wagon
A6 allroad
911 Carrera 4 GTS Cabriolet
F-PACE 30t
XE 25t AWD
Grand Cherokee 4X4
Highlander AWD (Start/Stop System)
Corolla XSE
RVR
Civic Hatchback Sport
Niro
Golf GTI
Journey FFV
EcoSport AWD
CTS AWD
Cooper S Convertible
AMG E 53 4MATIC+ Coupe
Cooper S Countryman ALL4
M4 Coupe
S5 Sportback
Range Rover Evoque P250
A5 Cabriolet quattro
Cooper Countryman ALL4
Ridgeline AWD
Nautilus AWD
Boxster
Mirage
Range Rover Velar P300
Tucson AWD
911 Turbo Cabriolet
Cruze Premier
Macan
X5 xDrive40i
Sportage AWD
Camry Hybrid XLE/SE
AMG C 43 4MATIC Cabriolet
Pathfinder
TT Coupe quattro
Forester AWD
Highlander Hybrid AWD Limited/Platinum
Metris Passenger
Pilot
440i xDrive Coupe
X1 xDrive28i
F-TYPE Coupe R-Dynamic
AMG S 63 4MATIC+ Coupe
AMG GLA 45 4MATIC
XTS AWD
X4 xDrive30i
Panamera 4
CX-30 4WD (Cylinder Deactivation)
Sedona
Panamera 4 ST
Cooper Convertible
Panamera 4S

Mazda3 5-Door
Santa Fe XL AWD
ILX
A5 Sportback quattro
Q50 AWD Red Sport
Tiguan 4MOTION
A7 quattro
Highlander Hybrid AWD
Trax AWD
Colorado 4WD
Fusion
M240i xDrive Coupe M Performance
XT4
CLA 250 4MATIC
F-TYPE Coupe R-Dynamic AWD
S7
1500 Classic
Escape Hybrid AWD
A8L
911 Carrera S
Mazda3 4-Door 4WD
HR-V AWD
XC90 T6 AWD
XT4 AWD
CT4 AWD
UX 200
S 560 Cabriolet
Maybach S 560 4MATIC
Renegade
V60 CC T5 AWD
E 450 4MATIC Wagon
XT5
LaCrosse
Civic Hatchback
IS 300
CX-30 4WD
X3 xDrive30i
RAV4
AMG CLS 53 4MATIC+
F-TYPE Coupe
AMG GLE 43 4MATIC
CX-9
Mustang (Performance Pkg)
911 Carrera 4S Cabriolet
Veloster
IS 300 AWD
F-150 4X4 XL/XLT
Compass 4X4
Atlas
Trax
M850i xDrive Gran Coupe
Envision AWD
440i xDrive Cabriolet
E 300 4MATIC
Insight EX
DB11 V8
XC60 T6 AWD
Panamera Turbo Executive
911 Carrera Cabriolet
MX-5 (SIL)
Versa
Giulia Quadrifoglio
Z4 M40i
XE 35t AWD
Eclipse Cross 4WD
300
Challenger
Panamera 4S Executive
CLA 250
Pilot AWD
M850i xDrive Coupe
Pacifica (Stop-Start)
Mustang Convertible (Performance Pkg)
F-TYPE Convertible R-Dynamic
Escape
CT4
Vantage V8
Avalon
CX-5 (Cylinder Deactivation)
A6 quattro
X2 M35i
1500 Classic FFV
CX-5 Diesel 4WD
S60 T6 AWD
Sentra Nismo
V60 T5
Mazda6 Turbo

911 Carrera GTS
911 Targa 4S
Jetta GLI
Golf SportWagen
AMG C 63
Civic Type R
911 Carrera 4 GTS
Terrain
Crosstrek AWD
F-150 FFV 4X4 (LT Tire Pkg)
530i xDrive Sedan
F-TYPE P300 Convertible
Atlas 4MOTION
MDX SH-AWD
Beetle Dune
Impreza 5-Door AWD
SLC 300
Sentra (Turbo)
Q3 quattro
CX-5
Cooper 3 Door
440i Coupe
GS 350 AWD
430i xDrive Gran Coupe
Mazda6
S 450 4MATIC SWB
Cooper S 3 Door
X6 xDrive35i
Highlander
911 Targa 4 GTS
Edge
Renegade 4X4 Trailhawk
CT4-V AWD
CT5 AWD
XF S AWD
430i xDrive Coupe
Micra
AMG SLC 43
Tacoma 4WD
Veloster N
S90 T6 AWD
Mazda3 4-Door (Cylinder Deactivation)
Terrain AWD
M550i xDrive Sedan
Spark
Equinox
F-150 (LT Tire Pkg)
F-150 4X4 (Payload Pkg)
Impala
RS 3
86
Aviator
Escape FFV
Jetta
TLX A-SPEC
RX 350 L AWD
Ghibli
370Z Roadster
Sorento AWD
Q60 AWD Red Sport
LS 500 AWD
Panamera 4 Executive
E 450 4MATIC
Fiesta
S90 T5 AWD
Canyon
Beetle
V60 T6 AWD
Yaris
Regal AWD
S5 Cabriolet
C 300 4MATIC
Camry XLE/XSE
Altima
Ghibli S
Forte
M240i xDrive Coupe
QX50 AWD
Tacoma
V90 T6 AWD
Giulia AWD
LS 500h AWD
Silverado WT
Corolla LE Eco
Kona AWD
Prius c
Wrangler JL 4X4

