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Problem Sheet #8

Date: 31/10/2023

Problem 8.1: Compute the following Integrals

(a) $\int \frac{1}{(x+1)\cdot(2x-3)}\,dx$ I prefer to write it like this from the beginning

$$\int -\frac{1}{5(x+1)} + \frac{2}{5(2x-3)}\,dx$$

$$-\int \frac{1}{5(x+1)}\,dx + \int \frac{2}{5(2x-3)}\,dx$$

$$-\frac{1}{5}\cdot\ln(|x+1|) + \frac{1}{5}\cdot\ln(|2x-3|) + C$$

(b) $\int \frac{1}{x^2+2x+3}\,dx$

$$\int \frac{1}{x^2+2x+1+2}\,dx$$

$$\int \frac{1}{(x+1)^2+2}\,dx$$

$t = x+1$

$$\int \frac{1}{t^2+2}\,dt$$

$$\frac{1}{\sqrt{2}}\cdot\arctan\left(\frac{t}{\sqrt{2}}\right)$$

Simplify are substitute "t" with "x+1"

$$\frac{\sqrt{2}\cdot\arctan\left(\frac{\sqrt{2}x+\sqrt{2}}{2}\right)}{2} + C$$

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Problem 8.1: c) $\int \frac{1}{\sqrt{x}(x-1)}\,dx$

$$\int \frac{1}{\sqrt{x}x - \sqrt{x}}\,dx$$

$$\int \frac{1}{x^{\frac{1}{2}} \cdot x - \sqrt{x}}\,dx \Rightarrow \int \frac{1}{x^{\frac{3}{2}} - x^{\frac{1}{2}}}\,dx$$

$t = \sqrt{x} = x^{\frac{1}{2}}$

$$\int \frac{2}{t^2-1}\,dt$$

$$2 \cdot \int \frac{1}{t^2-1}\,dt$$

$$2 \cdot \frac{1}{2} \cdot \ln\left(\left|\frac{t-1}{t+1}\right|\right)$$

$$\ln\left(\left|\frac{x^{\frac{1}{2}}-1}{x^{\frac{1}{2}}+1}\right|\right)$$

$$\ln(|\sqrt{x}-1|) - \ln(\sqrt{x}+1) + C$$

Problem 8.2: Compute the following Integrals:

ⓐ $\int \frac{2x^2+41x-91}{(x-1)(x+3)(x-4)}$

$$\int \frac{4}{x-1} - \frac{7}{x+3} + \frac{5}{x-4}\,dx$$

$$\int \frac{4}{x-1}\,dx - \int \frac{7}{x+3}\,dx + \int \frac{5}{x-4}\,dx$$

$$4\ln(|x-1|) - 7\ln(|x+3|) + 5\ln(|x-4|) + C$$

ⓑ $\int \frac{x^2}{(x+2)^2 \cdot (x+4)^2}\,dx$

$$\int -\frac{2}{x+2} + \frac{1}{(x+2)^2} + \frac{2}{x+4} + \frac{4}{(x+4)^2}\,dx$$

$$-\int \frac{2}{x+2}\,dx + \int \frac{1}{(x+2)^2}\,dx + \int \frac{2}{x+4}\,dx + \int \frac{4}{(x+4)^2}\,dx$$

$$-2\ln(|x+2|) + \left(-\frac{1}{x+2}\right) + 2\ln(|x+4|) + \left(-\frac{4}{x+4}\right)$$

$$-2\ln(|x+2|) - \frac{5x+12}{x^2+6x+8} + 2\ln(|x+4|) + C$$

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Problem 8.2: (c) $\int \frac{2x^2-3x-3}{(x-1)(x^2-2x+5)} dx$

$$\int -\frac{1}{x-1} + \frac{3x-2}{x^2-2x+5} dx$$

$$-\int \frac{1}{x-1} dx + \int \frac{3x-2}{x^2-2x+5} dx$$

$$-\ln(|x-1|) + \frac{3}{2} \cdot \ln(|x^2-2x+5|) + \frac{\arctan\left(\frac{x-1}{2}\right)}{2} + C$$

Problem 8.3:

a) $\int \frac{x+1}{x\sqrt{x-2}} dx$

$$\int \frac{2(u^2+3)\,du}{u^2+2}$$

$$2 \cdot \int \frac{u^2+3}{u^2+2} du$$

$$2 \cdot \int \frac{u^2}{u^2+2} du + \int \frac{3}{u^2+2} du$$

Substitute $u = \sqrt{x-2}$

$$2\left(-\sqrt{2}\arctan\left(\frac{u}{\sqrt{2}}\right) + u + \frac{3}{\sqrt{2}}\arctan\left(\frac{u}{\sqrt{2}}\right)\right)$$

$$2\left(-\sqrt{2}\arctan\left(\sqrt{\frac{1}{2}(x-2)}\right) + \sqrt{x-2} + \frac{3}{\sqrt{2}}\arctan\left(\sqrt{\frac{1}{2}(x-2)}\right)\right) + C$$

