STAT4870 HW4

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Problem 3a

```
set.seed(123)
library(astsa)

x <- rnorm(150, mean = 5)

arima(x, order = c(2, 0, 1))
```

```
##
## Call:
## arima(x = x, order = c(2, 0, 1))
##
## Coefficients:
##           ar1      ar2     ma1  intercept
##       -0.9315  -0.0910  0.9749     4.9766
## s.e.   0.0898   0.0873  0.0475     0.0732
##
## sigma^2 estimated as 0.8417:  log likelihood = -200.36,  aic = 410.71
```

We can see that $\hat{\phi_1} = -0.9315$ and $\hat{\theta} = 0.9749$ and both are significant. The data are independent, but the estimation implies a different result that the data are highly dependent. Therefore, we need ARMA model to be reduced to its simplest form.

Consider the model 1, $x_t = 0.8x_{t-1} - 0.15x_{t-2} + w_t - 0.3w_{t-1}$:

```
ARi <- c(1, -0.8, 0.15)
polyroot(ARi)
```

```
## [1] 2.000000-0i 3.333333+0i
```

```
MAi <- c(1, -0.3)
polyroot(MAi)
```

```
## [1] 3.333333+0i
```

The means that the model 1 can be reduced to:

$$x_t(2 - B) = w_t$$

$$\implies 2x_t - x_{t-1} = w_t$$
$$\implies 2x_t = x_{t-1} + w_t$$
$$\implies x_t = 0.5x_{t-1} + 0.5w_t$$

Consider the model 2, $x_t = x_{t-1} - 0.5x_{t-2} + w_t - w_{t-1}$.

```
ARii<-c(1, -1, 0.5)
polyroot(ARii)
```

```
## [1] 1+1i 1-1i
```

```
MAii<-c(1, -1)
polyroot(MAii)
```

```
## [1] 1+0i
```

We can see that the AR and MA part don't match, so model 2 is in its simplest form.

3b

For reduced model 1, $x_t = 0.5x_{t-1} + 0.5w_t$. We can rewrite this as:

$$x_t - 0.5x_{t-1} = 0.5w_t$$

Let $\phi(B) = (1 - 0.5B)x_t$ and $\theta(B) = 0.5$. We can see the root for $\phi(B)$ is 2, and this lies outside of unit circle. Therefore, it is causal. Additionally, since $\theta(B)$ doesn't have a root, it does not lie in a unit circle. Therefore, it is invertible.

Thus, model 1 is both causal and invertible.

For the model 2, $x_t = x_{t-1} - 0.5x_{t-2} + w_t - w_{t-1}$. We can rewrite it as:

$$x_t - x_{t-1} + 0.5x_{t-2} = w_t - w_{t-1}$$

Let $\phi(B) = (1 - B + 0.5B^2)x_t$ and the roots are 1+i and 1-i (shown in part a). Since the root is complex, we can check the distance of the root from the origin (0,0).

$$|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$
$$|1 - i| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Since they are greater than 1, which means it is outside of the unit circle, it is causal.

Let $\theta(B) = (1 - B)w_t$ and the root is 1. Since the root is exactly 1, it is not invertible.

Thus, model 2 is causal but not invertible.

Problem 3c

```
round(ARMAtoMA(ar = 0.5, ma = -0.5, lag.max = 50), 2)
```

```
##  [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## [39] 0 0 0 0 0 0 0 0 0 0 0 0
```

```r
round(ARMAtoAR(ar = 0.5, ma = -0.5, lag.max = 50), 2)
```

```
##  [1] 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
## [39] 0 0 0 0 0 0 0 0 0 0 0 0
```

For model 1, we can say that the both AR and MA converges to 0, so model 1 is causal and invertible.

```r
round(ARMAtoMA(ar = c(1,-.5), ma = -1, lag.max = 50),2)
```

```
##  [1]  0.00 -0.50 -0.50 -0.25  0.00  0.12  0.12  0.06  0.00 -0.03 -0.03 -0.02
## [13]  0.00  0.01  0.01  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
## [25]  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
## [37]  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00  0.00
## [49]  0.00  0.00
```