Cherokee 4X4 Active Drive Lock
Corvette
Sierra
Sienna
Range Rover Velar P340
Range Rover Sport TD6 Diesel
Golf Alltrack
X7 xDrive40i
Renegade 4X4
Charger FFV
Wrangler 4X4 eTorque
Q50 AWD
1500 HFE eTorque
Camaro
XF 35t AWD
540i xDrive Sedan
300 AWD FFV
911 Carrera S Cabriolet
Sienna AWD
TLX SH-AWD
Regal
UX 250h
Pathfinder 4WD
Range Rover Velar P250
C 300 4MATIC Wagon
XC60 T5 AWD
M550i xDrive
Trax 4WD
Panamera
430i xDrive Cabriolet
AMG CLA 45 4MATIC
Atlas Cross Sport 4MOTION
Sierra WT 4WD
Sportage
Cadenza
F-150 FFV 4X4
RX 350 AWD
Transit Connect Van FFV
IS 350 AWD
Altima SR/Platinum
Encore AWD (SIDI with Stop/Start)
AMG E 53 4MATIC+
Range Rover Sport 3.0
Santa Fe AWD
Altima AWD
MX-5
C 300 4MATIC Cabriolet
Veloster Turbo
Tahoe FFV
B 250
CT5
Soul
Niro FE
Cooper S Clubman ALL4
LaCrosse AWD
F-PACE S
Odyssey Touring
A 250
C-HR
Sentra
230i xDrive Coupe
MKC AWD (Start/Stop)
Golf SportWagen 4MOTION
John Cooper Works Countryman ALL4
F-150 FFV
A4 quattro
Transit Connect Wagon LWB
ES 350
Highlander AWD
Blazer
G90 AWD
911 Turbo S
XJ R-Sport AWD
Fit
Discovery Sport P290
Cayman GTS
Transit Connect Van LWB
Civic Sedan
Panamera Turbo ST
Range Rover TD6 Diesel
Pathfinder 4WD Platinum
M2 Competition
CX-3 (SIL)
E 450 4MATIC Cabriolet
Charger AWD
Fiesta ST
UX 250h AWD

Acadia AWD
XC90 T5 AWD
Blazer AWD
GLC 300 4MATIC Coupe
1500 Classic 4X4 FFV
Quattroporte S
NV200 Cargo Van
GLE 400 4MATIC
Mazda3 4-Door
Stelvio
Range Rover Velar D180
MKZ AWD
Ascent AWD
F-150 FFV (LT Tire Pkg)
F-150
Enclave
S 560 4MATIC SWB
LC 500h
F-TYPE P300 Coupe
Qashqai AWD
G80 AWD
Wrangler Unlimited 4X4
Optima Hybrid
Continental AWD
F-150 4X4 (LT Tire Pkg)
Durango AWD
M240i Coupe
Silverado
Encore GX
1500 eTorque
ES 300h
Journey
Golf
TT RS
300 FFV
Malibu Hybrid
Wrangler Unlimited 4X4 EcoDiesel
911 Carrera 4
Canyon 4WD
Forte 5
124 Spider
ProMaster City
Metris Cargo LWB
S60 T5
SL 450
Wrangler 4X4
CX-3 4WD
TT Roadster quattro
S3
Suburban FFV
Passport AWD
Ranger 4WD
Camry Hybrid LE
AMG E 53 4MATIC+ Cabriolet
1500 4X4 EcoDiesel
AMG C 63 S Coupe
Q5
Boxster GTS
Rio
Golf R
XF 25t AWD
Continental GT
911 Carrera 4S
RC 300 AWD
XC40 T4 AWD
Altima AWD SR/Platinum
CT4-V
M240i Cabriolet
Acadia
Explorer Hybrid AWD
Yukon XL FFV
M4 Cabriolet
Wrangler Unlimited 4X4 eTorque
Telluride AWD
CX-30
500
Grand Cherokee 4X4 EcoDiesel
440i xDrive Gran Coupe
1500 Classic 4X4 EcoDiesel
Outlander 4WD
Accord Sport/Touring
S6
Cruze Hatchback
Cayman S
M4 Cabriolet Competition
Escape AWD
Traverse AWD

F-TYPE Convertible R-Dynamic AWD
SQ5
XC40 T5 AWD
Sonata
Encore AWD
F-TYPE Convertible
G70 AWD
BRZ tS
Macan S
Sierra 4WD AT4
QX60 AWD
Voyager (Stop-Start)
RAV4 Hybrid AWD
Sentra SR
RX 450h AWD
Cooper Clubman ALL4
Charger
Santa Fe
A3 Cabriolet quattro
Encore (SIDI with Stop/Start)
Silverado 4WD
Legacy AWD
Explorer AWD
Elantra GT
500X AWD
XF 20d AWD
1500 Classic EcoDiesel
Murano AWD
Kona
Cherokee 4X4 Active Drive II
Mazda3 5-Door (SIL)
MDX SH-AWD A-SPEC
F-PACE 20d
Corolla XLE
Taurus FFV AWD
Grand Caravan
Accord
A 220
RDX AWD
Continental GT Convertible
Cayenne S
Cruze
S 560 4MATIC
XF 30t AWD
1500 EcoDiesel
WRX AWD
Range Rover Evoque P300
XE P300 AWD
MKZ Hybrid
Mirage G4
A3 quattro
M240i Coupe M Performance
E 450 4MATIC Coupe
M240i xDrive Cabriolet
Impreza 4-Door AWD
Frontier
X6 xDrive40i
Equinox AWD
Q60 AWD
E-PACE P300
Civic Coupe Si
Yaris (SIL)
MKC AWD
S4
Stinger AWD
Civic Sedan Si
Corsair AWD
Cruze Diesel
Colorado ZR2 4WD
370Z
RAV4 AWD
Grand Caravan FFV
RVR 4WD
Passat
911 Carrera
Prius
Cherokee 4X4 Active Drive I
230i xDrive Cabriolet
Charger AWD FFV
Malibu
911 Carrera 4 Cabriolet
Corolla Hatchback
Discovery TD6 Diesel
Frontier 4WD
Cayman
IONIQ Blue
F-150 FFV (Payload Pkg)

