

A. Reserves

GAIN

Gain = Actual Profit – Expected Profit

$$\text{Profit} = ({}_tV + G_t - e_t)(1 + i) - q_{x+t}(S_t + E_t) - p_{x+t} \cdot {}_{t+1}V$$

Analysis of Surplus for Insurance:

$$\text{expenses: } (e_t - e'_t)(1 + i) + (E_t - E'_t)q_{x+t}$$

$$\text{interest: } (i' - i)({}_tV + G_t - e'_t)$$

$$\text{mortality: } (q_{x+t} - q'_{x+t})(S_t + E'_t - {}_{t+1}V)$$

Order matters. Use actual experience if you have already accounted for that source.

Formulas would be different for an annuity. Don't memorize these formula, instead understand the process.

THIELE'S DIFFERENTIAL EQN

$$\frac{d}{dt}{}_tV = \delta_t \cdot {}_tV + G_t - e_t - (S_t + E_t - {}_tV)\mu_{[x]+t}$$

Using Euler's Method forward:

$$\frac{d}{dt}{}_tV \approx \frac{1}{h}({}_{t+h}V - {}_tV)$$

Using Euler's Method backward:

$$\frac{d}{dt}{}_tV \approx \frac{1}{h}({}_tV - {}_{t-h}V)$$

Select between forward/backward based on available given information