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ICS #5

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Problem: 5.1:

a) Find smallest and largest if $b=3$ and $n=6$

$$\text{Total Number of Elements: } b^n = 3^6 = 729 - 1 \Rightarrow \frac{728}{2} = 364$$

↑
for "0"

364 positive elements (not including 0)

0 takes 1 spot

{+364 ... 0 ... -364}

364 negative elements

$$3^5 \ 3^4 \ 3^3 \ 3^2 \ 3^1 \ 3^0$$

$$364 \rightarrow 1 \ 1 \ 1 \ 1 \ 1 \ 1_{-3} \Rightarrow 364_{-10}$$

For "-364" we revert and add "1" so:

$$\begin{array}{r} 1 \ 1 \ 1 \ 1 \ 1 \\ 0 \ 0 \ 0 \ 0 \ 1 \\ \hline 1 \ 1 \ 1 \ 1 \ 1_2 \end{array}_{-3}$$

$$3^5 \ 3^4 \ 3^3 \ 3^2 \ 3^1 \ 3^0$$

$$\text{So } "-364" \Rightarrow 1 \ 1 \ 1 \ 1 \ 1 \ 1_{-3} \Rightarrow -364_{-10}$$

b) for "-1":

$$3^5 \ 3^4 \ 3^3 \ 3^2 \ 3^1 \ 3^0$$

First we find positive "1" which is: 0 0 0 0 0 1

Now to find the negative we use " $\alpha'_i = (b-1) - \alpha_i$ " formula and then

$$\text{add "1": } 2 - 0 = 2$$

$$2 - 0 = 2$$

$$2 - 0 = 2$$

$$2 - 0 = 2$$

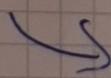
$$2 - 0 = 2$$

$$2 - 1 = 1$$

$$\Rightarrow 2 \ 2 \ 2 \ 2 \ 2 \ 1$$

$$\begin{array}{r} 0 \ 0 \ 0 \ 0 \ 0 \ 1 \\ \boxed{2 \ 2 \ 2 \ 2 \ 2 \ 2} \\ \hline -3 \end{array}$$

which is "-1"



for "-gg":

$$3^5 \ 3^4 \ 3^3 \ 3^2 \ 3^1 \ 3^0$$

First we find "gg" which is: 010200

Now to find the negative of "gg" we use " $a'_i = (b-1) - a_i$ " and then we add "1": $2 - 0 = 2$

$$2 - 1 = 1$$

$$2 - 0 = 2 \Rightarrow 2\overset{1}{1}2022$$

$$2 - 2 = 0$$

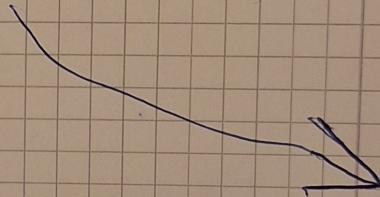
$$\begin{array}{r} 000001 \\ 212100 - 3 \\ \hline \end{array}$$

$$2 - 0 = 2$$

$$2 - 0 = 2$$

which is "-gg"

Problem 5.1: ⑦ is on the next page.



⑥ Add " -1 " and " -99 " and express it back in decimal

"Overload"
and is not counted.

$$\begin{array}{r}
 3^5 3^4 3^3 3^2 3^1 3^0 \\
 + 1 \\
 \hline
 2 2 2 2 2 2 \rightarrow "-1"_{-10} \\
 2 1 2 1 0 0 \rightarrow "-99"_{-10} \\
 \hline
 2 1 \underline{2} 0 2 2 \underline{-3} \rightarrow "-100"_{-10}
 \end{array}$$

To check we convert it to positive and check if it is "100":

* We use " $a_i' = (b-1) \cdot a_i$ "; $2-2=0$

$$2-1=1$$

$$2-2=0$$

$$2-0=2$$

$$2-2=0$$

$$2-2=0$$

$$\Rightarrow 010200_3$$

Now we add "1":

$$\begin{array}{r}
 010200 \\
 + 000001 \\
 \hline
 010201 \underline{-3}
 \end{array}$$

$$0 \cdot 3^5 = 0$$

$$1 \cdot 3^4 = 81$$

$$0 \cdot 3^3 = 0$$

$$2 \cdot 3^2 = 18$$

$$0 \cdot 3^1 = 0$$

$$1 \cdot 3^0 = 1$$

$$\boxed{\text{Total: } 100_{-10}}$$

Hence it is proven that since the positive
of 212022_{-3} is 010201_{-3}

which is equal to 100_{-10} then it is

also proven that 212022_{-3} is
equal to its negative which is -100_{-10} .

