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Problem 12.1: Find equations of the following lines:

a) Passing through points (1, 4) and (-3, 5)

$m = \frac{5-4}{-3-1} = -\frac{1}{4}$

$y - 4 = \frac{-1}{4}(x-1)$

$y - 4 = -\frac{1}{4}x + \frac{1}{4} \quad / \cdot 4$

$4y - 16 = -x + 1$

$\boxed{4y + x - 17 = 0}$

b) Parallel to vector $\binom{3}{2}$ and containing the point $M_0(4, 1)$:

$r = a + \lambda b$ — direction vector

So we can just substitute known values

Position vector of points on the line

Position vector of a specific point on the line.

Scalar parameter

$r = \begin{bmatrix} 4 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ 2 \end{bmatrix}$

now the equations that describe the line are: $\boxed{x = 4 + 3t}$ $\boxed{y = 1 + 2t}$

c) Perpendicular to line $\frac{x-2}{3} = \frac{y+4}{5}$ and passing through the origin:

$\frac{x-2}{3} = \frac{y+4}{5}$

$5(x-2) = 3(y+4)$

$5x - 10 = 3y + 12$

$5x - 22 = 3y \quad / \div 3$

$y = \boxed{\frac{5}{3}}x - \frac{22}{3}$

perpendicular of this is $\boxed{-\frac{3}{5}}$

Given that it passes through the origin (0, 0)

$y - 0 = \frac{-3}{5}(x - 0)$

$\boxed{y = -\frac{3}{5}x}$ — Final Answer

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Problem 12.2: Find the equations of the following planes:

a) The plane containing the point $(6,2,1)$ and perpendicular to the vector $\begin{pmatrix}1\\1\\1\end{pmatrix}$

equation is: $k_x(x-x_A)+k_y(y-y_B)+k_z(z-z_C)=0$

now we plug in the known information:

$1(x-6)+1(y-2)+1(z-1)=0$

$x-6+y-2+z-1=0$

$\boxed{x+y+z=9}$

b) The plane containing points $(1,2,-3)$, $(0,1,-2)$, and $(1,2,-2)$.

$$\begin{vmatrix} x-x_A & y-y_A & z-z_A \\ x_B-x_A & y_B-y_A & z_B-z_A \\ x_C-x_A & y_C-y_A & z_C-z_A \end{vmatrix}=0$$

$$\begin{vmatrix} x-1 & y-2 & z-(-3) \\ 0-1 & 1-2 & -2-(-3) \\ 1-1 & 2-2 & -2-(-3) \end{vmatrix}=0$$

$$\begin{vmatrix} x-1 & y-2 & z-(-3) \\ -1 & -1 & 1 \\ 0 & 0 & 1 \end{vmatrix}$$

$(x-1)(-1\cdot1-1\cdot0)-(y-2)((-1)\cdot1-1\cdot0)+(z-(-3))((-1)\cdot0-(-1)\cdot0)=0$

$(x-1)(-1)+(y-2)+(z+3)\cdot0=0$

$-x+1+y-2+0=0$

$-x-1+y=0$

$-x=1-y$

$\boxed{x=y-1}$ Final Answer

Problem 12.3: Consider the following vectors:

$$V_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 2 \end{pmatrix} \quad V_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 1 \end{pmatrix} \quad V_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 2 \end{pmatrix}$$

ⓐ Are vectors (v_1, v_2, v_3) linearly dependent or independent? Do they form a basis of $\mathbb{R}^4$?

If the dimension of the basis is less than the dimension of the set, the set is lineary dependent, otherwise it is linearly independent.

Reduced to row echelon form of $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$

Subtract row 1 from row 2: $R_2 = R_2 - R_1$

$$\begin{bmatrix} 1 & 1 & 1 & 2 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

Subtract row 2 from row 1: $R_1 = R_1 - R_2$

$$\begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 1 & 1 & 2 \end{bmatrix}$$

Subtract row 2 from row 3: $\begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 3 & 3 \end{bmatrix}$

Divide row 3 by 3: $\begin{bmatrix} 1 & 0 & 3 & 3 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

Subtract 3·Row 3 from Row 1:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & -2 & -1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

Finally add 2·Row 3 to Row 2:

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} A.$$

Reduced Echelon Form.

The row space of the matrix is:

$$\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} \cdot A$$

This is one basis and another one can be found using the column space form but we chose this because it is simpler.

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The basis has a dimension (aka. number of vectors in it) of 3.

Since the dimension of the basis = dimension of the set, the set is linearly independant.

$\boxed{\text{The set is linearly independant and has a basis of } \mathbb{R}^4.}$

ⓑ Consider vector $v_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$. Are vectors (v_1, v_2, v_3, v_4) linearly dependent or independent? Can they be a basis?

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*To save space I wrote the reduced echelon form of $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 2 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ and the steps that were taken.