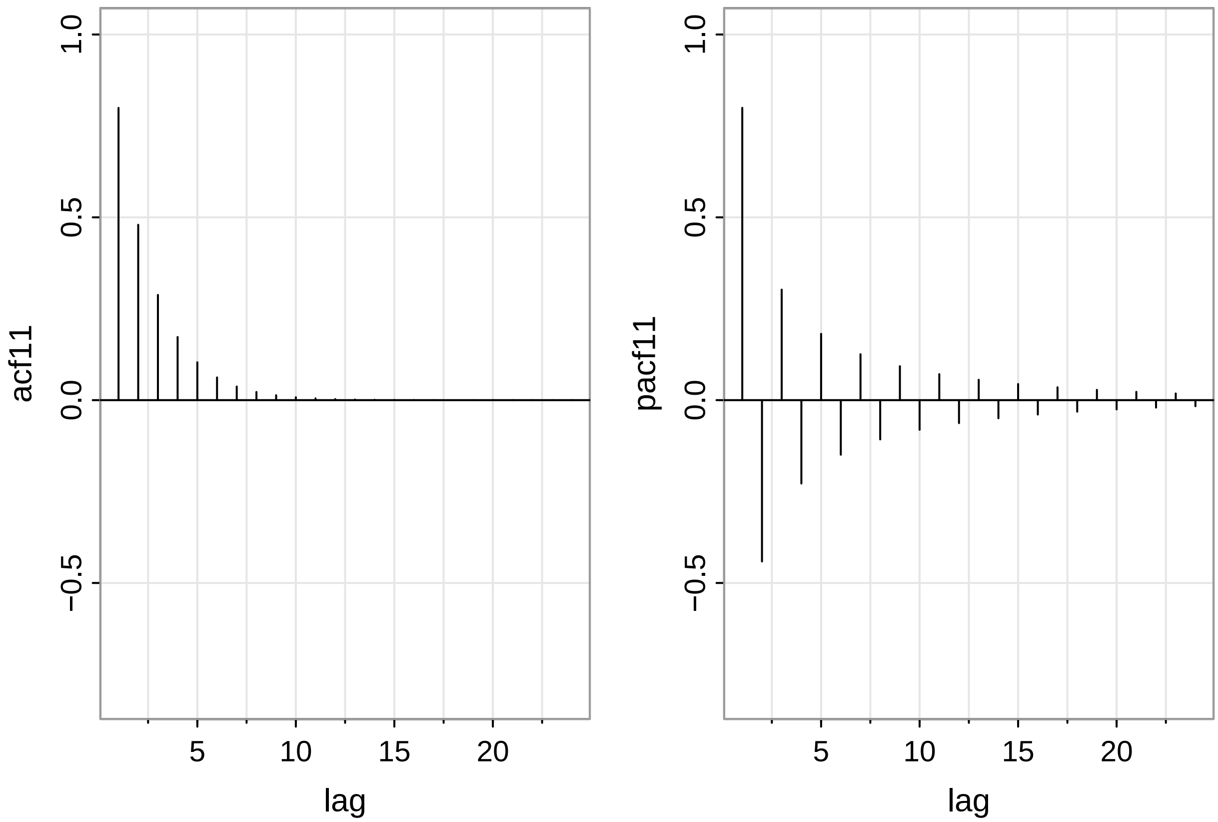
```r
round(ARMAtoAR(ar=c(1,-.5), ma=-1, 50), 2)
```

```
##  [1] 0.0 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5
## [20] 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5
## [39] 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5 0.5
```

