STAT4870 HW6

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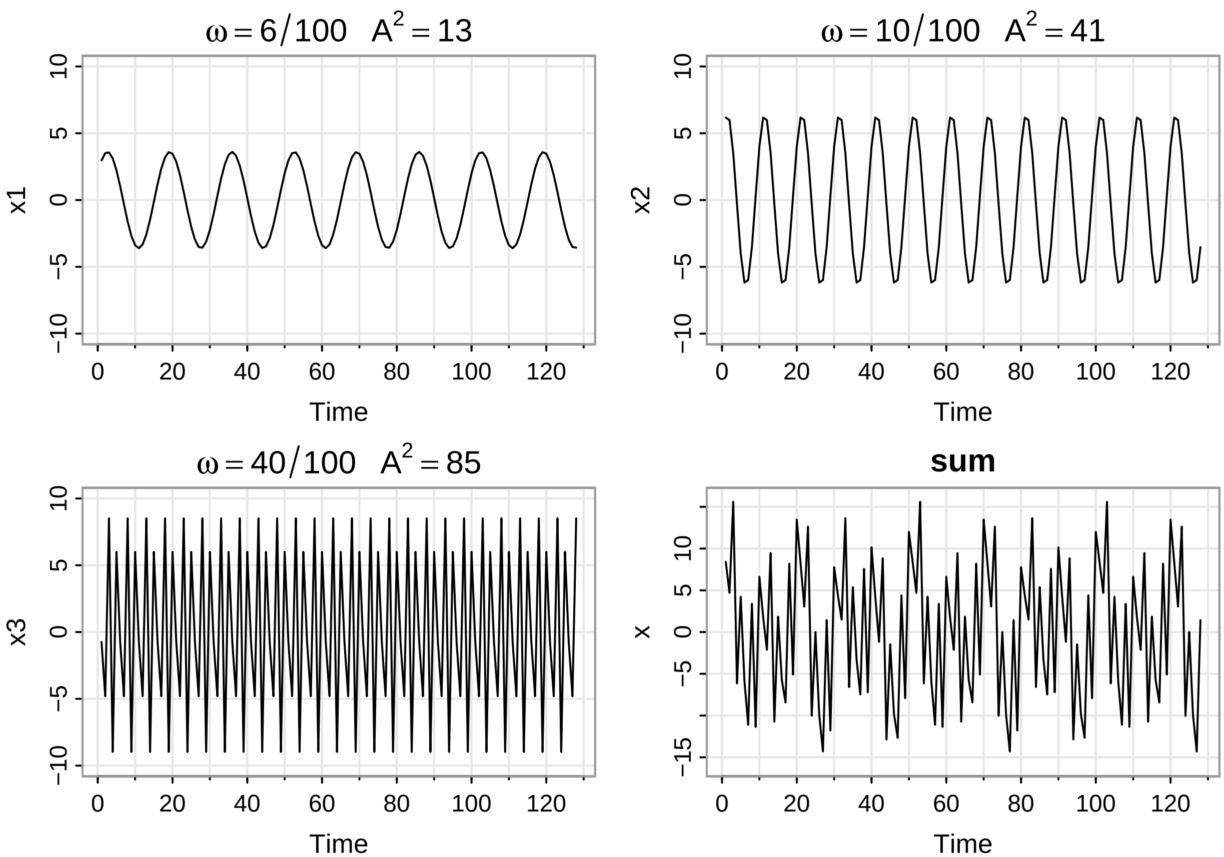
Problem 6.1a

```
library(astsa)

x1 = 2*cos(2*pi*1:128*6/100) + 3*sin(2*pi*1:128*6/100)
x2 = 4*cos(2*pi*1:128*10/100) + 5*sin(2*pi*1:128*10/100)
x3 = 6*cos(2*pi*1:128*40/100) + 7*sin(2*pi*1:128*40/100)
x =x1+x2+x3
par(mfrow=c(2,2))
tsplot(x1, ylim=c(-10,10), main=expression(omega==6/100~~~A^2==13))
tsplot(x2, ylim=c(-10,10), main=expression(omega==10/100~~~A^2==41))
tsplot(x3, ylim=c(-10,10), main=expression(omega==40/100~~~A^2==85))
tsplot(x,  ylim=c(-16,16), main="sum")
```

