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多传感器融合定位
第九章作业思路

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纲要

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- 及格要求思路提示
- 良好要求思路提示
- 优秀要求思路提示

及格要求

目标：推导imu预积分雅可比

残差定义：

与 VINS-MONO 代码相同，红色表示预积分量

$$
\begin{bmatrix} \mathbf{r}_p \\ \mathbf{r}_q \\ \mathbf{r}_v \\ \mathbf{r}_{b_a} \\ \mathbf{r}_{b_g} \end{bmatrix} = \begin{bmatrix} \mathbf{q}_{wb_i}^{-1}(\mathbf{p}_{wb_j} - \mathbf{p}_{wb_i} - \mathbf{v}_i^w \Delta t + \frac{1}{2}\mathbf{g}^w \Delta t^2 - \alpha_{b_j}^{b_i}) \\ 2[\mathbf{q}_{b_ib_j}^{-1} \otimes (\mathbf{q}_{wb_i}^{-1} \otimes \mathbf{q}_{wb_j})]_{xyz} \\ \mathbf{q}_{wb_i}^{-1}(\mathbf{v}_j^w - \mathbf{v}_i^w + \mathbf{g}^w \Delta t) - \beta_{b_j}^{b_i} \\ \mathbf{b}_j^a - \mathbf{b}_i^a \\ \mathbf{b}_j^g - \mathbf{b}_i^g \end{bmatrix}
$$

及格要求

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位置残差相对于状态量的雅可比:

$$\frac{\partial \mathbf{r}_p}{\partial \delta \mathbf{p}_i} = -\mathbf{R}_{b_i}^{w\top} \qquad \frac{\partial \mathbf{r}_p}{\partial \delta \mathbf{p}_j} = \mathbf{R}_{b_i}^{w\top}$$

$$\frac{\partial \mathbf{r}_p}{\partial \delta \boldsymbol{\theta}_i} = \left(\mathbf{R}_{b_i}^{w\top}(\mathbf{p}_{b_j}^{w} - \mathbf{p}_{b_i}^{w} - \mathbf{v}_i^{w}\Delta t + \frac{1}{2}\mathbf{g}^{w}\Delta t^2)\right)^{\wedge} \qquad \frac{\partial \mathbf{r}_p}{\partial \delta \boldsymbol{\theta}_j} = 0$$

$$\frac{\partial \mathbf{r}_p}{\partial \delta \mathbf{v}_i} = -\mathbf{R}_{b_i}^{w\top}\Delta t \qquad \frac{\partial \mathbf{r}_p}{\partial \delta \mathbf{v}_j} = 0$$

$$\frac{\partial \mathbf{r}_p}{\partial \delta \tilde{\mathbf{b}}_{a,i}} = -\mathbf{J}_{b_i^a}^{\alpha} \qquad \frac{\partial \mathbf{r}_p}{\partial \delta \tilde{\mathbf{b}}_{a,j}} = 0$$

$$\frac{\partial \mathbf{r}_p}{\partial \delta \tilde{\mathbf{b}}_{g,i}} = -\mathbf{J}_{b_i^g}^{\alpha} \qquad \frac{\partial \mathbf{r}_p}{\partial \delta \tilde{\mathbf{b}}_{g,j}} = 0$$

及格要求

速度残差相对于状态量的雅可比：

$$\frac{\partial \mathbf{r}_v}{\partial \delta \mathbf{p}_i} = 0 \qquad \frac{\partial \mathbf{r}_v}{\partial \delta \mathbf{p}_j} = 0$$

$$\frac{\partial \mathbf{r}_v}{\partial \delta \boldsymbol{\theta}_i} = \left(\mathbf{R}_{b_i}^{w}{}^{\top}(\mathbf{v}_j^w - \mathbf{v}_i^w + \mathbf{g}^w \Delta t)\right)^{\wedge} \qquad \frac{\partial \mathbf{r}_v}{\partial \delta \boldsymbol{\theta}_j} = 0$$

$$\frac{\partial \mathbf{r}_v}{\partial \delta \mathbf{v}_i} = -\mathbf{R}_{b_i}^{w}{}^{\top} \qquad \frac{\partial \mathbf{r}_v}{\partial \delta \mathbf{v}_j} = \mathbf{R}_{b_i}^{w}{}^{\top}$$

