Name: Flori Kusari | **Problem 4.2: Recurrences** | Date: 03/03/2024

(a) $T(n) = 36T\left(\frac{n}{6}\right) + 2n$

This recurrence is solved using Master Theorem. Here $a=36$, $b=6$, and $f(n)=2n$.

$n^{\log_b a} = n^{\log_6 36} = n^2$

$f(n) = 2n$ grows slower than $n^{\log_b a} = n^2$ so it is case 1 of the Master Theorem.

$f(n) = O(n^c)$ for $c < \log_b a$. Therefore, $T(n) = \Theta(n^{\log_b a}) = \Theta(n^2)$.

(b) $T(n) = 5T\left(\frac{n}{3}\right) + 17n^{1.2}$

This recurrence is solved using Master Theorem. Here $a=5$, $b=3$ and $f(n) = 17n^{1.2}$.

$n^{\log_b a} = n^{\log_3 5} = 1.465$

Since $1.2 < 1.465$, $f(n)$ grows slower than $n^{\log_b a}$ so we are in case 1 of the Master Theorem

Therefore, $T(n) = \Theta(n^{\log_3 5})$

(c) $T(n) = 12T\left(\frac{n}{2}\right) + n^2 \log_2 n$

$n^{\log_b a} = n^{\log_2 12} = n^{3.58496\ldots}$

So $n^{\log_2 12}$ grows faster than $n^2 \log_2 n$ seeing as this is true we can conclude that $T(n) = \Theta(n^{\log_2 12})$ as the exponential growth is larger.

(d) $T(n) = 3T\left(\frac{n}{5}\right) + T\left(\frac{n}{2}\right) + 2^n$

This does not fit the Master Theorem however from a standard analysis we can argue that for larger "n" the exponential growth takes precedence. So, $T(n) = \Theta(2^n)$.

(e) $T(n) = T(2n/5) + T(3n/5) + \Theta(n)$.

* I have no idea how to solve this one but an intuitive guess based on the fact that it obviously grows faster than linear would be $T(n) = \Theta(n \log n)$ or $\Theta(n)$