Mathematics 4450

Introduction to Complex Numbers

Professor Nat Thiemn

Homework #1

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Written problems

I ran out of time to tex all the homework all the questions from the book are attached.

Problems

1. Express the following in the forms $a + ib$, $\begin{bmatrix} a \\ b \end{bmatrix}$, $re^{i\theta}$ and $\begin{bmatrix} r\cos\theta & r\sin\theta \\ -r\sin\theta & r\cos\theta \end{bmatrix}$.

   (a) $\frac{1}{(1+i)}$

   $\frac{1}{(1+i)} = \frac{1}{1+i}\frac{1+i}{1+i} = \frac{1+i}{(1+i)(1+i)} = \frac{1+i}{(1^2+i^2+2i)} = \frac{1+i}{(1+(-1)+2i)} = \frac{1+i}{2i} = \frac{1}{2i} + \frac{i}{2i} = \frac{1}{2i}\frac{i}{i} + \frac{1}{2} = \frac{i}{2i^2} + \frac{1}{2}$
   $= \frac{i}{-2} + \frac{1}{2}$ So we know $a = \frac{1}{2}$ and $b = \frac{-1}{2}$.
   Finding r and θ
   $r = \sqrt{a^2+b^2} = \sqrt{\frac{1}{4}+\frac{1}{4}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
   $\tan\theta = \frac{b}{a} = \frac{-1(2)}{2(1)} = -1$
   $\theta = \frac{3\pi}{4}$
   $a + ib = \frac{1}{2} - i\frac{-1}{2}$ $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 1/2 \\ -1/2 \end{bmatrix}$ $re^{i\theta} = \frac{\sqrt{2}}{2}e^{i\frac{3\pi}{4}}$ $\begin{bmatrix} \frac{\sqrt{2}}{2}\cos\frac{3\pi}{4} & \frac{\sqrt{2}}{2}\sin\frac{3\pi}{4} \\ -\frac{\sqrt{2}}{2}\sin\frac{3\pi}{4} & \frac{\sqrt{2}}{2}\cos\frac{3\pi}{4} \end{bmatrix}$

   (b) $\left(\frac{2+i}{6i-(1-2i)}\right)^2$

   $\left(\frac{2+i}{6i-(1-2i)}\right)^2 = \frac{2+i}{6i-(1-2i)}\frac{2+i}{6i-(1-2i)} = \frac{4+i^2+4i}{1+8^2i^2-28i} = \frac{3+4i}{-63-16i} = \frac{3+4i}{-63-16i}\frac{-63-16i}{-63-16i} = \frac{-567+48i-252i+64i^2}{63^2-16^2i^2}$
   $= \frac{-631-204i}{4225}$
   $a = \frac{-631}{4225}$ $b = \frac{-204}{4225}$
   $r = 0.0246$ $\theta = 0.312$
   $a + ib = \frac{-631}{4225} - i\frac{204}{4225}$ $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \frac{-631}{4225} \\ \frac{-204}{4225} \end{bmatrix}$ $re^{i\theta} = 0.0246e^{i0.312}$ $\begin{bmatrix} 0.0246\cos 0.312 & 0.0246\sin 0.312 \\ -0.0246\sin 0.312 & 0.0246\cos 0.312 \end{bmatrix}$

   (c) e^{e^i}

   $e^{e^i} = e^{ie}$
   $r = 1$ $\theta = e$ $a = r\cos\theta = \cos e = -0.911$
   $a = r\sin\theta = \sin e = 0.411$
   $a + ib = -0.911 + i0.411$ $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -0.911 \\ 0.411 \end{bmatrix}$ $re^{i\theta} = 1e^{ie}$ $\begin{bmatrix} \cos e & \sin e \\ \sin e & \cos e \end{bmatrix}$

   (d) $\frac{e^{i\theta}-e^{-i\theta}}{i(e^{i\theta}+e^{-i\theta})}$

   $\frac{e^{i\theta}-e^{-i\theta}}{i(e^{i\theta}+e^{-i\theta})} = \frac{\cos\theta+i\sin\theta-(\cos-\theta+i\sin-\theta)}{i(\cos\theta+i\sin\theta+\cos-\theta+i\sin-\theta)} = \frac{\cos\theta+i\sin\theta-(\cos\theta-i\sin\theta)}{i(\cos\theta+i\sin\theta+\cos\theta-i\sin\theta)} = \frac{2i\sin\theta}{2i\cos\theta} = \tan\theta$
   $a = \tan\theta$ $b = 0$
   $r = \tan\theta$ $\theta = 0$

   $a + ib = \tan\theta + i0$ $\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} \tan\theta \\ 0 \end{bmatrix}$ $re^{i\theta} = \tan\theta e^{i0}$ $\begin{bmatrix} \tan\theta\cos 0 & \tan\theta\sin 0 \\ \tan\theta\sin 0 & \tan\theta\cos 0 \end{bmatrix}$

2. Completely factor $z^4 - 16$.

   $z^4 = 16$
   $z^4 = 16e^{0i}$

$z = (16e^{0i})^{1/4}$
$z = 2e^{0i} = 2e^{2/4\pi i} = 2e^{4/4\pi i} = 2e^{6/4\pi i}$
$z = 2 = 2i = -2 = -2i$

3. 1.2: 8, 17
   (see attached)

4. 1.3: 4, 8, 17
   (see attached)

5. 1.4: 7, 11
   (see attached)