Challenger AWD
Range Rover Evoque Convertible
CX-5 4WD
AMG C 43 4MATIC Wagon
Range Rover Velar P380
Corolla Hybrid
NX 300 AWD
Corolla
S5
M850i xDrive Cabriolet
Compass
Camry
Prius AWD
HR-V
Encore
CX-9 4WD
Traverse
Taurus FFV
CX-5 Turbo 4WD
Escape Hybrid
Fusion Hybrid
Arteon 4MOTION
Transit Connect Van
Camaro SS
Cooper S 5 Door
AMG C 63 S Cabriolet
CR-V AWD
CR-V
Insight EX/Touring
Accord Hybrid
Challenger (MDS)
F-PACE 25t
GLA 250 4MATIC
Sierra 4WD
Range Rover Evoque
Palisade AWD
911 Carrera GTS Cabriolet
BRZ
Cruze Hatchback Premier
TLX SH-AWD A-SPEC/Limited Edition
Tiguan
Mazda3 4-Door (SIL)
Tucson
RDX AWD A-SPEC
Highlander AWD LE
C 300 4MATIC Coupe
Insight Touring
Yaris Hatchback
GR Supra
MDX Hybrid AWD
Sorento
Outback AWD
B 250 4MATIC
1500 4X4 eTorque
AMG C 63 S
911 Carrera T
M340i xDrive Sedan
Qashqai
Panamera Turbo
A 250 4MATIC
John Cooper Works 3 Door
Murano
Discovery Sport
Edge AWD
XT6 AWD
XTS
Stelvio AWD
LaCrosse eAssist
AMG GLC 43 4MATIC Coupe
Cayenne
4C Spider
X3 M40i
Mustang
Camry TRD
XJL Portfolio AWD
Transit Connect Wagon LWB FFV
Kicks
A5 quattro
AMG GLC 43 4MATIC
X2 xDrive28i
NX 300h AWD
CX-5 4WD (Cylinder Deactivation)
Palisade
RX 450h L AWD
CX-3
Metris Cargo
ES 350 F SPORT

```
X4 M40i
RC 350 AWD
Elantra
Accent
Sonata SE
Yukon FFV
AMG C 43 4MATIC
IONIQ
Silverado WT 4WD
Camry LE/SE
300 AWD
911 Turbo S Cabriolet
Optima
CT6 AWD
Sonata Hybrid
Santa Fe XL
F-150 4X4
Mustang Convertible
XE 20d AWD
F-PACE 35t
A3
Panamera 4S ST
Niro Touring
V90 CC T6 AWD
GLC 300 4MATIC
A4 allroad quattro
Mazda3 5-Door 4WD
500L
CTS
XT5 AWD
M4 Coupe Competition
Rogue AWD
Camry XSE
RLX Hybrid
Venue
CT5-V AWD
Challenger GT AWD
Cruze Hatchback Diesel
F-150 (Payload Pkg)
CT5-V
Cooper 5 Door
AMG C 43 4MATIC Coupe
Maxima
Pacifica
```

In [ ]:

```python
best_cars_mlp_no_fuel = getBestCars(car_models,prediction_no_fuel_mlp[len(prediction_no_fuel_mlp)-1])
best_cars_naive_bayes_no_fuel = getBestCars(car_models,prediction_no_fuel_naive_bayes)
best_cars_knn_no_fuel = getBestCars(car_models,prediction_no_fuel_knn[len(prediction_no_fuel_knn)-1])
best_cars_random_forest_no_fuel = getBestCars(car_models,prediction_no_fuel_random_forest[len(prediction_no_fuel_random_forest
all_cars_no_fuel = set.union(best_cars_mlp_no_fuel,best_cars_naive_bayes_no_fuel,best_cars_knn_no_fuel,best_cars_random_forest
print("Best Cars predicted without considering fuel consumption: ")
for  u in all_cars_no_fuel:
  print(u)
```

Best Cars predicted without considering fuel consumption:
Civic Coupe
Ghibli S Q4
Boxster S
Beetle Dune Convertible
John Cooper Works Convertible
911 GT2 RS
A 220 4MATIC
Colorado
Cherokee
NX 300 AWD F SPORT
E-PACE P250
911 Turbo
Nautilus
AMG GT 53 4MATIC+ Coupe
Odyssey
G70
Wrangler JL Unlimited 4X4
Sierra WT
Discovery Sport P250
Rogue
John Cooper Works Clubman ALL4
Q70 AWD
Explorer FFV AWD
911 Targa 4
SL 550
Beetle Convertible
AMG S 63 4MATIC+
Enclave AWD
Sonata Hybrid SE
CLS 450 4MATIC
AMG E 53 4MATIC+ Wagon
A6 allroad
911 Carrera 4 GTS Cabriolet
F-PACE 30t
XE 25t AWD
Grand Cherokee 4X4
Highlander AWD (Start/Stop System)
Corolla XSE
RVR
Giulia
Civic Hatchback Sport
Niro
Golf GTI
Journey FFV
Corvette ZR1
EcoSport AWD
911 Turbo S Exclusive Cabriolet
CTS AWD
Cooper S Convertible
AMG E 53 4MATIC+ Coupe
Cooper S Countryman ALL4
M4 CS
XE P250 AWD
M4 Coupe
S5 Sportback
Range Rover Evoque P250
230i Cabriolet
Cooper Countryman ALL4
Ridgeline AWD
Nautilus AWD
Boxster
M5 Competition
Range Rover Velar P300
Mirage
Tucson AWD
911 Turbo Cabriolet
Cruze Premier
Macan
X5 xDrive40i
Sportage AWD
Camry Hybrid XLE/SE
AMG C 43 4MATIC Cabriolet
Pathfinder
TT Coupe quattro
Forester AWD
Highlander Hybrid AWD Limited/Platinum
Silverado 4WD Custom Trail Boss
Metris Passenger
Pilot
440i xDrive Coupe
X1 xDrive28i
F-TYPE Coupe R-Dynamic
AMG S 63 4MATIC+ Coupe
AMG GLA 45 4MATIC
XTS AWD
X4 xDrive30i

Panamera 4
CX-30 4WD (Cylinder Deactivation)
Stelvio AWD Quadrifoglio
Sedona
Panamera 4 ST
Cooper Convertible
Panamera 4S
Mazda3 5-Door
AMG GLE 43 4MATIC Coupe
Santa Fe XL AWD
ILX
Q50 AWD Red Sport
Tiguan 4MOTION
A7 quattro
M8 Gran Coupe Competition
Highlander Hybrid AWD
Colorado 4WD
Fusion
M240i xDrive Coupe M Performance
XT4
Panamera GTS ST
CLA 250 4MATIC
F-TYPE Coupe R-Dynamic AWD
S7
1500 Classic
Escape Hybrid AWD
A8L
911 Carrera S
Mazda3 4-Door 4WD
HR-V AWD
XC90 T6 AWD
XT4 AWD
CT4 AWD
UX 200
S 560 Cabriolet
Maybach S 560 4MATIC
Renegade
V60 CC T5 AWD
E 450 4MATIC Wagon
911 GT3
XT5
LaCrosse
M5 Sedan
Civic Hatchback
Panamera GTS
IS 300
CX-30 4WD
X3 xDrive30i
RAV4
AMG CLS 53 4MATIC+
F-TYPE Coupe
F-150 4X4 Limited
AMG GLE 43 4MATIC
CX-9
Mustang (Performance Pkg)
911 Carrera 4S Cabriolet
Veloster
IS 300 AWD
F-150 4X4 XL/XLT
Compass 4X4
Atlas
Trax
M850i xDrive Gran Coupe
AMG E 63 S 4MATIC+
Envision AWD
440i xDrive Cabriolet
E 300 4MATIC
Insight EX
DB11 V8
XC60 T6 AWD
Panamera Turbo Executive
911 Carrera Cabriolet
MX-5 (SIL)
X3 M
Versa
Giulia Quadrifoglio
Z4 M40i
XE 35t AWD
Eclipse Cross 4WD
Gladiator 4X4
300
Challenger
Panamera 4S Executive
CLA 250
Pilot AWD
Pacifica (Stop-Start)
Mustang Convertible (Performance Pkg)