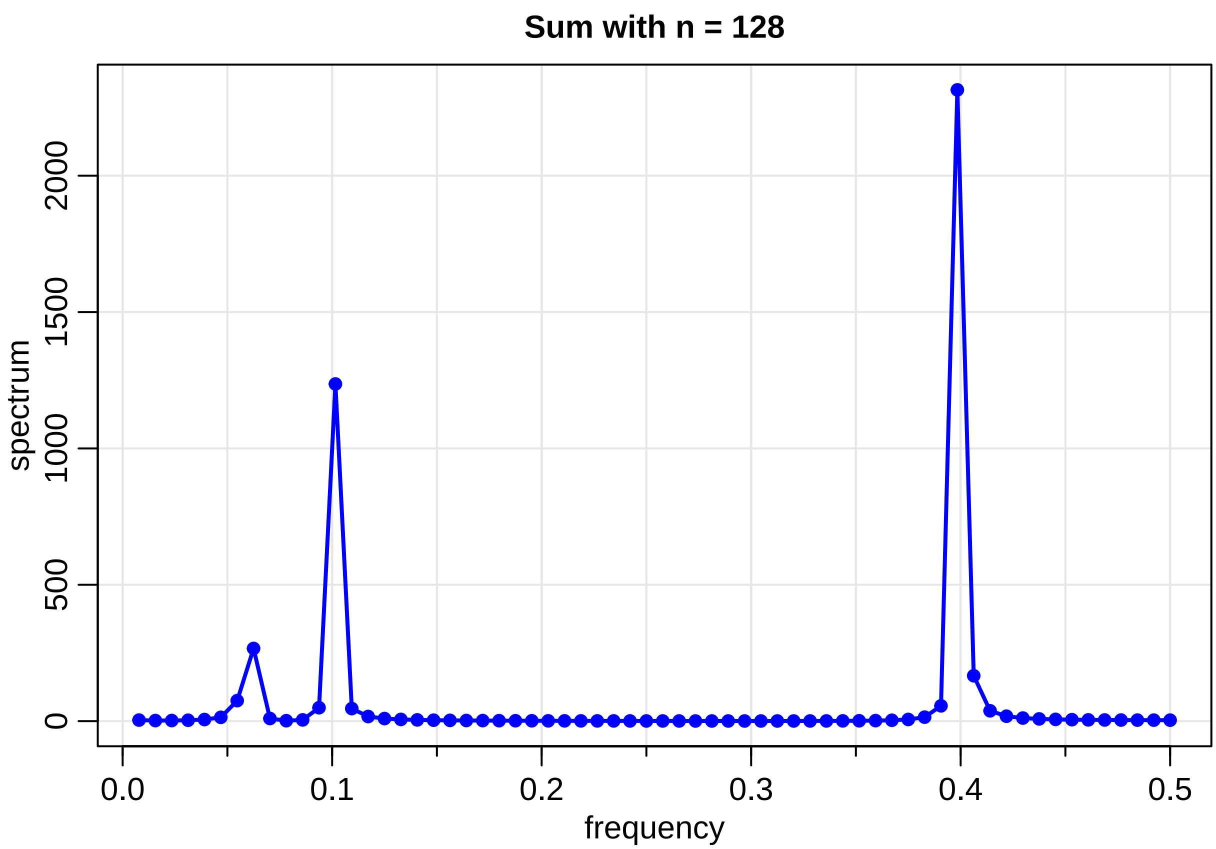


The major difference between the series generated in Example 6.1 is that the series generated is longer. This means that it has more observations and this is important for the analysis and the estimate of the spectrum and other properties of the series.

Problem 6.1b

```
mvspec(x, type = "o", lwd = 2, pch = 20, col = "blue", main = "Sum with n = 128")

periodogram<-mvspec(x, type = "o", lwd = 2, pch = 20, col = "blue", main = "Sum with n = 128")
```




```
periodogram$freq
```

```
##  [1] 0.0078125 0.0156250 0.0234375 0.0312500 0.0390625 0.0468750 0.0546875
##  [8] 0.0625000 0.0703125 0.0781250 0.0859375 0.0937500 0.1015625 0.1093750
## [15] 0.1171875 0.1250000 0.1328125 0.1406250 0.1484375 0.1562500 0.1640625
## [22] 0.1718750 0.1796875 0.1875000 0.1953125 0.2031250 0.2109375 0.2187500
## [29] 0.2265625 0.2343750 0.2421875 0.2500000 0.2578125 0.2656250 0.2734375
## [36] 0.2812500 0.2890625 0.2968750 0.3046875 0.3125000 0.3203125 0.3281250
## [43] 0.3359375 0.3437500 0.3515625 0.3593750 0.3671875 0.3750000 0.3828125
## [50] 0.3906250 0.3984375 0.4062500 0.4140625 0.4218750 0.4296875 0.4375000
## [57] 0.4453125 0.4531250 0.4609375 0.4687500 0.4765625 0.4843750 0.4921875
## [64] 0.5000000
```

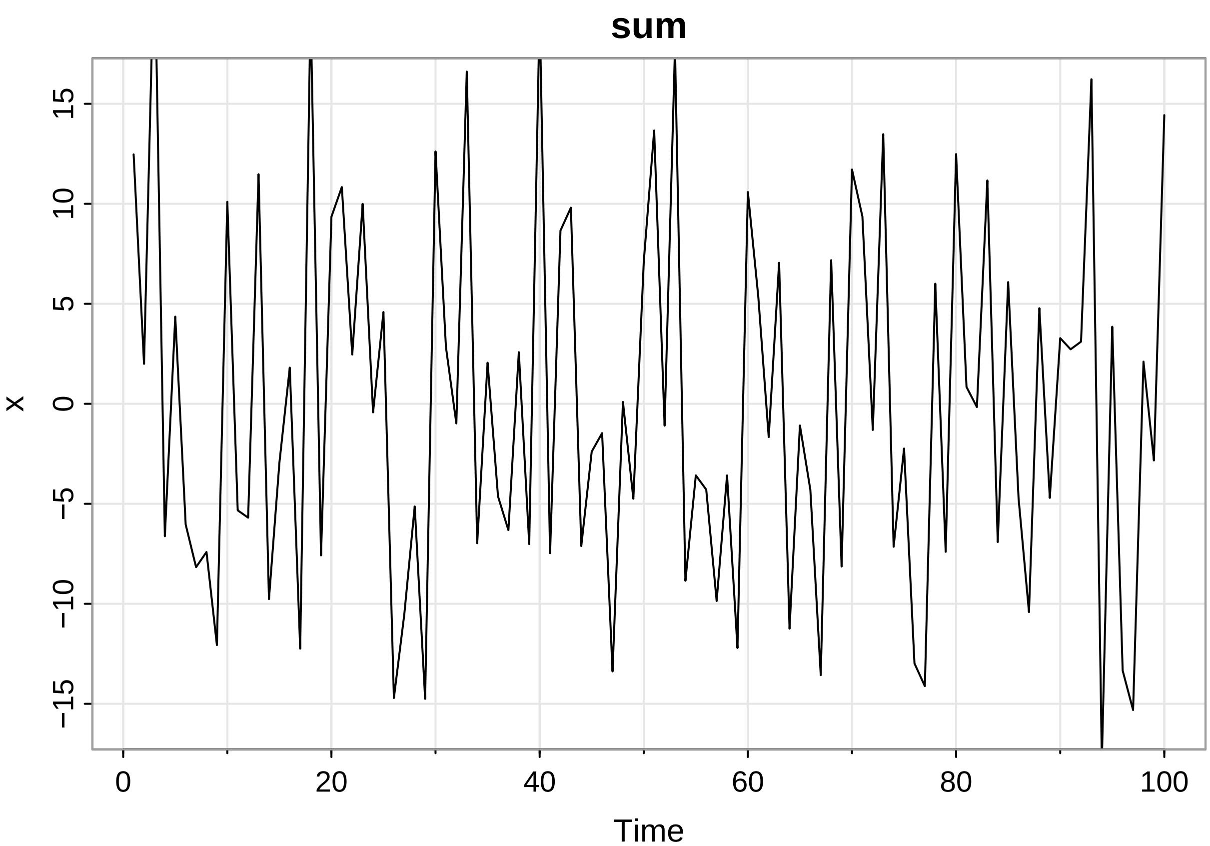
```
periodogram$spec
```

```
##  [1]    3.8421039    1.9879058    2.1687863    3.1923768    5.7674348
##  [6]   13.8547188   75.1229859  266.4820652    9.5657532    1.5260361
## [11]    4.4964835   49.1899227 1236.3567795   46.0232436   16.8714958
## [16]    9.3473250    6.1574862    4.4569866    3.4219595    2.7351236
## [21]    2.2507897    1.8935117    1.6206415    1.4064086    1.2344041
## [26]    1.0937325    0.9769099    0.8786539    0.7951575    0.7236381
## [31]    0.6620522    0.6089138    0.5631823    0.5242031    0.4916912
## [36]    0.4657607    0.4470118    0.4367002    0.4370412    0.4517486
## [41]    0.4870065    0.5532900    0.6689483    0.8677349    1.2159856
## [46]    1.8562112    3.1347259    6.0596053   14.5612898   55.9924779
## [51] 2314.8626825  166.3997784   37.8082085   17.9614852   11.2001506
## [56]    8.0549595    6.3267844    5.2776915    4.6012631    4.1513980
## [61]    3.8517152    3.6600539    3.5529597    3.5184856
```

