Physics 611

Electromagnetic Theory II

Professor Christopher Pope

Homework #1

Joe Becker
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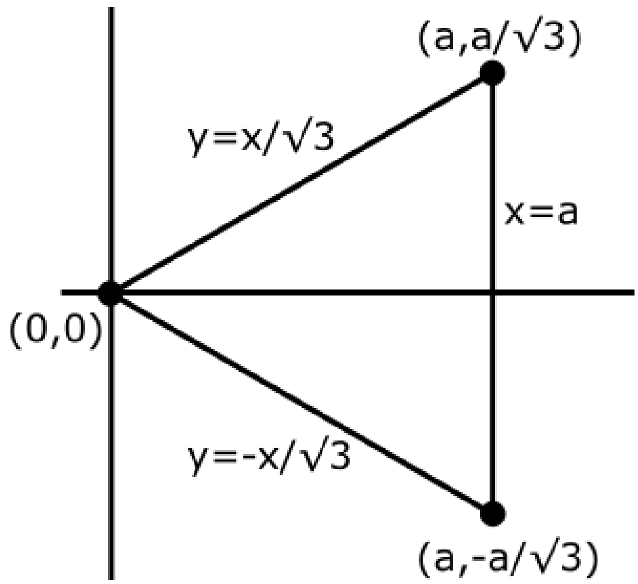

Figure 1: Equilateral triangle waveguide geometry.

1 Problem #1

(a) For a waveguide whose cross-section is an equilateral triangle whose vertices are at

$$(x,y) = \left\{(0,0),(a,a/\sqrt{3}),(a,-a/\sqrt{3})\right\}$$

we can verify that the function

$$\psi_{mn} = \sin\frac{l\pi x}{a}\sin\frac{(m-n)\pi y}{a\sqrt{3}} + \sin\frac{m\pi x}{a}\sin\frac{(n-l)\pi y}{a\sqrt{3}} + \sin\frac{n\pi x}{a}\sin\frac{(l-m)\pi y}{a\sqrt{3}}$$

where $l \equiv -m-n$ satisfy the TM boundary conditions as shown in figure 1 If we recall the result from homework 7 we see that

$$\psi_{nm}(a,y) = \cancelto{0}{\sin l\pi}\sin\frac{(m-n)\pi y}{a\sqrt{3}} + \cancelto{0}{\sin m\pi}\sin\frac{(n-l)\pi y}{a\sqrt{3}} + \cancelto{0}{\sin n\pi}\sin\frac{(l-m)\pi y}{a\sqrt{3}} = 0$$

and

$$\begin{aligned}\psi_{nm}(x,x/\sqrt{3}) &= \sin\frac{l\pi x}{a}\sin\frac{(m-n)\pi x}{3a} + \sin\frac{m\pi x}{a}\sin\frac{(n-l)\pi x}{3a} + \sin\frac{n\pi x}{a}\sin\frac{(l-m)\pi x}{3a} \\ &= \frac{1}{2}\Bigg[\cos\left(\frac{\pi x}{3a}(4m+2n)\right) - \cos\left(\frac{\pi x}{3a}(4m+2n)\right) \\ &\quad + \cos\left(\frac{\pi x}{3a}(2m-2n)\right) - \cos\left(\frac{\pi x}{3a}(2m-2n)\right) \\ &\quad + \cos\left(\frac{\pi x}{3a}(4n+2m)\right) - \cos\left(\frac{\pi x}{3a}(4n+2m)\right)\Bigg] = 0\end{aligned}$$

Note that for the final boundary condition we can use the result from above and the fact that sine is a odd function to see

$$\begin{aligned}\psi_{nm}(x,-x/\sqrt{3}) &= \sin\frac{l\pi x}{a}\sin\frac{-(m-n)\pi x}{3a} + \sin\frac{m\pi x}{a}\sin\frac{-(n-l)\pi x}{3a} + \sin\frac{n\pi x}{a}\sin\frac{-(l-m)\pi x}{3a} \\ &= -\left(\sin\frac{l\pi x}{a}\sin\frac{(m-n)\pi x}{3a} + \sin\frac{m\pi x}{a}\sin\frac{(n-l)\pi x}{3a} + \sin\frac{n\pi x}{a}\sin\frac{(l-m)\pi x}{3a}\right) \\ &= -\psi_{nm}(x,x/\sqrt{3}) = 0\end{aligned}$$

So we see that all TM boundary conditions are met.

(b) There exists a further set of TM modes, but not the modes described by the bisected equilateral triangular waveguide described in part (a). Note that we already have met the boundary condition for the full equilateral triangle with an additional boundary condition that $\psi_{mn}(0, y) = 0$.

(c)

2 Problem #2

(a)

(b)