C. Multiple Life Models

JOINT-LIFE STATUS

$T_{xy} = \min[T_x, T_y]$

Failure on the first death.

${}_tq_{xy}$ – probability first death of (x) and (y) occurs within t years

${}_tp_{xy}$ – probability that (x) will attain $x+t$ and (y) will attain $y+t$

${}_tp_{xy} = {}_tp_x \cdot {}_tp_y$ (work with p's)

${}_tq_{xy} = 1 - {}_tp_{xy}$

$f_{xy}(t) = {}_tp_{xy} \cdot \mu_{x+t:y+t}$

$\mu_{x+t:y+t} = \mu_{x+t} + \mu_{y+t}$

$${}_nq_{xy} = \int_0^n {}_tp_{xy}\, \mu_{x+t:y+t}\, dt$$

$$\mathring{e}_{xy} = \int_0^\infty {}_tp_{xy}\, dt$$

$$e_{xy} = \sum_{k=1}^{\infty} {}_kp_{xy}$$

$e_{xy} = p_{xy}\,(1 + e_{x+1:y+1})$

$$\bar{a}_{xy} = \int_0^\infty v^t\, {}_tp_{xy}\, dt$$

$$\bar{A}_{xy} = \int_0^\infty v^t\, {}_tp_{xy}\, \mu_{x+t:y+t}\, dt$$

$\bar{A}_{xy} = 1 - \delta \bar{a}_{xy}$

$P_{xy} = \frac{1}{\ddot{a}_{xy}} - d$

LAST-SURVIVOR STATUS

$T_{\overline{xy}} = \max[T_x, T_y]$

$T_{\overline{xy}} = T_x + T_y - T_{xy}$

that for $(\overline{xy})$ = that for (x) + that for (y) − that for (xy)

${}_tq_{\overline{xy}}$ – probability that last death happens within t years

${}_tq_{\overline{xy}} = {}_tq_x \cdot {}_tq_y$ (work the the q's)

${}_tp_{\overline{xy}}$ – probability at least one of (x) or (y) survives for t years

${}_tp_{\overline{xy}} = 1 - {}_tq_{\overline{xy}}$

${}_tp_{\overline{xy}} = {}_tp_x + {}_tp_y - {}_tp_{xy}$

$f_{\overline{xy}}(t) = {}_tp_x\, \mu_{x+t} + {}_tp_y\, \mu_{y+t} - {}_tp_{xy}\, \mu_{x+t:y+t}$

${}_{n|}q_{\overline{xy}} = {}_{n+1}q_{\overline{xy}} - {}_nq_{\overline{xy}}$

${}_{n|}q_{\overline{xy}} = {}_{n|}q_x + {}_{n|}q_y - {}_{n|}q_{xy}$

$\mathring{e}_{\overline{xy}} = \mathring{e}_x + \mathring{e}_y - \mathring{e}_{xy}$

$\bar{a}_{\overline{xy}} = \bar{a}_x + \bar{a}_y - \bar{a}_{xy}$

$\bar{A}_{\overline{xy}} = \bar{A}_x + \bar{A}_y - \bar{A}_{xy}$

$A^{\,1}_{\overline{xy}:\overline{n}|} = A^{\,1}_{x:\overline{n}|} + A^{\,1}_{y:\overline{n}|} - A^{\,1}_{\overline{xy}:\overline{n}|}$

EXACTLY ONE STATUS

${}_tp^{[1]}_{\overline{xy}}$ – probability exactly 1 of (x) and (y) live t years

${}_tp^{[1]}_{\overline{xy}} = {}_tp_x + {}_tp_y - 2\,{}_tp_{xy}$

$\bar{a}^{[1]}_{\overline{xy}} = \bar{a}_x + \bar{a}_y - 2\bar{a}_{xy}$

CONTINGENT PROBABILITIES

${}_nq^{\,1}_{xy}$ – probability (x) dies before (y) and before n years from now

$${}_nq^{\,1}_{xy} = \int_0^n \left(\int_s^\infty f_{xy}(s,t)\, dt \right) ds$$

${}_nq^{\;\;2}_{xy}$ – probability that (y) dies after (x) and before n years from now

