$\boxed{T5}$

$\xi \sim R[\theta, 2\theta]$ $\quad$ n – выборка

а) ОММ:

$$\xi \sim p(x) = \frac{1}{2\theta-\theta}\{[\theta,2\theta]\} = \frac{1}{\theta}\{[\theta,2\theta]\}$$

$$\alpha_1 = M[\xi] = \int_\theta^{2\theta} \frac{1}{\theta} x\,dx = \frac{1}{\theta}\frac{x^2}{2}\Big|_\theta^{2\theta} = \frac{1}{2\theta}(4\theta^2-\theta^2) = \frac{3\theta}{2}$$

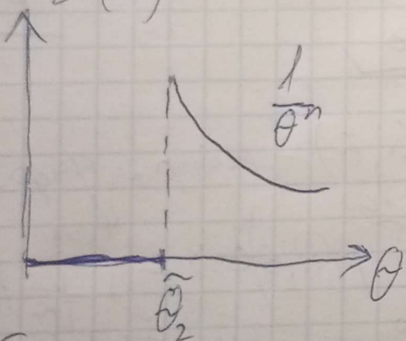
$$\hat{\alpha}_1 = \frac{1}{n}\sum_{i=1}^n x_i = \alpha_1 = \frac{3\theta}{2} = \bar{x} \Rightarrow \tilde{\theta}_1 = \frac{2}{3}\bar{x}$$

ОМП:

$$L(\vec{x}_n, \theta) = \frac{1}{\theta^n}\{[\theta,2\theta]\}$$

$L(\theta)$ $\quad$ $0 \le x \le 2\theta \Rightarrow x_{max} = max(x_i) \le 2\theta$

$$\tilde{\theta}_2 = \frac{x_{max}}{2}$$



б) 1) $M[\tilde{\theta}_1] = M[\frac{2}{3}\bar{x}] = \frac{2}{3}M[\xi] = \frac{2}{3}\cdot\frac{3\theta}{2} = \theta \Rightarrow$ Несмещённ

2) $M[\tilde{\theta}_2] = M[\frac{x_{max}}{2}] = \frac{1}{2}M[x_{max}]$

$\Psi(x) = (F(x))^n = \left(\frac{x}{\theta}-1\right)^n$ – з-н распр. max $\qquad$ $F(x) = \int_\theta^x \frac{1}{\theta}dx = \frac{x}{\theta}-1$

$$f(x) = \frac{n}{\theta}\left(\frac{x}{\theta}-1\right)^{n-1}$$

$$M[x_{max}] = \int_\theta^{2\theta} \frac{n}{\theta}\left(\frac{x}{\theta}-1\right)^{n-1} x\,dx = \left[\begin{matrix} u = x & du = dx \\ dv = \left(\frac{x}{\theta}-1\right)^{n-1}dx & \frac{\theta\left(\frac{x}{\theta}-1\right)^n}{n} = v \end{matrix}\right]$$

$$= \frac{n}{\theta}\cdot\frac{\theta}{n}\cdot x\left(\frac{x}{\theta}-1\right)^n\Big|_\theta^{2\theta} - \frac{n}{\theta}\int_\theta^{2\theta}\frac{\theta}{n}\left(\frac{x}{\theta}-1\right)^n dx =$$

$$= 2\theta - \theta\int_\theta^{2\theta}\left(\frac{x}{\theta}-1\right)^n d\left(\frac{x}{\theta}-1\right) = 2\theta - \frac{\theta\cdot\left(\frac{x}{\theta}-1\right)^{n+1}}{n+1}\Big|_\theta^{2\theta} =$$

$$= 2\theta - \frac{\theta}{n+1} = \frac{2\theta n + 2\theta - \theta}{n+1} = \frac{2n+1}{n+1}\theta$$

$$M[\tilde{\theta}_2] = \frac{(n+\frac{1}{2})\theta}{n+1} \Rightarrow \text{Смещ.} \Rightarrow \tilde{\theta}_2' = \frac{1}{2}\frac{n+1}{n+\frac{1}{2}} x_{max}$$

1) $$D[\tilde{\theta}_1] = \frac{4}{9} D\left[\frac{1}{n}\sum_{i=1}^{n} x_i\right] = \frac{4}{9}\cdot\frac{1}{n} D[\xi] =$$

$$= \frac{4}{9n}\left(M[\xi^2] - M^2[\xi]\right) = \frac{4}{9n}\left(\frac{1}{\theta}\frac{x^3}{3}\Big|_{\theta}^{2\theta} - \theta^2\right) =$$

$$= \frac{4}{9n}\left(\frac{7}{3}\theta^2 - \frac{9}{4}\theta^2\right) = \frac{4\theta^2}{9\cdot 12 n} = \frac{\theta^2}{27n} \xrightarrow[n\to\infty]{} 0 \Rightarrow$$ Оценка состоятел. по д. у.