F-TYPE Convertible R-Dynamic
Escape
CT4
Vantage V8
Avalon
CX-5 (Cylinder Deactivation)
A6 quattro
X2 M35i
1500 Classic FFV
S60 T6 AWD
Sentra Nismo
CX-5 Diesel 4WD
V60 T5
Mazda6 Turbo
911 Carrera GTS
911 Targa 4S
Quattroporte SQ4
Jetta GLI
Golf SportWagen
AMG C 63
Civic Type R
911 Carrera 4 GTS
Terrain
Crosstrek AWD
AMG S 63 4MATIC+ Cabriolet
F-150 FFV 4X4 (LT Tire Pkg)
Atlas 4MOTION
AMG E 63 S 4MATIC+ Wagon
MDX SH-AWD
AMG GT 63 4MATIC+ Coupe
Beetle Dune
Impreza 5-Door AWD
SLC 300
Sentra (Turbo)
Q3 quattro
CX-5
Cooper 3 Door
440i Coupe
GS 350 AWD
Mazda6
S 450 4MATIC SWB
Cooper S 3 Door
X6 xDrive35i
Highlander
911 Targa 4 GTS
Edge
CT4-V AWD
CT5 AWD
XF S AWD
Micra
AMG SLC 43
Tacoma 4WD
Veloster N
S90 T6 AWD
Mazda3 4-Door (Cylinder Deactivation)
Terrain AWD
911 Speedster
M550i xDrive Sedan
Spark
Equinox
F-150 (LT Tire Pkg)
F-150 4X4 (Payload Pkg)
Impala
RS 3
86
Aviator
Escape FFV
TLX A-SPEC
Jetta
RX 350 L AWD
Ghibli
370Z Roadster
Sorento AWD
Q60 AWD Red Sport
LS 500 AWD
AMG GLC 63 S 4MATIC+ Coupe
Panamera 4 Executive
E 450 4MATIC
Fiesta
S90 T5 AWD
Range Rover Velar SVAutobiography Dynamic
Canyon
Beetle
V60 T6 AWD
Yaris
Regal AWD
S5 Cabriolet

C 300 4MATIC
Camry XLE/XSE
Altima
Ghibli S
Forte
M240i xDrive Coupe
QX50 AWD
Tacoma
V90 T6 AWD
Giulia AWD
LS 500h AWD
Silverado WT
Corolla LE Eco
Kona AWD
Prius c
Wrangler JL 4X4
Cherokee 4X4 Active Drive Lock
Corvette
Sierra
Sienna
Range Rover Velar P340
Range Rover Sport TD6 Diesel
Golf Alltrack
X7 xDrive40i
Renegade 4X4
Charger FFV
Wrangler 4X4 eTorque
Q50 AWD
X3 M Competition
1500 HFE eTorque
Camaro
XF 35t AWD
540i xDrive Sedan
300 AWD FFV
911 Carrera S Cabriolet
Sienna AWD
TLX SH-AWD
Regal
UX 250h
Pathfinder 4WD
Range Rover Velar P250
C 300 4MATIC Wagon
XC60 T5 AWD
M550i xDrive
Trax 4WD
Panamera
430i xDrive Cabriolet
MKT Livery AWD
AMG CLA 45 4MATIC
Atlas Cross Sport 4MOTION
Sierra WT 4WD
Sportage
Cadenza
F-150 FFV 4X4
RX 350 AWD
Transit Connect Van FFV
IS 350 AWD
Altima SR/Platinum
Encore AWD (SIDI with Stop/Start)
AMG E 53 4MATIC+
Santa Fe AWD
Altima AWD
MX-5
C 300 4MATIC Cabriolet
230i Coupe
Q8
Veloster Turbo
X4 M
Tahoe FFV
B 250
CT5
Soul
Niro FE
Cooper S Clubman ALL4
LaCrosse AWD
F-PACE S
Odyssey Touring
A 250
C-HR
Sentra
230i xDrive Coupe
MKC AWD (Start/Stop)
John Cooper Works Countryman ALL4
Golf SportWagen 4MOTION
F-150 FFV
Transit Connect Wagon LWB
ES 350

Highlander AWD
Blazer
G90 AWD
911 Turbo S
F-PACE SVR
XJ R-Sport AWD
Fit
Discovery Sport P290
Cayman GTS
Ghibli SQ4
Transit Connect Van LWB
Civic Sedan
Panamera Turbo ST
Range Rover TD6 Diesel
Pathfinder 4WD Platinum
M2 Competition
CX-3 (SIL)
E 450 4MATIC Cabriolet
Charger AWD
Fiesta ST
UX 250h AWD
Acadia AWD
XC90 T5 AWD
Blazer AWD
GLC 300 4MATIC Coupe
1500 Classic 4X4 FFV
Quattroporte S
NV200 Cargo Van
GLE 400 4MATIC
Mazda3 4-Door
Stelvio
1500 Classic 4X4
Range Rover Velar D180
MKZ AWD
Ascent AWD
F-150 FFV (LT Tire Pkg)
Journey AWD
F-150
Enclave
Quattroporte S Q4
S 560 4MATIC SWB
LC 500h
Qashqai AWD
G80 AWD
Wrangler Unlimited 4X4
T-150 Wagon FFV
Continental AWD
F-150 4X4 (LT Tire Pkg)
Optima Hybrid
Durango AWD
M240i Coupe
Silverado
1500 eTorque
ES 300h
Journey
Golf
TT RS
300 FFV
Malibu Hybrid
Wrangler Unlimited 4X4 EcoDiesel
911 Carrera 4
Canyon 4WD
Forte 5
F-150 Raptor 4X4
M5
124 Spider
ProMaster City
Metris Cargo LWB
S60 T5
SL 450
Wrangler 4X4
CX-3 4WD
TT Roadster quattro
Suburban FFV
Passport AWD
Ranger 4WD
Camry Hybrid LE
AMG E 53 4MATIC+ Cabriolet
1500 4X4 EcoDiesel
S8
AMG C 63 S Coupe
Q5
Boxster GTS
Rio
Golf R
RAV4 AWD TRD Off-Road
XF 25t AWD