$$\frac{\partial \mathbf{r}_v}{\partial \delta \tilde{\mathbf{b}}_{a,i}} = -\mathbf{J}_{b_i^a}^{\beta} \qquad \frac{\partial \mathbf{r}_v}{\partial \delta \tilde{\mathbf{b}}_{a,j}} = 0$$

$$\frac{\partial \mathbf{r}_v}{\partial \delta \tilde{\mathbf{b}}_{g,i}} = -\mathbf{J}_{b_i^g}^{\beta} \qquad \frac{\partial \mathbf{r}_v}{\partial \delta \tilde{\mathbf{b}}_{g,j}} = 0$$

及格要求

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姿态残差相对于状态量的雅可比:

$$\frac{\partial \mathbf{r}_q}{\partial \delta \mathbf{p}_i} = 0 \qquad \frac{\partial \mathbf{r}_q}{\partial \delta \mathbf{p}_j} = 0$$

$$\frac{\partial \mathbf{r}_q}{\partial \delta \boldsymbol{\theta}_i} = -2 \begin{bmatrix} \mathbf{0}_{3\times 1} & \mathbf{I}_{3\times 3} \end{bmatrix} [\mathbf{q}_{wb_j}^{-1} \otimes \mathbf{q}_{wb_i}]_L [\hat{\mathbf{q}}_{b_i b_j}]_R \begin{bmatrix} \mathbf{0}_{1\times 3} \\ \frac{1}{2}\mathbf{I}_{3\times 3} \end{bmatrix} \qquad \frac{\partial \mathbf{r}_q}{\partial \delta \boldsymbol{\theta}_j} = 2 \begin{bmatrix} \mathbf{0}_{3\times 1} & \mathbf{I}_{3\times 3} \end{bmatrix} [\hat{\mathbf{q}}_{b_i b_j}^{-1} \otimes \mathbf{q}_{wb_i}^{-1} \otimes \mathbf{q}_{wb_j}]_L \begin{bmatrix} \mathbf{0}_{1\times 3} \\ \frac{1}{2}\mathbf{I}_{3\times 3} \end{bmatrix}$$

$$\frac{\partial \mathbf{r}_q}{\partial \delta \mathbf{v}_i} = 0 \qquad \frac{\partial \mathbf{r}_q}{\partial \delta \mathbf{v}_j} = 0$$

$$\frac{\partial \mathbf{r}_q}{\partial \delta \tilde{\mathbf{b}}_{a,i}} = 0 \qquad \frac{\partial \mathbf{r}_q}{\partial \delta \tilde{\mathbf{b}}_{a,j}} = 0$$

$$\frac{\partial \mathbf{r}_q}{\partial \delta \tilde{\mathbf{b}}_{g,i}} = -2 \begin{bmatrix} \mathbf{0}_{3\times 1} & \mathbf{I}_{3\times 3} \end{bmatrix} [\mathbf{q}_{wb_j}^{-1} \otimes \mathbf{q}_{wb_i} \otimes \hat{\mathbf{q}}_{b_i b_j}]_L \begin{bmatrix} \mathbf{0}_{1\times 3} \\ \frac{1}{2}\mathbf{J}^q_{b_i^g} \end{bmatrix} \qquad \frac{\partial \mathbf{r}_q}{\partial \delta \tilde{\mathbf{b}}_{g,j}} = 0$$

及格要求

零偏残差相对于状态量的雅可比：

$$\frac{\partial \mathbf{r}_{b_a}}{\partial \delta \mathbf{p}_i} = 0 \qquad \frac{\partial \mathbf{r}_{b_a}}{\partial \delta \mathbf{p}_j} = 0$$

$$\frac{\partial \mathbf{r}_{b_a}}{\partial \delta \boldsymbol{\theta}_i} = 0 \qquad \frac{\partial \mathbf{r}_{b_a}}{\delta \boldsymbol{\theta}_j} = 0$$

$$\frac{\partial \mathbf{r}_{b_a}}{\partial \delta \mathbf{v}_i} = 0 \qquad \frac{\partial \mathbf{r}_{b_a}}{\partial \delta \mathbf{v}_j} = 0$$