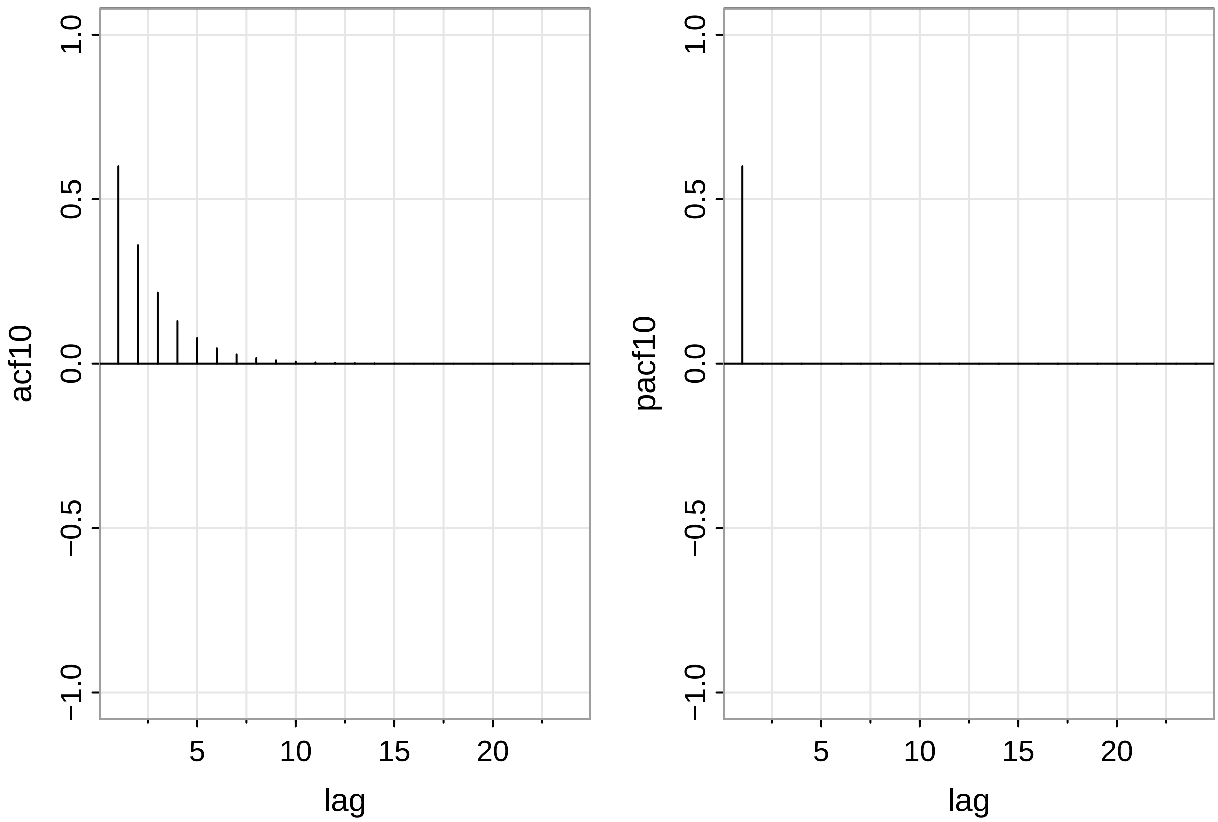
We can see that ARMAtoMA converges to 0, so it is causal. We can also see that ARMAtoAR does not converges to 0, so it is not invertible. Therefore, model2 is causal and not invertible.

Problem 4a

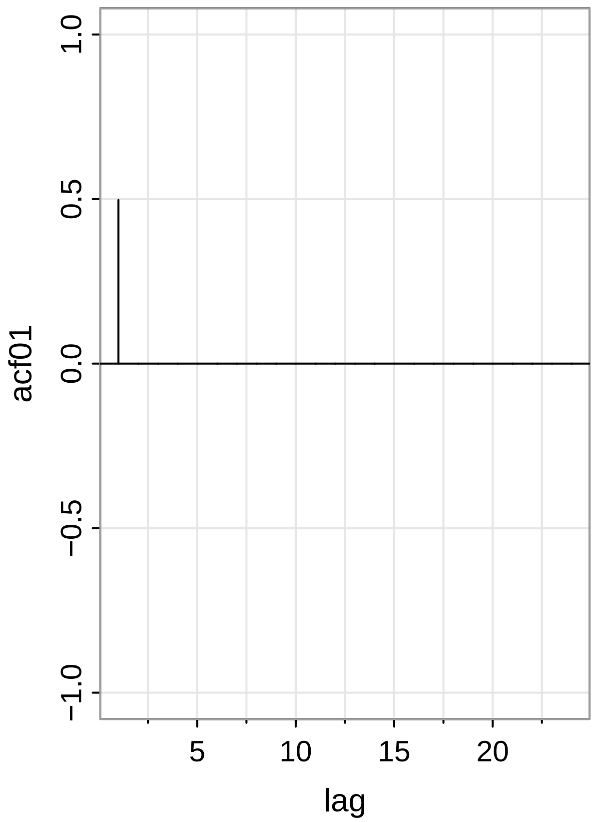
```r
phi<-0.6
theta<-0.9
acf11<-ARMAacf(ar = phi, ma = theta, 24)[-1]
pacf11<-ARMAacf(ar = phi, ma = theta, 24, pacf = TRUE)
par(mfrow=1:2)
tsplot(acf11, type="h", xlab="lag", ylim=c(-.8,1))
abline(h=0)
tsplot(pacf11, type="h", xlab="lag", ylim=c(-.8,1))
abline(h=0)
```