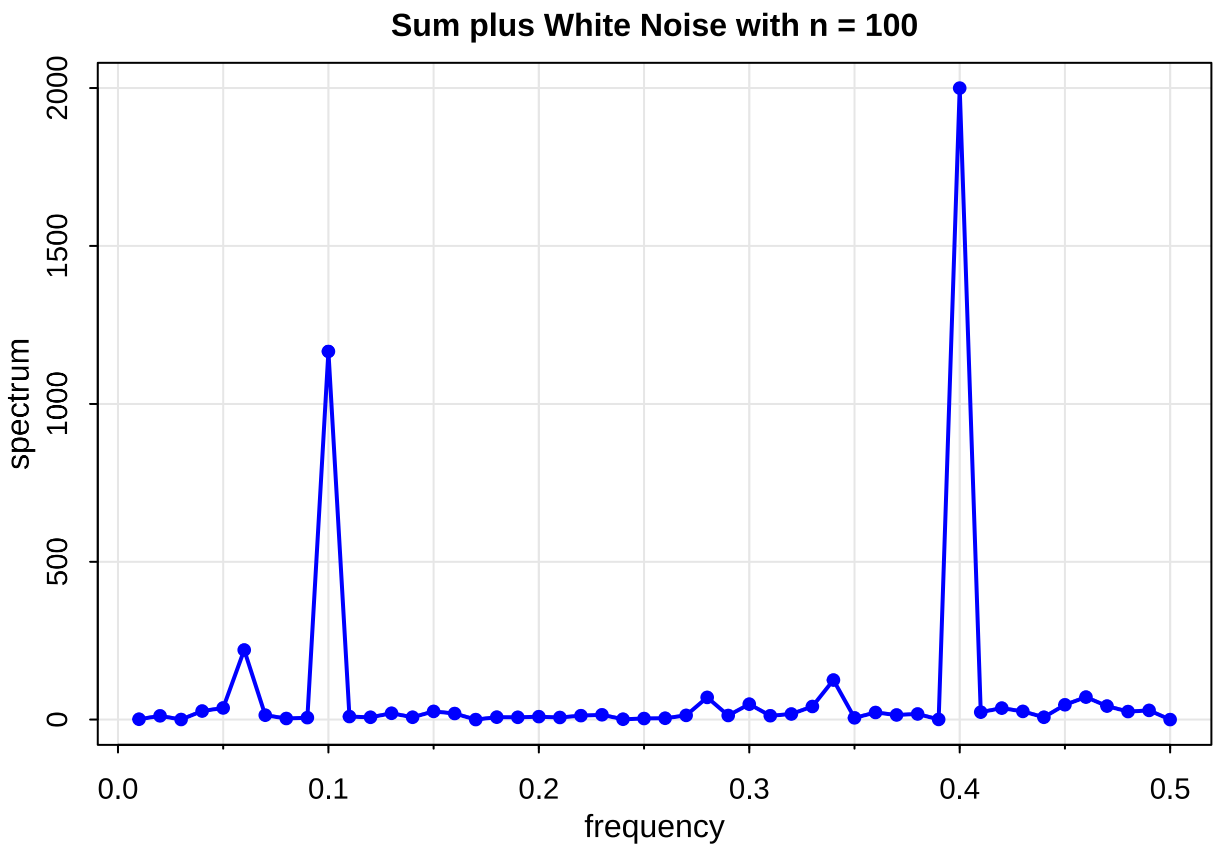
We can see that there are 3 peaks in the Periodogram. We can see that one of the peak is at frequency 0.0625, which means the period for this values is 16. That is, it takes 16 time periods for a complete cycle. Additionally, the largest peak happens at the frequency of 0.3984375.