$$\frac{\partial \mathbf{r}_{b_a}}{\partial \delta \tilde{\mathbf{b}}_{a,i}} = -\mathbf{I}_{3\times 3} \qquad \frac{\partial \mathbf{r}_{b_a}}{\partial \delta \tilde{\mathbf{b}}_{a,j}} = \mathbf{I}_{3\times 3}$$

$$\frac{\partial \mathbf{r}_{b_g}}{\partial \delta \tilde{\mathbf{b}}_{g,i}} = 0 \qquad \frac{\partial \mathbf{r}_{b_g}}{\partial \delta \tilde{\mathbf{b}}_{a,j}} = 0$$

$$\frac{\partial \mathbf{r}_{b_g}}{\partial \delta \mathbf{p}_i} = 0 \qquad \frac{\partial \mathbf{r}_{b_g}}{\partial \delta \mathbf{p}_j} = 0$$

$$\frac{\partial \mathbf{r}_{b_g}}{\partial \delta \boldsymbol{\theta}_i} = 0 \qquad \frac{\partial \mathbf{r}_{b_a}}{\delta \boldsymbol{\theta}_j} = 0$$

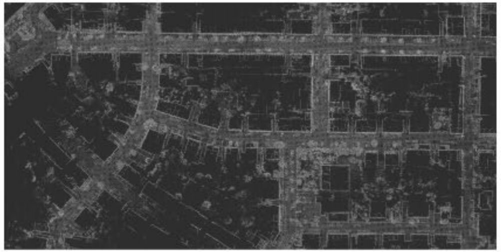
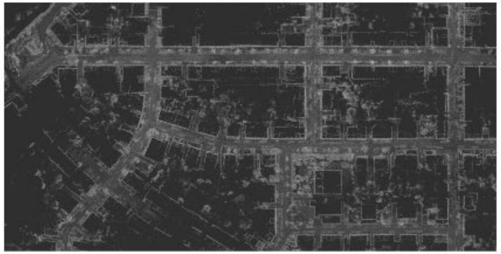
$$\frac{\partial \mathbf{r}_{b_g}}{\partial \delta \mathbf{v}_i} = 0 \qquad \frac{\partial \mathbf{r}_{b_g}}{\partial \delta \mathbf{v}_j} = 0$$

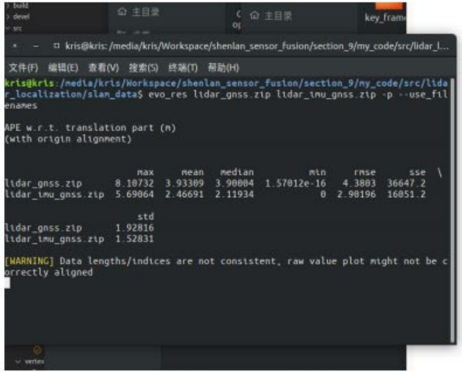
$$\frac{\partial \mathbf{r}_{b_g}}{\partial \delta \tilde{\mathbf{b}}_{a,i}} = 0 \qquad \frac{\partial \mathbf{r}_{b_g}}{\partial \delta \tilde{\mathbf{b}}_{a,j}} = 0$$

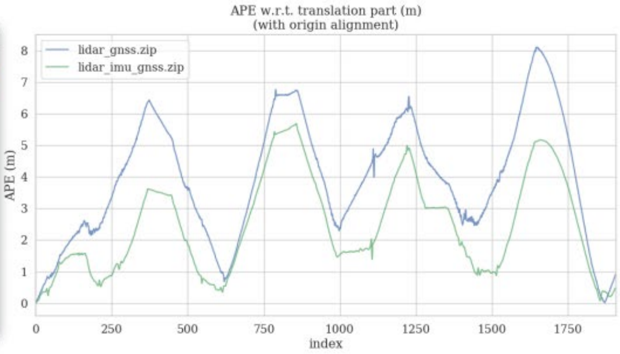
$$\frac{\partial \mathbf{r}_{b_g}}{\partial \delta \tilde{\mathbf{b}}_{g,i}} = -\mathbf{I}_{3\times 3} \qquad \frac{\partial \mathbf{r}_{b_g}}{\partial \delta \tilde{\mathbf{b}}_{g,j}} = \mathbf{I}_{3\times 3}$$