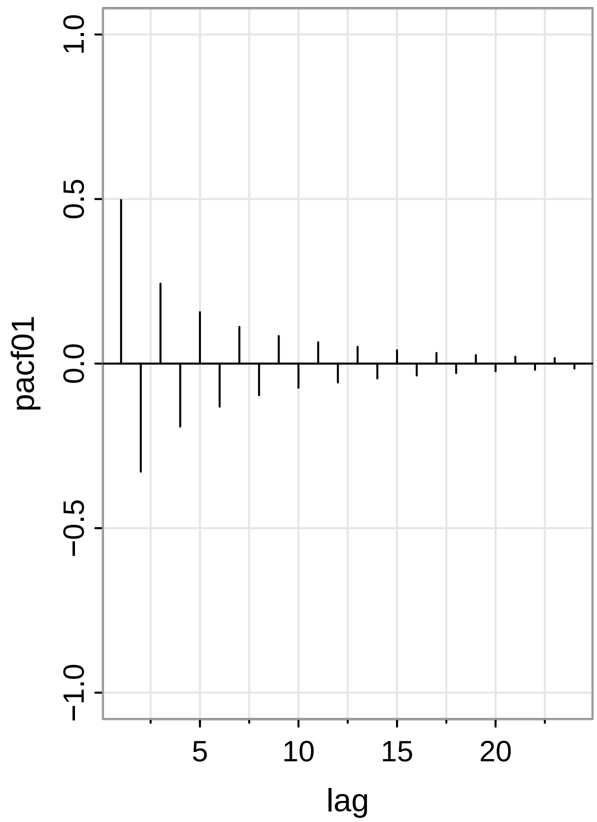



```
acf10 = ARMAacf(ar=phi, ma=0, 24)[-1]
pacf10 = ARMAacf(ar=phi, ma=0, 24, pacf=TRUE)
par(mfrow=1:2)
tsplot(acf10, type="h", xlab="lag", ylim=c(-1,1))
abline(h=0)
tsplot(pacf10, type="h", xlab="lag", ylim=c(-1,1))
abline(h=0)
```




```r
acf01 = ARMAacf(ar=0, ma=theta, 24)[-1]
pacf01 = ARMAacf(ar=0, ma=theta, 24, pacf=TRUE)
par(mfrow=1:2)
tsplot(acf01, type="h", xlab="lag", ylim=c(-1,1))
abline(h=0)
tsplot(pacf01, type="h", xlab="lag", ylim=c(-1,1))
abline(h=0)
```





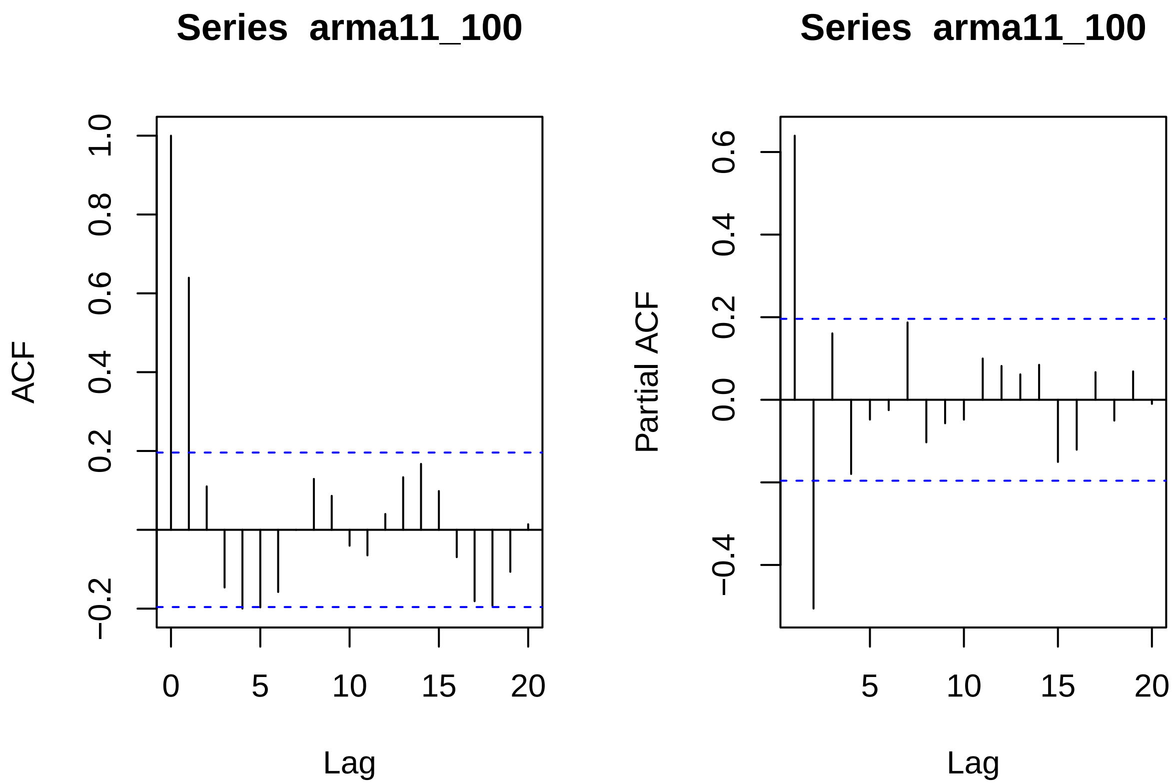
ARMA(1,1) tails off in both ACF and PACF. ARMA(0,1) tails off in ACF but PACF cuts off past lag 1. ARMA(0,1) ACF cuts off past lag 1 and PACF tails off. This matches the behavior with the table 4.1.

