**School of Applied Mathematics**
Discipline: Discrete Structures (CSCI 1102)
Fall Semester 2022: Final Examination
**Duration: 150 minutes (2 hours and 30 minutes)**
**The overall score is 40 points**

**Examination card № 1**

1. Sets, Relations and Functions.
   **(a)** (*5 points*) Basic definitions and properties (set; subset; universal set; empty set; disjoint sets; set operations; countable sets; power set; relation; product sets; inverse relation; composition of relations; types of relations; function; types of functions). **Give an exhaustive definition of each notion.**
   **(b)** (*5 points*) Related theorems (theorems on subsets; duality law; inclusion-exclusion principle; principle of mathematical induction; theorem on transitive relation; theorem on invertible function). **Demonstrate your understanding of these theorems by examples.**

**2.** (*5 points*) Find all solutions of the system of congruencies $x \equiv 1(mod\ 5)$, $x \equiv 3(mod\ 6), and\ x \equiv 7(mod\ 11)$. **Give a full solution.**

**3.** (*5 points*) There are 12 students in the class. They need to be divided into two groups (first and second), consisting of an even number of students. How many ways can this be done? **Give a full explanation of your answer.**

4. (*5 points*) It turned out that at some university any two freshmen have no common friends, and any two non-friend students have exactly two common friends. **Prove** that all freshmen at this university have the same number of friends.

5. (*5 points*) Solve the word equation
$$x101 = 001x$$
over the alphabet $\{0,1\}$; that is, find the set of strings $x$ over $\{0,1\}$ which satisfy the equation. **Prove your answer.**

*On the next page, you will find test problems!*

**Teacher** ____________________ **A.A.Srazhdinova**

**Dean of School of Applied Mathematics** ____________________ **A.A. Issakhov**

**Approved by protocol №9 of the 30th of November 2022**

**Attention!!!** ***Some test problems have several correct answers.***

1. *(1 point)* If $A$ is the set of roots of the equation $x^3 - 3x + 2 = 0$, then $A$ is equal to
a) $\{1; -3; 2\}$; b) $\{1; -2\}$; c) $\{-1; 2\}$; d) $\{-2; 1; -2\}$; e) $\{-1; -2; 3\}$

2. *(1 point)* For what sets $A, B, C$ the equation $A \cap B = A \cap C$ will be true?
a) $A = \{1,2,3,3\}, B = \{3,3,4\}, C = \{3,4,4\}$
b) $A = \{3,5,7,9\}, B = \{2,5,5,6,7\}, C = \{1,5,8,5,1,6,7,6,7\}$
c) $A = \{2,3,5,3,2\}, B = \{2,4,6,8\}, C = \{2,9,2,3,6,8,4\}$
d) $A = \{7,7,7,77\}, B = \{77,71,17\}, C = \{1,77,7,0,7\}$
e) $A = \{1,5,1,4,8,3,9\}, B = \{1,1,6,4,6\}, C = \{3,1,2,2,4,7\}$

3. *(1 point)* Find true equations for any sets $A, B$ and $C$
a) $A \times (B \cup C) = (A \times B) \cap (A \times C)$
b) $(A \cup B) \times C = (A \times C) \cup (B \times C)$
c) $A \times (B \backslash C) = (A \times B) \cup (A \times C)$
d) $(A \cup B) \times (C \cup D) = (A \times C) \cup (B \times C)$
e) $(A \backslash B) \times C = (A \times C) \backslash (B \times C)$

4. *(1 point)* Find sets equal to a domain of the relation $R = \{(x, y) | x, y \in \mathbb{R} \; and \; 2x \geq 3y\}$
a) $\{x \,| x \in \mathbb{R}, x^2 + 1 \geq 0\}$; b) $\{x \,| x \in \mathbb{R}, x \geq 0\}$; c) $\{x \,| x \in \mathbb{R}, -\infty < x < +\infty\}$;
d) $\{x \,| x \in \mathbb{R}, x \geq \frac{3}{2}\}$; e) $\{x \,| x \in \mathbb{R}, -\frac{3}{2} \leq x \leq \frac{3}{2}\}$

5. *(1 point)* Which are the correct translations for the following statement into propositional logic: You can successfully complete the Discrete Mathematics course only if you have passed all quizzes well or you have not failed the final exam. Where $s$ is "you can successfully complete the Discrete Mathematics course", $q$ is "you pass all quizzes well" and $f$ is "you have failed the final exam".
a) $(q \vee \neg f) \to s$; b) $s \to (q \vee \neg f)$; c) $(s \to q) \vee \neg f$; d) $s \to q \vee \neg f$; e) $q \vee \neg f \to s$

6. *(1 point)* Find tautologies (by using truth tables).
a) $[p \wedge (p \to q)] \to \neg q$; b) $[\neg p \wedge (p \vee q)] \to q$; c) $\neg p \to \neg(p \to q)$;
d) $(p \wedge \neg q) \to (p \to q)$; e) $\neg(p \to q) \to p$

7. *(1 point)* Let $F(x, y, z) = x\bar{y}z + (\overline{xyz})$ be the Boolean function. Find correct equations.
a) $F(0,1,1) = 1$; b) $F(1,1,1) = 1$; c) $F(1, y, 1) = \bar{y}$; d) $F(1,0, z) = z$; e) $F(x, 1,1) = 1 + x$

8. *(1 point)* For which of these expressions $\bar{x}$ is minimal expansion?
a) $\bar{x}\bar{y} + \bar{x}y + x\bar{y} + xy$; b) $\bar{x}y + \bar{x}\bar{y}$; c) $\bar{x}yz + \bar{x}y\bar{z} + \bar{x} + xyz$;
d) $\bar{x}\bar{y}z + \bar{x}\bar{y}\bar{z} + \bar{x} + \bar{x}y$; e) $x\bar{y}z + x\bar{y}\bar{z} + xy\bar{z} + xyz$

9. *(1 point)* Find a recurrence relation for the number of ternary strings of length $n$ that do not contain three consecutive 0's.
a) $a_n = 2a_{n-1} + 2a_{n-2}$; b) $a_n = 3a_{n-1} + 3a_{n-2}$; c) $a_n = 2a_{n-1} + 2a_{n-2} + 2a_{n-3}$;
d) $a_n = a_{n-1} + a_{n-2} + a_{n-3}$; e) $a_n = 3a_{n-2} + 2a_{n-3}$

10. *(1 point)* What is the solution of the recurrence relation $a_n = 4a_{n-1} + 5a_{n-2}$ with $a_0 =$ 2 and $a_1 = 4$?

a) $a_n = 1 + (-5)^n$; b) $a_n = (-1)^n + (-5)^n$; c) $a_n = (-1)^n + 5^n$; d) $a_n = [\frac{\sqrt{33}()}{\sqrt{33}}\backslash \cdot [\frac{*\$\sqrt{33}}{2}\backslash^n + [\frac{\sqrt{33}\$)}{\sqrt{33}}\backslash \cdot [\frac{*(\sqrt{33}}{2}\backslash^n$; e) $a_n = [\frac{2\sqrt{3*}\$1*}{\sqrt{3*}}\backslash \cdot [\frac{\$+\$\sqrt{3*}}{2}\backslash^n + [\frac{2\sqrt{3*}(1*}{\sqrt{3*}}\backslash \cdot [\frac{\$+(\sqrt{3*}}{2}\backslash^n$