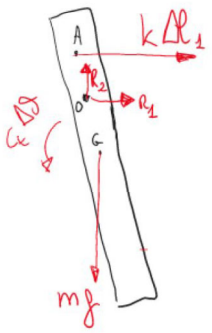
1. FBD



2. Newton's law

$I_0\ddot{\vartheta} = -mgd\sin\vartheta - k\Delta\ell_1 a\cos\vartheta + c_t\dot{\vartheta}$

3. Reaction forces: N/A

4. Kinematics: $\Delta\ell_1 = a\sin\vartheta$

$\Delta\dot{\vartheta} = -\dot{\vartheta}$

$I_0\ddot{\vartheta} = -mgd\sin\vartheta - ka^2\cos\vartheta\sin\vartheta - c_t\dot{\vartheta}$

$I_0\ddot{\vartheta} + c_t\dot{\vartheta} + ka^2\cos\vartheta\sin\vartheta + mgd\sin\vartheta = 0$

$\ddot{\vartheta} + \dot{\vartheta} + \cos\vartheta\sin\vartheta + \sin\vartheta = 0$ (part a)

5. Equilibrium: setting $\dot{\vartheta}=0, \ddot{\vartheta}=0 \rightarrow -mgd\sin\vartheta_{eq} - ka^2\cos\vartheta_{eq}\sin\vartheta_{eq} = 0$

So: $\sin\vartheta_{eq}\left[mgd + ka^2\cos\vartheta_{eq}\right] = 0$

for $\vartheta_{eq}=0$, $\sin\vartheta_{eq}=0$ so the equation above is satisfied and $\vartheta_{eq}=0$ is a position of equilibrium

6. Stability: $V = mgh_G + \frac{1}{2}k\Delta\ell_1^2 = -mgd\cos\vartheta + \frac{1}{2}ka^2\sin^2\vartheta$

$\frac{dV}{d\vartheta} = mgd\sin\vartheta + ka^2\sin\vartheta\cos\vartheta$

$\frac{d^2V}{d\vartheta^2} = mgd\cos\vartheta + ka^2\left[\cos^2\vartheta - \sin^2\vartheta\right]$

Substituting $\vartheta_{eq}=0$

$\left.\frac{d^2V}{d\vartheta^2}\right|_{\vartheta_{eq}=0} = mgd + ka^2 = \ldots\ \frac{J}{rad^2} > 0$

So $\vartheta_{eq}=0$ is a STABLE EQUILIBRIUM POSITION (part b)

7. Linearization: $\sin\vartheta = \sin(0) + \cos(0)\tilde{\vartheta} = \tilde{\vartheta}$

$\cos\vartheta\sin\vartheta = \cos(0)\sin(0) + \left[\cos^2(0) - \sin^2(0)\right]\tilde{\vartheta} = \tilde{\vartheta}$

$I_0\ddot{\tilde{\vartheta}} = -mgd\tilde{\vartheta} - ka^2\tilde{\vartheta} - c_t\dot{\tilde{\vartheta}}$

8. Rearranging: $I_0\ddot{\tilde{\vartheta}} + c_t\dot{\tilde{\vartheta}} + \left(mgd + ka^2\right)\tilde{\vartheta} = 0$

$I^* = I_0 = \ldots\ kgm^2$ $\quad c_t^* = c_t = \ldots\ \frac{Nms}{rad}$ $\quad k_t^* = ka^2 + mgd = \ldots\ \frac{Nm}{rad}$ (part c)

$\omega_n = \sqrt{\frac{k_t^*}{I^*}} = \ldots\ \frac{rad}{s}$ $\quad \zeta = \frac{c_t^*}{2I^*\omega_n} = \ldots = \ldots\ 100\ \%$ (part d)