STAT4870 HW2

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2.12

a.) Simulate a series of n = 500 moving average observations as in Example 1.8 and compute the sample ACF, $\rho(h)$, to lag 20. Compare the sample ACF you obtain to the actual ACF $\rho(h)$. [Recall Example 2.18.]

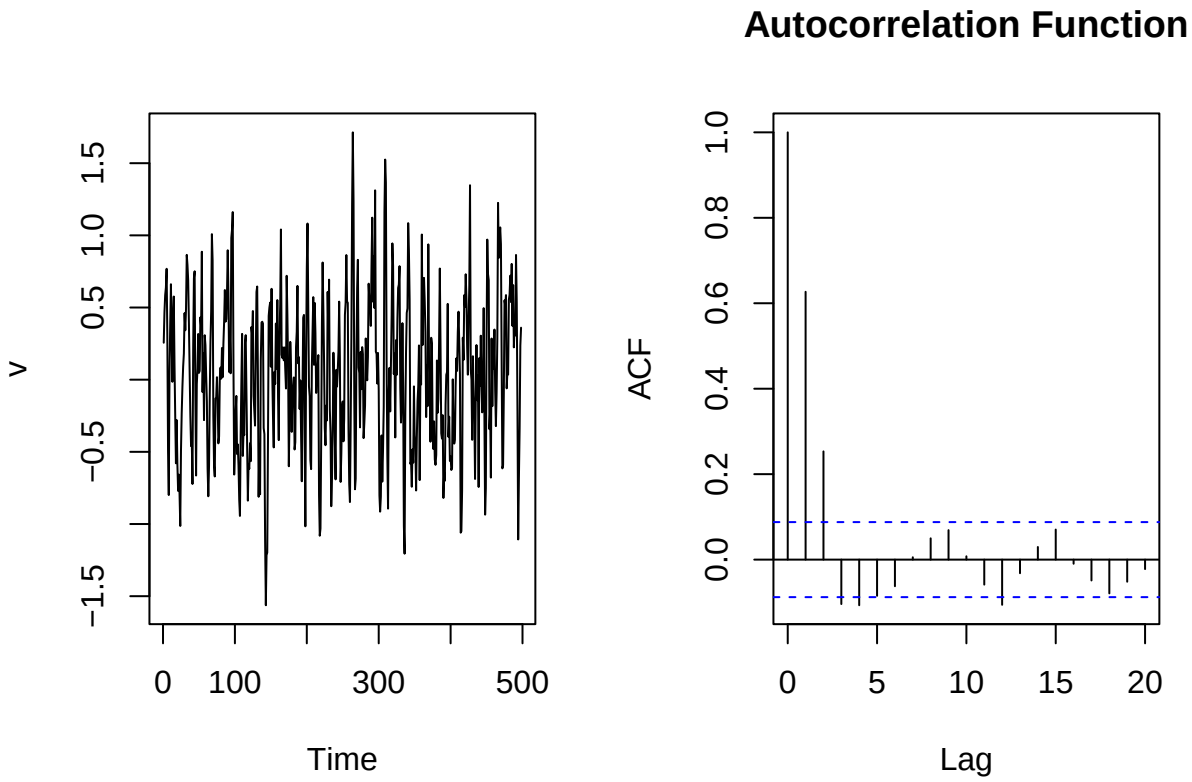
```
library(astsa)
library(xts)
```

```
## Loading required package: zoo
```

```
##
## Attaching package: 'zoo'
```

```
## The following objects are masked from 'package:base':
##
##     as.Date, as.Date.numeric
```