Problem 6.1c

```
x1 = 2*cos(2*pi*1:100*6/100) + 3*sin(2*pi*1:100*6/100)
x2 = 4*cos(2*pi*1:100*10/100) + 5*sin(2*pi*1:100*10/100)
x3 = 6*cos(2*pi*1:100*40/100) + 7*sin(2*pi*1:100*40/100)
x = x1+x2+x3+rnorm(n = 100, mean = 0, sd = 5)
tsplot(x,  ylim=c(-16,16), main="sum")
```



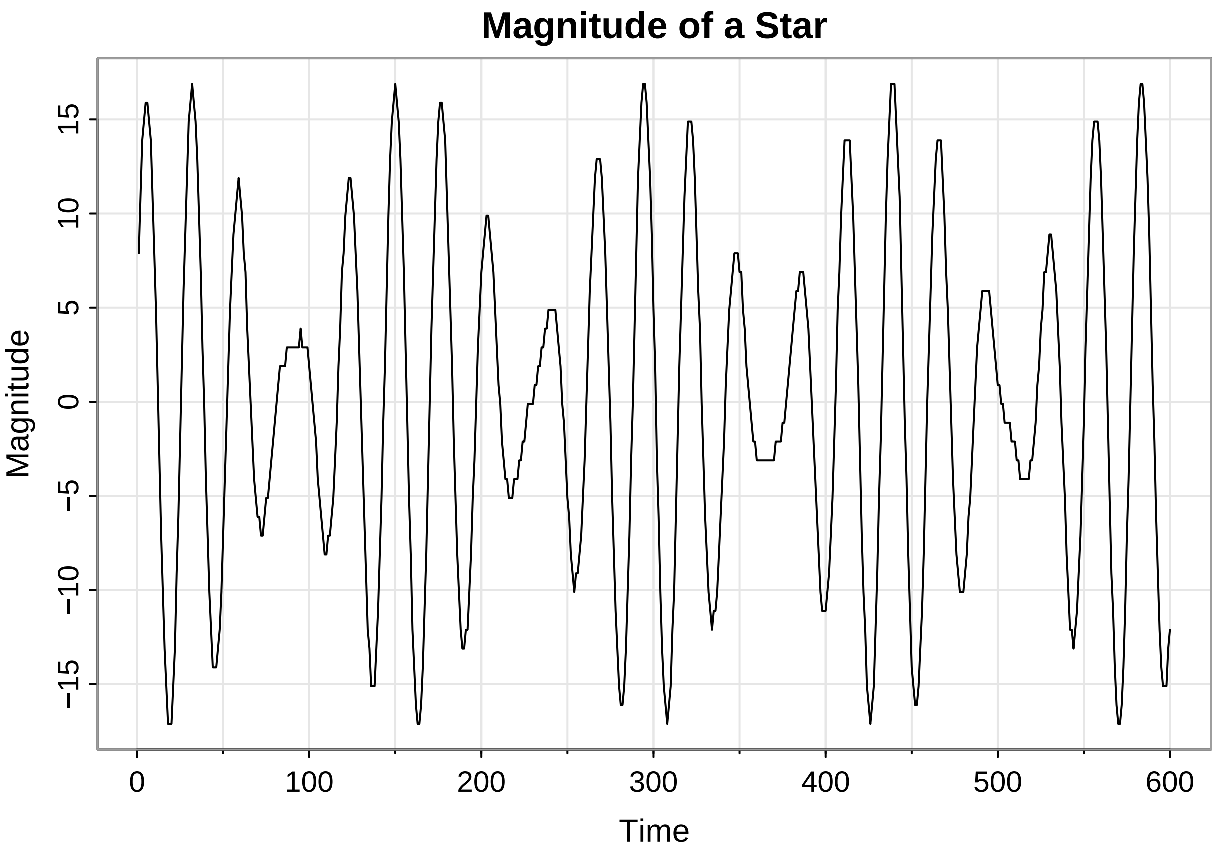

```
mvspec(x, type = "o", lwd = 2, pch = 20, col = "blue", main = "Sum plus White Noise with n = 100")
```



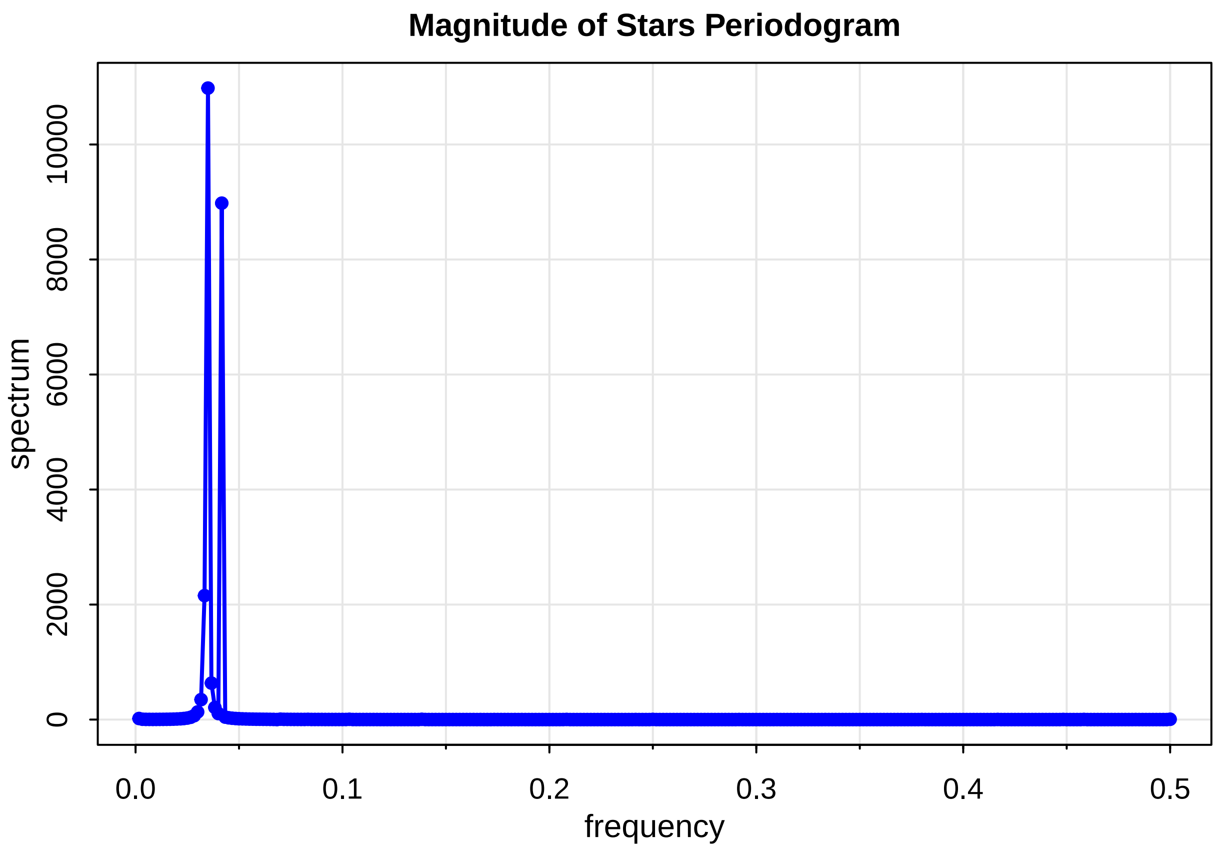
Based on the Periodogram analysis, the largest peak aligns with that of the series without white noise. However, the frequencies with lower spectral density appear less stable compared to the series without white noise. Additionally, while the seasonal pattern is less distinct in the presence of white noise, the overall trend of the series is still visible