Problem 4b

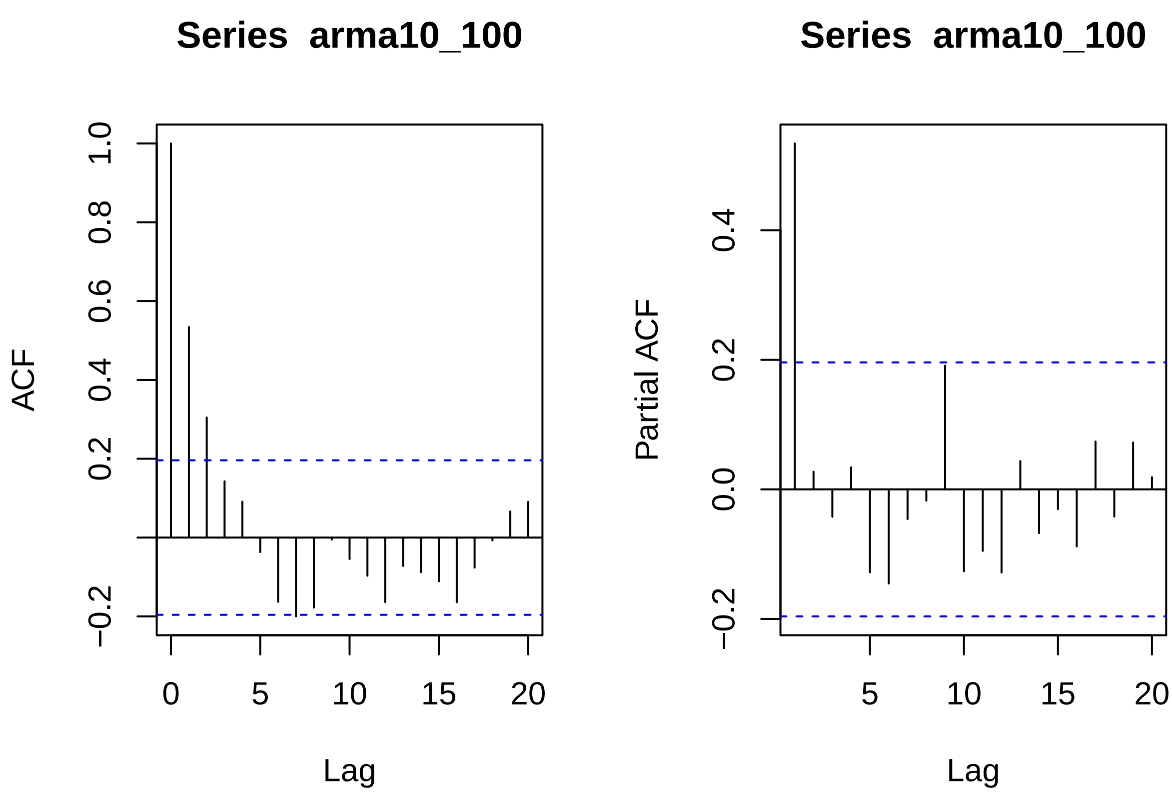
```
n<-100
arma11_100 <- arima.sim(model = list(ar = phi, ma = theta), n = n)
arma10_100 <- arima.sim(model = list(ar = phi, ma = 0), n = n)
arma01_100 <- arima.sim(model = list(ar = 0, ma = theta), n= n)
```

```
## Warning in min(Mod(polyroot(c(1, -model$ar)))): no non-missing arguments to min;
## returning Inf
```

