Expected Values

CS231

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Probability Axioms

- Let E be an event in a sample space S. The probability of the complement of E is:
$$P(\bar{E}) = 1 - P(E)$$
- The probability of the union of two events E_1 and E_2 is:
$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

Example

- If you choose a number between 1 and 100, what is the probability that it is divisible by 2 or 5 or both?
- Let n be the number chosen
  - $p(2|n) = 50/100$ (all the even numbers)
  - $p(5|n) = 20/100$
  - $p(2|n)$ and $p(5|n) = p(10|n) = 10/100$
  - $p(2|n)$ or $p(5|n) = p(2|n) + p(5|n) - p(10|n)$

$$= 50/100 + 20/100 - 10/100$$

$$= 3/5$$

When is gambling worth it?

- This is a *statistical* analysis, not a moral/ethical discussion

- What if you gamble \$1, and have a ½ probability to win \$10?
- What if you gamble \$1 and have a 1/100 probability to win \$10?
- One way to determine if gambling is worth it:
  - probability of winning * payout $\geq$ amount spent per play

Expected Value

- The expected value of a process with outcomes of values $a_1, a_2, \ldots, a_n$ which occur with probabilities $p_1, p_2, \ldots, p_n$ is:

$$\sum_{i=1}^{n} a_i p_i$$

Expected values of gambling

- Gamble \$1, and have a ½ probability to win \$10
  - $(10-1)*0.5+(-1)*0.5 = 4$
- Gamble \$1 and have a 1/100 probability to win \$10?
  - $(10-1)*0.01+(-1)*0.99 = -0.9$

- Another way to determine if gambling is worth it: Expected value > 0

When is lotto worth it?

- In many older lotto games (Pick-6) you have to choose 6 numbers from 1 to 48
  - Total possible choices (order does not matter) are C(48,6) = 12,271,512
  - Total possible winning numbers is C(6,6) = 1
  - Probability of winning is 0.0000000814
    - Or 1 in 12.3 million

- If you invest $1 per ticket, it is only statistically worth it if the payout is > $12.3 million

Powerball lottery

- Modern powerball lottery: you pick 5 numbers from 1-55
  - Total possibilities: C(55,5) = 3,478,761
- You then pick one number from 1-42 (the powerball)
  - Total possibilities: C(42,1) = 42
- You need to do both - apply the product rule,
  - Total possibilities are 3,478,761* 42 = 146,107,962
- The probability for the jackpot is about 1 in 146 million
- If you count in the other (sub)prizes, then you will “break even” if the jackpot is $121M