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Formalization of Phase I of the Recursive Structural Model

The Recursive Structural Model (RSM) posits that reality arises from the impossibility of an absolute void. This document formalizes the core mathematical structures underpinning the model's first phase. All definitions and results are stated in structural terms—no appeal is made to agency or intention. We employ neutral mathematical language and avoid anthropomorphic metaphors.

1 Paradox, Logic and Co-Emergence

1.1 Paraconsistent logic and the void

At the logical level, the starting point of the RSM is an **impossible** state, denoted P_0, which represents “absolute nothingness”. In a classical logic, the proposition “P_0 exists” would be straightforwardly false. However, the act of naming nothing introduces its negation: if there is a term for “no-thing”, there is also a term for “some-thing”. If one reasons in an explosive logic, the simultaneous assertion of P_0 and its negation would trivialise the system. To avoid this, we appeal to a **paraconsistent logic**—for example, the **Logic of Paradox (LP)** introduced by Priest. In LP, contradictions are tolerated: both P_0 and $\neg P_0$ can be true without entailing that every proposition is true. This framework allows a formal **co-emergence**: the void and its complement are both affirmed.

We do not attempt a full axiomatization here, but adopt the following working principle: if a proposition $ \alpha$ and its negation $\neg\alpha$ both hold, they generate a **pair** of mutually constraining states rather than collapse into triviality. The minimal such pair is represented geometrically by positive coordinates (x,y)—neither coordinate can vanish without removing the pair entirely.

1.2 Reciprocal symmetry and the product constraint

To model co-emergence structurally, we embed the state space in the positive quadrant

$$\mathbb{R}^2_{>0} = \{(x, y) \mid x > 0,\ y > 0\},$$

interpreting the axes as **dimension** and **contrast**. A **paradoxical balance point** is the symmetric point

$$P_1 := (1, 1) \in \mathbb{R}^2_{>0},$$

at which neither coordinate dominates. The model demands that the balance be preserved under arbitrary rescalings: increasing one coordinate must be compensated by a corresponding decrease in the other. Formally, for any scaling factor $\lambda>0$ we require

$$(x, y) \sim (\lambda x, \lambda^{-1} y).$$

This relation defines an equivalence class under the action of the **reciprocal scaling group**

$$\mathrm{RS} := \{(\lambda,\lambda^{-1}) \mid \lambda>0\}, \qquad (x,y)\mapsto(\lambda x,\lambda^{-1}y).$$

It is straightforward to verify that if (x_1,y_1) and (x_2,y_2) lie on the same orbit of this group, then $x_1 y_1 = x_2 y_2$. Conversely, if $x_1 y_1 = x_2 y_2$, there exists λ such that $(x_2,y_2) = (\lambda x_1, \lambda^{-1} y_1)$. Hence the equivalence classes are precisely the level sets of the function $(x,y)\mapsto x\,y$. Preservation of co-emergence therefore requires **invariance under reciprocal scaling**, which forces

$$x\,y = k$$

for some constant $k>0$. We call

$$G_k := \{(x,y)\in\mathbb{R}_{>0}^2 \mid x\,y = k\}$$

the **gradient field** of product k. In particular, G_1 contains P_1 and represents the minimal hyperbolic grammar.

1.3 From logical pairs to geometric coordinates

The passage from logical co-emergence to a geometric representation merits clarification. In the paraconsistent setting of Section 1.1, both a proposition and its negation are present. To **realise** this pair structurally, we assign a positive coordinate to each component. Positivity encodes existence: a coordinate equal to zero or negative would correspond to the absence of one aspect and thus to collapse of the paradox. Hence the minimal geometric container for a pair of mutually constraining states is the positive quadrant $\mathbb{R}_{>0}^2$. The reciprocal scaling group acts on this space by $(x,y)\mapsto(\lambda x, \lambda^{-1} y)$, expressing the idea that any increase in one aspect must be balanced by a proportional decrease in the other. This geometric framework provides a concrete bridge from the abstract logical pair to the hyperbolic structure of co-emergence.

