Name: Flori Kusari Probability and Random Processes Date: 14/09/2024

① $A = 30$ $O = 40$

$\frac{1}{3}A = 10$ $\frac{1}{2}O = 20$

$\#\Omega = 70$

$P[S] = \frac{30}{70} = \boxed{\frac{3}{7}}$

sweet fruits $= 30$

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② Top & Bottom Corner Possibilities : $4 \cdot 3 = 12$
Middle : 8
Others : $4 \cdot 5 = 20$

Total : 40

All Possibilities : $\boxed{9!}$

Order does matter so : $\frac{7!}{9!}$ but we must multiply it by the possibilities.

$$\frac{40 \cdot 7!}{9!} = \frac{40}{9 \cdot 8} = \frac{40}{98} = \frac{8 \cdot 5}{9 \cdot 8} = \boxed{\frac{5}{9}} \leftarrow \text{Answer}$$

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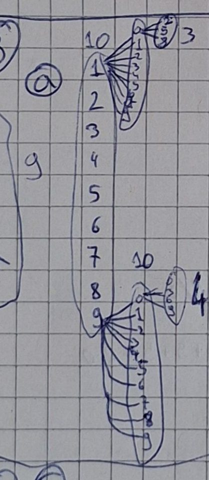
③ ⓐ



* We don't start with "0" because it isn't 3-digit if it has a "0" infront *

For first number already divisible by 3 we have : $4 \times 10 = 40$
For other starting numbers we have : $3 \times 10 = 30$

So : $30 + 30 + 40 + 30 + 30 + 40 + 30 + 30 + 40 = \boxed{300}$ Favorable outcomes

Total outcomes : $999 - 100 + 1 = \boxed{900}$ Total outcomes

$P[A] = \frac{300}{900} = \boxed{\frac{1}{3}}$

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③ ⓑ Total number of outcomes is $\boxed{900}$ Same as last question

First perfect square is 100 which is the perfect square of 10
The largest square possible is 961 which is the perfect square of 31.

Favorable Outcomes : $31 - 10 + 1 = \boxed{22}$

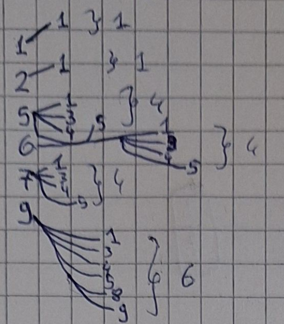
$P[B] = \frac{22}{900} = \boxed{\frac{11}{450}}$

4

A = 1, 2, 5, 6, 7, 9
B = 1, 3, 4, 5, 8, 9
C = 2, 3, 4, 6, 7, 8

Show that $A \geq B$ most of the time so: $P[A \geq B] > \frac{1}{2}$

Total outcomes: $6^2 = \boxed{36}$



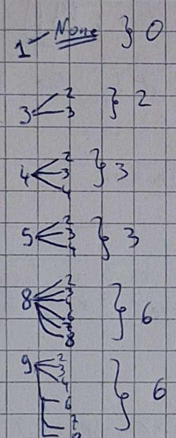
Favorable Outcomes: $1+1+4+4+4+6 = 20$

$P[A \geq B] = \frac{20}{36} = \boxed{\frac{5}{9}} \; \boxed{> 1}$

$\boxed{\text{A beats B}}$ ✓

Show that $B \geq C$ most of the time so: $P[B \geq C] > 1/2$

Total outcomes: $6^2 = 36$

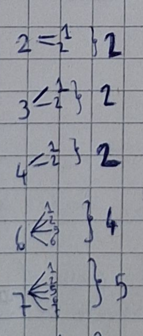


Favorable outcomes: $0+2+3+3+6+6 = 20$

$P[B \geq C] = \frac{20}{36} = \boxed{\frac{5}{9}} \; \boxed{> 1}$

$\boxed{\text{B beats C}}$ ✓

Show that $C \geq A$ most of the time so: $P[C \geq A] > 1/2$



Favorable outcomes: $2+2+2+4+5+5 = 20$

$P[C \geq A] = \frac{20}{36} = \boxed{\frac{5}{9}} \; \boxed{> 1}$

C beats A

⑤ Total subsets of $E = 2^{10} = 1024$

ⓐ M has at least 5 elements so 5, 6, 7, 8, 9, 10

$\binom{10}{5} = 252 \quad \binom{10}{6} = 210 \quad \binom{10}{7} = 120 \quad \binom{10}{8} = 45 \quad \binom{10}{9} = 10 \quad \binom{10}{10} = 1$

Compute using $\boxed{\frac{n!}{k!(n-k)!}}$

Favorable Out come: $252 + 210 + 120 + 45 + 10 + 1 = \boxed{638}$

$P[a] = \frac{638}{1024} = \boxed{\frac{319}{512}}$

ⓑ Probability of $M \cap F$ having 2 elements and $M \cap F^c$ has 3 elements

$M \cap F$ means elements possibly in M that aren't in F so {5, 6, 7, 8, 9, 10} from which we need it to have 2 elements so: $\binom{6}{2}$

$M \cap F^c$ means the elements possibly in M that are in F so {1, 2, 3, 4} from which we need it to have 3 elements: $\binom{4}{3}$

$$\binom{6}{2} \cdot \binom{4}{3} = 15 \cdot 4 = 60$$

$P[b] = \frac{60}{1024} = \boxed{\frac{15}{256}}$

ⓒ Probability of $M \cap F$ having at most 2 elements means

$\binom{6}{0} = 1 + \binom{6}{1} = 6 + \binom{6}{2} = 15$

Favorable outcome $= 1 + 6 + 15 = 22$

Now for the possible elements from $F = 2^4 = \boxed{16}$

$$22 \cdot 16 = 352$$

$P[c] = \frac{352}{1024} = \boxed{\frac{11}{32}}$

THE END