2) $$D[\tilde{\theta}_2'] = M[\tilde{\theta}_2'^2] - M[\tilde{\theta}_2']^2$$

$$M[\tilde{\theta}_2'^2] = \left(\frac{n+1}{2n+1}\right)^2 \int_{\theta}^{2\theta} x^2 \frac{n}{\theta}\left(\frac{x-\theta}{\theta}\right)^{n-1} dx = \frac{(n+1)^2}{(2n+1)^2}\cdot\left(4\theta - \frac{4\theta^2}{n+1} + \frac{2\theta^2}{(n+1)(n+2)}\right)$$

$$D[\tilde{\theta}_2'] = M[\tilde{\theta}_2'^2] - \theta^2 = \frac{n\theta^2}{(2n+1)^2(n+2)} \xrightarrow[n\to\infty]{} 0 \Rightarrow$$ по д. у. состоят.

c) $$D[\tilde{\theta}_1] = \frac{16\theta^2}{27n} \sim \frac{16}{27n},\quad n\to\infty$$

$$D[\tilde{\theta}_2] = \frac{n\theta^2}{(2n+1)^2(n+1)} \sim \frac{1}{4n^2},\quad n\to\infty$$

$\Rightarrow \tilde{\theta}_2$ – ~~эфф~~ еективнее

d) $$\exists f(\theta, \vec{x}_n) = \frac{x_{max}}{\theta} = \eta$$

$$\eta \sim P(y) = P\left(y > \frac{x_{max}}{\theta}\right) = P(\theta y > x_{max}) = F_{x_{max}}(\theta y) =$$

$$= R^n(\theta y)[\theta, 2\theta] = \left(\frac{\theta y - \theta}{\theta}\right)^n = (y-1)^n \Rightarrow$$

$$\theta \le \theta y \le 2\theta \Rightarrow 1 \le y \le 2$$

$\Rightarrow \varphi(y) = n(y-1)^{n-1} \ [1,2]$

$t_1 = \eta_{\frac{1-\beta}{2}} = \eta_{0{,}025} : \int_1^{t_1} \varphi(y)dy = \frac{1-\beta}{2} \Rightarrow (t_1-1)^n = \frac{1-\beta}{2} \Rightarrow$

$t_1 = 1 + \sqrt[n]{\frac{1-\beta}{2}}$

$t_2 = \eta_{0{,}975} : \int_1^{t_2} \varphi(y)dy = \frac{1+\beta}{2} \Rightarrow t_2 = 1 + \sqrt[n]{\frac{1+\beta}{2}}$

$1 + \sqrt[n]{\frac{1-\beta}{2}} < \frac{x_{max}}{\theta} < 1 + \sqrt[n]{\frac{1+\beta}{2}}$

$\frac{x_{max}}{1+\sqrt[n]{\frac{1+\beta}{2}}} < \theta < \frac{x_{max}}{1+\sqrt[n]{\frac{1-\beta}{2}}}$ ← точный доверительный интервал

e) $\int_\theta^{2\theta} \frac{\partial}{\partial\theta} \frac{1}{\theta} dx = \int_\theta^{2\theta} -\frac{1}{\theta^2} dx = -\frac{1}{\theta} \neq 0 \Rightarrow$ модель нерегулярн.

$\Rightarrow$ не можем строить асимпт. довер. интервал для ОМП

ОММ: $\tilde\theta = \frac{2}{3}\bar{x} \Rightarrow g(\tilde\alpha) = \frac{2}{3}\tilde\alpha_1$ $\quad g(\alpha) = \frac{2}{3}\alpha_1$

$\sigma(\alpha) = \sqrt{\nabla^T g(\alpha) K \nabla g(\alpha)} = \sqrt{\frac{2}{3}(\alpha_2 - \alpha_1^2)\frac{2}{3}}$

$\sqrt{n}\,\frac{g(\tilde\alpha) - g(\alpha)}{\sigma(\alpha)} \sim N(0,1)$

$\sqrt{n}\,\frac{\frac{2}{3}\tilde\alpha_1 - \frac{2}{3}\alpha_1}{\sqrt{\frac{2}{3}(\tilde\alpha_2 - \tilde\alpha_1^2)\frac{2}{3}}} = \sqrt{n}\,\frac{\tilde\alpha_1 - \alpha_1}{\sqrt{\tilde\alpha_2 - \tilde\alpha_1^2}} \sim N(0,1)$

$$-1{,}96 < \sqrt{n}\,\frac{\tilde{\alpha}_1 - \frac{3}{2}\theta}{\sqrt{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}} < 1{,}96$$

$$\boxed{\frac{2}{3}\left(+1{,}96\sqrt{\frac{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}{n}} + \tilde{\alpha}_1\right) < \theta < \frac{2}{3}\left(-1{,}96\left(\sqrt{\frac{\tilde{\alpha}_2 - \tilde{\alpha}_1^2}{n}}\right) + \tilde{\alpha}_1\right)}$$