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Problem 6.1:

$$(k\cdot 2^0)+(k\cdot 2^1)+(k+2^2)=1$$

(a) $k(1+2+4)=1$

$7k=1$

$\boxed{k=1/7}$

(b) $P[X \text{ is even}] \Rightarrow 0$ or 2

$$\Rightarrow \frac{1}{7}\cdot 2^0+\frac{1}{7}\cdot 2^2=\frac{1}{7}+\frac{4}{7}=\boxed{\frac{5}{7}}$$

(c) Determining $E(X)$ and $Var[X]$

$$E[X]=0\cdot\left(\frac{1}{7}\cdot 2^0\right)+1\left(\frac{1}{7}\cdot 2^1\right)+2\left(\frac{1}{7}\cdot 2^2\right)$$

$$=0+\frac{2}{7}+\frac{8}{7}$$

$$=\frac{10}{7}$$

$$E[X^2]=0^2\cdot\left(\frac{1}{7}\cdot 2^0\right)+1^2\left(\frac{1}{7}+2^1\right)+2^2\left(\frac{1}{7}\cdot 2^2\right)$$

$$=0+\frac{2}{7}+\frac{16}{7}$$

$$=\frac{18}{7}$$

$$Var(X)=\frac{18}{7}-\left(\frac{10}{7}\right)^2=\frac{18}{7}-\frac{100}{49}=\boxed{\frac{26}{49}}$$

Problem 6.2: Given $E[X]=10$; $Var[X]=15$

(a) $E[X^2]=Var(X)+E[X]^2$ → Comes from $Var[X]=E[X^2]-E[X]^2$

$=15+100$

$\boxed{=115}$

(b) $E[2X]=2\cdot E[X]=2\cdot 10=\boxed{20}$

(c) $Var[2X+1]=2^2\cdot Var[X]=4\cdot 15=\boxed{60}$

Problem 6.3: Value of X for each possible value.

$P[X=1] \to 9$ Possibilities $\to P[X=1] = 9/9000 = 1/1000$

$P[X=2] \to 108$ Possibilities $\to P[X=2] = 108/9000 = 3/250$

$P[X=3] \to 336$ Possibilities $\to P[X=3] = 336/9000 = 28/750$

$P[X=4] \to 3024$ Possibilities $\to P[X=4] = 3024/9000 = 168/250$

$E[X] = 1 \cdot P[X=1] + 2 \cdot P[X=2] + 3 \cdot P[X=3] + 4 \cdot P[X=4]$

$= 0.001 + 0.024 + 0.112 + 1.344 = 1.481$

$\boxed{E[X] \approx 1.481}$

Problem 6.4:

Geometric Distribution shows that $P[HH] = 0.25$ or $1/4$

$E[X]$ for Geometric Distribution is $E[X] = \frac{1}{P[HH]} = \frac{1}{1/4} = 4$

$\boxed{E[X] = 4}$

Problem 6.5:

(a) $E[N] = \underset{A}{\text{Nr. of Initial Block Tests}} + \underset{B}{\text{Expected Number of Tests in Positive Blocks}}$

A: $100/m$

B: $100 \cdot P[\text{At least one person in a block is positive}] = 100(1 - P(\text{All people are negative}))$

$= 100(1 - (0.99)^m)$

$E[N] = 100/m + 100(1 - (0.99)^m)$

(b) According to GeoGebra the minimization point is 11 for integers with value of "19.557083665".

Analysing the graph we find that the minimum is close to 10.516237300 with value of $\boxed{19.5389077}$.