

Phil 120, Review Notes

Stuff

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1 References

1.1 BasicLogic Operators

The followings are for $A * B$, where '*' is an operator, A is top row, B is left column.

\wedge	T	F	AND. Conjunction. $A \wedge B$ is true only when both A and B are true.
T	T	F	
F	F	F	
<hr/>			
\vee	T	F	OR. Disjunction. $A \vee B$ is true when either A or B, or Both are true.
T	T	T	
F	T	F	
<hr/>			
\rightarrow	T	F	IMPLIES. If A then B. A implies B. A implies B is true when A is true and B is true, or when A is false. Note: $A \rightarrow B = \neg A \vee B$
T	T	T	
F	F	T	
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\leftrightarrow	T	F	IFF, A if and only B. A is logically equivalent, two way implication. $A \leftrightarrow B$ is true exactly when the truth value of A is the same as B. Note: $A \leftrightarrow B = (A \rightarrow B) \wedge (B \rightarrow A) = (A \wedge B) \vee (\neg A \wedge \neg B)$
T	T	F	
F	F	T	

1.2 Some Logic Identities

$A \vee \neg A = T$	Excluded Middle, either A or not A must be true.
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$\neg(A \wedge \neg A)$	Non-contradiction. It is true that not both A and not A hold at the same time.
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$A \rightarrow B, A \implies B$	Modus ponenes, to prove. If A implies and B and A is true, then B is true.
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$A \rightarrow B, \neg B \implies \neg A$	Modus tollens, to disprove. If the conclusion is false, then the premise is false also.
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$A \vee B, \neg A \implies B$	Disjunctive syllogism. If at least one of A or B is true, then if one of them is false, the other must be true.
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$(A \rightarrow B) \iff (\neg B \rightarrow \neg A)$	Contrapositive. Similar to Modus tollens.
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$A, \neg A \implies B$	Explosion. From a false premise you can arrive at any conclusion.
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$\neg(A \vee B) \iff \neg A \wedge \neg B$ $\neg(A \wedge B) \iff \neg A \vee \neg B$	De Morgan's Law.
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$A \vee (B \wedge C) \iff (A \vee B) \wedge (A \vee C)$ $A \wedge (B \vee C) \iff (A \wedge B) \vee (A \wedge C)$	Distributability

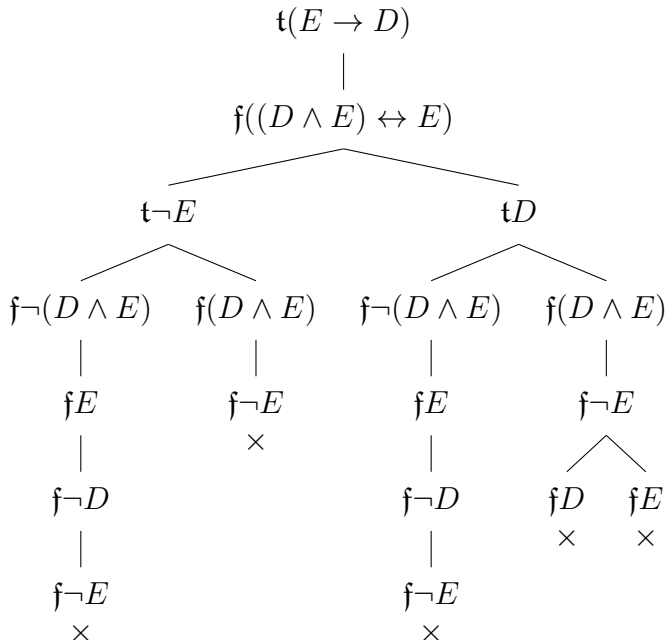
1.3 Tableaux Identities

$\mathbf{t}\wedge:$	$\begin{array}{c} \mathbf{t}A \wedge B \\ \\ \mathbf{t}A \\ \mathbf{t}B \end{array}$	$\mathbf{f}\wedge:$	$\begin{array}{c} \mathbf{f}A \wedge B \\ \swarrow \quad \searrow \\ \mathbf{f}A \quad \mathbf{f}B \end{array}$
$\mathbf{t}\vee:$	$\begin{array}{c} \mathbf{t}A \vee B \\ \swarrow \quad \searrow \\ \mathbf{t}A \quad \mathbf{t}B \end{array}$	$\mathbf{f}\vee:$	$\begin{array}{c} \mathbf{f}A \vee B \\ \\ \mathbf{f}A \\ \mathbf{f}B \end{array}$
$\mathbf{t}\rightarrow:$	$\begin{array}{c} \mathbf{t}A \rightarrow B \\ \swarrow \quad \searrow \\ \mathbf{t}\neg A \quad \mathbf{t}B \end{array}$	$\mathbf{f}\rightarrow:$	$\begin{array}{c} \mathbf{f}A \rightarrow B \\ \\ \mathbf{f}\neg A \\ \mathbf{f}B \end{array}$
$\mathbf{t}\leftrightarrow:$	$\begin{array}{c} \mathbf{t}A \leftrightarrow B \\ \swarrow \quad \downarrow \quad \searrow \quad \swarrow \\ \mathbf{t}\neg A \quad \mathbf{t}\neg A \quad \mathbf{t}\neg B \quad \mathbf{t}A \\ \quad \quad \quad \\ \mathbf{t}\neg B \quad \mathbf{t}A \quad \mathbf{t}B \quad \mathbf{t}B \end{array}$	$\mathbf{f}\leftrightarrow:$	$\begin{array}{c} \mathbf{f}A \leftrightarrow B \\ \swarrow \quad \searrow \\ \mathbf{f}\neg A \quad \mathbf{f}\neg B \\ \quad \\ \mathbf{f}B \quad \mathbf{f}A \end{array}$

Note:

- Use De Morgan's Law to deal with negations(\neg), also, $\neg(A \rightarrow B) = (\neg B \wedge A)$.
- To prove a consequence, set the premise to true, conclusion to false. This proves that there is no possible counter example, since the negation is never satisfied.
 - If there are atomic branches left open, then those are valid counter examples.
- A branch is closed any of following pairs occurs in a branch: $\{(\mathbf{f}A, \mathbf{t}A), (\mathbf{f}A, \mathbf{f}\neg A), (\mathbf{t}A, \mathbf{t}\neg A)\}$, use a ' \times ' to indicate a closed branch.

Example: Use tableaux to prove $(E \rightarrow D) \vdash_1 ((D \wedge E) \leftrightarrow E)$



Since all branches are closed, the negation of the conclusion is never satisfied, thus the relation always holds for all values D and E might take on.