Phil 120, Finals

Ray Gong

June 25, 2021

Q1

p	q	r	$(\neg p$	$\wedge$	$(q\vee$	$\neg r))$	$\leftrightarrow$	$(\neg q$	$\rightarrow p)$
F	F	F	T	T	T	T	F	T	F
F	F	T	T	F	F	F	T	T	F
F	T	F	T	T	T	T	T	F	T
F	T	T	T	T	T	F	T	F	T
T	F	F	F	F	T	T	F	T	T
T	F	T	F	F	F	F	F	T	T
T	T	F	F	F	T	T	T	F	T
T	T	T	F	F	T	F	T	F	T

so, the formula is not a) a logical validity, b) is contingent, c) is not a falsehood.

Q2

a)

let a be alfred, k be kurt, Lx if x proves a lemma, Tx if x proves a theorem, Px if x writes a paper.
$(Tk \vee Lk) \rightarrow Pa$

b)

let a be alfred, b be albert, Cx if x is cooking, Rx if it is raining where x is, Sx if x is shopping.
$\neg Sa \wedge (\neg Rb \rightarrow Cb)$

c)

let s be snow, let t be 2 + 2, f be 5, u be tuesday, Wx if x is white, Tx if today's weekday is x.
$Ws \rightarrow (t = f \vee \neg Tu)$

d)

let a be alfred, k be kurt, Hx if x is on a hike, Cxy if x is chatting with y.
$Ha \rightarrow \neg Cka$

Q3

$$\mathfrak{t}@w\exists x\Diamond Ax$$

$$|$$

$$\mathfrak{f}@w\Diamond\exists x\ Ax$$

$$|$$

$$\mathfrak{t}@w\mathbb{E}b$$

$$|$$

$$\mathfrak{t}@w \diamond Ab$$

$$|$$

$$\mathfrak{t}@w_1 Ab$$

$$|$$

$$\mathfrak{f}@w_1\exists x Ax$$

$$\begin{array}{cc} \mathfrak{f}@w_1\mathbb{E}b & \mathfrak{f}@w_1 Ab \\ \times_1 & \times_1 \end{array}$$

since all branches closed with rule 1, the statement holds in CL, k3, LP, and fde.

Q4

suppose there are 2 worlds in the universe, w1 and w2 such that C is true in w1, but false in w2.
since $\neg\Box C \iff \diamond\neg C$(proof is trivial from tableaux), $\Box\neg\Box C \iff \Box\diamond\neg C$. Then since $\neg C$ is satisfied in at least world, $\diamond\neg C$ holds in every worlds, which means $\Box\diamond\neg$ holds too. However, $\neg C$ does not hold in every world, so $\Box\neg C$ is false, and so this is a counter example for the implication statement($\rightarrow$).

Q5

a)

Suppose $A \subseteq B$, then for any $(a_1, a_2, ...a_i) \in \mathcal{P}(A), a_1, a_2...a_i \in A \subseteq B$, then $a_1, a_2...a_i \in B \implies (a_1, a_2...a_i) \in \mathcal{P}(B)$ by definition of powerset, and so a) holds.

b)

$|C \times C \times C| = |C|^3 = 27, 25 < 27 < 30$, so yes, the statement holds.

Q6

$\mathfrak{t} A \wedge (B \vee C)$

$\mathfrak{t} B \to D$

$\mathfrak{t} C \to \neg E$

$\mathfrak{f} E \to D$

$\mathfrak{t} A$

$\mathfrak{t} B \vee C$

$\mathfrak{f} \neg E$

$\mathfrak{f} D$

- $\mathfrak{t} \neg B$
  - $\mathfrak{t} B$
    $\times$
  - $\mathfrak{t} C$
    - $\mathfrak{t} \neg C$
      $\times$
    - $\mathfrak{t} \neg E$
      $\times$
- $\mathfrak{t} D$
  $\times$

All branches closes, the consequence relation holds in TV.