Problem 6.3

```
data<- star - mean(star)
tsplot(data, ylab = "Magnitude", main = "Magnitude of a Star")
```




```
mvspec(data, type = "o", lwd = 2, pch = 20, col = "blue", main = "Magnitude of Stars Periodogram")
periodogram<-mvspec(data, type = "o", lwd = 2, pch = 20, col = "blue", main = "Magnitude of Stars Perio
```




```r
periodogram$freq
```

```
##   [1] 0.001666667 0.003333333 0.005000000 0.006666667 0.008333333 0.010000000
##   [7] 0.011666667 0.013333333 0.015000000 0.016666667 0.018333333 0.020000000
##  [13] 0.021666667 0.023333333 0.025000000 0.026666667 0.028333333 0.030000000
##  [19] 0.031666667 0.033333333 0.035000000 0.036666667 0.038333333 0.040000000
##  [25] 0.041666667 0.043333333 0.045000000 0.046666667 0.048333333 0.050000000
##  [31] 0.051666667 0.053333333 0.055000000 0.056666667 0.058333333 0.060000000
##  [37] 0.061666667 0.063333333 0.065000000 0.066666667 0.068333333 0.070000000
##  [43] 0.071666667 0.073333333 0.075000000 0.076666667 0.078333333 0.080000000
##  [49] 0.081666667 0.083333333 0.085000000 0.086666667 0.088333333 0.090000000
##  [55] 0.091666667 0.093333333 0.095000000 0.096666667 0.098333333 0.100000000
##  [61] 0.101666667 0.103333333 0.105000000 0.106666667 0.108333333 0.110000000
##  [67] 0.111666667 0.113333333 0.115000000 0.116666667 0.118333333 0.120000000
##  [73] 0.121666667 0.123333333 0.125000000 0.126666667 0.128333333 0.130000000
##  [79] 0.131666667 0.133333333 0.135000000 0.136666667 0.138333333 0.140000000
##  [85] 0.141666667 0.143333333 0.145000000 0.146666667 0.148333333 0.150000000
##  [91] 0.151666667 0.153333333 0.155000000 0.156666667 0.158333333 0.160000000
##  [97] 0.161666667 0.163333333 0.165000000 0.166666667 0.168333333 0.170000000
## [103] 0.171666667 0.173333333 0.175000000 0.176666667 0.178333333 0.180000000
## [109] 0.181666667 0.183333333 0.185000000 0.186666667 0.188333333 0.190000000
## [115] 0.191666667 0.193333333 0.195000000 0.196666667 0.198333333 0.200000000
## [121] 0.201666667 0.203333333 0.205000000 0.206666667 0.208333333 0.210000000
## [127] 0.211666667 0.213333333 0.215000000 0.216666667 0.218333333 0.220000000
## [133] 0.221666667 0.223333333 0.225000000 0.226666667 0.228333333 0.230000000
## [139] 0.231666667 0.233333333 0.235000000 0.236666667 0.238333333 0.240000000
```

```
## [145] 0.241666667 0.243333333 0.245000000 0.246666667 0.248333333 0.250000000
## [151] 0.251666667 0.253333333 0.255000000 0.256666667 0.258333333 0.260000000
## [157] 0.261666667 0.263333333 0.265000000 0.266666667 0.268333333 0.270000000
## [163] 0.271666667 0.273333333 0.275000000 0.276666667 0.278333333 0.280000000
## [169] 0.281666667 0.283333333 0.285000000 0.286666667 0.288333333 0.290000000
## [175] 0.291666667 0.293333333 0.295000000 0.296666667 0.298333333 0.300000000
## [181] 0.301666667 0.303333333 0.305000000 0.306666667 0.308333333 0.310000000
## [187] 0.311666667 0.313333333 0.315000000 0.316666667 0.318333333 0.320000000
## [193] 0.321666667 0.323333333 0.325000000 0.326666667 0.328333333 0.330000000
## [199] 0.331666667 0.333333333 0.335000000 0.336666667 0.338333333 0.340000000
## [205] 0.341666667 0.343333333 0.345000000 0.346666667 0.348333333 0.350000000
## [211] 0.351666667 0.353333333 0.355000000 0.356666667 0.358333333 0.360000000
## [217] 0.361666667 0.363333333 0.365000000 0.366666667 0.368333333 0.370000000
## [223] 0.371666667 0.373333333 0.375000000 0.376666667 0.378333333 0.380000000
## [229] 0.381666667 0.383333333 0.385000000 0.386666667 0.388333333 0.390000000
## [235] 0.391666667 0.393333333 0.395000000 0.396666667 0.398333333 0.400000000
## [241] 0.401666667 0.403333333 0.405000000 0.406666667 0.408333333 0.410000000
## [247] 0.411666667 0.413333333 0.415000000 0.416666667 0.418333333 0.420000000
## [253] 0.421666667 0.423333333 0.425000000 0.426666667 0.428333333 0.430000000
## [259] 0.431666667 0.433333333 0.435000000 0.436666667 0.438333333 0.440000000
## [265] 0.441666667 0.443333333 0.445000000 0.446666667 0.448333333 0.450000000
## [271] 0.451666667 0.453333333 0.455000000 0.456666667 0.458333333 0.460000000
## [277] 0.461666667 0.463333333 0.465000000 0.466666667 0.468333333 0.470000000
## [283] 0.471666667 0.473333333 0.475000000 0.476666667 0.478333333 0.480000000
## [289] 0.481666667 0.483333333 0.485000000 0.486666667 0.488333333 0.490000000
## [295] 0.491666667 0.493333333 0.495000000 0.496666667 0.498333333 0.500000000
```

```
periodogram$spec
```

```
