

CMSC 231: Discrete Mathematics
Practice Exam #1
Fall 2017

- This review exam has *not* been tested for length. The real exam might be shorter or longer.
- You will be able to bring one standard sheet of US-letter-sized paper with notes. The notes may appear on both sides of the sheet.
- No calculators or other electronic devices will be allowed on the exam.
- Several questions ask you to write negations. Simply writing a “not” on the outside of a statement does not qualify as a correct answer; you must use De Morgan’s laws to propagate the negation.

Definitions:

- \mathbb{Z} is the set of all integers
- If S is a set, then S^+ is the members of S that are greater than 0.
- A *function* F from a set A to a set B is a relation with domain A and co-domain B that satisfies the following two properties:
 1. For every element x in A , there is an element y in B such that $(x, y) \in F$.
 2. For all elements x in A and y and z in B , if $(x, y) \in F$ and $(x, z) \in F$, then $y = z$.

1. For each pair of sets below, state whether they are equal or not. If you wish, you may include a short explanation of why you think your answer is correct, but a correct answer with no explanation still receives full credit.

(a) $\{x \in \mathbb{Z} \mid (1 < x < 10) \wedge (x \text{ is prime})\}$
 $\{2, 3, 5, 7\}$

(b) $\{1, 2, 3\}$
 $\{x \in \mathbb{Z} \mid 0 < x^2 < 10\}$

(c) $\{\{1\}, \{2\}\}$
 $\{2, 1\}$

(d) Assume $B = \{1, 2, 3\}$ and $C =$ the set of all possible sets of integers.
 $\{x \in C \mid x \subseteq B\}$
 $\{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

(e) Assume $M = \{1, 2, 3\}$ and $N = \{4, 5\}$.
 $\{(x, y) \in (M \times N) \mid (2 < x) \wedge (2 < y)\}$
 $\{(4, 3), (5, 3)\}$

2. Each problem below ends with a boxed statement. Indicate whether the boxed statement is true or false. If you wish, you may include a short explanation of why you think your answer is correct, but a correct answer with no explanation still receives full credit.

You may assume the following definitions:

$$R = \{1, 2, 3, 4\} \quad S = \{-2, -1, 0, 1, 2\} \quad T = \{1, 3, 5, 7, 9\}$$

(a) Assume $U = \{(x, y) \in (R \times S) \mid x + y = 0\}$. $\boxed{2 \ U \ -2}$

(b) Assume U as above. $\boxed{3 \ U \ -3}$

(c) Assume U as above. $\boxed{2 \ U \ 2}$

(d) Assume U as above. $\boxed{U \text{ is a function from } R \text{ to } S}$

(e) Assume $V = \{(x, y) \in (S \times T) \mid 2x = y\}$. $\boxed{0 \ V \ 1}$

(f) Assume V as above. $\boxed{1 \ V \ 1}$

(g) Assume V as above. $\boxed{V \text{ is a function from } S \text{ to } T}$

(h) Assume $W = \{(x, y) \in (R \times T) \mid 2x - 1 = y\}$. $\boxed{2 \ W \ 3}$

(i) Assume W as above. $\boxed{4 \ W \ 9}$

(j) Assume W as above. $\boxed{W \text{ is a function from } R \text{ to } T}$

3. Assuming the definitions below, write out each sentence using symbolic notation, with no words.

$$p = \text{"It's raining"} \quad q = \text{"It's Wednesday"} \quad r = \text{"It's October"}$$

(a) It's a rainy Wednesday.

(b) It's October, but it's neither Wednesday nor raining out.

(c) It's raining but not October.

4. Use De Morgan's laws to write negations for the following statements. Your answers should be complete English sentences.

(a) I have math class and computer science class today.

(b) I love to eat cookies but not peanut butter.

(c) I am wearing red or green socks today.

5. Give the *converse*, *inverse*, *contrapositive* and *negation* (using De Morgan's laws) of the following:

$$(p \wedge \sim q) \rightarrow r$$

(a) Converse:

(b) Inverse:

(c) Contrapositive:

(d) Negation:

6. Prove the following logical equivalence, using the rules of Boolean algebra (a truth table is not an acceptable answer):

$$(p \vee q) \wedge \sim p \equiv (q \wedge \sim p)$$

7. Draw a digital circuit corresponding to the Boolean formula below:

$$(\sim p \wedge q) \vee (p \wedge (q \vee r))$$

8. (a) Add the following binary numbers. Write the result as a 4-bit number, dropping any bits that overflow.

$$\begin{array}{r} 1\ 0\ 1\ 1 \\ + 0\ 1\ 1\ 0 \\ \hline \end{array}$$

- (b) Rewrite the addition problem above in decimal notation, interpreting any number greater than 7_{10} as a two's-complement negative number. (This is 4-bit two's-complement.) If the sum in decimal does not match the sum in binary, explain why.

9. For each of the following statements, rewrite the statement formally (using \forall and \exists), and write the negation of the statement both in English and in formal mathematics. You may assume these definitions:

C = the set of all Bryn Mawr classes
 $\text{len}(x)$ = the meeting length of a class x , in hours
 $\text{cs}(x)$ = class x is a computer science class
 $\text{math}(x)$ = class x is a math class
 $\text{prereq}(x, y)$ = class x is a prerequisite for class y

- (a) All classes at Bryn Mawr meet for at least 2 hours.

Written formally:

Negated formally:

Negated informally:

- (b) Every class at Bryn Mawr has at least one prerequisite class at Bryn Mawr.

Written formally:

Negated formally:

Negated informally:

- (c) One computer science class at Bryn Mawr has every math class as prerequisites.

Written formally:

Negated formally:

Negated informally:

- (d) There is a math class at Bryn Mawr that is the prerequisite for all computer science courses.

Written formally:

Negated formally:

Negated informally:

Definition: If r is a real number, then r is *rational* iff:

$$\exists(a \in \mathbb{Z}), \exists(b \in \mathbb{Z}), (r = \frac{a}{b}) \wedge (b \neq 0)$$

10. Prove: The product of any two rational numbers is a rational number.

Definition: If n is an integer, then n is *odd* iff:

$$\exists(k \in \mathbb{Z}), n = 2k + 1$$

Definition: If x is a real number, then the *ceiling* of x , written $\lceil x \rceil$, is defined as follows:

$$\lceil x \rceil = n \quad \text{means} \quad n - 1 < x \leq n$$

11. Prove: For all odd integers n , $\lceil n/2 \rceil = (n + 1)/2$.