Continental GT
911 Carrera 4S
RC 300 AWD
Q7
T-150 Wagon FFV 4WD
XC40 T4 AWD
Altima AWD SR/Platinum
CT4-V
M240i Cabriolet
Acadia
GLS 450 4MATIC
Explorer Hybrid AWD
M4 Cabriolet
Wrangler Unlimited 4X4 eTorque
M8 Gran Coupe
Telluride AWD
CX-30
500
Grand Cherokee 4X4 EcoDiesel
440i xDrive Gran Coupe
1500 Classic 4X4 EcoDiesel
Outlander 4WD
Accord Sport/Touring
S6
Cruze Hatchback
Cayman S
M4 Cabriolet Competition
Escape AWD
Traverse AWD
F-TYPE Convertible R-Dynamic AWD
SQ5
XC40 T5 AWD
Sonata
Encore AWD
F-TYPE Convertible
G70 AWD
BRZ tS
Macan S
T-150 Wagon
Sierra 4WD AT4
QX60 AWD
Voyager (Stop-Start)
RAV4 Hybrid AWD
Sentra SR
Cooper Clubman ALL4
Charger
Santa Fe
A3 Cabriolet quattro
Encore (SIDI with Stop/Start)
Silverado 4WD
Taurus AWD
Legacy AWD
Explorer AWD
Elantra GT
XF 20d AWD
1500 Classic EcoDiesel
Murano AWD
AMG GLC 63 S 4MATIC+
Kona
Cherokee 4X4 Active Drive II
Mazda3 5-Door (SIL)
MDX SH-AWD A-SPEC
F-PACE 20d
Corolla XLE
Taurus FFV AWD
Grand Caravan
A 220
Accord
RDX AWD
Continental GT Convertible
Cayenne S
Cruze
S 560 4MATIC
XF 30t AWD
1500 EcoDiesel
911 GT3 Touring
WRX AWD
Range Rover Evoque P300
XE P300 AWD
MKZ Hybrid
Mirage G4
A3 quattro
M240i Coupe M Performance
E 450 4MATIC Coupe
M240i xDrive Cabriolet
Impreza 4-Door AWD
Frontier

X6 xDrive40i
Equinox AWD
Q60 AWD
E-PACE P300
Civic Coupe Si
Yaris (SIL)
MKC AWD
S4
Stinger AWD
Civic Sedan Si
Corsair AWD
Cruze Diesel
Colorado ZR2 4WD
370Z
RAV4 AWD
Grand Caravan FFV
RVR 4WD
Passat
911 Carrera
Prius
Cherokee 4X4 Active Drive I
230i xDrive Cabriolet
X4 M Competition
Charger AWD FFV
Malibu
911 Carrera 4 Cabriolet
Corolla Hatchback
Discovery TD6 Diesel
Frontier 4WD
Cayman
IONIQ Blue
Challenger AWD
Range Rover Evoque Convertible
CX-5 4WD
AMG C 43 4MATIC Wagon
Corvette Z06
Range Rover Velar P380
Corolla Hybrid
NX 300 AWD
Corolla
S5
Camry
Compass
Prius AWD
HR-V
Encore
CX-9 4WD
Traverse
Taurus FFV
Santa Fe XL Ultimate AWD
CX-5 Turbo 4WD
Escape Hybrid
Silverado 4WD Trail Boss
Fusion Hybrid
Arteon 4MOTION
Transit Connect Van
Cooper S 5 Door
AMG C 63 S Cabriolet
CR-V AWD
CR-V
Insight EX/Touring
Accord Hybrid
WRX STI AWD
F-PACE 25t
GLA 250 4MATIC
Sierra 4WD
Range Rover Evoque
911 Carrera GTS Cabriolet
BRZ
Cruze Hatchback Premier
TLX SH-AWD A-SPEC/Limited Edition
Tiguan
Mazda3 4-Door (SIL)
Tucson
RDX AWD A-SPEC
Highlander AWD LE
C 300 4MATIC Coupe
Insight Touring
Yaris Hatchback
GR Supra
MDX Hybrid AWD
Sorento
Outback AWD
B 250 4MATIC
1500 4X4 eTorque
AMG C 63 S
911 Carrera T

```
M340i xDrive Sedan
Qashqai
Panamera Turbo
A 250 4MATIC
John Cooper Works 3 Door
Murano
Discovery Sport
Edge AWD
XT6 AWD
XTS
LaCrosse eAssist
Stelvio AWD
AMG GLC 43 4MATIC Coupe
Cayenne
4C Spider
X3 M40i
Mustang
Camry TRD
AMG GT 63 S 4MATIC+ Coupe
XJL Portfolio AWD
Silverado 4WD FFV
Transit Connect Wagon LWB FFV
Kicks
AMG GLC 43 4MATIC
X2 xDrive28i
NX 300h AWD
CX-5 4WD (Cylinder Deactivation)
Suburban 4WD FFV
Tahoe 4WD FFV
CX-3
Metris Cargo
ES 350 F SPORT
X4 M40i
RC 350 AWD
Accent
Elantra
Silverado FFV
Sonata SE
AMG C 43 4MATIC
IONIQ
Sierra FFV
Silverado WT 4WD
Camry LE/SE
300 AWD
911 Turbo S Cabriolet
Optima
CT6 AWD
Sonata Hybrid
Santa Fe XL
F-150 4X4
Mustang Convertible
XE 20d AWD
F-PACE 35t
A3
Panamera 4S ST
Niro Touring
V90 CC T6 AWD
GLC 300 4MATIC
A4 allroad quattro
Mazda3 5-Door 4WD
500L
CTS
XT5 AWD
M4 Coupe Competition
Rogue AWD
Camry XSE
RLX Hybrid
Venue
CT5-V AWD
Challenger GT AWD
Cruze Hatchback Diesel
Sierra 4WD FFV
F-150 (Payload Pkg)
CT5-V
Cooper 5 Door
AMG C 43 4MATIC Coupe
Maxima
Pacifica
```