##   [1] 1.882436e+01 6.159546e+00 3.810464e+00 3.299023e+00 3.091250e+00
##   [6] 3.384484e+00 3.655361e+00 4.367529e+00 5.085605e+00 6.371012e+00
##  [11] 7.852783e+00 1.026367e+01 1.343364e+01 1.860826e+01 2.651111e+01
##  [16] 4.072164e+01 6.793406e+01 1.327575e+02 3.473861e+02 2.155696e+03
##  [21] 1.098030e+04 6.342813e+02 2.100519e+02 1.050165e+02 8.979785e+03
##  [26] 4.290180e+01 3.111462e+01 2.367724e+01 1.870232e+01 1.518591e+01
##  [31] 1.259114e+01 1.064040e+01 9.087791e+00 7.881546e+00 6.848786e+00
##  [36] 6.030070e+00 5.262050e+00 4.622470e+00 3.901734e+00 3.059436e+00
##  [41] 6.974170e-01 7.224920e+00 4.543513e+00 3.838938e+00 3.381001e+00
##  [46] 3.108779e+00 2.826621e+00 2.659817e+00 2.443911e+00 3.963054e+00
##  [51] 2.151197e+00 2.070171e+00 1.917266e+00 1.860267e+00 1.726327e+00
##  [56] 1.688232e+00 1.570264e+00 1.549495e+00 1.449917e+00 1.456742e+00
##  [61] 1.428556e+00 4.923378e+00 1.010502e+00 1.092969e+00 1.023718e+00
##  [66] 1.043599e+00 9.610111e-01 9.776003e-01 8.944588e-01 9.125700e-01
##  [71] 8.303688e-01 8.506406e-01 7.691089e-01 7.911744e-01 7.092014e-01
##  [76] 7.319639e-01 6.470609e-01 6.677191e-01 5.729313e-01 5.813049e-01
##  [81] 4.450582e-01 3.971453e-01 4.647490e+00 1.031076e+00 7.758408e-01
##  [86] 7.327494e-01 6.393736e-01 6.501884e-01 5.756108e-01 6.000422e-01
##  [91] 5.309522e-01 5.613505e-01 4.942757e-01 5.280089e-01 4.613053e-01
##  [96] 4.970149e-01 4.291437e-01 4.655923e-01 3.937071e-01 6.809489e-01
## [101] 3.433230e-01 3.678053e-01 3.876840e-01 1.132920e+00 5.724599e-01
## [106] 5.199222e-01 4.407973e-01 4.598915e-01 3.985481e-01 4.300507e-01
## [111] 3.727392e-01 4.090097e-01 3.532660e-01 3.920794e-01 3.372057e-01
## [116] 3.776447e-01 3.234434e-01 3.650618e-01 3.116197e-01 3.542600e-01
## [121] 3.022629e-01 3.466531e-01 3.026376e-01 5.214664e-01 2.291657e+00
```

```
## [126] 3.136786e-01 2.606562e-01 3.082354e-01 2.544732e-01 3.011196e-01
## [131] 2.475008e-01 2.939562e-01 2.406368e-01 2.870533e-01 2.341443e-01
## [136] 2.805191e-01 2.282043e-01 2.744538e-01 2.231444e-01 2.691293e-01
## [141] 2.200697e-01 2.660103e-01 2.268090e-01 2.999142e-01 1.023863e+00
## [146] 2.697002e-01 1.925269e-01 2.484097e-01 1.879021e-01 2.830917e+00
## [151] 1.840048e-01 2.378350e-01 1.801510e-01 2.338859e-01 1.764539e-01
## [156] 2.305151e-01 1.730604e-01 2.279335e-01 1.702370e-01 2.268243e-01
## [161] 1.687890e-01 2.298268e-01 1.731564e-01 2.594387e-01 3.673382e-01
## [166] 3.285225e-01 1.649830e-01 1.945872e-01 1.494465e-01 1.937040e-01
## [171] 1.456249e-01 1.925301e-01 1.427785e-01 1.909795e-01 8.697624e-01
## [176] 1.893578e-01 1.379246e-01 1.879155e-01 1.359427e-01 1.870011e-01
## [181] 1.345752e-01 1.874998e-01 1.349544e-01 1.937288e-01 1.482178e-01
## [186] 5.739414e-01 1.169888e-01 1.579606e-01 1.161029e-01 1.628076e-01
## [191] 1.158264e-01 1.632994e-01 1.146720e-01 1.626464e-01 1.132318e-01
## [196] 1.615927e-01 1.117190e-01 1.604338e-01 1.102589e-01 4.953048e-01
## [201] 1.090064e-01 1.588004e-01 1.083629e-01 1.598936e-01 1.104994e-01
## [206] 1.757794e-01 1.797881e-01 1.323946e-01 9.321789e-02 1.402588e-01
## [211] 9.381514e-02 1.402292e-01 9.256255e-02 1.385986e-01 9.061106e-02
## [216] 1.360685e-01 8.809697e-02 1.326071e-01 8.486071e-02 1.277642e-01
## [221] 8.042474e-02 1.203487e-01 7.359785e-02 1.069461e-01 6.127119e-02
## [226] 7.601122e-02 1.444681e-01 1.058795e+00 2.220739e-01 2.272912e-01
## [231] 1.347336e-01 1.847933e-01 1.162704e-01 1.699211e-01 1.080310e-01
## [236] 1.621321e-01 1.031829e-01 1.572183e-01 9.989478e-02 1.537924e-01
## [241] 9.749353e-02 1.513066e-01 9.572748e-02 1.496269e-01 9.468039e-02
## [246] 1.493582e-01 9.614545e-02 1.751484e-01 8.169279e-02 1.915868e+00
## [251] 8.636717e-02 1.389918e-01 8.610635e-02 1.383849e-01 8.533059e-02
## [256] 1.374152e-01 8.438393e-02 1.362644e-01 8.331182e-02 1.349210e-01
## [261] 8.206060e-02 1.332486e-01 8.045232e-02 1.308479e-01 7.793507e-02
## [266] 1.262479e-01 7.180206e-02 1.066112e-01 2.239600e+00 1.647627e-01
## [271] 9.320054e-02 1.437150e-01 8.665472e-02 1.389020e-01 2.475208e+00
## [276] 1.362686e-01 8.217580e-02 1.342082e-01 8.057035e-02 1.321638e-01
## [281] 7.881972e-02 1.296873e-01 7.650737e-02 1.260136e-01 7.270396e-02
## [286] 1.189534e-01 6.401069e-02 9.687277e-02 1.887855e-02 4.466139e-01
## [291] 1.345880e-01 1.733370e-01 1.036495e-01 1.574629e-01 9.679927e-02
## [296] 1.522155e-01 9.412693e-02 1.500499e-01 9.309405e-02 5.894263e+00
```

