$\boxed{T6}$

$$p(x)=\begin{cases}\dfrac{\theta-1}{x^{\theta}}, & x\geq 1\\ 0, & x<1\end{cases}\qquad \theta>1$$

а) $L(x_n,\theta)=\prod_{i=1}^{n}\left(\dfrac{\theta-1}{x_i^{\theta}}\right)=(\theta-1)^n\prod_{i=1}^{n}\left(\dfrac{1}{x_i^{\theta}}\right)$; sup-?

$\ln L(x_n,\theta)=n\ln(\theta-1)-\theta\sum_{i=1}^{n}\ln x_i$

$\dfrac{\partial\ln(L(x_n,\theta))}{\partial\theta}=\dfrac{n}{\theta-1}-\sum_{i=1}^{n}\ln x_i=0$, $\quad\tilde{\theta}=\dfrac{n}{\sum_{i=1}^{n}\ln x_i}+1$

$\dfrac{\partial\ln^2 L(x_n,\theta)}{(\partial\theta)^2}=-\dfrac{n}{(\theta-1)^n}<0\Rightarrow$ sup

б) $\int_{-\infty}^{\tilde{x}}p(x)d(x)=\dfrac{1}{2}$; $\int_{1}^{\tilde{x}}\dfrac{\theta-1}{x^{\theta}}dx=\dfrac{1}{2}$; $(\theta-1)\int_{1}^{\tilde{x}}\dfrac{dx}{x^{\theta}}=\dfrac{1}{2}$

$\dfrac{(\theta-1)}{(-\theta+1)}\cdot\dfrac{1}{x^{\theta-1}}\Big|_{1}^{\tilde{x}}=\dfrac{1}{2}$; $-x^{1-\theta}\Big|_{1}^{\tilde{x}}=\dfrac{1}{2}$

$-\tilde{x}^{1-\theta}+1=\dfrac{1}{2}$; $\tilde{x}^{1-\theta}=\dfrac{1}{2}$ $\quad\tilde{x}^{\theta-1}=2$ $\quad\tilde{x}=2^{\frac{1}{\theta-1}}$

$med(\theta)=2^{\frac{1}{\theta-1}}$

ОМП: Регулярна ли модель?

1) $p(x,\theta)$ — непр. дифф. по θ на Θ

2) $\int_{1}^{\infty}\dfrac{\partial}{\partial\theta}\dfrac{\theta-1}{x^{\theta}}dx=\int_{1}^{\infty}\dfrac{x^{\theta}-(\theta-1)\theta x^{\theta-1}}{x^{2\theta}}dx=\dots$

$=-\theta+1+\theta-1=0\Rightarrow$ перестановочно

3) $I(\theta)$ – ?

$$\ln p(x,\theta) = \ln(\theta-1) - \theta \ln x \qquad \frac{\partial \ln p(x,\theta)}{\partial\theta} = \frac{1}{\theta-1} - \ln x$$

$$I(\theta) = \int_1^{+\infty} \left(\frac{1}{\theta-1} - \ln x\right)^2 \frac{\theta-1}{x^\theta} dx = \int_1^{+\infty} \left(\frac{1}{(\theta-1)x^\theta} - \frac{2\ln x}{x^\theta} + \frac{(\theta-1)\ln^2 x}{x^\theta}\right) dx = \frac{1}{(\theta-1)^2}$$ – непр. на Θ

$I(\theta) > 0$ на Θ

$\Rightarrow$ модель регулярная

$$\sqrt{n}\,\frac{f(\tilde\theta) - f(\theta)}{\sigma(\theta)} \rightsquigarrow N(0,1) \qquad \sigma = \sqrt{\nabla^T f(\theta)\, I^{-1}(\theta)\, \nabla f(\theta)}$$

$\exists\, f(\theta) = \text{med}\,\theta \Rightarrow \nabla f(\theta) = 2^{\frac{1}{\theta-1}} \ln 2 \left(-\frac{1}{(\theta-1)^2}\right)$

$$\sigma = \sqrt{\left(2^{\frac{1}{\theta-1}} \ln 2\, \frac{1}{(\theta-1)^2}\right)^2 \cdot (\theta-1)^2} = \frac{2^{\frac{1}{\theta-1}} \ln 2}{\theta-1}$$

$$\sqrt{n}\,\frac{2^{\frac{1}{\tilde\theta-1}} - 2^{\frac{1}{\theta-1}}}{\frac{2^{\frac{1}{\tilde\theta-1}} \ln 2}{\tilde\theta-1}} \rightsquigarrow N(0,1)$$

$$\cancel{2^{\frac{1}{\tilde\theta-1}} - \frac{2^{\frac{1}{\tilde\theta-1}} \ln 2}{(\tilde\theta-1)\sqrt{n}}}$$

$$2^{\frac{1}{\tilde\theta-1}} - \frac{2^{\frac{1}{\tilde\theta-1}} \cdot \ln 2}{(\tilde\theta-1)\sqrt{n}}\, t_2 < \underset{\text{med}}{\hat{x}} < 2^{\frac{1}{\tilde\theta-1}} - \frac{2^{\frac{1}{\tilde\theta-1}} \cdot \ln 2}{(\tilde\theta-1)\sqrt{n}}\, t_1$$

$\frac{1}{\log_2 \text{med}} =$

$t_1 = N_{\frac{1-\beta}{2}}$

$t_2 = N_{\frac{1+\beta}{2}}$

Асимптот. доверит. интервал ~~для~~ для медианы

c) $f(\theta) = \theta \qquad f(\tilde{\theta}) = \tilde{\theta}$

$$\sigma(\theta) = \sqrt{I^{-1}(\theta)} = (\theta - 1) \Rightarrow \qquad \sqrt{n}\,\frac{\tilde{\theta} - \theta}{\tilde{\theta} - 1} \rightarrow N(0,1)$$

$$t_1 < \sqrt{n}\,\frac{\tilde{\theta} - \theta}{\tilde{\theta} - 1} < t_2$$

$$\tilde{\theta} - \frac{t_2(\tilde{\theta} - 1)}{\sqrt{n}} < \theta < \tilde{\theta} - \frac{t_1(\tilde{\theta} - 1)}{\sqrt{n}}$$