2 Derivation and Uniqueness of the Hyperbolic Constraint

We now derive the hyperbolic relation directly from reciprocal symmetry. Suppose $y = f(x)$ is a differentiable, strictly positive function on $(0,\infty)$ such that the graph of f is invariant under the transformation $(x,y)\mapsto(\lambda x,\lambda^{-1} y)$ for all $\lambda>0$. That is,

$$f(\lambda x) = \lambda^{-1} f(x) \quad \text{for all } x>0,\ \lambda>0.$$

Fix $x>0$ and differentiate both sides with respect to λ at $\lambda=1$. By the chain rule we obtain

$$x f'(x) = -f(x).$$

Solving this linear ordinary differential equation yields $f(x) = c/x$ for some constant $c>0$. Substituting back shows that the orbit of $(x,f(x))$ under the reciprocal scaling group consists precisely of the points (u,v) satisfying $uv = c$. Therefore any differentiable curve preserved by reciprocal symmetry must lie on a hyperbola $x y = k$, and conversely each hyperbola is preserved. This establishes the structural uniqueness of the hyperbolic constraint.

2.1 Alternative forms revisited

Although other relations may be definable on the positive quadrant, they do not respect reciprocal symmetry or positivity. We illustrate this by considering common examples.

- **Linear relations.** Take $x+y = k$ with $k>0$. Invariance under reciprocal scaling demands $\lambda x + \lambda^{-1} y = k$ for all $\lambda>0$. Differentiating at $\lambda=1$ implies $x - y = 0$, so $x=y=k/2$. Thus the linear relation is preserved only at this single point; away from it, the orbit of a point under scaling does not remain on the line. Furthermore, for $x>k$ the expression $y=k-x$ becomes negative, leaving the domain.

- **Circular relations.** Consider $x^2+y^2 = k^2$. Applying reciprocal scaling produces $\lambda^2 x^2 + \lambda^{-2} y^2$, which varies with λ unless x and y are both zero. The circle also approaches the coordinate axes at $x=0$ or $y=0$, which would correspond to one coordinate vanishing and thus the disappearance of co-emergence.

- **Exponential and trigonometric relations.** Functions such as $y=e^{x}$ or $y=\sin x$ lack any reciprocal scaling symmetry. Under $(x,y)\mapsto(\lambda x,\lambda^{-1} y)$ their graphs are not mapped to themselves; exponential functions grow or decay faster than any reciprocal compensation, and trigonometric functions oscillate and attain negative values, violating the positivity requirement.

These examples show that the requirement of reciprocal symmetry uniquely selects the hyperbolic relation among smooth curves on the positive quadrant. Any alternative either fails to be invariant under the scaling group or exits the domain.

It is important to stress why **exiting the domain**—for example, by crossing a coordinate axis—violates the structural requirements. The positive quadrant represents the simultaneous presence of both sides of the paradox; if either coordinate were zero or negative, one aspect would vanish, collapsing the co-emergent pair. Thus, relations that allow trajectories to approach or cross the axes do not preserve the paradox. The hyperbola $x y = k$ is the only smooth curve that remains entirely within the positive quadrant and respects the required symmetry.

3 Circulation as a One-Parameter Group

To maintain the paradoxical state, a configuration must not become static; otherwise, one coordinate would monotonically increase or decrease, driving the system toward an asymptote where one coordinate diverges and the other tends to zero. We formalise **circulation** as a flow preserving the hyperbola.

