Name: Flori Kusari Problem Sheet #4 Date: 06/10/2024

① $P[A] = ?$ A → Probability of first 4 cards being Red.

Red = 26

Deck = 52

$$P[A] = \frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50} \cdot \frac{23}{49} = \frac{13800}{249900} = \boxed{\frac{46}{833}} \approx 0.055221$$

$\frac{26}{52} \rightarrow \frac{1}{2}$

② Hat = 100 coins $H_F = 80$ $H_D = 20$

ⓐ $P[A] = \frac{1}{16} \cdot \frac{80}{100} + \frac{1}{1} \cdot \frac{20}{100} = \frac{1}{20} + \frac{2}{10} = \boxed{\frac{5}{20} = 0.25}$

$\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$ Possibility of 4 Heads Normal Coin

# of Normal Coins

Possibility of Double-Headed being Heads

Of Double-Headed Coins

ⓑ B - Final Solution

H → 4 Heads

C → Probability of Double-Headed

$$P[B] = \frac{P(H|C) \cdot P(C)}{P(H)} = \frac{P(C)}{P(H)} = \frac{\frac{20}{100}}{5/20} = \frac{20}{100} \cdot \frac{20}{5} = \frac{400}{500} = \boxed{\frac{4}{5} = 0.8}$$

Possibility of Coin being Heads 4 times given that it is Double Headed which is always.

$B = (C|H)$

↓

By definition

③ Spam = 20% % Spam containing "winner" = 25% Non-Spam containing "Winner" = 10%

ⓐ Percentage of Emails containing Winner

Spam "Winner"

Spam Total

Winner

Not spam

$P[A] = P[SW] \cdot P[S] + P[WNS] \cdot P[NS] = 0.25 \cdot 0.2 + 0.1 \cdot 0.8 = 0.05 + 0.08 = \boxed{0.13}$

$\boxed{13\%}$

(b) $P[S/A] = \frac{P[A/S]\,P(S)}{P(A)} = \frac{0.25 \cdot 0.20}{0.13} = \frac{0.05}{0.13} = 0.384615$

S: Spam; A: Contains "Winner" from part (a)

given

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(4) n in {1, 2, ..., 120}. Suppose A_k denote the number n is divisible by k.

(a) Find $A_1, A_2, A_3, A_4, A_4, A_5$

$A_1 = 1$ because every number is divisible by 1 //

$A_2 = 1/2$ since 120 is divisible by 2 all numbers in between hold the same idea so $\frac{60}{120}$

$A_3 = 1/3$ same logic but $40/120$

$A_4 = 1/4$ same logic but $30/120$

$A_5 = 1/5$ same logic but $24/120$

(b) For the following to be true: A_2, A_3, A_5 are independent.

we must prove: $A_2 \cap A_3 = A_2 \cdot A_3$

$A_3 \cap A_5 = A_3 \cdot A_5$

$A_2 \cap A_5 = A_2 \cdot A_5$

$A_2 \cap A_3 \cap A_5 = A_2 \cdot A_3 \cdot A_5$

$\boxed{A_2 \cap A_3 = A_2 \cdot A_3}$: $A_2 \cap A_3$ => Divisible by both so anything divisible by 6 should do the trick.

$\frac{120}{6} = \boxed{20}$ $\frac{20}{120} = \boxed{\frac{1}{6}} = A_6$ $\frac{1}{6} = \frac{1}{2} \cdot \frac{1}{3}$

$\boxed{\frac{1}{6} = \frac{1}{6}}$ // Independent

$A_3 \cap A_5 = A_3 \cdot A_5$: Least Common Multiple is 15

$120 \div 15 = 8$ $\quad \frac{8}{120} = \frac{1}{15} = A_{15}$ $\quad A_{15} = A_3 \cdot A_5$

$$\frac{1}{15} = \frac{1}{3} \cdot \frac{1}{5}$$

$$\boxed{\frac{1}{15} = \frac{1}{15}}$$ // Independent

$A_2 \cap A_5 = A_2 \cdot A_5$: Least Common Multiple is 10

$A_{10} = \frac{1}{10}$ $\quad A_{10} = A_2 \cdot A_5$

$$\frac{1}{10} = \frac{1}{2} \cdot \frac{1}{5}$$

$$\boxed{\frac{1}{10} = \frac{1}{10}}$$ // Independent

$A_2 \cap A_3 \cap A_5 = A_2 \cdot A_3 \cdot A_5$: Least Common Multiple is 30.

$\frac{120}{30} = 4$ $\quad \frac{4}{120} = \frac{1}{30}$ $\quad A_{30} = A_2 \cdot A_3 \cdot A_5$

$$\frac{1}{30} = \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{5}$$

$$\boxed{\frac{1}{30} = \frac{1}{30}}$$ // Independent Fully

(c) $A_4 \cap A_6 = A_4 \cdot A_6$??? Independent ???

Least Common Multiple = 12

$\frac{120}{12} = 10$ $\quad \frac{10}{120} = \frac{1}{12}$ $\quad A_{12} = A_4 \cdot A_6$

$$\frac{1}{12} = \frac{1}{4} \cdot \frac{1}{6}$$

$$\boxed{\frac{1}{12} \neq \frac{1}{24}}$$ XX Not Independent

(5) Suppose A, B, C are independent and $P[A] = P[B] = P[C] = \frac{1}{3}$.
Compute $P[A \cup B \cup C]$.

3 Event Union:

$$P[A \cup B \cup C] = P[A] + P[B] + P[C] - \overset{\frac{1}{3}\cdot\frac{1}{3}}{\overset{\uparrow}{P[A \cap B]}} - \overset{\frac{1}{3}\cdot\frac{1}{3}}{\overset{\uparrow}{P[B \cap C]}} - \overset{\frac{1}{3}\cdot\frac{1}{3}}{\overset{\uparrow}{P[A \cap C]}} + \overset{\frac{1}{3}\cdot\frac{1}{3}\cdot\frac{1}{3}}{\overset{\uparrow}{P[A \cap B \cap C]}}$$

$$P[A \cup B \cup C] = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} + \frac{1}{27}$$

$$P[A \cup B \cup C] = \frac{9 + 9 + \cancel{9} \cancel{-3-3-3} + 1}{27} = \boxed{\frac{19}{27}} \approx 0.703\overline{703}$$

THE END