Problem 8.3: b

$$\int \frac{1}{1+\sqrt[3]{1+x}}\,dx$$

$$\int \frac{1}{1+\sqrt[3]{u}}\,du$$

$$\int \frac{3v^2}{1+v}\,dv$$

$$3\cdot\int \frac{v^2}{1+v}\,dv$$

$$\boxed{w=(1+\sqrt[3]{1+x})}$$

$$3\cdot\int \frac{(w-1)^2}{w}\,dw \implies 3\cdot\int w-2+\frac{1}{w}$$

$$3\left(\int w\,dw-\int 2\,dw+\int \frac{1}{w}\,dw\right)$$ Applying the Sum Rule

$$3\left(\frac{w^2}{2}-2w+\ln|w|\right)$$

Substitute back

$$\boxed{3\left(\frac{(1+\sqrt[3]{1+x})^2}{2}-2(1+\sqrt[3]{1+x})+\ln\left|1+\sqrt[3]{1+x}\right|\right)+C}$$

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Problem 8.3: c) $\int \frac{1}{\sqrt{1+e^x}} dx$

$$\int \frac{2}{w^2-1} dw \qquad \boxed{w = \sqrt{1+e^x}}$$

$$2 \int \frac{1}{w^2-1} dw$$

$$2 \cdot \left(- \int \frac{1}{-w^2+1} dw \right)$$

$$2 \cdot \left(- \left(\frac{\ln(|w+1|)}{2} - \frac{\ln(|w-1|)}{2} \right) \right)$$

$$-\ln(|w+1|) + \ln(|w-1|)$$

$$-\ln\left(\left|\sqrt{1+e^x}+1\right|\right) + \ln\left(\left|\sqrt{1+e^x}-1\right|\right) + C$$

Problem 8.4: a) $\int_0^1 \frac{x}{(x^2+1)^2}\,dx$ $\qquad t = x^2+1$

$$\int_0^1 \frac{1}{2t^2}\,dt$$

$$\frac{1}{2}\int_0^1 \frac{1}{t^2}\,dt$$

$$\frac{1}{2}\cdot\left(-\frac{1}{t}\right)$$

$$-\frac{1}{2x^2+2}\Big|_0^1$$

$$\Downarrow$$

$$-\frac{1}{2\cdot 1^2+2}-\left(-\frac{1}{2\cdot 0^2+2}\right)=-\frac{1}{4}+\frac{2}{4}$$

$$\Rightarrow \boxed{\frac{1}{4}}$$

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Problem 8.4: b $\displaystyle\int_0^{16} \frac{1}{\sqrt{x-9}-\sqrt{x}}\,dx$

$\Downarrow$

$$\int_0^{16} \frac{1 \cdot (\sqrt{x+9}+\sqrt{x})}{(\sqrt{x-9}-\sqrt{x}) \cdot (\sqrt{x+9}+\sqrt{x})}\,dx$$

$$\int_0^{16} \frac{\sqrt{x+9}+\sqrt{x}}{9}\,dx$$

$$\frac{1}{9}\left(\int_0^{16} \sqrt{x+9}\,dx + \int x^{\frac{1}{2}}\,dx\right)$$

$$\frac{1}{9}\left(\frac{2\sqrt{x+9}\cdot|x+9|}{3} + \frac{2\sqrt{x}\cdot|x|}{3}\right)$$

$$\left.\frac{2\sqrt{x+9}\cdot|x+9| + 2\sqrt{x}\cdot|x|}{27}\right|_0^{16} = \boxed{12}$$

Problem 8.4: C $\int_0^{\pi/2} x \cos(x)\, dx$

$$x \cdot \sin(x) - \int \sin(x)\, dx$$

$$x \cdot \sin(x) - (-\cos(x))$$

$$\{x \cdot \sin(x) + \cos(x)\} \Big|_0^{\pi/2}$$

$$\Downarrow$$

$$\frac{\pi}{2} \cdot \sin\left(\frac{\pi}{2}\right) + \cos\left(\frac{\pi}{2}\right) - \left(0 \sin(0) + \cos(0)\right)$$

$$\Rightarrow \boxed{\frac{\pi}{2} - 1}$$

THE END

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