

## Problem 8.1: Quine-McCluskey Algorithm

A Boolean function  $F$  is defined by the following sum of minterms:

$$F(x_4, x_3, x_2, x_1, x_0) = m_1 + m_5 + m_7 + m_8 + m_9 + m_{10} + m_{11} + m_{13} + m_{16} + m_{18} + m_{24} + m_{29}$$

Step 0:

1-00001

5-00101

7-00111

8-01000

9-01001

10-01010

11-01011

13-01101

16-10000

18-10010

24-11000

29-11101

Group them based on the number of "1"s they have:

$$A_1 : m_1 \ m_8 \ m_{16}$$

$$A_2 : m_5 \ m_9 \ m_{10} \ m_{18} \ m_{24}$$

$$A_3 : m_7 \ m_{11} \ m_{13}$$

$$A_4 : m_{29}$$

Step 1:

Group	Minterm	$x_4$	$x_3$	$x_2$	$x_1$	$x_0$	Used
$A_1$	$m_1$	0	0	0	0	1	✓
	$m_8$	0	1	0	0	0	✓
	$m_{16}$	1	0	0	0	0	✓
$A_2$	$m_5$	0	0	1	0	1	✓
	$m_9$	0	1	0	0	1	✓
	$m_{10}$	0	1	0	1	0	✓
	$m_{18}$	1	0	0	1	0	✓
	$m_{24}$	1	1	0	0	0	✓
$A_3$	$m_7$	0	0	1	1	1	✓
	$m_{11}$	0	1	0	1	1	✓
	$m_{13}$	0	1	1	0	1	✓
$A_4$	$m_{29}$	1	1	1	0	1	✓

Step 2:

Group	Minterm	$x_4$	$x_3$	$x_2$	$x_1$	$x_0$	Used
$B_1$ ( $A_1 - A_2$ )	$m_{1,5}$	0	0	-	0	1	✓
	$m_{1,9}$	0	-	0	0	1	✓
	$m_{8,9}$	0	1	0	0	-	✓
	$m_{8,10}$	0	1	0	-	0	✓
	$m_{8,24}$	-	1	0	0	0	✓
	$m_{16,18}$	1	0	0	-	0	✓
$B_2$ ( $A_2 - A_3$ )	$m_{5,7}$	0	0	1	-	1	✓
	$m_{5,13}$	0	-	1	0	1	✓
	$m_{9,11}$	0	1	0	-	1	✓
	$m_{9,13}$	0	1	-	0	1	✓
	$m_{10,11}$	0	1	0	1	-	✓
$B_3(A_3 - A_4)$	$m_{13,29}$	-	1	1	0	1	✓



### Step 3: Identify all Primary Implicants

Group	Minterms	$X_4, X_3, X_2, X_1, X_0$	Used
C	$m_{1,5,9,13}$	0 - - 0 1	N/A
	$m_{1,9,5,13}$	0 - - 0 1	N/A
(B <sub>1</sub> -B <sub>2</sub> )	$m_{8,9,10,11}$	0 1 0 - -	N/A
	$m_{8,10,9,11}$	0 1 0 - -	N/A

$m_{1,5,9,13}$  and  $m_{8,9,10,11}$  are Primary Implicants.

In addition we have:  $m_{8,24}$ ;  $m_{16,18}$ ;  $m_{16,24}$ ;  $m_{5,7}$ ;  $m_{13,29}$ , as Primary Implicants.

### Step 4:

Primary Implicant	1	5	7	8	9	10	11	13	16	18	24	29	Is Essential Primary Implicant
$m_{1,5,9,13}$	⊗	X			X			X					Yes
$m_{8,9,10,11}$				X	X	⊗	⊗						Yes
$m_{8,24}$				X							X		No
$m_{16,18}$									X	⊗			Yes
$m_{16,24}$									X		X		No
$m_{5,7}$		X	⊗										Yes
$m_{13,29}$							X					⊗	Yes

### Step 5: Simplified Boolean Expression:

$$F(X_4, X_3, X_2, X_1, X_0) = (\neg X_4 \wedge \neg X_1 \wedge X_0) \vee (\neg X_4 \wedge X_3 \wedge \neg X_2) \vee (X_4 \wedge \neg X_3 \wedge \neg X_2 \wedge \neg X_0) \\ \vee (\neg X_4 \wedge \neg X_3 \wedge X_2 \wedge X_0) \vee (X_3 \wedge X_2 \wedge \neg X_1 \wedge X_0)$$

### Step 6: Calculate the Cost:

Original Function:  $12 \cdot 4 + 11 = 48 + 11 = 59$

New Function: 17