Т11

$H_0: p_0(x) = \begin{cases} 1, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$ $\quad H_1: p_1(x) = \begin{cases} \frac{e^{1-x}}{e-1}, & x \in (0,1) \\ 0, & x \notin (0,1) \end{cases}$

а) $\underline{n=1}$ $\quad \ell = \frac{L_1}{L_0} = \frac{p_1(x)}{p_0(x)} = \frac{e^{1-x}}{e-1} \{(0,1)\} \geq C$

$1 - x \geq \ln C(e-1)$; $\quad x \leq 1 - \ln C(e-1) = A$

$G_{кр}: x \leq A$; $P(x \leq A | H_0) = \alpha$, т.е. $\int_0^A p_0(x)dx = \alpha \Rightarrow \underline{A = \alpha}$

$G_{кр}: x \leq \alpha$. $\quad \underline{\underline{\alpha_1 = \alpha}}$ – ошибка I рода

$W = P(x \leq A | H_1) = \int_0^\alpha p_1(x)dx = \int_0^\alpha \frac{e^{1-x}}{e-1}dx = -\frac{e^{1-x}}{e-1}\Big|_0^\alpha =$

$= \frac{e}{e-1}(1 - e^{-\alpha})$; $\quad \underline{W = \frac{e}{e-1}(1-e^{-\alpha})}$ – мощность критерия

$\underline{\underline{\alpha_2 = 1 - \frac{e}{e-1}(1 - e^{-\alpha})}}$ – ошибка II рода

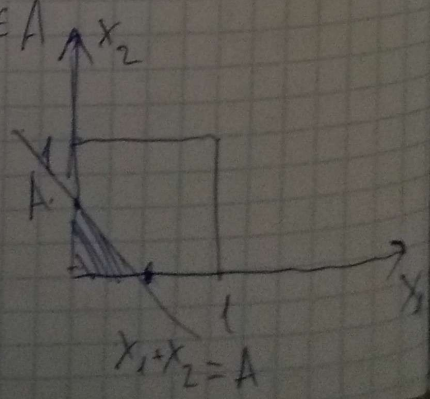
б) $\underline{n=2}$ $\quad \ell = \frac{L_1}{L_0} = \frac{p_1(x_1)\,p_1(x_2)}{p_0(x_1)\,p_0(x_2)} = \frac{e^{1-x_1} \cdot e^{1-x_2}}{(e-1)^2} \geq C$

$e^{-x_1-x_2} \geq \frac{C(e-1)^2}{e^2}$, $\quad -x_1 - x_2 \geq \ln\left(\left(\frac{e-1}{e}\right)^2 C\right) \Rightarrow$

$\Rightarrow G_{кр}: x_1 + x_2 \leq A$

$P(x_1 + x_2 \leq A | H_0) = \alpha$; $\quad I = \iint dx_1\,dx_2 = \alpha$

$P = \frac{1}{2}A^2 = \alpha \Rightarrow A = \sqrt{2\alpha}$



Имеем:

$G_{кр}: x_1 + x_2 \leq \sqrt{2\alpha}$, $\quad \underline{\underline{\alpha_1 = \alpha}}$

$$W = P\left(x_1 + x_2 \le A \mid H_1\right) = \iint\limits_{x_1+x_2\le A} \frac{e^{1-x_1} e^{1-x_2}}{(e-1)^2}\, dx_1 dx_2 =$$

$$= \frac{e^2}{(e-1)^2} \int\limits_0^A dx_1 \int\limits_0^{A-x_1} e^{-x_1} e^{-x_2} dx_2 = \frac{e^2}{(e-1)^2} \int\limits_0^A e^{-x_1}\left(1 - e^{-A+x_1}\right) dx_1 =$$

$$= \frac{e^2}{(e-1)^2}\left(1 - e^{-A} - e^{-A}\cdot A\right); \quad W = \frac{e^2}{(e-1)^2}\left[1 - e^{-A}(1+A)\right]$$

$$\alpha_2 = 1 - \frac{e^2}{(e-1)^2}\left[1 - e^{-A}(1+A)\right]$$

c) Асимптотический критерий: $n = n$

$$l = \frac{L_1}{L_0} = \frac{\prod_{i=1}^{n} p_1(x_i)}{\prod_{i=1}^{n} p_0(x_i)} = \prod_{i=1}^{n} \frac{p_1(x_i)}{p_0(x_i)} \ge C \Rightarrow$$

$$\Rightarrow \sum_{i=1}^{n} \ln\left(\frac{p_1(x_i)}{p_0(x_i)}\right) \ge \ln C; \quad \text{ЦПТ}: \frac{\sum \eta_i - nM_{\eta_i}}{\sqrt{nD\eta_i}} \sim N(0,1)$$