In [ ]:

```
best_cars_mlp_only_fuel = getBestCars(car_models,prediction_only_fuel_mlp[len(prediction_only_fuel_mlp)-1])
best_cars_naive_bayes_only_fuel = getBestCars(car_models,prediction_only_fuel_naive_bayes)
best_cars_knn_only_fuel = getBestCars(car_models,prediction_only_fuel_knn[len(prediction_only_fuel_knn)-1])
best_cars_random_forest_only_fuel = getBestCars(car_models,prediction_only_fuel_random_forest[len(prediction_only_fuel_random_
all_cars_only_fuel = set.union(best_cars_mlp_only_fuel,best_cars_naive_bayes_only_fuel,best_cars_knn_only_fuel,best_cars_rando
print("Best Cars predicted considering fuel consumption alone: ")
for  u in all_cars_only_fuel:
  print(u)
```

Best Cars predicted considering fuel consumption alone:
Civic Coupe
Boxster S
Beetle Dune Convertible
John Cooper Works Convertible
A 220 4MATIC
Colorado
Cherokee
NX 300 AWD F SPORT
E-PACE P250
911 Turbo
Nautilus
AMG GT 53 4MATIC+ Coupe
Odyssey
Range Rover 3.0
G70
Wrangler JL Unlimited 4X4
Sierra WT
Discovery Sport P250
Rogue
John Cooper Works Clubman ALL4
Q70 AWD
RC F
911 Targa 4
TTS Coupe
SL 550
Beetle Convertible
AMG S 63 4MATIC+
Enclave AWD
Sonata Hybrid SE
CLS 450 4MATIC
AMG E 53 4MATIC+ Wagon
A6 allroad
911 Carrera 4 GTS Cabriolet
F-PACE 30t
XE 25t AWD
Grand Cherokee 4X4
Highlander AWD (Start/Stop System)
Corolla XSE
RVR
Giulia
Civic Hatchback Sport
Niro
Golf GTI
Journey FFV
EcoSport AWD
CTS AWD
Cooper S Convertible
AMG E 53 4MATIC+ Coupe
Cooper S Countryman ALL4
XE P250 AWD
M4 Coupe
S5 Sportback
Range Rover Evoque P250
A5 Cabriolet quattro
230i Cabriolet
Cooper Countryman ALL4
Ridgeline AWD
Nautilus AWD
Boxster
Mirage
Range Rover Velar P300
Tucson AWD
911 Turbo Cabriolet
Cruze Premier
Macan
X5 xDrive40i
Sportage AWD
Camry Hybrid XLE/SE
AMG C 43 4MATIC Cabriolet
Pathfinder
TT Coupe quattro
Forester AWD
Highlander Hybrid AWD Limited/Platinum
Metris Passenger
Pilot
440i xDrive Coupe
X1 xDrive28i
F-TYPE Coupe R-Dynamic
AMG S 63 4MATIC+ Coupe
AMG GLA 45 4MATIC
XTS AWD
X4 xDrive30i
Panamera 4
CX-30 4WD (Cylinder Deactivation)
Stelvio AWD Quadrifoglio
Sedona

Panamera 4 ST
Cooper Convertible
Panamera 4S
Mazda3 5-Door
Santa Fe XL AWD
ILX
A5 Sportback quattro
Q50 AWD Red Sport
Tiguan 4MOTION
A7 quattro
Highlander Hybrid AWD
Trax AWD
Colorado 4WD
Fusion
M240i xDrive Coupe M Performance
XT4
CLA 250 4MATIC
F-TYPE Coupe R-Dynamic AWD
S7
1500 Classic
Escape Hybrid AWD
A8L
911 Carrera S
Mazda3 4-Door 4WD
HR-V AWD
XC90 T6 AWD
XT4 AWD
CT4 AWD
UX 200
S 560 Cabriolet
Maybach S 560 4MATIC
Renegade
V60 CC T5 AWD
E 450 4MATIC Wagon
XT5
LaCrosse
Civic Hatchback
IS 300
CX-30 4WD
X3 xDrive30i
RAV4
AMG CLS 53 4MATIC+
F-TYPE Coupe
Camry AWD XLE/XSE
CX-9
Mustang (Performance Pkg)
911 Carrera 4S Cabriolet
Veloster
IS 300 AWD
F-150 4X4 XL/XLT
Compass 4X4
Atlas
Trax
330i xDrive Sedan
Envision AWD
440i xDrive Cabriolet
E 300 4MATIC
Insight EX
DB11 V8
XC60 T6 AWD
Panamera Turbo Executive
911 Carrera Cabriolet
MX-5 (SIL)
Versa
Giulia Quadrifoglio
Z4 M40i
XE 35t AWD
Eclipse Cross 4WD
Gladiator 4X4
300
Challenger
Panamera 4S Executive
CLA 250
Pilot AWD
M850i xDrive Coupe
Pacifica (Stop-Start)
Mustang Convertible (Performance Pkg)
F-TYPE Convertible R-Dynamic
Escape
CT4
Vantage V8
Avalon
CX-5 (Cylinder Deactivation)
A6 quattro
X2 M35i
1500 Classic FFV
S60 T6 AWD

Sentra Nismo
CX-5 Diesel 4WD
V60 T5
Mazda6 Turbo
911 Carrera GTS
911 Targa 4S
Jetta GLI
Golf SportWagen
AMG C 63
Civic Type R
911 Carrera 4 GTS
Terrain
Crosstrek AWD
530i xDrive Sedan
F-TYPE P300 Convertible
Atlas 4MOTION
MDX SH-AWD
Beetle Dune
Impreza 5-Door AWD
SLC 300
Sentra (Turbo)
Q3 quattro
CX-5
Cooper 3 Door
440i Coupe
GS 350 AWD
430i xDrive Gran Coupe
Mazda6
S 450 4MATIC SWB
Cooper S 3 Door
X6 xDrive35i
Highlander
911 Targa 4 GTS
Edge
Renegade 4X4 Trailhawk
CT4-V AWD
CT5 AWD
XF S AWD
430i xDrive Coupe
Micra
AMG SLC 43
Tacoma 4WD
Veloster N
S90 T6 AWD
Mazda3 4-Door (Cylinder Deactivation)
Terrain AWD
M550i xDrive Sedan
Spark
Equinox
F-150 (LT Tire Pkg)
F-150 4X4 (Payload Pkg)
Impala
RS 3
86
Aviator
Escape FFV
Jetta
TLX A-SPEC
RX 350 L AWD
Ghibli
370Z Roadster
Sorento AWD
Q60 AWD Red Sport
LS 500 AWD
Panamera 4 Executive
E 450 4MATIC
Fiesta
S90 T5 AWD
Canyon
Beetle
V60 T6 AWD
Yaris
Regal AWD
S5 Cabriolet
C 300 4MATIC
Camry XLE/XSE
Altima
Ghibli S
Forte
M240i xDrive Coupe
QX50 AWD
Tacoma
V90 T6 AWD
Giulia AWD
LS 500h AWD
Silverado WT
Corolla LE Eco

