Name: Flori Kusari | Home work

Date: 05/09/2023

Homework 1

Problem 1.1: Simplify the expression:

a) $(2+3i)(4-5i)+(2-3i)(4+5i)=$

$8\cancel{-10i}\cancel{+12i}-15i^2+8\cancel{+10i}\cancel{-12i}-15i^2=$

$8+8-15(-1)-15(-1)=$

$16+15+15=$

$=46$

Problem 1.1: Simplify the expression:

b) $(\sqrt{3}-i)(\sqrt{2}+i\sqrt{3})=$

$\sqrt{6}+3i-i\sqrt{2}-(i)^2\sqrt{3}=$

$\sqrt{6}+3i-i\sqrt{2}-(-1)\sqrt{3}=$

$\sqrt{6}+3i-i\sqrt{2}+\sqrt{3}=$

$\sqrt{6}+(3-\sqrt{2})i+\sqrt{3}=$

$\sqrt{6}+\sqrt{3}+(3-\sqrt{2})i$

Problem 1.2: Compute the complex fractions:

a) $\dfrac{(-2+i)\cdot(1-3i)}{(1+3i)\cdot(1-3i)}=\dfrac{-2+6i+i-3(i)^2}{1\cancel{-3i}\cancel{+3i}-9(i)^2}=\dfrac{-2+7i-3(-1)}{1-9(-1)}=$

$=\dfrac{-2+7i+3}{1+9}=\dfrac{1+7i}{10}=\boxed{\dfrac{1}{10}+\dfrac{7}{10}i}$

Problem 1.2: Compute the complex fractions:

b) $$\frac{2+3i}{(2-3i)^2} = \frac{2+3i}{(2-3i)(2-3i)} = \frac{2+3i}{4-6i-6i+9(i)^2} = \frac{2+3i}{4-12i+9(-1)}$$

$$= \frac{2+3i}{4-9+12i} = \frac{2+3i}{-5-12i} = \frac{(2+3i)(-5+12i)}{(-5-12i)(-5+12i)} =$$

$$= \frac{-10+24i-15i+36(i)^2}{25-\cancel{60i}+\cancel{60i}-144(i)^2} = \frac{-46+9i}{25-144(-1)} = \frac{-46+9i}{25+144} =$$

$$= \frac{-46+9i}{169}$$

$$\boxed{= \frac{-46}{169} + \frac{9}{169}i}$$

Problem 1.3: Find all the roots of the equations:

a) $x^2+36=0$

$x^2=-36 \quad /\sqrt{\ }$

$x = \pm 6i$

$\underline{x_1 = -6i}$ $\quad$ $\underline{x_2 = 6i}$

Answer: $x=-6i$
$x=6i$
a)

b) $x^4+5x^2+4=0$

$(x^2+1)(x^2+4)=0$

$x^2+1=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-0 \pm \sqrt{0^2-4\cdot 1\cdot 1}}{2\cdot 1}$

$x = \frac{0 \pm \sqrt{-4}}{2}$

$x = \frac{\pm 2i}{2}$

$x = \pm i$

$\boxed{x_1 = i}$ $\boxed{x_2 = -i}$

$x^2+4=0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-0 \pm \sqrt{0^2-4\cdot 1\cdot 4}}{2\cdot 1}$

$x = \frac{0 \pm \sqrt{-16}}{2}$

$x = \frac{\pm 4i}{2}$

$x = \pm 2i$

$\boxed{x_3 = 2i}$ $\boxed{x_4 = -2i}$

Answer: $x=i$
$x=-i$
$x=2i$
$x=-2i$
b)

Problem 1.4: Prove that $(z^*)^n = (z^n)^*$, where $z \in C$, $n \in N$.

Base Case: Assume that $\boxed{n = 1}$:

$$(z^1)^* = z^* = (z^*)^1$$

We need to conclude $n = x + 1$

Proving through Induction: We make the assumption that for some positive integer "x" the expressions are equal: $(z^x)^* = (z^*)^x$

$$(z^{x+1})^* = (z^x \cdot z)^* = (z^x)^* z^* = (z^*)^x z^* = z^* \cdot z^* \ldots z^* \cdot z^* = (z^*)^{x+1}$$

Now, it has been concluded that $(z^*)^{x+1}$ is the conjugate of (z^{x+1}).

This proves that since [redacted] $n = x + 1$, [redacted] through mathematical induction:

$(z^*)^n = (z^n)^*$ is true for $n \in N$ and $z \in C$ where N represents the Natural Numbers and C represents Complex Numbers.

THE END