For $t\in\mathbb{R}$ define

$$\phi_t(x,y) := (e^t x, e^{-t} y), \quad (x,y)\in\mathbb{R}^2_{>0}.$$

Then ϕ_t is a diffeomorphism of $\mathbb{R}_{>0}^2$ for each t. Direct computation shows

$$\phi_t(x,y)\in G_k \iff (e^t x)(e^{-t} y) = xy = k.$$

The family $(\phi_t)*{t\in\mathbb{R}}$ is a* ***one-parameter group****: $\phi* = \phi_t\circ\phi_s$ and ϕ_0 is the identity map. To see that ϕ_t is generated by the vector field

$$V(x,y) = \left.\frac{\mathrm{d}}{\mathrm{d}t}\right|_{t=0} \phi_t(x,y) = (x,-y),$$

we compute the derivatives

$$\frac{\mathrm{d}}{\mathrm{d}t}(e^t x) = e^t x\big|_{t=0} = x, \qquad \frac{\mathrm{d}}{\mathrm{d}t}(e^{-t} y) = -e^{-t} y\big|_{t=0} = -y.$$

Thus the system of ordinary differential equations

$$\dot{x}(t) = x(t), \quad \dot{y}(t) = -y(t)$$

has solution $x(t) = e^t x_0$, $y(t) = e^{-t} y_0$ for initial data (x_0,y_0). These **integral curves** follow the hyperbola $x y = x_0 y_0$ and define the circulation flow. They never converge to a fixed point; as $t\to\pm\infty$ one coordinate diverges and the other vanishes. The only way to remain near P_1 is to continually move along the group in both positive and negative directions. Static equilibrium at P_1 is structurally unstable: linearising around $(1,1)$ yields eigenvalues $+1$ and -1, revealing a saddle.

4 Dimensional Necessity for Sustained Circulation

The RSM requires persistent, non-periodic circulation to maintain paradox. In two dimensions, continuous dynamical systems do not admit such behaviour. The Poincaré–Bendixson theorem states that any non-empty compact ω-limit set of a plane flow is a fixed point, a periodic orbit, or a finite union of these [1]. Consequently, two-dimensional continuous systems cannot possess strange attractors or aperiodic recurrent orbits.

In contrast, three-dimensional systems evade this restriction. The Lorenz equations provide a canonical example: when certain parameter values are chosen, solutions neither settle into fixed points nor periodic orbits but instead approach a chaotic attractor of fractal dimension ≈ 2.06 [2]. Such a system demonstrates persistent circulation without repetition. More generally, non-integrable deterministic flows are possible only in three or more dimensions; an abstract summary notes that “nonintegrable dynamical systems are confined to $n\geq 3$ dimensions” [3]. Even-dimensional spaces facilitate perfect pairwise cancellation of degrees of freedom; odd dimensions provide the minimal asymmetry required for a residual direction to sustain circulation.

4.1 Paradox maintenance as a dynamical flow

The one-parameter family ϕ_t introduced in Section 3 describes motion along a hyperbola at a constant rate: $\dot{x} = x$, $\dot{y} = -y$. If t only increases or decreases monotonically, the orbit drifts to infinity or collapses, undermining the co-emergent balance. To **maintain** the paradox near P_1, the parameter t itself must vary in a way that keeps the trajectory within a bounded region of state space. This requires coupling t to additional degrees of freedom. A simple way to express this is to extend the state by a third coordinate z and consider a three-dimensional vector field $(\dot{x},\dot{y},\dot{z})$ that reduces to $(x,-y)$ when projected onto the (x,y)-plane but whose evolution for z feeds back into the rates of change of x and y. The Lorenz system is a classic example of such an extension; it couples three variables in a way that yields a bounded **strange attractor**, ensuring that trajectories neither converge to a fixed point nor escape to infinity [2]. In two dimensions, the Poincaré–Bendixson theorem prohibits such chaotic recurrence [1]. Thus, any dynamical realisation of paradox maintenance that keeps the orbit from collapsing must live in dimension three or higher, where non-integrable behaviour and chaotic attractors can emerge.