From the periodogram, we can see that the most prominent frequency occurs at 0.035 with a spectrum of 10980.30, which means that it takes about period of 28.57 to complete the cycle. This is consistent with the time series plot. The second most prominent frequency occurs at 0.0416667, which has a spectrum of about 8979.785. This means that it takes period of 24 to complete the cycle, which also makes sense.

Problem 6.5a

By definition,

$$f(\omega) = \sum_{h=-\infty}^{\infty} \gamma(h) e^{-2\pi i \omega h}$$

We can compute the spectral density using Property 6.8, which $f(\omega) = \sigma_\omega^2 |\theta e^{-2\pi i \omega}|^2$. We have $\theta(z) = 1 + \theta z$, we have

$$|\theta(e^{-2\pi i \omega})|^2 = |1 + \theta e^{-2\pi i \omega}|^2 = (1 + \theta e^{-2\pi i \omega})(1 + \theta e^{2\pi i \omega})$$

$$= 1 + \theta e^{-2\pi i\omega} + \theta e^{2\pi i\omega} + \theta^2 e^{-2\pi i\omega} e^{2\pi i\omega}$$
$$= 1 + \theta(e^{-2\pi i\omega} + e^{2\pi i\omega}) + \theta^2$$

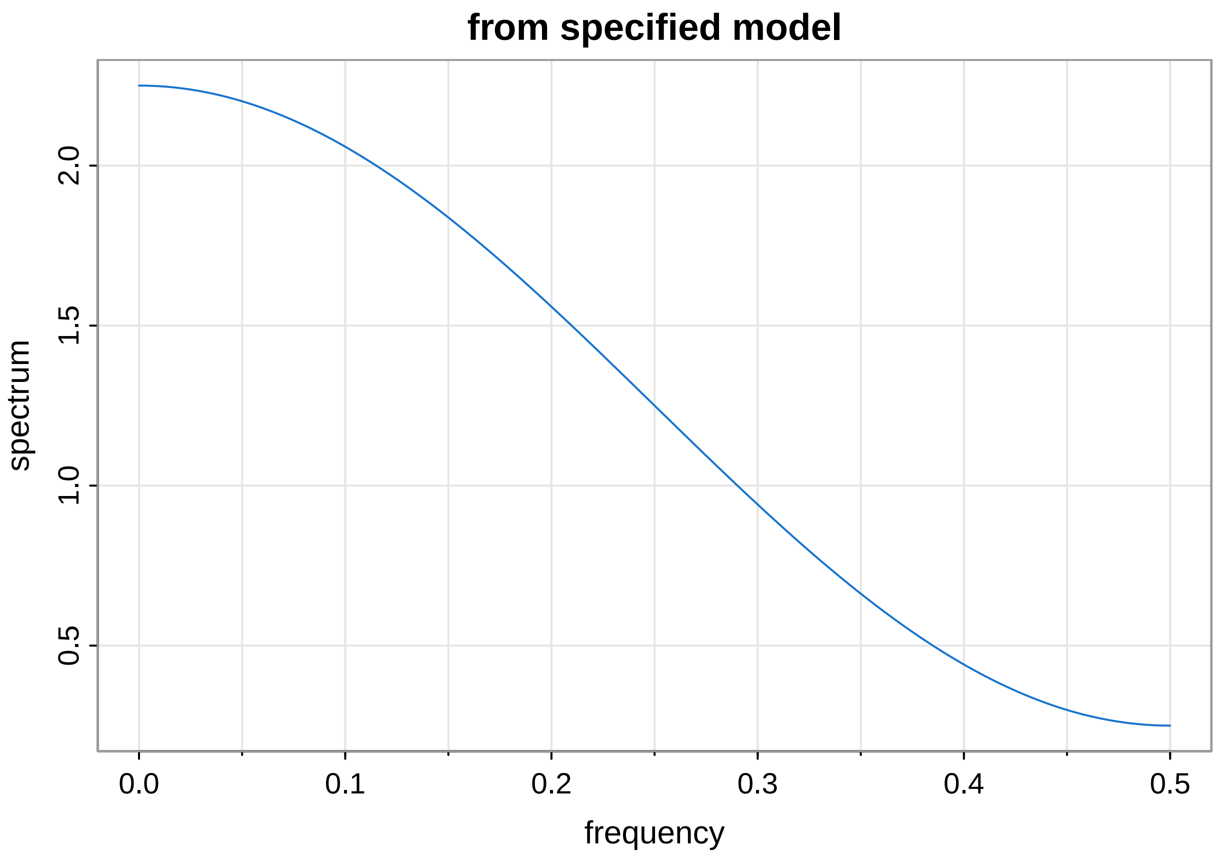
Therefore, we have a power spectrum:

$$f(\omega) = \sigma_\omega^2[1 + \theta(e^{-2\pi i\omega} + e^{2\pi i\omega}) + \theta^2]$$