良好要求

目标：加和不加imu的效果对比

lidar+imu+gnss	lidar+gnss
	





优秀要求

目标： 融合编码器时预积分公式推导

状态量更新：

$$\begin{bmatrix} p_{wb_j} \\ q_{wb_j} \\ b_j^g \end{bmatrix} = \begin{bmatrix} p_{wb_i} + q_{wb_i} \alpha_{b_i b_j} \\ q_{wb_i} q_{b_i b_j} \\ b_i^g \end{bmatrix}$$

预积分量更新：

$$\alpha_{b_i b_j} = \bar{\alpha}_{b_i b_j} + J_{b_i^g}^{\alpha} \delta b_i^g, \quad \frac{\partial \alpha_{b_i b_j}}{\partial \delta b_i^g} = J_{b_i^g}^{\alpha}$$

$$q_{b_i b_j} = \bar{q}_{b_i b_j} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} J_{b_i^g}^{q} \end{bmatrix}, \quad \frac{\partial q_{b_i b_j}}{\partial \delta b_i^g} = J_{b_i^g}^{q}$$

残差定义：

$$\begin{bmatrix} r_p \\ r_q \\ r_{bg} \end{bmatrix} = \begin{bmatrix} q_{wb_i}^{*} (p_{wb_j} - p_{wb_i}) - \alpha_{b_i b_j} \\ 2 [q_{b_i b_j}^{*} \otimes (q_{wb_i}^{*} \otimes q_{wb_j})]_{xyz}. \\ b_j^g - b_i^g \end{bmatrix}$$

优秀要求

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姿态误差离散形式:

姿态误差的微分方程:

$$\delta\dot{\theta}_t^{b_k} = -[w_t - b_{wt}]_\times \delta\theta_t^{b_k} + n_w - \delta b_{wt}$$

离散形式:

$$\delta\theta_{k+1} = (I - [\bar{w}]_\times \delta t)\delta\theta_k + \frac{\delta t}{2} n_{w_k} + \frac{\delta t}{2} n_{w,k+1} - \delta b_{w_k} \cdot \delta t$$

$$\bar{w} = \frac{w_k + w_{k+1}}{2} - b_{w_k}$$

优秀要求

位置误差离散形式：

位置误差的微分方程：

$$\delta\dot{p} = -R_{wb}[V^b]_\times \delta\theta + R_{wb} n_v$$

离散形式：

$$\begin{aligned}\delta\alpha_{k+1} = \delta\alpha_k & - \frac{\delta t}{2}\left[R_k[V_k]_\times + R_{k+1}[V_{k+1}]_\times (I - [\bar{\omega}]_\times \delta t)\right]\delta\theta_k \\ & - \frac{\delta t^2}{4} R_{k+1}[V_{k+1}]_\times \cdot n_{w_k} \\ & - \frac{\delta t^2}{4} R_{k+1}[V_{k+1}]_\times n_{w_{k+1}} \\ & + \frac{\delta t^2}{2} R_{k+1}[V_{k+1}]_\times \delta b_{w,k} \\ & + \frac{\delta t}{2} R_k n_{v_k} + \frac{\delta t}{2} R_{k+1} \cdot n_{v_{k+1}}\end{aligned}$$

零偏误差离散形式：

$$\delta b_{g,k+1} = n_{bg,k} \cdot \delta t + \delta b_{g,k}$$

优秀要求

离散形式状态方程：

$$x_{k+1} = F_k x_k + B_k \cdot w_k$$

$$x_k = \begin{bmatrix} \delta\alpha_k \\ \delta\theta_k \\ \delta b_{g_k} \end{bmatrix} \qquad w_k = \begin{bmatrix} n_{vk} \\ n_{gk} \\ n_{v,k+1} \\ n_{g,k+1} \\ n_{bg,k} \end{bmatrix}$$

$$F_k = \begin{array}{c} \\ \delta\alpha \\ \delta\theta \\ \\ \end{array} \begin{array}{c} \begin{array}{ccc} \delta\alpha & \delta\theta & \qquad \end{array} \\ \begin{bmatrix} I & -\frac{\delta t}{2}\left[R_k [V_k]_\times + R_{k+1}[V_{k+1}]_\times (I - [\bar{w}]_\times \delta t)\right] & \frac{\delta t^2}{2} R_{k+1} [V_{k+1}]_\times \\ 0 & (I - [\bar{w}]_\times \delta t) & -I\delta t \\ 0 & 0 & I \end{bmatrix} \end{array}$$

