CONJ: Dual-Pair-Based 1+2 Decomposition with Residual Invariance

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What problem does this address? (pipeline crosstalk) Many image/video pipelines apply a *1+2* structure: a 3D color signal is split into a *1D tone axis* (luma/lightness-like) and a *2D chroma residual*. In practice, a tone nonlinearity (gamma, tone mapping, roll-off, etc.) is often applied on the 1D axis *after* converting to a nonlinear representation, or *before* a decomposition defined in linear light. This ordering mismatch commonly produces unintended changes in the chroma residual (*chroma shift / crosstalk*).

Proposed design rule (implementation-friendly statement) Pick: (i) a *tone axis* vector u (e.g., achromatic direction), and (ii) a *readout* $\ell(x)$ that extracts the tone coordinate (e.g., luma-like dot product), and enforce the single consistency condition $\ell(u) = 1$ (a normalized dual pair).
Then define the residual by

$$P(x) := x - \ell(x)u \quad \text{(hence } P(x) \in \ker \ell\text{)}.$$

For any scalar tone curve $\phi : \mathbb{R} \to \mathbb{R}$, apply it *only* to the tone coordinate:

$$T_\phi(x) := \phi(\ell(x))\, u + P(x).$$

Guarantee (no crosstalk):

$$P(T_\phi(x)) = P(x) \qquad \text{for any } \phi.$$

Engineering interpretation: once you fix u and ℓ and normalize $\ell(u) = 1$, you can change the tone curve freely without moving the chroma residual.

How we quantify crosstalk (metrics used in this 1-pager).
We report two simple, implementation-facing metrics computed on a sample set D in a chosen working space: (i) mean squared change of the residual, $C(F;D) = \frac{1}{|D|}\sum_{x\in D} \|P(F(x)) - P(x)\|_2^2$, and (ii) residual covariance change, $\Delta\Sigma(F) = \|\Sigma(P(F(D))) - \Sigma(P(D))\|_F$ (Frobenius norm). Both are zero if the residual is preserved exactly.

Why two metrics (difference in what they detect).
The two metrics are *complementary*: $C(F;D)$ measures the *pointwise* mean-squared change of the residual ($\Delta r(x) := P(F(x)) - P(x)$), whereas $\Delta\Sigma(F)$ measures the change in *second-order* chroma statistics (the residual covariance) over the set D. Both become zero under exact residual invariance, but $\Delta\Sigma(F) = 0$ does *not* generally imply $C(F;D) = 0$.

Numerical evidence (example summary).

Setting	Conventional pipelines	Proposed (CONJ)
Uniform random inputs	$\sim 10^{-2}$	rounding-error level (FP64)
Real photographs	$\sim 10^{-3}$	rounding-error level (FP64)

Notes on baselines. Examples include $Y'CbCr$-style processing after gamma correction and lightness operations in CIELAB, where the effective 1+2 decomposition is not preserved under subsequent nonlinear steps.

Key points (for a quick technical read).

- **Bottom line:** a design rule that keeps the chroma residual unchanged when applying tone nonlinearities only along a chosen tone axis.
- **Mechanism:** the identity $P(T_\phi(x)) = P(x)$, which holds for *any* scalar tone curve ϕ.
- **Practical instantiation:** examples of choosing u (tone axis) and ℓ (tone readout).
- **How to read results:** the gap to conventional orderings, and how $C(F;D)$ / $\Delta\Sigma(F)$ reflect residual drift.

Links.
PDF: `https://github.com/goldkiss2010-ai/conj-transform-paper/tree/master/paper`
Code: `https://github.com/goldkiss2010-ai/conj-transform-paper`

All numbers are reproducible from the accompanying code and data (see links).