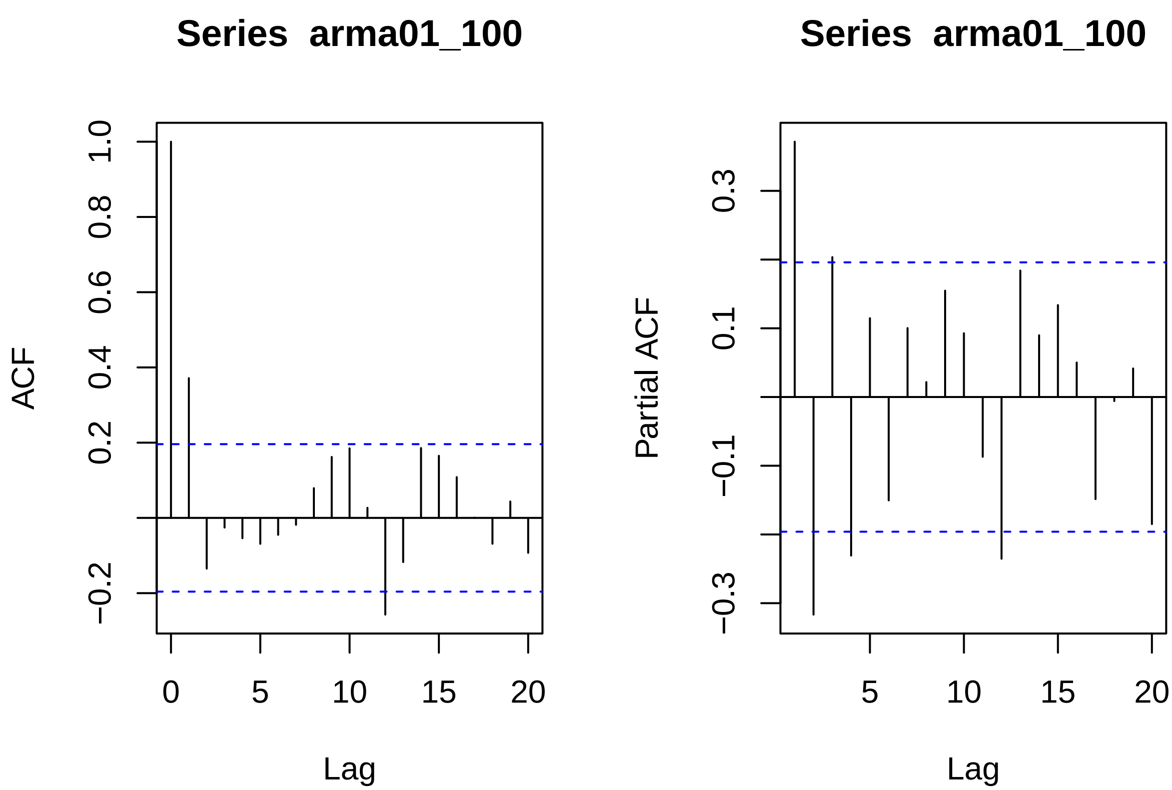
```
par(mfrow=1:2)
acf(arma11_100)
pacf(arma11_100)
```




```
par(mfrow=1:2)
acf(arma10_100)
pacf(arma10_100)
```



```r
par(mfrow=1:2)
acf(arma01_100)
pacf(arma01_100)
```



