Q_1: Part1

$$L\frac{di(t)}{dt}+Ri(t)+\frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau = v_{in}(t) \xrightarrow{\frac{d}{dt}} L\frac{d^2i(t)}{dt^2}+R\,\frac{di(t)}{dt}+\frac{1}{C}i(t)=\frac{dv_{in}(t)}{dt}$$

Part2

$$L\frac{d^2i(t)}{dt^2}+R\,\frac{di(t)}{dt}+\frac{1}{C}i(t)=\frac{dv_{in}(t)}{dt} \xrightarrow{\mathcal{L}} Ls^2\,I(s)+Rs\,I(s)+\frac{1}{C}I(s)=sV_{in}(s)$$

$$\Longrightarrow I(s)=\frac{sV_{in}(s)}{Ls^2+Rs+\frac{1}{C}}=\frac{V_{in}(s)}{Ls+R+\frac{1}{Cs}}$$

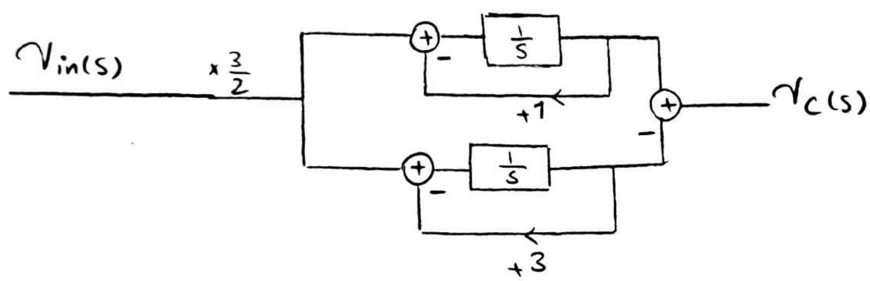
Part3

we know: $1)\ x(t)\xrightarrow{\mathcal{L}}X(s) \rightarrow \int_{-\infty}^{t} x(\tau)d\tau \xrightarrow{\mathcal{L}} \frac{1}{s}X(s)$

$$v_c(t)=\frac{1}{C}\int_{-\infty}^{t} i(\tau)d\tau \xrightarrow{\mathcal{L}} V_c(s)=\frac{1}{Cs}I(s)=\frac{1}{Cs}\times\frac{V_{in}(s)}{Ls+R+\frac{1}{Cs}}=\frac{V_{in}(s)}{LCs^2+RCs+1}$$

Part4

$$V_c(s)=\frac{V_{in}(s)}{\frac{s^2}{3}+\frac{4}{3}s+1}\Longrightarrow\frac{V_c(s)}{V_{in}(s)}=\frac{1}{\left(\frac{s}{3}+1\right)(s+1)}=\frac{3}{2}\times\left(\frac{1}{s+1}-\frac{1}{s+3}\right)$$



Part5

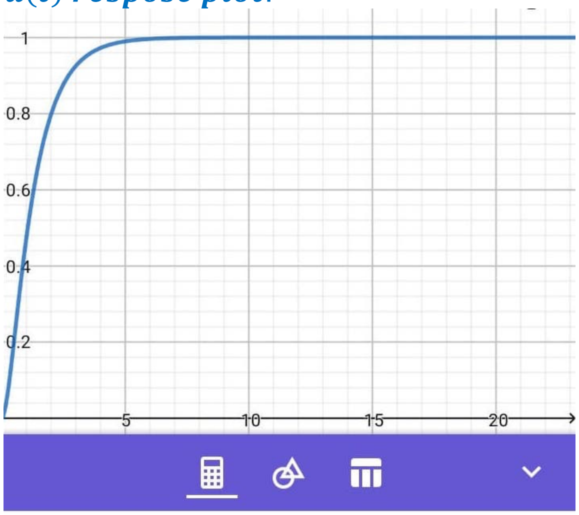
$$v_{in}(t)=u(t)\xrightarrow{\mathcal{L}}\frac{1}{s}\ ;\ \ V_c(s)=\frac{3}{2}\times\frac{1}{s}\times\left(\frac{1}{s+1}-\frac{1}{s+3}\right)\rightarrow\frac{3}{2}\left(\frac{1}{s(s+1)}-\frac{1}{s(s+3)}\right)$$

$$\frac{3}{2}\left(\frac{1}{s}-\frac{1}{s+1}-\frac{1}{3}\left(\frac{1}{s}-\frac{1}{s+3}\right)\right)=\frac{3}{2}\left(\frac{1}{3}\times\frac{1}{s+3}-\frac{1}{s+1}+\frac{2}{3}\times\frac{1}{s}\right)=\frac{1}{2}\times\frac{1}{s+3}+\frac{1}{s}-\frac{3}{2}\times\frac{1}{s+1}$$

Cause $u(t)$ is right − sided there for its response must be right − sided as well:

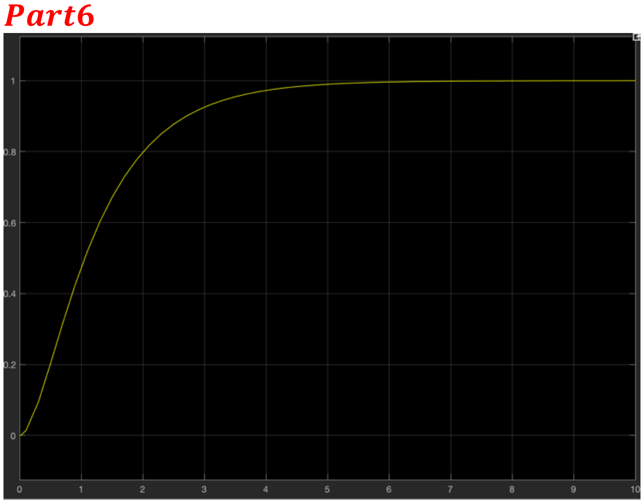
$$\frac{1}{2}\times\frac{1}{s+3}+\frac{1}{s}-\frac{3}{2}\times\frac{1}{s+1}\xrightarrow{\mathcal{L}^{-1}}\frac{1}{2}e^{-3t}u(t)+u(t)-\frac{3}{2}e^{-t}u(t)$$

u(t) respose plot:



$$s(x) = \frac{1}{2} e^{-3x} + 1 - \frac{3}{2} e^{-x}$$

Part6



As we can see the results of part5 and part6 are the same.

Q_2: *Part1*

$$set\ M = K = 1 \rightarrow x(t) - y(t) + B\left(\frac{dx(t)}{dt} - \frac{dy(t)}{dt}\right) = \frac{d^2y(t)}{dt^2}$$

$$\Longrightarrow B\frac{dx(t)}{dt} + x(t) = \frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + y(t)$$

Part2

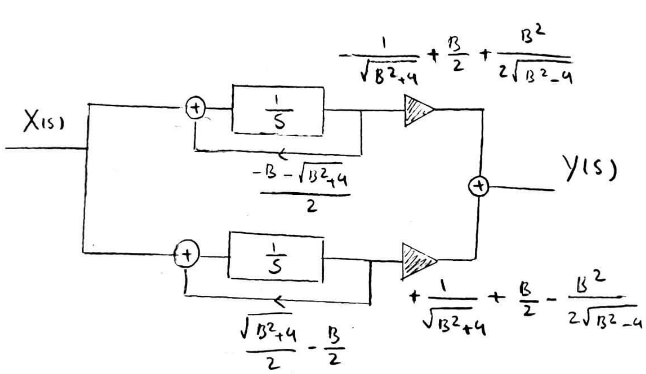
$$B\frac{dx(t)}{dt} + x(t) = \frac{d^2y(t)}{dt^2} + B\frac{dy(t)}{dt} + y(t) \xrightarrow{\mathcal{L}} BsX(s) + X(s) = s^2Y(s) + BsY(s) + Y(s)$$

Assume $B \neq 0$:

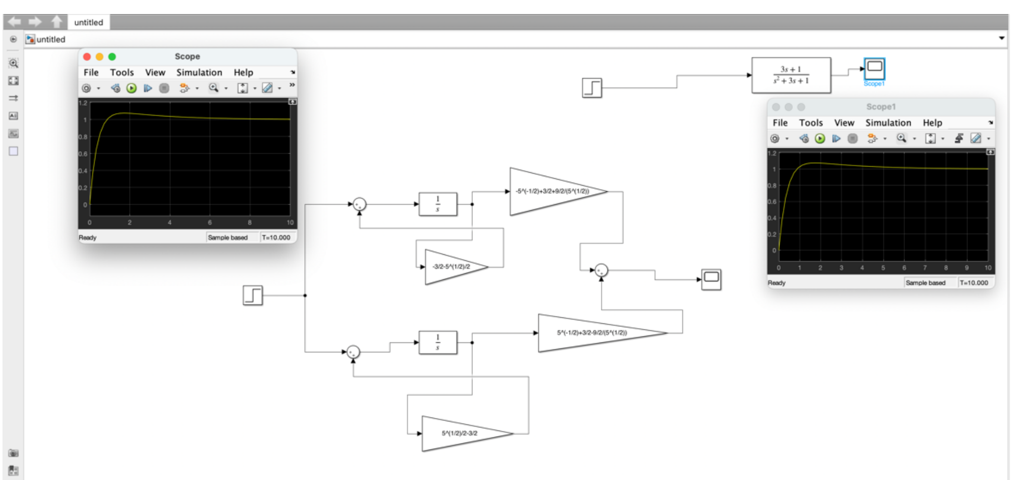
$$(Bs+1)X(s) = (s^2+Bs+1)Y(s) \Longrightarrow \frac{Y(s)}{X(s)} = \frac{Bs+1}{s^2+Bs+1} = \frac{Bs+1}{\left(s - \frac{-B+\sqrt{B^2-4}}{2}\right)\left(s - \frac{-B-\sqrt{B^2-4}}{2}\right)}$$

