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Problem 2.1:

$\# \Omega: 6^6 = 46656$

Favorable outcomes: $\binom{6}{3} = \frac{6!}{3! \times (6-3)!} = 20$

We need 3 distinct ones

For arrangement: $\frac{6!}{2! \cdot 2! \cdot 2!} = \frac{720}{8} = \boxed{90}$

used tool for this simplification

$$\frac{90 \cdot 20}{6^6} = \frac{1800}{46656} = \boxed{\frac{25}{648}}$$

Problem 2.2:

A → Not Born on Sunday

B → Born on Sunday

(a)

$P[A] = 1 - \frac{1}{7} = \boxed{\frac{6}{7}}$

Now for 6 people: $\boxed{\left(\frac{6}{7}\right)^6}$

For at least 1 born on Sunday: $P[B] = 1 - \left(\frac{6}{7}\right)^6 \approx 0.6651$

(b) Probability for weekend is $2/7$.

$P[A] = \left(\frac{2}{7}\right)^6 = \frac{64}{117649} \approx \boxed{0.00544}$

(c) We need to find probability of the opposite then subtract from 1.

$P[B] = \frac{7}{7} \cdot \frac{6}{7} \cdot \frac{5}{7} \cdot \frac{4}{7} \cdot \frac{3}{7} \cdot \frac{2}{7}$ Event B

$= \frac{5040}{117649} \approx 0.0429$

$P[A] = 1 - P[B] = \boxed{0.9571}$

→

Problem 2.3:

Total possibilities: $6^3 = 216$

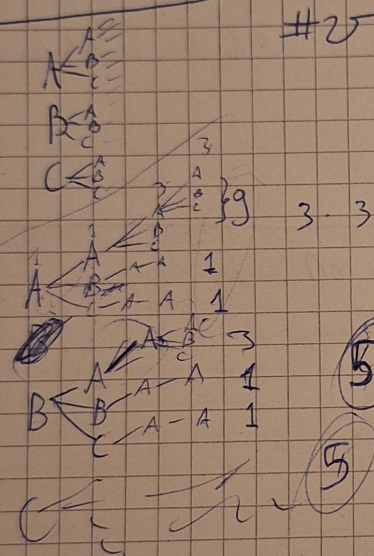
Non winning probability happens when: First $\geq$ second $\geq$ third

so [redacted] $\binom{8}{3} = 56$

$$P[A] = 1 - P[B] = 1 - \frac{56}{216} = \frac{160}{216} = \boxed{\frac{20}{27}}$$

Problem 2.4

$\# \Omega = 3^4 = 81$



$3 \cdot 3 \cdot 1 \cdot 1 = 9$

A → starting → 11
B → starting → 5
[redacted] C → starting → 5

Favorable outcomes:
$1 + 1 + 9 + 1 + 1 + 3 + 1 + 1 + 3 = \boxed{21}$

$$P[A] = \frac{21}{81} = \frac{7}{27} \approx \boxed{0.2592}$$

Problem 2.5:

ⓐ Prove that $P(A \cup B) \leq P(A) + P(B)$

$$P(A \cup B) = P(A) + P[B] - P[A \cap B]$$

$$\cancel{P(A)} + \cancel{P[B]} - P[A \cap B] \leq \cancel{P(A)} + \cancel{P(B)}$$

$$-P[A \cap B] \leq 0$$

Since Probability cannot be negative we know this is proved true as $P[A \cap B] \geq 0$.

6) $P[A] + P[B] \leq 1 + P[A \cap B]$

$P[A] + P[B] - P[A \cap B] \leq 1$

$P[A \cup B] \leq 1$

Since all probabilities obey this rule as long as it is a rational event we can assume this is correct and proven true

THE END