If (x) and (y) independent:

$${}_nq^{\,1}_{xy} = \int_0^n {}_tp_x\, \mu_{x+t} \cdot {}_tp_y\, dt$$

$${}_nq^{\;\;2}_{xy} = \int_0^n {}_tp_y\, \mu_{y+t} \cdot {}_tq_x\, dt$$

${}_nq_y = {}_nq^{\;\;1}_{xy} + {}_nq^{\;\;2}_{xy}$

${}_nq^{\,1}_{xy} = {}_nq^{\;\;2}_{xy} + {}_nq_x \cdot {}_np_y$

Probability (x) dies more than n years after death of (y):

${}_np_x \cdot {}_\infty q^{\;\;2}_{x+n:y}$

CONTINGENT INSURANCE

$$\bar{A}^{\,1}_{xy} = \int_0^\infty v^t \cdot {}_tp_x\, \mu_{x+t} \cdot {}_tp_y\, dt$$

$$\bar{A}^{\;\;2}_{xy} = \int_0^\infty v^t \cdot {}_tp_y\, \mu_{y+t} \cdot {}_tq_x\, dt$$

$$\bar{A}^{\;\;2}_{xy} = \int_0^\infty v^t \cdot {}_tp_x\, \mu_{x+t} \cdot {}_tp_y\, \bar{A}_{y+t}\, dt$$

$\bar{A}_y = \bar{A}^{\;\;1}_{xy} + \bar{A}^{\;\;2}_{xy}$

SBP for a payment of 1 at the death of (x) if he dies more than n years after the death of (y):

${}_nE_x \cdot \bar{A}^{\;\;2}_{x+n:y}$

REVERSIONARY ANNUITY

(y) gets money after (x) dies:

$\bar{a}_{x|y} = \bar{a}_y - \bar{a}_{xy}$

Other versions:

$\bar{a}_{x|y:\overline{n}|} = \bar{a}_{y:\overline{n}|} - \bar{a}_{xy:\overline{n}|}$

$\bar{a}_{x:\overline{n}||y} = \bar{a}_y - \bar{a}_{xy:\overline{n}|}$

In other words:

$\bar{a}_{u|v} = \bar{a}_v - \bar{a}_{uv}$

COMMON SHOCK

Find the total force for each status:
total force on $(x) = \mu_x^* + \lambda$
total force on $(y) = \mu_y^* + \lambda$
total force on $(xy) = \mu_x^* + \mu_y^* + \lambda$

$\Pr[(x) \text{ dies first}] = \frac{\mu_x^*}{\mu_x^* + \mu_y^* + \lambda}$

$\Pr[(y) \text{ dies first}] = \frac{\mu_y^*}{\mu_x^* + \mu_y^* + \lambda}$

$\Pr[T_x = T_y] = \frac{\lambda}{\mu_x^* + \mu_y^* + \lambda}$

$\bar{A}_x = \frac{\mu_x^* + \lambda}{\mu_x^* + \lambda + \delta}$

$\bar{A}_y = \frac{\mu_y^* + \lambda}{\mu_y^* + \lambda + \delta}$

CONSTANT FORCE

$\bar{A}_{xy} = \frac{\mu_x + \mu_y}{\mu_x + \mu_y + \delta}$

$\bar{a}_{xy} = \frac{1}{\mu_x + \mu_y + \delta}$

${}_nq^{\,1}_{xy} = \frac{\mu_x}{\mu_x + \mu_y}\, {}_nq_{xy}$

$\bar{A}^{\,1}_{xy} = \frac{\mu_x}{\mu_x + \mu_y + \delta}$

DE MOIVRE'S LAW

$\mathring{e}_{xx} = \frac{\omega-x}{3}$

$\mathring{e}_{xy} = {}_{y-x}p_x \cdot \mathring{e}_{yy} + {}_{y-x}q_x \cdot \mathring{e}_y$

${}_nq_{xy}^{\ 2} = \frac{1}{2}\, {}_nq_{\overline{xy}}$

$\bar{A}^1_{xy} = \dfrac{\bar{a}_{y:\overline{\omega-x}|}}{\omega-x}$