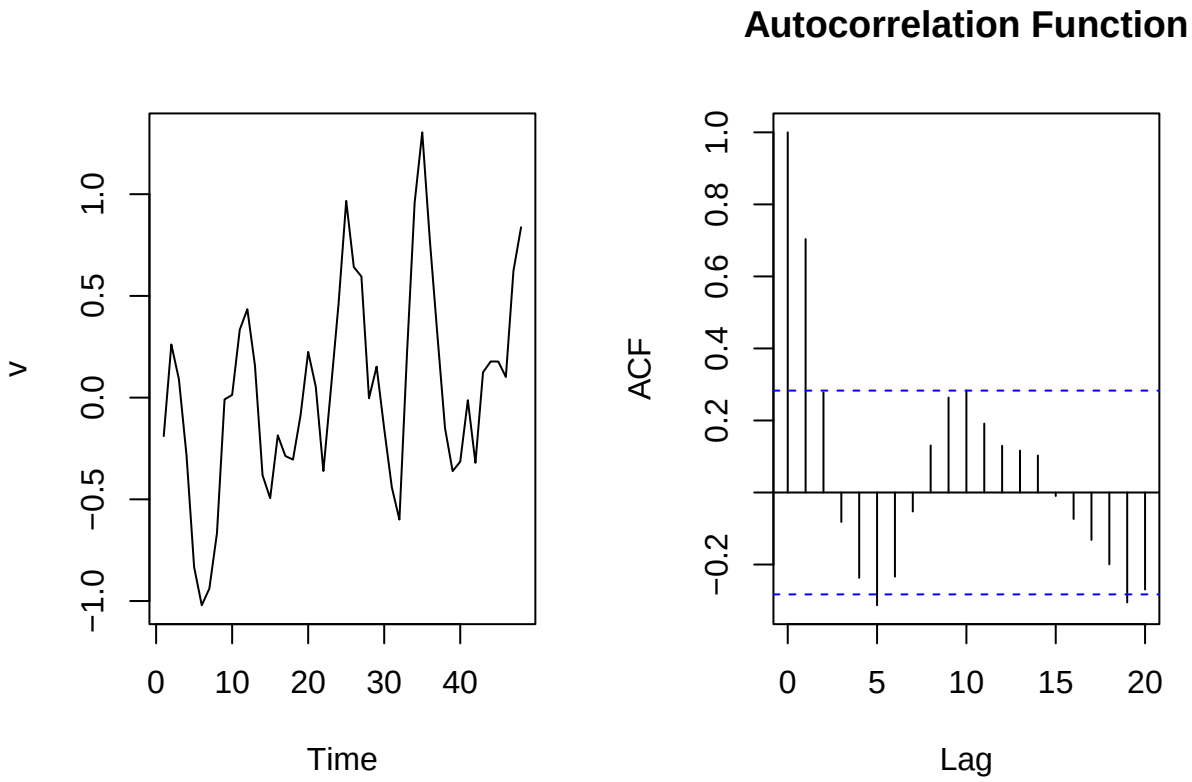
```
set.seed(123)
w<-rnorm(500)
v<-filter(w, sides = 2, filter = rep(1/3,3))
#Remove the first and the last observations since they are NA
v<-v[c(-1,-500)]
par(mfrow = c(1,2))
ts.plot(v)
acf(v, 20, main = "Autocorrelation Function")
```



The autocorrelation function shows that the series is truncated in lag 2, and lag 3 also shows statistically different than zero (probably due to small sampling). The theoretical ACF for a moving average process suggests that the graph be truncated at lag 2, but after lag 2 should have 0 autocorrelation. However, the graph doesn't quite show that autocorrelations be near to 0 after lag 2.

b.) Repeat part (a) using only n = 50. How does n affect the results?

```
w<-rnorm(50)
v<-filter(w, sides = 2, filter = rep(1/3,3))
#Remove the first and the last observations since they are NA
v<-v[c(-1,-50)]
par(mfrow = c(1,2))
ts.plot(v)
acf(v, 20, main = "Autocorrelation Function")
```