a) 321.123

* Determine the sign bit (S): 321.123 is positive so $\Rightarrow S = 0$

* Convert integer part to binary: 321 \Rightarrow binary

$$321 \bmod 2 = 1 \Rightarrow 1$$

$$160 \bmod 2 = 0 \Rightarrow 01$$

$$80 \bmod 2 = 0 \Rightarrow 001$$

$$40 \bmod 2 = 0 \Rightarrow 0001$$

$$20 \bmod 2 = 0 \Rightarrow 00001$$

$$10 \bmod 2 = 0 \Rightarrow 000001$$

$$5 \bmod 2 = 1 \Rightarrow 1000001$$

$$2 \bmod 2 = 0 \Rightarrow 01000001$$

$$1 \bmod 2 = 1 \Rightarrow 101000001$$

$$321 \Rightarrow \text{binary} \Rightarrow 101000001$$

* Convert the fractional part to binary: .123 \Rightarrow binary

$$0.123 \cdot 2 = 0.246 \Rightarrow 0$$

$$0.246 \cdot 2 = 0.492 \Rightarrow 00$$

$$0.492 \cdot 2 = 0.984 \Rightarrow 000$$

$$0.984 \cdot 2 = 1.968 \Rightarrow 0001$$

$$0.968 \cdot 2 = 1.936 \Rightarrow 00011$$

$$0.936 \cdot 2 = 1.872 \Rightarrow 0001111$$

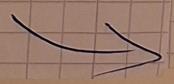
$$0.872 \cdot 2 = 1.744 \Rightarrow 00011111$$

$$0.744 \cdot 2 = 1.488 \Rightarrow 000111111$$

$$0.488 \cdot 2 = 0.976 \Rightarrow 0001111110$$

$$0.976 \cdot 2 = 1.952 \Rightarrow 00011111101$$

$$0.952 \cdot 2 = 1.904 \Rightarrow 000111111011$$



$$0.904 \cdot 2 = 1.808 \Rightarrow 00011110111$$

$$0.808 \cdot 2 = 1.616 \Rightarrow 00011110111$$

$$0.616 \cdot 2 = 1.232 \Rightarrow 00011110111$$

$$0.232 \cdot 2 = 0.464 \Rightarrow 00011110111$$

This is close enough with "single precision floating point number" for this case.

The binary representation of the fraction part is "0 001111011110"

* Normalize the Binary Representation:

101000001.0001111011110
↑
Move by an exponent of 8

1.010000010001111011110

* Calculate the exponent and turn it to binary

$$8 + 127 = 135$$

↑
Bias 127

$$135 \bmod 2 = 1 \Rightarrow 1$$

$$67 \bmod 2 = 1 \Rightarrow 11$$

$$33 \bmod 2 = 1 \Rightarrow 111$$

$$16 \bmod 2 = 0 \Rightarrow 0111$$

$$8 \bmod 2 = 0 \Rightarrow 00111$$

$$4 \bmod 2 = 0 \Rightarrow 000111$$

$$2 \bmod 2 = 0 \Rightarrow 0000111$$

$$1 \bmod 2 = 1 \Rightarrow 10000111$$

End of Problem 5.2

Binary Representation of the exponent is 10000111

Part a

* Now that we have everything we can:

0 1 2 3 4 5 6 7 8 | 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
↓ s | exponent | mantissa (23 bits) |
0 1 0 0 0 0 1 1 | 0 1 0 0 0 0 0 1 0 0 0 1 1 1 1 0 1 1 1 1 0 1

Problem 5.2:

⑥ The value actually stored is:

Value Entered: 321.123

Value Actually Stored: 321.12298583984375

Absolute Error : $321.123 - \text{Value Actually Stored} = 0.0001416015625$

Problem 5.3 is submitted separately as a text file. (as requested).

THE END