F. Universal Life Insurance

UNIVERSAL LIFE - NOTATION

Universal life is renewable term insurance with a savings account.

FA − face amount
AV_t − account value at end of year t
CV_t − cash value at end of year t
DB_t − death benefit for year t
ADB_t − additional death benefit for year t
CoI_t − cost of insurance for year t
SC_t − surrender charge for year t
i_t^c − credited interest rate in year t
I_t − amount of credited interest in year t
v_q − discount factor used in CoI calc
q_{x+t-1}^* − mort. rate used in CoI calc for year t
EC_t − expense charge for year t

$$CV_t = \max[AV_t - SC_t, 0]$$

UNIVERSAL LIFE - COI

$$DB_t = AV_t + ADB_t$$

Type A:

$$ADB_t = FA - AV_t$$
$$DB_t = FA$$

Type B:

$$ADB_t = FA$$
$$DB_t = FA + AV_t$$

$$CoI_t = ADB_t \, A_{\overline{x+t-1:\overline{1}|}}^{\,1} = ADB_t \cdot v_q \, q_{x+t-1}^*$$

Type A:

$$CoI_t^A =$$

$$\left[FA - \left(AV_{t-1} + P_t - EC_t - CoI_t^A\right)\left(1 + i_t^c\right)\right] v_q \, q_{x+t-1}^*$$

$$CoI_t^A = \frac{\left[FA - \left(AV_{t-1} + P_t - EC_t\right)\left(1 + i_t^c\right)\right] v_q \, q_{x+t-1}^*}{1 - v_q \, q_{x+t-1}^* \left(1 + i_t^c\right)}$$

Type B:

$$CoI_t^B = FA \cdot v_q \, q_{x+t-1}^*$$

UNIVERSAL LIFE - ROLL FORWARD

Account Value Roll Forward:

Starting AV (AV_{t-1})

+ Premium (P_t)

− Expense Charge (EC_t)

− Mortality Charge (CoI_t)

+ Credited Interest (I_t)

= Ending Account Value (AV_t)

$$AV_t = \left(AV_{t-1} + P_t - EC_t - CoI_t\right)\left(1 + i_t^c\right)$$

$$AV_t^A = \frac{\left(AV_{t-1} + P_t - EC_t - FA \, v_q \, q_{x+t-1}^*\right)\left(1 + i_t^c\right)}{1 - v_q \, q_{x+t-1}^* \left(1 + i_t^c\right)}$$

$AV_t^B =$ same as numerator for AV_t^A

UNIVERSAL LIFE - ANNUAL PROFIT

The profit for year 0 is the negative of the expenses incurred at time 0.

Annual Profit =

Previous Rsv (usually AV)

+ Premium Collected

− Expenses Incurred

+ Interest Earned on
    Prev Rsv plus Premium less Expenses

− Expected Cost of Benefits

− Expected Surrender Benefits

− Expected Cost of Ending Rsv

CORRIDOR FACTORS

To qualify as life insurance the death benefit must be at least a certain multiple (γ_t for year t) of the account value.

$$ADB_t^c = (\gamma_t - 1) AV_t$$

$$ADB_t^f = DB_t - AV_t$$

$$ADB_t = \max[ADB_t^c, ADB_t^f]$$

$$CoI_t = ADB_t \cdot v_q \, q_{x+t-1}^*$$

Alternatively:

$$CoI_t^c = (\gamma_t - 1) \, AV_t \, v_q q_{x+t-1}^*$$

$$CoI_t^f = (DB_t - AV_t) \, v_q q_{x+t-1}^*$$

$$CoI_t = \max(CoI_t^c, CoI_t^f)$$

If you just need the AV:

$$AV_t = \min(AV_t^c, AV_t^f)$$

NO-LAPSE GUARANTEE

If EPV of the guarantee is greater than the AV, then the insurer should hold a reserve for the difference.

$$_tV^{nlg} = \max(\text{EPV-NLG}_t - AV_t, 0)$$

$$_tV = AV_t + {}_tV^{nlg}$$

RETROSPECTIVE AND CREATION

If the premium, expense charges and credited interest rate are level:

Type A use retrospective formula:

$$AV_t = \text{AAVPP} - \text{AAVPB} - \text{AAVPEC}$$

Type B use reserve creation:

$$AV_t = \sum_{k=0}^{t-1} \left(P - FAv q_{x+k} - EC_{k+1}\right)\left(1 + i\right)^{t-k}$$