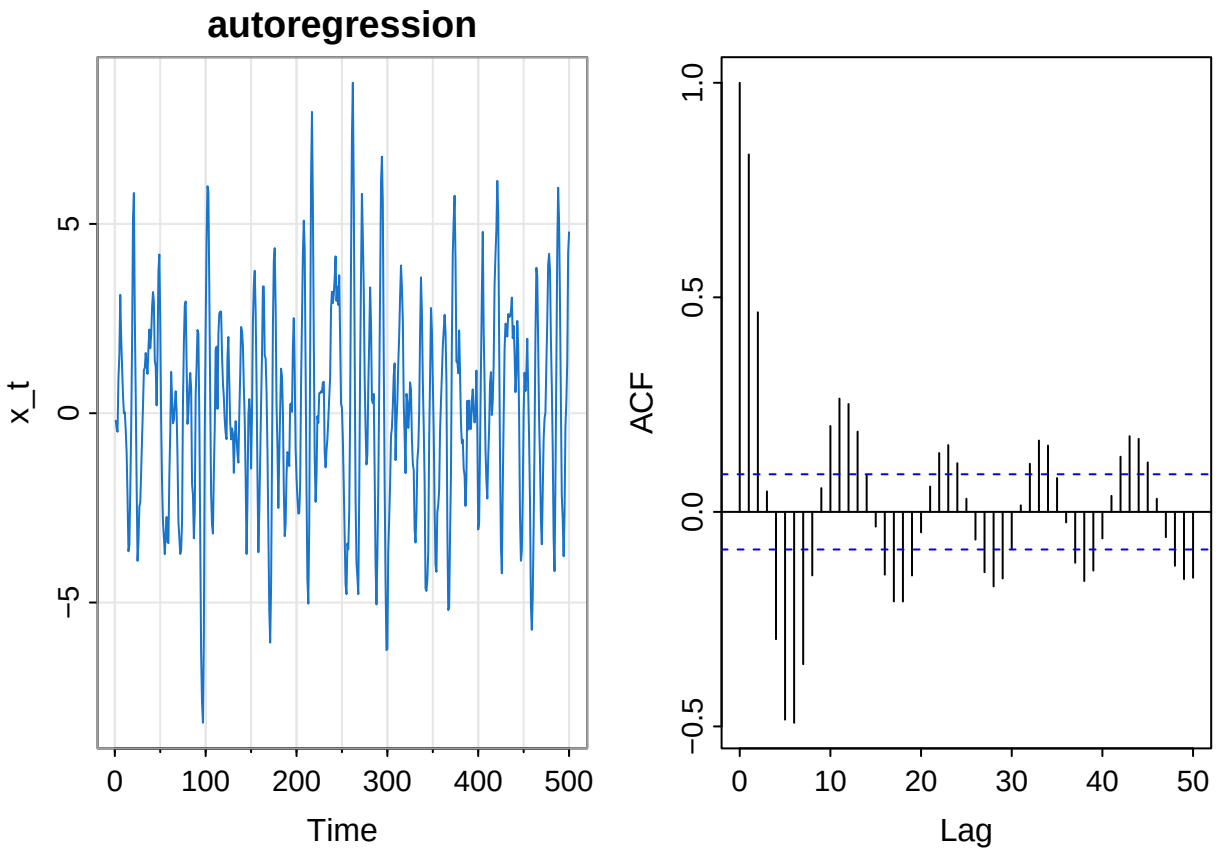


We can see that the autocorrelation is small after lag 4 for n = 500, and autocorrelation is larger and the cyclic pattern of autocorrelation is more apparent for n = 50. We also can see that the graph is farther away from our theoretical result, as the autocorrelations after lag 2 are larger, which is inconsistent with our theoretical result due to small sampling.

2.13

Simulate 500 observations from the AR model specified in Example 1.9 and then plot the sample ACF to lag 50. What does the sample ACF tell you about the approximate cyclic behavior of the data? Hint: Recall Example 2.32.

```
set.seed(123)
w<-rnorm(500 + 50)
x_t<-filter(w, filter = c(1.5, -0.75), method = "recursive")[-(1:50)]
par(mfrow = c(1,2))
tsplot(x_t, main = "autoregression", col = 4)
acf(x_t, 50, main = "Autocorrelation Function")
```



The cyclic behaviors indicate that there are periodicity corresponding to the correlation between values separated by 5 or 6 observations. We can also see that the highest correlation highest correlation all happens at the center of the period, and this indicates that there are some correlation pattern.