Q7

a)

let Cx if x is car, Sx if x is small, Fx if x is fast.
$\exists x(Cx \wedge Sx \wedge Fx)$

b)

let $\mathbb{N}, \mathbb{R} - \mathbb{Q}, \mathbb{R}$ be the set of natural, irrational, and real numbers. let $x \in A$ be the preicate that x is in the set A. $x < y$ if x is less than y.

$$\forall n \in \mathbb{N}(((2n+1) \in \mathbb{R} - \mathbb{Q}) \vee (\exists x \; (x \in \mathbb{R} \wedge (2n+1) < x)))$$

c)

let Sx if x is student, Bx if x is bike, Oxy if x owns y, Rxy if x rides y daily.
$\forall x(Sx \to (\exists y(By \wedge Oxy) \to Rxy))$

Q8

$$\mathfrak{t}\forall(Px \to (Qx \vee \neg Rx))$$

$$\mathfrak{t}\forall x((Qx \wedge Rx) \to \neg Px)$$

$$\mathfrak{t}\exists x(Qx \wedge Rx)$$

$$\mathfrak{f}\forall((Px \wedge Qx) \to (Px \wedge \neg Rx))$$

$$\mathfrak{t}Qa \wedge Ra$$

$$\mathfrak{t}Qa$$

$$\mathfrak{t}Ra$$

$$\mathfrak{f}(Pb \wedge Qb) \to (Pb \wedge \neg Rb)$$

$$\mathfrak{t}(Qa \wedge Ra) \to \neg Pa$$

$$\mathfrak{t}Pa \to (Qa \vee \neg Ra)$$

$$\begin{array}{cc} \mathfrak{t}\neg Pa & \mathfrak{t}Qa \vee \neg Ra \\ \begin{array}{cc} \mathfrak{t}\neg(Qa \wedge Ra) & \mathfrak{t}\neg Pa \\ \begin{array}{cc} \mathfrak{t}\neg Qa & \mathfrak{t}\neg Ra \\ \times & \times \end{array} & \end{array} & \begin{array}{cc} \mathfrak{t}Qa & \mathfrak{t}\neg Ra \\ & \times \end{array} \end{array}$$

looks like i choses the wrong variable names, but i dont have time to redo this question. most likely the first 2 'forall' statements should take the same variable name as the name chosen for the conclusion.

Q9

let D = {o1, o2}, E = {o1}, $\delta(a) = o1, \delta(b) = o2$
+(F) = {o1}, -(F) = {o2}, +(G) = {o1, o2}, -(G) = ∅.

So, for all x, x = a since x can only be assigned o1, which denotes a. Fa is true so $Favee\neg Gb$ is true, $\forall x(Fx \to Gx)$ holds since x can only be o1, and o1 is in the extension set of both F and G. however the conclusion is false since on existing object to be assigned to y satisfy both $\neg Fy$ and $\neg Gy$

Q10

p	q	$(\neg p)$	$\leftrightarrow$	$\neg$	$(q \vee p)$	$\rightarrow$	$(p\wedge$	$\neg q)$
F	F	T	T	T	F	F	F	T
F	T	T	F	F	T	T	f	F
F	N	T	N	N	N	N	F	N
T	F	F	T	F	T	T	T	T
T	T	F	T	F	T	F	F	F
T	N	F	T	F	T	N	N	N
N	F	N	N	N	N	N	N	T
N	T	N	N	F	T	N	F	F
N	N	N	N	N	N	N	N	N

Q11

$\mathfrak{t}\neg(A \wedge \neg C) \wedge B$ all branches closed under rule 1, relation holds in fde.

$\mathfrak{f}(\neg A \vee C) \vee D$

$\mathfrak{f}D$

$\mathfrak{f}\neg A$

$\mathfrak{f}C$

$\mathfrak{t}B$

$\mathfrak{t}\neg A$ $\quad$ $\mathfrak{t}\neg\neg C$

$\times_1$ $\quad$ $\mathfrak{t}C$

$\times_1$

Q12

a)false. since there are no theorems in K3, its not possible to find a formula that is a theorem in both K3 and LP.

b) true. the shared content is that if the premise is satisfied(true or both), then the conclusion be be satisfied also.

c) true. truth table and tableaux both provide ways to determine truth value of a formula in all 4 logic theories.

d) true.

e)false. if a predicate is that ‘at most five of some thing’, then in particular, it could be 0 or fewer, so ‘at least one...’ is not implied here.