Date: 12/10/2023 ICS #5 Name: Flori Kusari

Problem: 5.1:

a) Find smallest and largest if b=3 and n=6

Total Number of Elements: $b^n = 3^6 = 729 \underset{\text{for "0"}}{\underline{-1}} \Rightarrow \frac{728}{2} = 364$

364 positive elements (not icluding 0)
0 takes 1 spot
364 negative elements

$\{+364 \ldots 0 \ldots -364\}$

$$\begin{array}{cccccc} 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 3^0 \end{array}$$

$364 \rightarrow 111111_{-3} \Rightarrow 364_{-10}$

For "-364" we revert and add "1" so:

$$\begin{array}{r} 111111 \\ 000001 \\ \hline 111112_{-3} \end{array}$$

$$\begin{array}{cccccc} 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 3^0 \end{array}$$

So "-364" $\Rightarrow 111112_{-3} \Rightarrow -364_{-10}$

b) for "-1":

First we find positive "1" which is: $\begin{array}{cccccc} 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 3^0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{array}$

Now to find the negative we use "$a_i' = (b-1) - a_i$" formula and then add "1":

$2-0=2$
$2-0=2$
$2-0=2$
$2-0=2$
$2-0=2$
$2-1=1$

$$\Rightarrow \begin{array}{r} 222221 \\ +000001 \\ \hline \boxed{222222}_{-3} \end{array}$$

which is "-1"

for "-99":

First we find "99" which is: $\overset{3^5}{0}\overset{3^4}{1}\overset{3^3}{0}\overset{3^2}{2}\overset{3^1}{0}\overset{3^0}{0}$

Now to find the negative of "99" we use "$a_i' = (b-1) - a_i$" and then we add "1":

$$\begin{aligned} 2-0&=2 \\ 2-1&=1 \\ 2-0&=2 \\ 2-2&=0 \\ 2-0&=2 \\ 2-0&=2 \end{aligned} \Longrightarrow \begin{array}{r} \overset{}{2}\overset{}{1}\overset{}{2}\overset{1}{0}\overset{1}{2}2 \\ \underline{000001} \\ \boxed{212100_3} \end{array}$$

which is "-99"

Problem 5.1: (c) is on the next page.

© Add "-1" and "-99" and express it back in decimal

"Over load" and is not counted.

$$
\begin{array}{r|cccccc|l}
 & 3^5 & 3^4 & 3^3 & 3^2 & 3^1 & 3^0 & \\
 & {}^1 & {}^1 & {}^1 & & & & \\
 & 2 & 2 & 2 & 2 & 2 & 2 & \rightarrow \text{"}-1\text{"}_{10} \\
+\ 1 & 2 & 1 & 2 & 1 & 0 & 0 & \rightarrow \text{"}-99\text{"}_{10} \\
\hline
 & 2 & 1 & 2 & 0 & 2 & 2_{3} & \rightarrow \text{"}-100\text{"}_{10}
\end{array}
$$

To check we convert it to positive and check if it is "100":

* We use "$a_i' = (b-1) - a_i$" :

$2-2=0$
$2-1=1$
$2-2=0$
$2-0=2$
$2-2=0$
$2-2=0$

$\Longrightarrow$ "010200_3"

Now we add "1":

$$
\begin{array}{r}
010200 \\
+\ 000001 \\
\hline
\boxed{010201_3}
\end{array}
$$

$0 \cdot 3^5 = 0$
$1 \cdot 3^4 = 81$
$0 \cdot 3^3 = 0$
$2 \cdot 3^2 = 18$
$0 \cdot 3^1 = 0$
$1 \cdot 3^0 = 1$

$\boxed{\text{Total}: 100_{10}}$

Hence it is proven that since the positive of "212022_3" is "010201_3" which is equal to "100_{10}" then it is also proven that "212022_3" is equal to its negative which is "-100_{10}".