5 Information and Constant Accuracy / Variable Precision

Let $(X,\mathcal{B},\mu)$ be the measure space where $X = \mathbb{R}_{>0}^2$, $\mathcal{B}$ is the Borel σ-algebra and μ is the **multiplicative measure** defined by

$$\mathrm{d}\mu(x,y) = \frac{1}{xy}\,\mathrm{d}x\,\mathrm{d}y.$$

This choice reflects the reciprocal symmetry. To see why μ is preserved, note that the Jacobian matrix of ϕ_t is diagonal with entries e^t and e^{-t}, so its determinant satisfies $\det D\phi_t = e^t e^{-t} = 1$. Writing $(x',y')=\phi_t(x,y)$, we have

$$\mathrm{d}x'\,\mathrm{d}y' = \det D\phi_t\,\mathrm{d}x\,\mathrm{d}y = \mathrm{d}x\,\mathrm{d}y.$$

Hence

$$\mathrm{d}\mu(\phi_t(x,y)) = \frac{1}{x'y'}\,\mathrm{d}x'\,\mathrm{d}y' = \frac{1}{(e^t x)(e^{-t} y)}\,\mathrm{d}x\,\mathrm{d}y = \frac{1}{xy}\,\mathrm{d}x\,\mathrm{d}y = \mathrm{d}\mu(x,y),$$

so the measure is invariant. A **state** of the system is a non-negative function $\rho\colon X\to [0,\infty)$ with $\int_X \rho\,\mathrm{d}\mu = 1$; it plays the role of a probability density with respect to μ. The **entropy** of such a density is

$$H(\rho) = -\int_X \rho(x,y)\,\ln\rho(x,y)\,\mathrm{d}\mu(x,y).$$

If $T\colon X\to X$ is a diffeomorphism preserving the measure μ (that is, $\mu(T^{-1}(A)) = \mu(A)$ for all measurable A), then $(\rho\circ T)$ is also a density, and a change of variables shows

$$H(\rho\circ T) = H(\rho).$$

This invariance [4] expresses the idea that **structural information**—captured by entropy relative to μ —remains constant under unimodular transformations such as reciprocal scaling. Refinement of measurement corresponds to coarse-graining ρ, which can change H by discarding fine details but does not alter the underlying hyperbolic relation.

The **Constant Accuracy / Variable Precision (CAVP) principle** states that the proportional relation $x y = k$ is exact and invariant, but the energetic cost of accessing finer structure increases as measurements resolve smaller scales. Physical examples illustrate this general tendency:

- **Instrumentation.** Transmission electron microscopy achieves higher resolution only by using thinner samples and higher incident electron energies [5]. Finer spatial resolution corresponds to smaller effective radii and greater energy expenditure.
- **Biological scaling.** Metabolic rates scale with body mass to the power $3/4$ across species [6]. Smaller organisms, which operate at smaller characteristic lengths, expend more energy per unit mass.

Determining the exact functional relationship between energy and scale is an open problem. The RSM posits that, in a suitably normalised setting, the energetic cost of probing the field at scale r grows as r decreases, and conjectures an inverse dependence $E(r) \propto 1/r$. Deriving this scaling from first principles remains a subject for further work; the examples above suggest only that finer resolution entails greater expenditure.

6 Conclusion

The formalism above rigorously characterises the minimal structure required by the Recursive Structural Model. The hyperbola arises uniquely from the demand of reciprocal symmetry: dimension and contrast co-emerge and are preserved only by keeping their product constant. Circulation is modelled by a one-parameter group of unimodular scalings, whose integral curves never settle and whose generator exhibits a saddle structure around the balance point. Persistent non-periodic circulation necessitates at least three dimensions, as lower-dimensional flows are constrained by the Poincaré–Bendixson theorem [1] and cannot support a strange attractor [3]. Structural information, captured by entropy relative to the multiplicative measure, is invariant under these transformations, while accessing finer precision requires ever greater energetic investment. The exact scaling of this energy with resolution remains conjectural; nevertheless, the interplay of symmetry, dynamics and information provides a mathematically precise foundation for subsequent phases of the RSM without invoking agency or design.

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