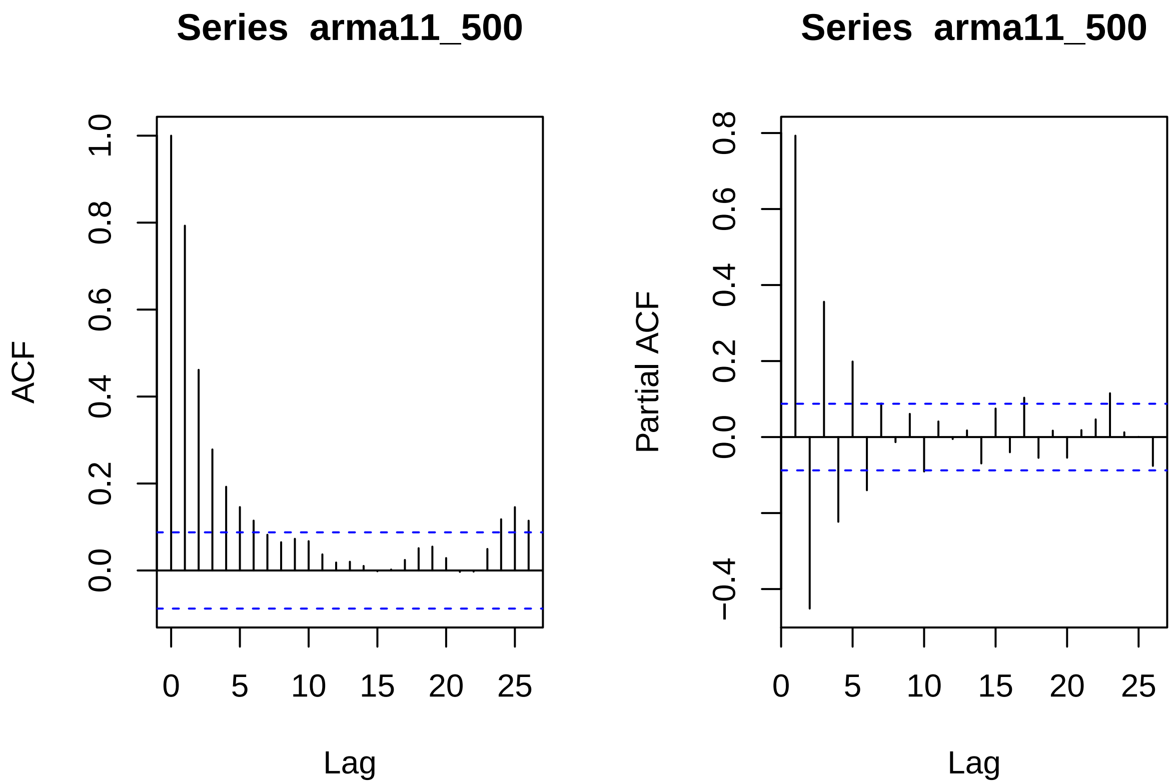
The begavior of the ACF and PACF of the simulated models is different for the AR and MA described in table 4.1. This is because of the small sample size.

Problem 4c

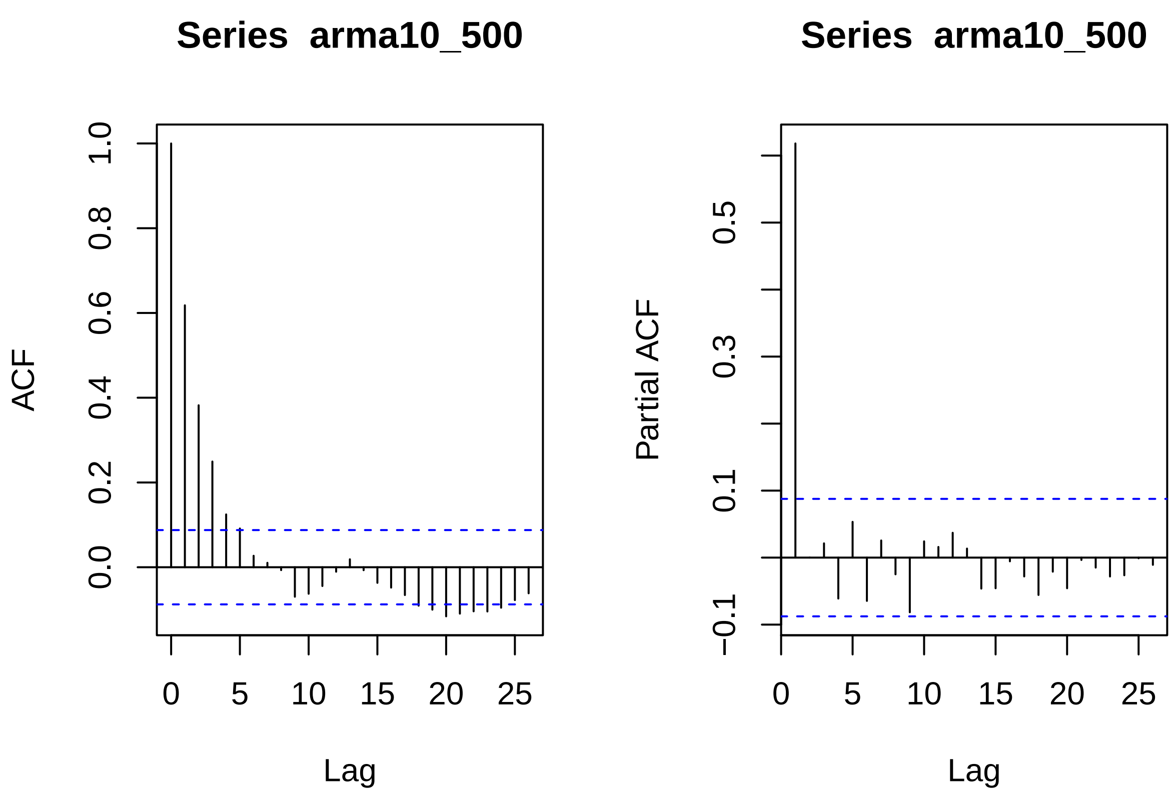
```r
n <- 500
arma11_500 <- arima.sim(model = list(ar = phi, ma = theta), n = n)
arma10_500 <- arima.sim(model = list(ar = phi, ma = 0), n = n)
arma01_500 <- arima.sim(model = list(ar = 0, ma = theta), n= n)
```

```
## Warning in min(Mod(polyroot(c(1, -model$ar)))): no non-missing arguments to min;
## returning Inf
```