a) 321.123

* Determine the sign bit (S): 321.123 is positive so $\Rightarrow S = 0$

* Convert integer part to binary: 321 $\Rightarrow$ binary

$321 \bmod 2 = 1 \Rightarrow 1$

$160 \bmod 2 = 0 \Rightarrow 01$

$80 \bmod 2 = 0 \Rightarrow 001$

$40 \bmod 2 = 0 \Rightarrow 0001$

$20 \bmod 2 = 0 \Rightarrow 00001$

$10 \bmod 2 = 0 \Rightarrow 000001$

$5 \bmod 2 = 1 \Rightarrow 1000001$

$2 \bmod 2 = 0 \Rightarrow 01000001$

$1 \bmod 2 = 1 \Rightarrow 101000001$

321 $\Rightarrow$ binary $\Rightarrow$ 101000001

* Convert the fractional part to binary: .123 $\Rightarrow$ binary

$0.123 \cdot 2 = 0.246 \Rightarrow 0$

$0.246 \cdot 2 = 0.492 \Rightarrow 00$

$0.492 \cdot 2 = 0.984 \Rightarrow 000$

$0.984 \cdot 2 = 1.968 \Rightarrow 0001$

$0.968 \cdot 2 = 1.936 \Rightarrow 00011$

$0.936 \cdot 2 = 1.872 \Rightarrow 000111$

$0.872 \cdot 2 = 1.744 \Rightarrow 0001111$

$0.744 \cdot 2 = 1.488 \Rightarrow 00011111$

$0.488 \cdot 2 = 0.976 \Rightarrow 000111110$

$0.976 \cdot 2 = 1.952 \Rightarrow 0001111101$

$0.952 \cdot 2 = 1.904 \Rightarrow 00011111011$

$0.904 \cdot 2 = 1.808 \Longrightarrow 0\,0\,0\,1\,1\,1\,1\,1\,0\,1\,1\,1$

$0.808 \cdot 2 = 1.616 \Longrightarrow 0\,0\,0\,1\,1\,1\,1\,1\,0\,1\,1\,1\,1$

$0.616 \cdot 2 = 1.232 \Longrightarrow 0\,0\,0\,1\,1\,1\,1\,1\,0\,1\,1\,1\,1\,1$

$0.232 \cdot 2 = 0.464 \Longrightarrow 0\,0\,0\,1\,1\,1\,1\,1\,0\,1\,1\,1\,1\,1\,0$

This is close enough with "single precision floating point number" for this case.

The binary representation of the fraction part is "0 0 0 1 11110111110"

* Normalize the Binary Representation:

$$\underbrace{1.0100000001}_{\text{Move by an exponent of } 8}.000111110111110$$

Move by an exponent of 8

$$1.010000001000111110111110$$

* Calculate the exponent and turn it to binary

$8 + 127 = 135$

Bias 127

135 mod 2 = 1 ⟹	1
67 mod 2 = 1 ⟹	11
33 mod 2 = 1 ⟹	111
16 mod 2 = 0 ⟹	0111
8 mod 2 = 0 ⟹	00111
4 mod 2 = 0 ⟹	000111
2 mod 2 = 0 ⟹	0000111
1 mod 2 = 1 ⟹	10000111

Binary Representation of the exponent is 10000111

* Now that we have everything we can:

End of Problem 5.2 Part a

0	1 2 3 4 5 6 7 8	9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
S	exponent	mantissa (23 bits)
0	1 0 0 0 0 1 1 1	0 1 0 0 0 0 0 0 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 0

Problem 5.2:

(b) The value actually stored is:

0 | 1 0 0 0 0 1 1 1 | 0 1 0 0 0 0 0 1 0 0 0 1 1 1 1 1 0 1 1 1 1 1 0 |

mantissa

sign is positive

exponent is 135

1.25438666343688.......

Value Entered: 321.123

Value Actually Stored: 321.12298583984375

Absolute Error: 321.123 - Value Actually Stored = -0.00001416015625

Problem 5.3 is submitted separately as a text file. (as requested).

THE END