Kona AWD
Prius c
Wrangler JL 4X4
Cherokee 4X4 Active Drive Lock
Corvette
Sierra
Sienna
Range Rover Velar P340
Range Rover Sport TD6 Diesel
Golf Alltrack
X7 xDrive40i
Renegade 4X4
Charger FFV
Wrangler 4X4 eTorque
Q50 AWD
1500 HFE eTorque
Camaro
XF 35t AWD
540i xDrive Sedan
300 AWD FFV
911 Carrera S Cabriolet
Sienna AWD
TLX SH-AWD
Regal
UX 250h
Pathfinder 4WD
Range Rover Velar P250
C 300 4MATIC Wagon
XC60 T5 AWD
M550i xDrive
Trax 4WD
Panamera
430i xDrive Cabriolet
AMG CLA 45 4MATIC
Atlas Cross Sport 4MOTION
Sierra WT 4WD
Sportage
Cadenza
Camry AWD LE/SE
F-150 FFV 4X4
RX 350 AWD
Transit Connect Van FFV
IS 350 AWD
Encore AWD (SIDI with Stop/Start)
Altima SR/Platinum
AMG E 53 4MATIC+
Range Rover Sport 3.0
Santa Fe AWD
Altima AWD
MX-5
C 300 4MATIC Cabriolet
230i Coupe
Veloster Turbo
B 250
CT5
Soul
Niro FE
Cooper S Clubman ALL4
LaCrosse AWD
F-PACE S
Odyssey Touring
A 250
C-HR
Sentra
230i xDrive Coupe
MKC AWD (Start/Stop)
Golf SportWagen 4MOTION
John Cooper Works Countryman ALL4
F-150 FFV
A4 quattro
Transit Connect Wagon LWB
ES 350
Highlander AWD
Blazer
G90 AWD
911 Turbo S
XJ R-Sport AWD
Fit
430i Coupe
Discovery Sport P290
Cayman GTS
Transit Connect Van LWB
Civic Sedan
Panamera Turbo ST
Range Rover TD6 Diesel
Pathfinder 4WD Platinum
M2 Competition

CX-3 (SIL)
E 450 4MATIC Cabriolet
Charger AWD
Fiesta ST
UX 250h AWD
Acadia AWD
XC90 T5 AWD
Blazer AWD
GLC 300 4MATIC Coupe
Quattroporte S
NV200 Cargo Van
GLE 400 4MATIC
Mazda3 4-Door
Stelvio
MKZ AWD
Range Rover Velar D180
Z4 sDrive30i
Ascent AWD
F-150 FFV (LT Tire Pkg)
F-150
Enclave
S 560 4MATIC SWB
LC 500h
F-TYPE P300 Coupe
Qashqai AWD
G80 AWD
Wrangler Unlimited 4X4
Optima Hybrid
Continental AWD
F-150 4X4 (LT Tire Pkg)
Durango AWD
M240i Coupe
Silverado
Encore GX
1500 eTorque
ES 300h
Journey
Golf
TT RS
300 FFV
Malibu Hybrid
Wrangler Unlimited 4X4 EcoDiesel
911 Carrera 4
Canyon 4WD
Forte 5
124 Spider
ProMaster City
Metris Cargo LWB
S60 T5
SL 450
Wrangler 4X4
CX-3 4WD
TT Roadster quattro
S3
Passport AWD
Ranger 4WD
Camry Hybrid LE
AMG E 53 4MATIC+ Cabriolet
1500 4X4 EcoDiesel
AMG C 63 S Coupe
Q5
Boxster GTS
Rio
Golf R
EcoSport
RAV4 AWD TRD Off-Road
XF 25t AWD
911 Carrera 4S
RC 300 AWD
XC40 T4 AWD
Altima AWD SR/Platinum
CT4-V
M240i Cabriolet
Acadia
Explorer Hybrid AWD
M4 Cabriolet
Wrangler Unlimited 4X4 eTorque
Telluride AWD
CX-30
500
Grand Cherokee 4X4 EcoDiesel
440i xDrive Gran Coupe
1500 Classic 4X4 EcoDiesel
Outlander 4WD
Accord Sport/Touring
S6
Cruze Hatchback

Cayman S
M4 Cabriolet Competition
Escape AWD
Traverse AWD
F-TYPE Convertible R-Dynamic AWD
SQ5
XC40 T5 AWD
Sonata
Encore AWD
F-TYPE Convertible
G70 AWD
BRZ tS
Macan S
Sierra 4WD AT4
QX60 AWD
Voyager (Stop-Start)
RAV4 Hybrid AWD
Sentra SR
RX 450h AWD
Cooper Clubman ALL4
Charger
Santa Fe
A3 Cabriolet quattro
Encore (SIDI with Stop/Start)
Silverado 4WD
Legacy AWD
Explorer AWD
Elantra GT
500X AWD
XF 20d AWD
1500 Classic EcoDiesel
Murano AWD
Kona
Cherokee 4X4 Active Drive II
Mazda3 5-Door (SIL)
MDX SH-AWD A-SPEC
F-PACE 20d
Corolla XLE
Taurus FFV AWD
Grand Caravan
Accord
A 220
RDX AWD
Cayenne S
Cruze
S 560 4MATIC
Charger (MDS)
XF 30t AWD
1500 EcoDiesel
WRX AWD
Range Rover Evoque P300
XE P300 AWD
MKZ Hybrid
Mirage G4
A3 quattro
M240i Coupe M Performance
E 450 4MATIC Coupe
M240i xDrive Cabriolet
Impreza 4-Door AWD
Frontier
Encore GX AWD
X6 xDrive40i
Equinox AWD
Q60 AWD
E-PACE P300
Civic Coupe Si
Yaris (SIL)
MKC AWD
S4
Stinger AWD
Civic Sedan Si
Corsair AWD
Cruze Diesel
Colorado ZR2 4WD
370Z
RAV4 AWD
Grand Caravan FFV
RVR 4WD
Passat
911 Carrera
Prius
Cherokee 4X4 Active Drive I
230i xDrive Cabriolet
Charger AWD FFV
Malibu
911 Carrera 4 Cabriolet
Corolla Hatchback

