

Home work

Name: Flora Kusari
Date: 25/09/2023

Problem 3.1: Compute the following limits:

$$a) \lim_{x \rightarrow 0} \frac{x^2 - 1}{2x^2 - x - 1} = \frac{0^2 - 1}{2 \cdot 0^2 - 0 - 1} = \frac{-1}{-1} = \boxed{1}$$

$$b) \lim_{x \rightarrow 1} \frac{x^2 - 1}{2x^2 - x - 1} = \frac{(x-1) \cdot (x+1)}{(2x+1) \cdot (x-1)} = \frac{x+1}{2x+1} = \frac{1+1}{2 \cdot 1 + 1} = \boxed{\frac{2}{3}}$$

$$c) \lim_{x \rightarrow \infty} \frac{x^2 - 1}{2x^2 - x - 1} = \frac{x^2 \cdot \left(1 - \frac{1}{x^2}\right)}{x^2 \cdot \left(2 - \frac{1}{x} - \frac{1}{x^2}\right)} = \frac{1 - \frac{1}{x^2}}{2 - \frac{1}{x} - \frac{1}{x^2}} = \frac{1 - 0}{2 - 0 - 0} = \boxed{\frac{1}{2}}$$

Problem 3.2: Compute the limits:

$$a) \lim_{x \rightarrow 0} \frac{(1+x) \cdot (1+2x) \cdot (1+3x) - 1}{x} = \frac{(1+2x+x+2x^2) \cdot (1+3x) - 1}{x} =$$

$$= \frac{(1+3x+2x^2) \cdot (1+3x) - 1}{x} = \frac{1+3x+3x+9x^2+2x^2+6x^3-1}{x} = \frac{6x+11x^2+6x^3}{x}$$

$$= \frac{x(6+11x+6x^2)}{x} = (6+11x+6x^2) = 6 \cdot 0^2 + 11 \cdot 0 + 6 = \boxed{6}$$

$$b) \lim_{x \rightarrow 0} \frac{(1+x)^5 - (1+5x)}{x^2 + x^5} = \frac{1+5x+10x^2+10x^3+5x^4+x^5 - (1+5x)}{x^2 + x^5} =$$

$$= \frac{10x^2+10x^3+5x^4+x^5}{x^2 + x^5} = \frac{x^2 \cdot (10+10x+5x^2+x^3)}{x^2 \cdot (1+x^3)} = \frac{10+10x+5x^2+x^3}{1+x^3}$$

$$= \frac{10+10 \cdot 0 + 5 \cdot 0^2 + 0^3}{1+0^3} = \frac{10}{1} = \boxed{10}$$



Problem 3.3: Compute the limits:

$$a) \lim_{x \rightarrow 3} \frac{x^2 - 5x + 6}{x^2 - 8x + 15} = \frac{x \cdot (x-2) - 3(x-2)}{x \cdot (x-3) - 5(x-3)} = \frac{(x-2)(x-3)}{(x-3)(x-5)} = \frac{x-2}{x-5} =$$

$$= \frac{3-2}{3-5} = \boxed{-\frac{1}{2}}$$

$$b) \lim_{x \rightarrow 2} \frac{x^3 - 2x^2 - 4x + 8}{x^4 - 8x^2 + 16} = \frac{x^2 \cdot (x-2) - 4(x-2)}{(x^2-4)^2} = \frac{(x-2) \cdot (x^2-4)}{(x-2)^2 \cdot (x+2)^2} =$$

$$= \frac{(x+2) \cdot (x-2)^2}{(x-2)^2 \cdot (x+2)^2} = \frac{x+2}{(x+2)^2} = \frac{1}{x+2} = \frac{1}{2+2} = \boxed{\frac{1}{4}}$$

Problem 3.4: Compute the limit:

$$\lim_{x \rightarrow 2} \frac{(x^2 - x - 2)^{20}}{(x^3 - 12x + 16)^{10}} = \frac{(x^2 + x - 2x - 2)^{20}}{(x^3 - 16x + 4x + 16)^{10}} = \frac{(x \cdot (x+1) - 2(x+1))^{20}}{(x + (x^2 - 16) + 4(x+4))^{10}} =$$

$$= \frac{((x+1) \cdot (x-2))^{20}}{(x \cdot (x-4)(x+4) + 4(x+4))^{10}} = \frac{(x+1)^{20} \cdot (x-2)^{20}}{((x+4) \cdot (x \cdot (x-4) + 4))^{10}} = \frac{(x+1)^{20} \cdot (x-2)^{20}}{((x+4) \cdot (x^2 - 4x + 4))^{10}} =$$

$$= \frac{(x+1)^{20} \cdot (x-2)^{20}}{((x+4) \cdot (x-2)^2)^{10}} = \frac{(x+1)^{20} \cdot (x-2)^{20}}{(x+4)^{10} \cdot (x-2)^{20}} = \frac{(x+1)^{20}}{(x+4)^{10}} = \frac{(2+1)^{20}}{(2+4)^{10}} = \frac{3^{20}}{6^{10}} =$$

$$\boxed{\left(\frac{3}{2} \right)^{10} \text{ or } 57 \frac{681}{1024} \text{ or } \frac{59049}{1024}}$$

THE END