3.4: a.) $X_t = \beta_0 + \beta_1 t + \omega_t$

$E[X_t] = \beta_0 + \beta_1 t$

$\boxed{\text{We can see that the expected value depends on time, So it is nonstationary.}}$

b.) $X_t = \beta_0 + \beta_1 t + \omega_t$

$\nabla X_t = X_t - X_{t-1}$

$= \beta_0 + \beta_1 t + \omega_t - \beta_0 - \beta_1(t-1) - \omega_{t-1}$

$= \beta_0 + \beta_1 t + \omega_t - \beta_0 - \beta_1 t + \beta_1 - \omega_{t-1}$

$= \beta_1 + \omega_t - \omega_{t-1}$

$E[\nabla X_t] = \beta_1$

Since $E[\nabla X_t]$ does not depend on t so it does meet the mean requirement of stationary.

$Cov[\nabla X_{t+h}, \nabla X_t] = Cov[\beta_1 + \omega_{t+h} - \omega_{t+h-1}, \beta_1 + \omega_t - \omega_{t-1}]$

$= Cov[\omega_{t+h}, \omega_t] - Cov[\omega_{t+h}, \omega_{t-1}] - Cov[\omega_{t+h-1}, \omega_t] + Cov[\omega_{t+h-1}, \omega_{t-1}]$

$$Cov[\nabla X_{t+h}, \nabla X_t] = \begin{cases} 2\sigma_\omega^2 & h=0 \\ -\sigma_\omega^2 & |h|=1 \\ 0 & |h|\geq 2 \end{cases}$$

This show that the covariance for stationarity is met.

$\boxed{\text{Thus } \nabla X_t \text{ is Stationary}}$

c.) $X_t = \beta_0 + \beta_1 t + Y_t$

$\nabla X_t = X_t - X_{t-1}$

$= \beta_0 + \beta_1 t + Y_t - \beta_0 - \beta_1(t-1) - Y_{t-1}$

$= \beta_1 + Y_t - Y_{t-1}$

$E[\nabla X_t] = E[\beta_1 + Y_t - Y_{t-1}]$

$= \beta_1 + \mu_Y + \mu_Y$

$= \beta_1 + 2\mu_Y$

Since expected value does not depend on time, it does satisfy the mean requirement for stationary.

$Cov(\nabla X_{t+h}, \nabla X_t) = Cov(\beta_1 + Y_{t+h} - Y_{t+h-1}, \beta_1 + Y_t - Y_{t-1})$

$= Cov(Y_{t+h}, Y_t) - Cov(Y_{t+h}, Y_{t-1}) - Cov(Y_{t+h-1}, Y_t) + Cov(Y_{t+h-1}, Y_{t-1})$

$= 2\gamma_Y(h) - \gamma_Y(h+1) - \gamma_Y(h-1)$

We can see that the covariance is independent of time and Y_t is stated to be stationary.

$\boxed{\text{Therefore } \nabla X_t \text{ with } Y_t \text{ instead of } \omega_t \text{ is stationary}}$