Constructor University
Assignment 5

14 October 2024, 11:59am

Probability and Random Processes

(**5.1**) A pair of fair dice are tossed. Let X denote the sum and Y be the minimum of the outcomes. For instance, if the outcomes are 1 and 4 then $X = 5$ and $Y = 1$. Assume also that every outcome (i, j) of the pair of dice occurs with probability $1/36$. Find the probability mass functions of X and the probability mass function of Y.

(**5.2**) Suppose X is a discrete random variable taking values $x \in \{1, 2, 3, 4\}$ such that

$$\mathbb{P}[X = x] = kx^2$$

for some k.

(a) Find the value of k

(b) Determine $\mathbb{P}[X > 2]$.

(**5.3**) Suppose X is a random variable with Binomial distribution with parameter (n, p) where $n = 4$. Show that the probability that X is even is given by $1 - 4p + 12p^2 - 16p^3 + 8p^4$.

(**5.4**) A company produces light bulbs that are packed into boxes of 100. A study shows that 1% of the bulbs are defective. Assume that the distribution of defective bulbs has a Poisson distribution. Suppose that random box is chosen.

(a) Find the probability of the event that it contains at most two defective bulb.

(b) Find the probability of the event that it contains at least two defective bulb.

(**5.5**) There are 10 problems in the final exam of a probability course. Each correct answer is worth 4 point and each incorrect answer is penalized with -1 point. Questions that are not answered receive zero points. Let $1 \leq m \leq 10$. A student answers questions from 1 to m randomly and leave the rest of questions unanswered. Suppose X be the number of questions answered correctly and G denote the grade of the student.

(a) Show that $G = 5X - m$.

(b)

$$\mathbb{P}[G > 0] = \begin{cases} 1 - (3/4)^m & \text{if } 1 \leq m \leq 4 \\ 1 - (3/4)^m - m(1/4)(3/4)^{m-1} & \text{if } 4 < m \leq 9 \\ 1 - (3/4)^{10} - (10/4)(3/4)^9 - (45/16)(3/4)^8 & \text{if } m = 10 \end{cases}$$

(c) (Bonus) Find the value of m for which $\mathbb{P}[G > 0]$ is maximized.