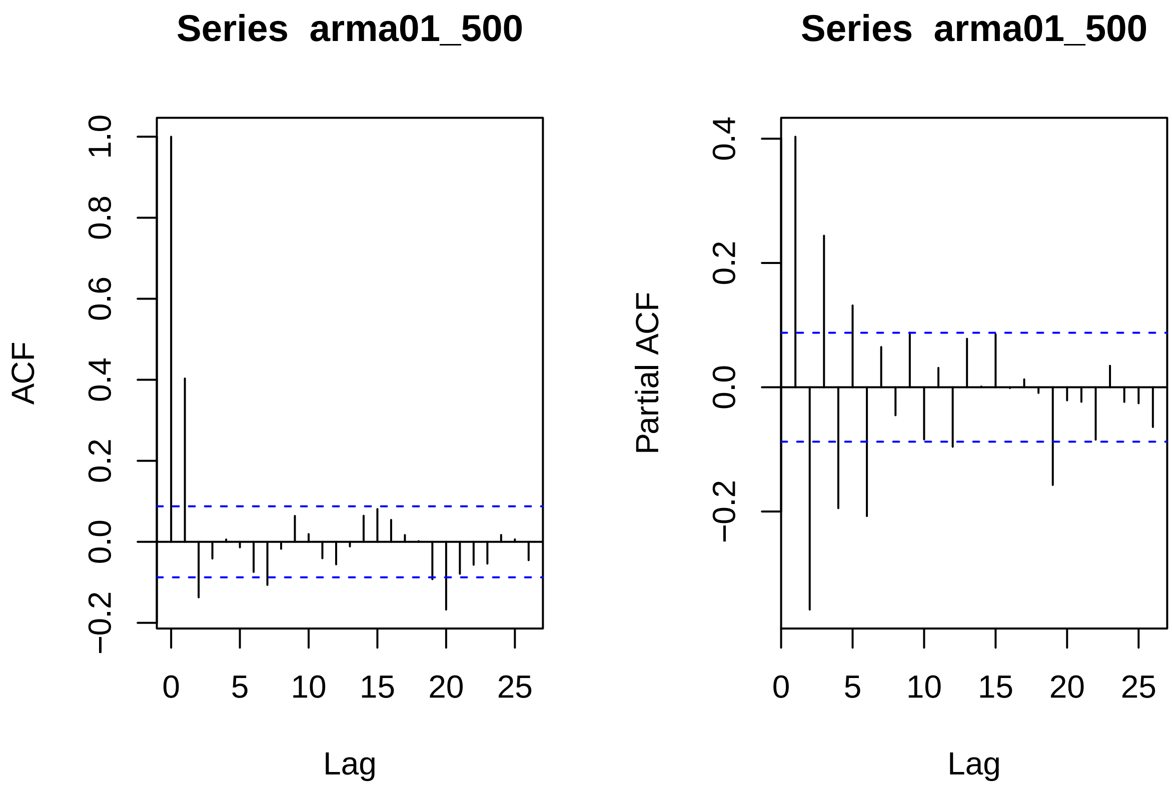
```r
par(mfrow = c(1,2))
acf(arma11_500)
pacf(arma11_500)
```




```
par(mfrow = c(1,2))
acf(arma10_500)
pacf(arma10_500)
```



```r
par(mfrow = c(1,2))
acf(arma01_500)
pacf(arma01_500)
```



We can see that the simulation is still different from the theoretical result, but it is better than sample size of 100. This shows that as the sample size increases, the result will be closer to the theoretical result.