$$B_k = \begin{array}{c} \\ \delta\alpha \\ \delta\theta \\ \delta b_g \end{array} \begin{array}{c} \begin{array}{ccccc} n_{v,k} & n_{w,k} & n_{v,k+1} & n_{w,k+1} & n_{bg,k} \end{array} \\ \begin{bmatrix} \frac{\delta t}{2} R_k & -\frac{\delta t^2}{4} R_{k+1} [V_{k+1}]_\times & \frac{\delta t}{2} R_{k+1} & -\frac{\delta t^2}{4} R_{k+1} [V_{k+1}]_\times & 0 \\ 0 & \frac{\delta t}{2} I_3 & 0 & \frac{\delta t}{2} I_3 & 0 \\ 0 & 0 & 0 & 0 & I_3 \delta t \end{bmatrix} \end{array}$$

优秀要求

状态量：　$[\delta P_{wb_i}, \delta\theta_{b_ib_i'}, \delta b_i^g], [\delta P_{wb_j}, \delta\theta_{b_jb_j'}, \delta b_j^g]$

位置残差对状态量的雅可比：

(1) $\frac{\partial r_p}{\partial \delta P_{wb_i}} = -R_{wb_i}^T$　　(2) $\frac{\partial r_p}{\partial \delta P_{wb_j}} = R_{wb_i}^T$

(3) $\frac{\partial r_p}{\partial \delta\theta_{b_ib_i'}} = [R_{wb_i}^T (P_{wb_j} - P_{wb_i})]_\times$　　(4) $\frac{\partial r_p}{\partial \delta\theta_{b_jb_j'}} = 0$

(5) $\frac{\partial r_p}{\partial \delta b_i^g} = -J_{b_i^g}^{\alpha}$　　(6) $\frac{\partial r_p}{\partial \delta b_j^g} = 0_{3\times3}$

优秀要求

姿态残差对状态量的雅可比：

$$(1)\quad \frac{\partial r_q}{\partial p_{wb_i}} = 0 \qquad\qquad (2)\quad \frac{\partial r_q}{\partial p_{wb_j}} = 0$$

$$(3)\quad \frac{\partial r_q}{\partial \delta\theta_{b_ib_i'}} = -2 \begin{bmatrix} 0 & I \end{bmatrix} \left[q_{wb_j}^{*} \otimes q_{wb_i} \right]_L \left[q_{b_ib_j} \right]_R \begin{bmatrix} 0 \\ \frac{1}{2} I \end{bmatrix}$$

$$(4)\quad \frac{\partial r_q}{\partial \delta\theta_{b_jb_j'}} = 2 \begin{bmatrix} 0 & I \end{bmatrix} \left[q_{b_ib_j}^{*} \otimes q_{wb_i}^{*} \otimes q_{wb_j} \right]_L \begin{bmatrix} 0 \\ \frac{1}{2} I \end{bmatrix}$$

$$(5)\quad \frac{\partial r_q}{\partial \delta b_i^g} = -2 \begin{bmatrix} 0 & I \end{bmatrix} \left[q_{wb_j}^{*} \otimes q_{wb_i} \otimes q_{b_ib_j} \right]_L \begin{bmatrix} 0 \\ \frac{1}{2} J^q_{b_i^g} \end{bmatrix}$$

$$(6)\quad \frac{\partial r_q}{\partial \delta b_j^g} = 0$$

优秀要求

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零偏残差对状态量的雅可比：

(1) $\dfrac{\partial r_{bg}}{\partial \delta p_{wbi}} = 0$ (2) $\dfrac{\partial r_{bg}}{\partial \delta p_{wbj}} = 0$ (3) $\dfrac{\partial r_{bg}}{\partial \delta \theta_{bi bi'}} = 0$ (4) $\dfrac{\partial r_{bg}}{\partial \delta \theta_{bj bj'}} = 0$.

(5) $\dfrac{\partial r_{bg}}{\partial \delta b_i^g} = -I$ (6) $\dfrac{\partial r_{bg}}{\partial \delta b_j^g} = I$.

在线问答

Q&A

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感谢各位聆听！

Thanks for Listening