Problem 6.5b

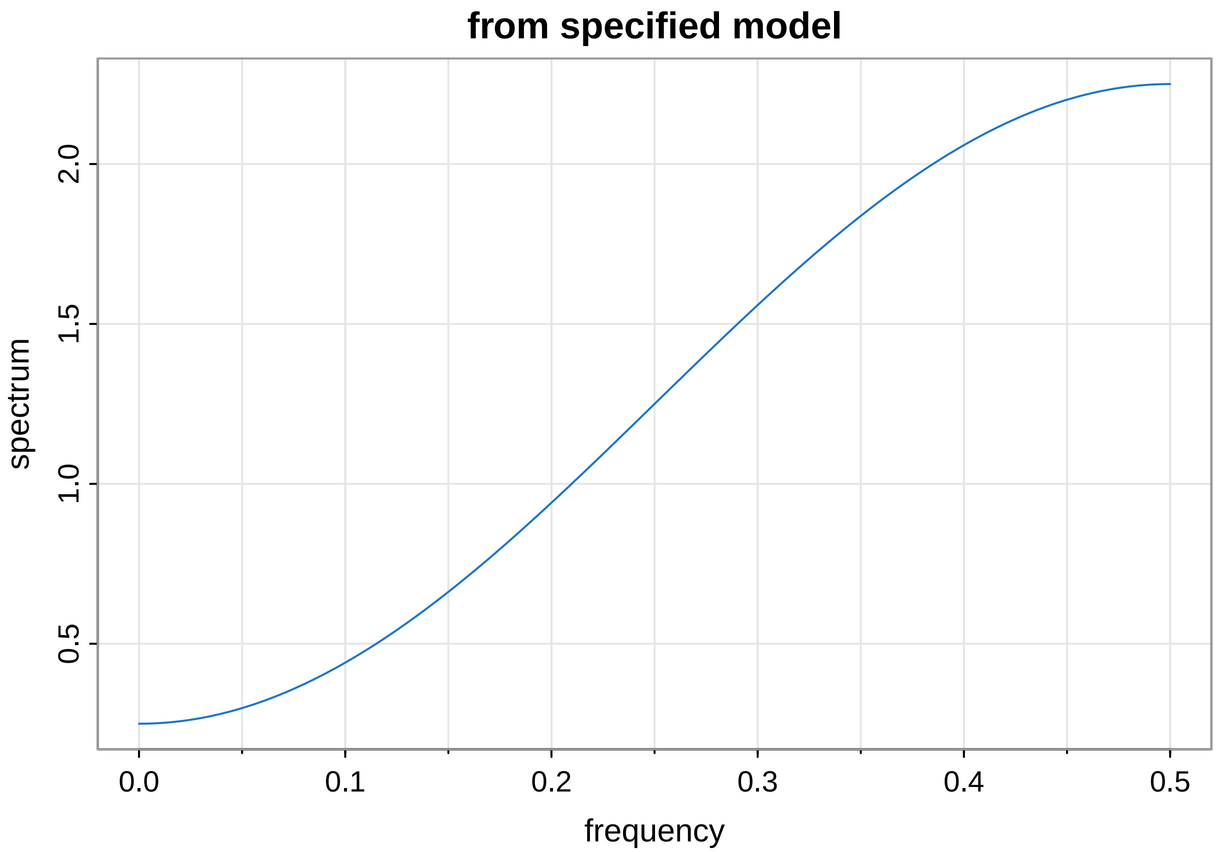
First, let $\theta = 0.5$

```
arma.spec(ma = 0.5, col = 4)
```



Now let $\theta = -0.5$:

```
arma.spec(ma = -0.5, col = 4)
```



Problem 6.5c

Based on part b, the spectral density shows more energy concentrated at lower frequencies with $\theta = 0.5$, and this suggests the time series has a stronger trend or long term correlation. However, with $\theta = -0.5$ shifts energy towards higher frequencies, and this indicates more oscillatory behavior or short-term fluctuations in the time series.

In summary, $\theta > 0$ will have the highest spectrum at a lower frequencies, and $\theta < 0$ will have the highest spectrum at the high frequencies. In other word, if $\theta > 0$, the MA(1) is a low-pass filter, and when $\theta < 0$, the MA(1) is a high-pass filter.

Problem 6.9a

From Property 6.11, $f_z(\omega) = |A_{zy}(\omega)|^2 f_y(\omega)$

Given $z_t = \sum_s b_s y_{t-s}$ and $y_t = \sum_r a_r x_{t-r}$. We can substitute the y_{t-s}:

$$z_t = \sum_s b_s (\sum_r a_r x_{t-s-r}) = \sum_s \sum_r a_r b_s x_{t-s-r}$$

We know that $A(\omega) = \sum_r b_s e^{-2\pi i \omega r}$ and $|A(\omega)|^2 = \sum_r a_r$ and same for $B(\omega)$. Now we can write z as the density:

$$f_z(\omega) = |A(\omega)|^2 |B(\omega)|^2 f_x(\omega)$$

by using Property 6.11.

Problem 6.9b

Given $u_t = x_t - x_{t-12}$ followed by $v_t = u_t - u_{t-1}$:

$$v_t = x_t - x_{t-12} - x_{t-1} + x_{t-13} = (x_t - x_{t-1}) - (x_{t-12} - x_{t-13}) = \Delta x_t - \Delta x_{t-12}$$

We can see that by applying the filter, we are differencing the observation x_t and x_{t-12} The first filter u_t is a seasonal differencing filter with period 12, which removes seasonal cycles of period 12 from the series. v_t is a differencing filter, removing the trend. Therefore, it would create a high-pass filter that allow the high frequencies to pass through. These filters can make the time series more stationary.

Problem 6.9c

```
omega <-seq(0,.5, length=1000)
par(mfrow=c(2,1))
FR12 <- abs(1-exp(2i*12*pi*omega))^2
tsplot(omega, FR12, main="12th Difference")
abline(v=1:6/12)
FR121 <- abs(1-exp(2i*pi*omega)-exp(2i*12*pi*omega)+exp(2i*13*pi*omega))^2
tsplot(omega, FR121, main="1st Difference and 12th Difference")
abline(v=1:6/12)
```