Physics 607

Statistical Physics and Thermodynamics

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Homework #8

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1 Problem #1

(1) To find the Debye screening length, λ_D, of a semiconductor with a dielectric constant $\varepsilon = 20$ and density of electrons $n_0 = 10^{18}\,\mathrm{cm}^{-3}$ at room temperature, $T = 300\,\mathrm{K}$ we can use the reciprocal length, κ, given as

$$\kappa^2 = \frac{4\pi e^2}{T}\sum_a n_{a0} z_a^2$$

Noting that for a semiconductor the ions are fixed so we only consider the electrons in the sum. This implies that

$$\lambda_D = \frac{1}{\kappa} = \sqrt{\frac{\varepsilon\varepsilon_0 k_B T}{4\pi n_0 e^2}}$$

note that we added the constants k_B and ε_0 to get the correct dimensionality. So for the semiconductor we calculate

$$\begin{aligned}\lambda_D = \sqrt{\frac{\varepsilon\varepsilon_0 k_B T}{4\pi n_0 e^2}} &= \sqrt{\frac{(20)(8.85\times10^{-12}\,\mathrm{C^2\ N^{-1}\ m^{-2}})(1.38\times10^{-23}\,\mathrm{J\ K^{-1}})(300\,\mathrm{K})}{4\pi(10^{24}\,\mathrm{m^{-3}})(1.6\times10^{-19}\,\mathrm{C})^2}} \\ &= 1.51\,\mathrm{nm}\end{aligned}$$

(2) For a semiconductor in the form of a rectangular slab with thickness L, where L is much smaller than the other two dimensions, we attach two electrodes to the faces of the slab and apply a bias voltage, V. To solve for the electric field we solve the *Poisson Equation* given by

$$\nabla^2\phi - \kappa^2\phi = 0 \tag{1.1}$$

where κ is defined above. Taking the directions that create the face as infinite we assume that ϕ is only dependent on z which yields a solution of the form

$$\phi(z) = C_1 e^{\kappa z} + C_2 e^{-\kappa z}$$

applying the boundary conditions

$$\phi(z=0) = 0 \qquad \phi(z=L) = V$$

we find that

$$\begin{aligned}\phi(z=0) = 0 &= C_1 e^{\kappa 0} + C_2 e^{-\kappa 0} \\ &\Downarrow \\ C_1 &= -C_2 \\ &\Downarrow \\ \phi(z) = 0 &= C\left(e^{\kappa z} - e^{-\kappa z}\right) = C\sinh(\kappa z)\end{aligned}$$

and

$$\begin{aligned}\phi(z=L) = V &= C\sinh(\kappa L) \\ &\Downarrow \\ C &= \frac{V}{\sinh(\kappa L)}\end{aligned}$$

So the potential as a function of z is

$$\phi(z) = \frac{V}{\sinh(\kappa L)}\sinh(\kappa z)$$

(3) Note if we take the limit as the thickness becomes infinite we have the following condition

$$\kappa L \to \infty$$

we see that in this limit we have

$$\lim_{\kappa L \to \infty} \frac{V}{\sinh(\kappa L)} = 0$$

So we see that as we take the thickness to be infinite the screening will completely cancel the bias voltage.

2 Problem #2

(a) Given the physical characterization of the triple point of water found experimentally as

$$T_t = 273.16\,\text{K} \qquad P_t = 612\,\text{Pa} \qquad \rho_L = 1\,\text{g cm}^{-3} \qquad \rho_S = 0.894\,\text{g cm}^{-3}$$

where ρ_L is the density of water in the liquid state and ρ_S is the density of water in the solid state. Noting the data for the freezing point of water at atmospheric pressure is

$$T = 273.15\,\text{K} \qquad P_t = 101\,\text{kPa} \qquad \rho_L = 1\,\text{g cm}^{-3} \qquad \rho_S = 0.894\,\text{g cm}^{-3}$$

we can find the heat of ice melting assuming that it does not change between triple point and the freezing point at normal pressure by taking the *Clapeyron-Clausius formula*

$$\frac{dP}{dT} = \frac{q}{T(v_2 - v_1)} \tag{2.1}$$

and solving for the heat, q. Note we can approximate the derivative by taking the two different data points from the freezing point to the triple point. So equation 2.1 becomes

$$\begin{aligned}
q &= \frac{dP}{dT} T(v_2 - v_1) \\
&= \frac{P_t - P}{T_t - T} T \left(\frac{1}{\rho_L} - \frac{1}{\rho_S} \right) \\
&= \frac{612\,\text{Pa} - 101000\,\text{Pa}}{273.16\,\text{K} - 273.15\,\text{K}} (273.15\,\text{K}) \left(\frac{1}{1 \times 10^6\,\text{g m}^{-3}} - \frac{1}{0.894 \times 10^6\,\text{g m}^{-3}} \right) \\
&= 325\,\text{J g}^{-1}
\end{aligned}$$

(b) Using equation 2.1 and the given value for the latent heat of vaporization

$$q = 2264.76\,\text{J g}^{-1}$$

we can calculate the derivative of boiling temperature

$$\begin{aligned}
\frac{dT_b}{dP} &= \frac{T(v_2 - v_1)}{q} \\
&= \frac{T(\rho_V^{-1} - \rho_L^{-1})}{q}
\end{aligned}$$

and the same follows for ice vaporization

$$\begin{aligned}
\frac{dT_v}{dP} &= \frac{T(v_2 - v_1)}{q} \\
&= \frac{T(\rho_V^{-1} - \rho_S^{-1})}{q}
\end{aligned}$$

where we take ρ_V as the density of water vapor.

(c) When we take water at the freezing temperature, $T = 273.15\,\text{K}$, and atmospheric pressure, $P = 103\,\text{kPa}$, and decrease the pressure until the pressure reaches $P' = 300\,\text{Pa}$ We see that this patch will pass through the triple point described in part (a). This implies that the water will skip the liquid phase and sublimate from solid to vapor.