Recursive Substrate Mechanics (RSM) Lagrangian

We define the total Lagrangian as:
$\mathscr{L}_{\text{RSM}} = \mathscr{L}_{\text{substrate}} + \mathscr{L}_{\text{emergence}} + \mathscr{L}_{\text{recursion}} + \mathscr{L}_{\text{constraint}} + \mathscr{L}_{\text{interaction}}$

1. Substrate Field Lagrangian —P_0

$$\mathscr{L}_{\text{substrate}} = -\frac{1}{2}(\partial_\mu P_0)(\partial^\mu P_0) - \frac{1}{2}m_0^2 P_0^2 + \frac{\lambda_0}{4!}P_0^4$$

- P_0 : Unframeable substrate field (scalar)
- M_0 : Substrate mass parameter (sets emergence threshold)
- λ_0 : Substrate self-consistency coupling (prevents unbounded growth)

2. Emergence Axis Lagrangian — O_1^i

$$\mathscr{L}_{\text{emergence}} = \sum_i \left[O_1^{i\dagger} \left(i\partial_t - \hat{H}_{\text{fold}} \right) O_1^i \right] - g_1 |P_0|^2 \sum_i |O_1^i|^2$$

where the Folding Operator $\hat{H}_{\text{fold}}$ is defined as:

$$\hat{H}_{\text{fold}} = -\frac{\hbar^2}{2m_{\text{fold}}}\nabla^2 + V_{\text{fold}}(r) \quad \text{with} \quad V_{\text{fold}}(r) = V_0 \left(1 - e^{-r/r}\text{fold} \right)$$

- O_1^i : Emergence axis field (complex scalar per dimension)
- m_{fold} : Dimensional folding mass parameter
- $V_{\text{fold}}(r)$: Folding potential stabilizing dimensions near radius r_{fold}
- g_1 : Coupling constant linking substrate and emergence axes

3. Recursion Field Lagrangian —$Z_{1,\mu}$

$$\mathscr{L}_{\text{recursion}} = -\frac{1}{4}F^{\mu\nu}_{\text{rec}}F^{\text{rec}}_{\mu\nu} + \frac{1}{2}\beta_{\text{rec}}Z_{1,\mu}Z^\mu$$

where the recursion field strength tensor is:

$$F^{\text{rec}}_{\mu\nu} = \partial_\mu Z_{1,\nu} - \partial_\nu Z_{1,\mu}$$

- $Z_{1,\mu}$: Recursion circulation vector field
- β_{rec} : Recursion field mass (circulation strength term)

4. Constraint Lagrangian (Paradox Stability Condition)

We enforce the paradox stability constraint $\frac{\partial P_n}{\partial t} = 0$ via a Lagrange multiplier λ_n :

$$\mathscr{L}_{\text{interaction}} = -g_1 |P_0|^2 \sum_i |O_1^i|^2 - g_{\text{rec}}Z_{1,\mu}J^\mu_{\text{fold}}$$

where:

- J^{μ}_{fold} is the current associated with O1 fold dynamics

Total RSM Lagrangian

$$
\begin{aligned}
\mathscr{L}_{\text{RSM}} = &-\frac{1}{2}(\partial_\mu P_0)(\partial^\mu P_0) - \frac{1}{2}m_0^2 P_0^2 + \frac{\lambda_0}{4!}P_0^4 \\
&+\sum_i \left[O_1^{i\dagger}\left(i\partial_t - \hat{H}_{\text{fold}}\right)O_1^i\right] - g_1 |P_0|^2 \sum_i |O_1^i|^2 \\
&-\frac{1}{4}F^{\mu\nu}_{\text{rec}}F_{\mu\nu}^{\text{rec}} + \frac{1}{2}\beta_{\text{rec}}Z_{1,\mu}Z^\mu \\
&+\lambda_n \cdot \frac{\partial P_n}{\partial t} - g_{\text{rec}}Z_{1,\mu}J^\mu_{\text{fold}}
\end{aligned}
$$