Steps: $R_2 = R_2 - R_1$

$R_1 = R_1 - R_2$

$R_3 = R_3 - R_2$

$R_4 = R_4 - R_2$

$R_3 = \frac{R_3}{3}$

$R_1 = R_1 - 3R_3$

$R_2 = R_2 + 2R_3$

$R_4 = R_4 - 2R_3$

$R_4 = -R_4$

$R_2 = R_2 - R_4$

$R_3 = R_3 - R_4$

Result: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ The row space of the matrix $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$. A

Also the basis

→

Since the dimension of the basis of the set equals the dimension of the set, the set is linearly independent.
Yes the basis can be formed

ⓒ Consider also vector $v_5 = \begin{bmatrix} 1 \\ -1 \\ 5 \\ 4 \end{bmatrix}$ Are vectors (v_1, v_2, v_3, v_5) linearly dependent or independent? Can they be a basis?

To save space I did the same thing as in problem ⓑ

Echelon Form of $\begin{bmatrix} 1 & 1 & 1 & 2 \\ 1 & 2 & -1 & 1 \\ 0 & 1 & 1 & 2 \\ -1 & -1 & 5 & 4 \end{bmatrix}$

Steps:
$R_2 = R_2 - R_1$
$R_4 = R_4 - R_1$
$R_1 = R_1 - R_2$
$R_3 = R_3 - R_2$
$R_4 = R_4 + 2R_2$
$R_3 = \frac{R_3}{3}$
$R_1 = R_1 - 3R_3$
$R_2 = R_2 + 2R_3$

The Reduced echeleon form is $\left\{ \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{matrix} \right\} A.$

The last Row is disregarded due to being 0 0 0 0.

The basis is $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\} .A$

Since the dimension of the basis less than the dimension of the set, the set is linearly dependent and cannot form a basis.

Problem 12.4: Solve the system of linear equations:

ⓐ
$$2a+2b-c+d=4$$
$$4a+3b-c+2d=6$$
$$8a+5b-3c+4d=12$$
$$3a+3b-2c+2d=6$$

Solving by using Cramer's Rule.

$$D=\begin{vmatrix} 2 & 2 & -1 & 1 \\ 4 & 3 & -1 & 2 \\ 8 & 5 & -3 & 4 \\ 3 & 3 & -2 & 2 \end{vmatrix}$$

$$D_1=\begin{vmatrix} 4 & 2 & -1 & 1 \\ 6 & 3 & -1 & 2 \\ 12 & 5 & -3 & 4 \\ 6 & 3 & -2 & 2 \end{vmatrix}$$

$$D_3=\begin{vmatrix} 2 & 2 & 4 & 1 \\ 4 & 3 & 6 & 2 \\ 8 & 5 & 12 & 4 \\ 3 & 3 & 6 & 2 \end{vmatrix} \qquad D_4=\begin{vmatrix} 2 & 2 & -1 & 4 \\ 4 & 3 & -1 & 6 \\ 8 & 5 & -3 & 12 \\ 3 & 3 & -2 & 6 \end{vmatrix}$$

$$D_2=\begin{vmatrix} 2 & 4 & -1 & 1 \\ 4 & 6 & -1 & 2 \\ 8 & 12 & -3 & 4 \\ 3 & 6 & -2 & 2 \end{vmatrix}$$

$D=2$ $\quad D_3=-2$

$D_1=2$ $\quad D_4=-2$

$D_2=2$

$a=\frac{D_1}{D}=1$ $\quad d=\frac{D_4}{D}=-1$

$b=\frac{D_2}{D}=1$

$c=\frac{D_3}{D}=-1$

$$2(1)+2(1)-(-1)+(-1)=4 \qquad 4=4$$
$$4(1)+3(1)-(-1)+2(-1)=6 \Rightarrow 6=6$$
$$8(1)+5(1)-3(-1)+4(-1)=12 \qquad 12=12$$
$$3(1)+3(1)-2(-1)+2(-1)=6 \qquad 6=6$$

So the solution works.

$\boxed{(a,b,c,d)=(1,1,-1,-1)}$ → Final Answer

ⓑ
$$a-3b-26c+22d=0$$
$$a-8c+7d=0$$
$$a+b-2c+2d=0$$
$$4a+5b-2c+3d=0$$

Transform the equations:

$-3b-18c+15d=0$ ⟶ Multiply equation 3 by 3 and add it to equation 1

$a-8c+7d=0$

$b+6c-5d=0$

$5b+30c-25d=0$ ⟶ Multiply equation 3 by -5 and add it to equation 4.

$0=0$

$a-8c+7d=0 \to a=8c-7d$

$b+6c-5d=0 \Rightarrow b=-6c+5d$

$0=0$

$\boxed{(a,b,c,d)=(8c-7d,-6c+5d,c,d)}$ ⇒ Solution

$(8c-7d)-3(-6c+5d)-26c+22d=0$
$(8c-7d)-8c+7d=0$
$(8c-7d)+(-6c+5d)-2c+2d=0$
$4(8c-7d)+5(-6c+5d)-2c+3d=0$

When simplified: $0=0$
$0=0$
$0=0$
$0=0$

Therefore the solution $(a,b,c,d)=(8c-7d,-6c+7d,c,d)$ for $(c,d)\in\mathbb{R}$ holds.

THE END