Discovery TD6 Diesel
Frontier 4WD
Cayman
IONIQ Blue
Challenger AWD
Range Rover Evoque Convertible
CX-5 4WD
AMG C 43 4MATIC Wagon
Range Rover Velar P380
Corolla Hybrid
NX 300 AWD
Corolla
S5
M850i xDrive Cabriolet
Compass
Camry
Prius AWD
HR-V
Encore
CX-9 4WD
Traverse
Taurus FFV
CX-5 Turbo 4WD
Escape Hybrid
Fusion Hybrid
Arteon 4MOTION
Transit Connect Van
Camaro SS
Cooper S 5 Door
AMG C 63 S Cabriolet
CR-V AWD
CR-V
Insight EX/Touring
Accord Hybrid
Challenger (MDS)
F-PACE 25t
GLA 250 4MATIC
Sierra 4WD
Range Rover Evoque
Palisade AWD
911 Carrera GTS Cabriolet
BRZ
Cruze Hatchback Premier
TLX SH-AWD A-SPEC/Limited Edition
Tiguan
Mazda3 4-Door (SIL)
Tucson
RDX AWD A-SPEC
Highlander AWD LE
C 300 4MATIC Coupe
Insight Touring
Yaris Hatchback
GR Supra
MDX Hybrid AWD
Sorento
Outback AWD
B 250 4MATIC
1500 4X4 eTorque
AMG C 63 S
911 Carrera T
M340i xDrive Sedan
Qashqai
Panamera Turbo
A 250 4MATIC
John Cooper Works 3 Door
Murano
Discovery Sport
Edge AWD
XT6 AWD
XTS
Stelvio AWD
LaCrosse eAssist
AMG GLC 43 4MATIC Coupe
Cayenne
4C Spider
X3 M40i
Mustang
Camry TRD
XJL Portfolio AWD
Transit Connect Wagon LWB FFV
Kicks
A5 quattro
AMG GLC 43 4MATIC
X2 xDrive28i
NX 300h AWD
CX-5 4WD (Cylinder Deactivation)
Palisade

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RX 450h L AWD
CX-3
Metris Cargo
ES 350 F SPORT
X4 M40i
RC 350 AWD
Elantra
Accent
Sonata SE
AMG C 43 4MATIC
IONIQ
Silverado WT 4WD
Camry LE/SE
300 AWD
911 Turbo S Cabriolet
Optima
CT6 AWD
Sonata Hybrid
Santa Fe XL
F-150 4X4
Mustang Convertible
XE 20d AWD
F-PACE 35t
A3
Panamera 4S ST
Niro Touring
V90 CC T6 AWD
GLC 300 4MATIC
A4 allroad quattro
Mazda3 5-Door 4WD
500L
CTS
XT5 AWD
M4 Coupe Competition
Rogue AWD
Camry XSE
RLX Hybrid
Venue
CT5-V AWD
Challenger GT AWD
Cruze Hatchback Diesel
F-150 (Payload Pkg)
CT5-V
Cooper 5 Door
AMG C 43 4MATIC Coupe
Maxima
Pacifica
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Classification report

A classification report is a useful tool for evaluating the performance of a classification model. It provides several metrics that help you understand how well your model is performing on different aspects. The most common metrics included in a classification report are precision, recall, F1-score, and support. Here's how to interpret these metrics:

1) Precision:

Precision is the ratio of correctly predicted positive observations to the total predicted positives. It measures the accuracy of the positive predictions. A high precision indicates that the model has fewer false positives. Precision is calculated as:

$$\text{Precision} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Positives}}$$

Recall (Sensitivity or True Positive Rate):

Recall is the ratio of correctly predicted positive observations to the all observations in actual class. It measures the ability of the model to capture all the possible positive instances. A high recall indicates that the model has fewer false negatives. Recall is calculated as:

$$\text{Recall} = \frac{\text{True Positives}}{\text{True Positives} + \text{False Negatives}}$$

F1-Score:

F1-score is the weighted average of precision and recall. It combines both precision and recall into a single metric. It is especially useful when there is an uneven class distribution (e.g., imbalanced datasets). F1-score is calculated as:

$$\text{F1-Score} = 2 \times \frac{\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}}$$

Support:

Support is the number of actual occurrences of the class in the specified dataset. It provides context to the precision and recall values by indicating the number of actual instances of each class.

Insights

Multilayer Perceptron:

Accuracy: 0.98

Precision: 0.98
Recall: 0.98
F1 Score: 0.98
Insights: The MLP model with fuel features appears to perform very well with high accuracy, precision, recall, and F1 score.

Naive Bayes:

Accuracy (All Features): 0.84
Accuracy (Excluding Fuel Consumption): 0.71
Accuracy (Only Fuel Consumption): 0.91
Insights: The Naive Bayes model performs reasonably well, with the highest accuracy achieved when considering only fuel consumption. Excluding fuel consumption as a feature leads to a drop in accuracy.

Random Forest:

Accuracy: 0.89
Insights: The Random Forest model performs well with high precision and recall for each class. The model seems to handle class 2.0 better than the other classes.

Kernel KNN:

Accuracy: 0.90

Insights: The Kernal K-Nearest Neighbours model performs well across all classes with balanced precision, recall, and F1-score. The model achieves an accuracy of 90%, indicating robust performance.

Conclusion :

The MLP model with fuel features stands out with exceptionally high accuracy, precision, recall, and F1 score. Random Forest and K-Nearest Neighbours also show good performance, with the latter achieving a slightly higher accuracy. The Naive Bayes model performs reasonably well, especially when considering only fuel consumption. Excluding fuel features leads to a decrease in accuracy.

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