

# The Simply Typed Lambda-Calculus

$\tau$	$::=$	Types
	$\mathbb{Z}$	Integer type
	$\tau_1 \rightarrow \tau_2$	Function
$e$	$::=$	Expressions
	$x$	Variable
	$e_1 e_2$	Application
	$\lambda x:\tau. e$	Abstraction
$\Gamma$	$::=$	Contexts
	$\bullet$	Empty context
	$\Gamma, x:\tau$	Variable binding

$\Gamma \vdash e : \tau$  Expression typing

$$\begin{array}{c}
 \frac{x:\tau \in \Gamma}{\Gamma \vdash x : \tau} \quad \text{E\_VAR} \\
 \\
 \frac{\Gamma \vdash e_1 : \tau_1 \rightarrow \tau_2 \quad \Gamma \vdash e_2 : \tau_1}{\Gamma \vdash e_1 e_2 : \tau_2} \quad \text{E\_APP} \\
 \\
 \frac{\Gamma, x:\tau_1 \vdash e : \tau_2}{\Gamma \vdash \lambda x:\tau_1. e : \tau_1 \rightarrow \tau_2} \quad \text{E\_ABS}
 \end{array}$$

Example:  $\bullet \vdash \lambda x:\mathbb{Z}. x + 1 : \mathbb{Z} \rightarrow \mathbb{Z}$

Example:  $\bullet \vdash \lambda x:\mathbb{Z} \rightarrow \mathbb{Z}. \lambda y:\mathbb{Z}. x y : \mathbb{Z}$