В нашем случае: $\eta_i = \ln \frac{e^{1-x_i}}{e-1} = \ln \frac{e}{e-1} - x_i$

$$H_0: M_{\eta_i} = M\left[\ln\frac{e}{e-1} - x_i\right] = \ln\frac{e}{e-1} - \frac{1}{2}$$

$$D_{\eta_i} = D\left[\ln\frac{e}{e-1} - x_i\right] = D[x_i] = 1/12$$

$$P(\ln L \ge \ln C \mid H_0) = \alpha$$

$$P\left(\sum \eta_i \ge \ln C \mid H_0\right) = P\left(\frac{\sum \eta_i - nM_{\eta_i}}{\sqrt{n\cdot\frac{1}{12}}} \ge A\right), \quad M_{\eta_i} = \ln\left(\frac{e}{e-1}\right) - \frac{1}{2}$$

$$\text{æ} \sim N(0,1)$$

$$P = \int\limits_A^{\infty} \frac{e^{-\frac{x^2}{2}}}{\sqrt{2\pi}} dx = \alpha;$$

$$\frac{\ln C - n\left(\ln\frac{e}{e-1} - \frac{1}{2}\right)}{\sqrt{n/12}} = u \qquad \ln C = n\ln\frac{e}{e-1} - \frac{n}{2} + u\sqrt{\frac{n}{12}}$$

$$\ln L = \sum_{i=1}^{n} \ln \frac{e}{e-1} - \sum_{i=1}^{n} x_i \qquad G_{кр}: \ln \ell \geq \ln C$$

$$-\sum_{i=1}^{n} x_i \geq -\frac{n}{2} + u \cdot \sqrt{\frac{n}{12}} \quad \Big| : (-1)$$

$$G_{кр}: \bar{x} \leq \frac{1}{2} - u \cdot \sqrt{\frac{1}{12 n}} \,, \quad \alpha_1 = \alpha$$

Асимптот. крит. область:

$$W = P(\ln \ell \geq \ln C \mid H_1) \,; \; P\Bigg(x \geq \underbrace{\frac{\ln C - n M_{\eta_i}}{\sqrt{n D_{\eta_i}}}}_{B}\Bigg)$$

$$\sum^{n} \eta_i$$

$$H_1: \; M_{\eta_i} = \ln \frac{e}{e-1} - M[x_i] = \ln \frac{e}{e-1} - \underbrace{\int_0^1 x \frac{e^{1-x}}{e-1} dx}_{\frac{e-2}{e-1}} =$$

$$= \ln \frac{e}{e-1} - \frac{e-2}{e-1}$$

$$D_{\eta_i} = \frac{e^2 - 3e + 1}{(e-1)^2} \qquad W = \int_B^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \,;$$

$$B = \frac{n\left(\frac{e-2}{e-1} - \frac{1}{2}\right) + u \cdot \sqrt{\frac{n}{12}}}{\sqrt{n \cdot \frac{e^2 - 3e + 1}{(e-1)^2}}}$$

$$W \xrightarrow[n \to +\infty]{} \int_{-\infty}^{+\infty} e^{-x^2/2} \cdot \frac{1}{\sqrt{2\pi}} dx = 1 \Rightarrow \text{критерий состоятельности}$$

$$\alpha_2 = 1 - W$$

d) $G_{кр}: X_{min} < C$ $\quad P(X_{min} < C | H_0) = \alpha$

$H_0: \xi \sim R(0,1)$ $\quad H_1: X_{min} \sim 1-(1-F(x))^n$

$1-(1-F(C))^n = \alpha \quad (1-F(C))^n = 1-\alpha \quad F(C) = 1-\sqrt[n]{1-\alpha} = C$

$G_{кр}: X_{min} < 1-\sqrt[n]{1-\alpha}$ $\quad \alpha_1 = \alpha$ (Простая гипотеза H_0)

$W = P(X_{min} < C | H_1)$; $\quad F_1(x) = \int_{-\infty}^{x} p_1(t)dt = \int_0^x \frac{e^{1-t}}{e-1}dt =$

$= \frac{e}{e-1}(1-e^{-x})$, $x \in (0,1)$

$$W = 1-(1-F(C))^n = 1-\left(1+\frac{e^{1-x}}{e-1} - \frac{e}{e-1}\right)^n\Bigg|_C =$$

$$= 1-\left(1+\frac{e^{1-1+\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n = 1-\left(1+\frac{e^{\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n$$

$$W = 1-\left(1+\frac{e^{\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n$$ — мощность критерия

$$\alpha_2 = 1-W = \left(1+\frac{e^{\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n$$

Состоятельность - ?

($W \to 1$ $n \to \infty$)

$W \to 0 \Leftrightarrow \alpha_2 \to 0$, рассм. $\left(1+\frac{e^{\sqrt[n]{1-\alpha}}}{e-1} - \frac{e}{e-1}\right)^n \xrightarrow[n\to\infty]{} ?$

$$e^{\sqrt[n]{1-\alpha}} = e^{e^{\frac{1}{n}\ln(1-\alpha)}} = e^{\left(1+\frac{\ln(1-\alpha)}{n}+o\left(\frac{1}{n}\right)\right)} = e\left(1+\frac{\ln(1-\alpha)}{n}+o\left(\frac{1}{n}\right)\right)$$

$$\alpha_2 = \left(1+\frac{e\left(1+\ln(1-\alpha)\cdot\frac{1}{n}+o\left(\frac{1}{n}\right)\right)}{e-1} - \frac{e}{e-1}\right)^n =$$

$$= \left(1+\frac{e\ln(1-\alpha)}{(e-1)\cdot n} + \underbrace{\frac{e}{e-1}\cdot o\left(\frac{1}{n}\right)}_{\to 0}\right)^n \to e^{\frac{e}{e-1}\ln(1-\alpha)} =$$

$= (1-\alpha)^{\frac{e}{e-1}} \neq 0 \Rightarrow$ Критерий не явл. состоят.