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Problem 7.1: Not-or is a universal boolean function

$$\text{AND}(\wedge):\ X \wedge Y \equiv (X \barwedge X)\ \bar{\vee}\ (Y \bar{\vee} \bar{Y})\ *$$

$$\text{OR}(\vee):\ X \vee Y \equiv (X \bar{\vee} Y)\ \bar{\vee}\ (X \bar{\vee} Y)\ **$$

$$\text{NOT}(\neg):\ \neg X \equiv X \bar{\vee} X\ *$$

** De Morgan's Law: The negation of a conjuction (AND) is equivalent to the disjunction (OR) of the negations of the individual terms.

* Using the logic from the "Idempotent Law" logic which states that a variable combined with itself using a particular operation leads in the return of the original variable.

Truth Table:

X	Y	AND($\bar{\vee}$)	OR($\bar{\vee}$)	NOT($\bar{\vee}$)	AND($\wedge$)	OR($\vee$)	NOT($\neg$)
0	0	0	0	1	0	0	1
0	1	0	1	1	0	1	1
1	0	0	1	0	0	1	0
1	1	1	1	0	1	1	0

and De Morgan's Law.

Since the truth tables give the same correct output for the inputs for both equivalent boolean functions for the original and the NOT-OR version of the functions combined with the logic followed by the "Idempotent Law" the boolean function NOT-OR is thus proven to be a universal boolean function by proving that it can implement the other classical universal Boolean functions $\wedge, \vee, \neg$. □

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Problem 7.2: Simplify a boolean expression using algebraic equivalence laws

$$F(X,Y,Z) = (((X \land Y) \lor (X \land \neg Z)) \lor (Z \land \neg 0))$$

It is given that the above expression is equivalent to $G(X,Y,Z) = (X \lor Z)$

X	Y	Z	$(((X \land Y) \lor (X \land \neg Z)) \lor (Z \land \neg 0))$	$X \lor Z$
0	0	0	0	0
0	0	1	1	1
0	1	0	0	0
0	1	1	1	1
1	0	0	1	1
1	0	1	1	1
1	1	0	1	1
1	1	1	1	1

With proof tables this is proven now we need to do so by applying the boolean equivalence laws.

$(((X \land Y) \lor (X \land \neg Z)) \lor (Z \land \neg 0))$ Original Expression

Separate into $((X \land Y) \lor (X \land \neg Z))$ and $(Z \land \neg 0)$ keep in mind the sign "$\lor$" between them.

$(Z \land \neg 0) \longrightarrow (Z \land 1)$ *Basic Negation Laws

$(Z \land 1) \longrightarrow Z$ *Identity Law.

$(((X \land Y) \lor (X \land \neg Z)) \lor (Z))$ *New Formula

$(((X \land Y) \lor (X \land (X \lor Y)) \lor Z)$ *Simplification using Negation Laws

$(((X \land Y) \lor (X \land X)) \lor Z)$ *Combination of Distributive and Indempotent Law and Basic Law of OR "$\lor$" operator.

$(((X \land Y) \lor X) \lor Z)$ * Indempotent Law

$\swarrow$

$(X \lor Z)$ * Absorption Law

Thus proved that $F(X, Y, Z) = (((X \land Y) \lor (X \land \neg Z)) \lor (Z \land \neg 0))$ can be turned into the $G(X, Y, Z) = (X \lor Z)$. $\square$

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* 7.3 has been submitted as a text file as requested. *

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