D. Pension Mathematics

REPLACEMENT RATIO

Replacement ratio:

$$R = \frac{\text{pension income in year after retirement}}{\text{salary in the year before retirement}}$$

SALARY PROJECTIONS

S_y – salary earned between ages y and $y+1$

$\bar{S}_y$ – rate of salary at exact age y

$S_y = \int_0^1 \bar{S}_{y+t}\,dt$

Rate of Salary Function ($\bar{s}_y$):

$$\bar{S}_y = \bar{S}_x \times \frac{\bar{s}_y}{\bar{s}_x}$$

Salary Scale (s_y):

$$S_y = S_x \times \frac{s_y}{s_x}$$

$s_y = \int_0^1 \bar{s}_{y+t}\,dt$

$s_{x-0.5} \approx 50\%(s_{x-1}) + 50\%(s_x)$

$\bar{s}_x \approx s_{x-0.5}$

DEFINED BENEFIT

Defined Benefit Pension Amount $= n\bar{S}\alpha$

where n - number of years of service
$\bar{S}$ - average salary
α - accrual rate

$$n\bar{S}\alpha = n_1\bar{S}\alpha \quad \text{(accrued benefit)}$$
$$+\, n_2\bar{S}\alpha \quad \text{(future service benefit)}$$

where n_1 - years of service to valuation
n_2 - years from valuation to retirement

Projected unit method – use salaries projected to exit date

Traditional unit method – use salaries up to valuation date

FUNDING THE BENEFITS

C_t - normal contribution for year $t+1$
- aka normal cost

$${}_tV + C_t = \text{EPV of benefits for mid-year exits} + v p_x^{00}\,{}_{t+1}V$$

${}_tV$ = EPV of accrued benefit at time t

${}_tV$ aka actuarial liability

VALUING HEALTH BENEFITS

Employer pays premiums to health care insurer at retirement

$B(x,t)$ = annual premium for age x at time t

Value at retirement age $x = B(x,t)\,\ddot{a}_B(x,t)$

$$\text{where } \ddot{a}_B(x,t) = \sum_{k=0}^{\infty} \frac{B(x+k,t+k)}{B(x,t)}\, v^k\,{}_kp_x$$

Typical Model:

$$B(x+k,t+k) = c^k(1+j)^k B(x,t)$$

$$\longrightarrow \ddot{a}_B(x,t) = \ddot{a}_{x|i^*}$$

$$\text{where } 1+i^* = \frac{1+i}{c(1+j)}$$

The actuarial value of the total health benefit (AVTHB) is

$$\sum_{k=0}^{65-x} \frac{r_{x+k}}{l_x}\, v^k B(x+k,k)\,\ddot{a}_B(x+k,k)$$

where r_{x+k} = retirements in year k

Under Typical Model,

$$AVTHB = \frac{B(x,0)}{l_x} \sum_{k=0}^{65-x} r_{x+k}\, v_{i*}^k\, \ddot{a}_{x+k|i^*}$$

FUNDING HEALTH BENEFITS

Employers not required to pre-fund health benefits, but many choose to do so.

Assume benefits accrue linearly over each employee's period of employment

APBO - accumulated post-retirement benefit obligation is the accrued EPV of benefits

Value all using earliest retirement date:

$${}_0V^h = \frac{x - x_0}{r^- - x_0} \cdot AVTHB$$

where x = current age, x_0 = starting age and r^- = earliest retirement age

Or accrue linearly using each retirement age

$${}_0V^h = \sum_{k=0}^{r^+ - x} \left(\frac{x - x_0}{x + k - x_0}\right)\left(\frac{r_{x+k}}{l_x}\right) v^k \times B(x+k,k)\,\ddot{a}_B(x+k,k)$$

Normal Cost, no mid-year exits

$$C_0^h = \frac{{}_0V^h}{x - x_0}$$

Normal Cost, with mid-year exits

A benefit at the beginning of the current year is already fully funded, thus there is no funding needed for this benefit.

For a mid-year benefit exit, we need to account for half-a-year of the total accrual period, thus we apply a factor of 0.5/(length-of-accrual period) to the actuarial value of health benefit.

For benefits beyond the current year, we need to account for a year of the total accrual period, thus we apply a factor of 1/(length-of-accrual period) to the actuarial value of the health benefit.