$$= \frac{B}{\sqrt{B^2-4}}\left(s+\frac{1}{B}\right)\left(-\frac{1}{\left(s - \frac{-B-\sqrt{B^2-4}}{2}\right)} + \frac{1}{\left(s - \frac{-B+\sqrt{B^2-4}}{2}\right)}\right) = \frac{B}{\sqrt{B^2-4}}\left(\frac{\left(s+\frac{1}{B}\right)}{\left(s - \frac{-B+\sqrt{B^2-4}}{2}\right)} - \frac{\left(s+\frac{1}{B}\right)}{\left(s - \frac{-B-\sqrt{B^2-4}}{2}\right)}\right)$$

$$= \frac{B}{\sqrt{B^2-4}}\left(\frac{\frac{1}{B} + \frac{-B+\sqrt{B^2-4}}{2}}{s - \frac{-B+\sqrt{B^2-4}}{2}} + \frac{-\frac{1}{B} + \frac{B+\sqrt{B^2-4}}{2}}{s - \frac{-B-\sqrt{B^2-4}}{2}}\right) = \frac{\left(\frac{1}{\sqrt{B^2-4}} + \frac{B}{2} - \frac{B^2}{2\sqrt{B^2-4}}\right)}{s - \frac{-B+\sqrt{B^2-4}}{2}} + \frac{\left(-\frac{1}{\sqrt{B^2-4}} + \frac{B}{2} + \frac{B^2}{2\sqrt{B^2-4}}\right)}{s - \frac{-B-\sqrt{B^2-4}}{2}}$$



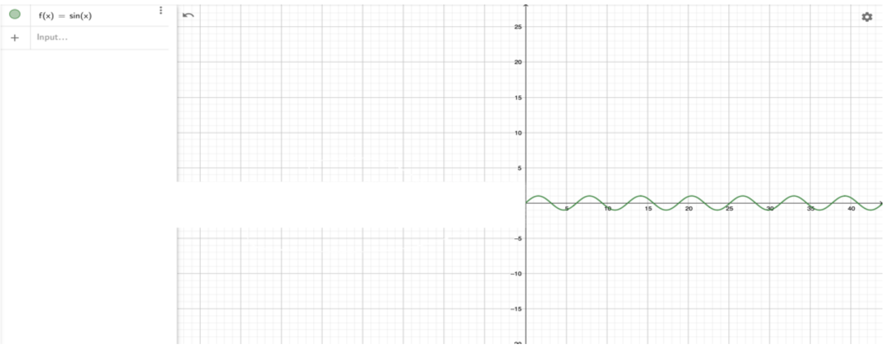
To check if the calculation above is correct, I substitude B as 3 and here is the simulation result:



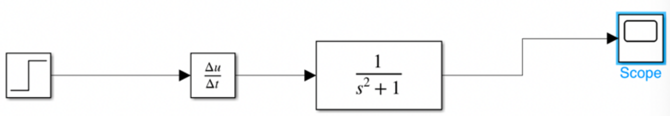
As we can see, the responses to $u(t)$ *are the same therefore the calculation is correct*

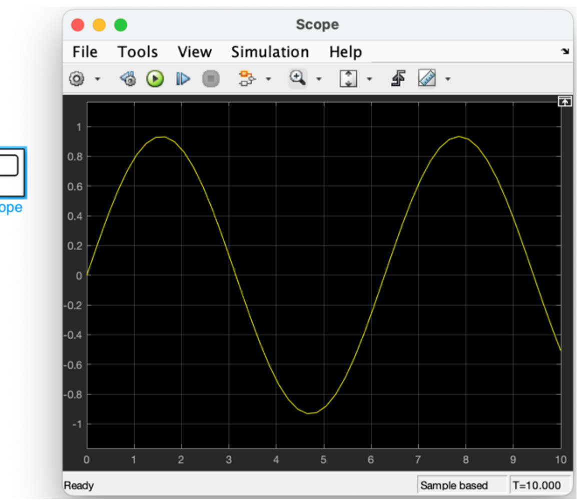
Part3

$$\frac{Y(s)}{X(s)} = \frac{Bs+1}{s^2+Bs+1} \xrightarrow{B=0} \frac{Y(s)}{X(s)} = \frac{1}{s^2+1} \Longrightarrow Y(s) = X(s)\frac{1}{s^2+1};\ X(s) = \mathcal{L}\{\delta(t)\} = 1 \Longrightarrow Y(s) = \frac{1}{s^2+1} \xrightarrow{\mathcal{L}^{-1}} \sin(t)\,u(t)$$



In the absence of damper, if we have a dirac input, then the car will be contantly turbulent.





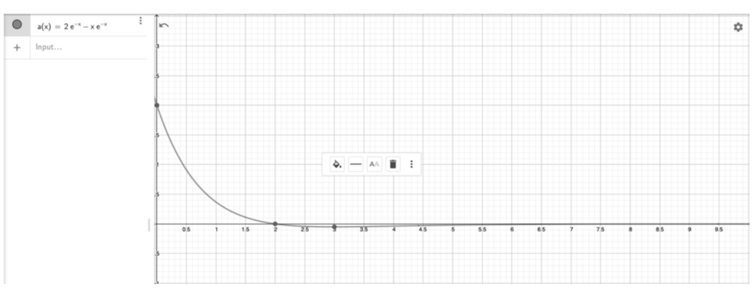
Part4

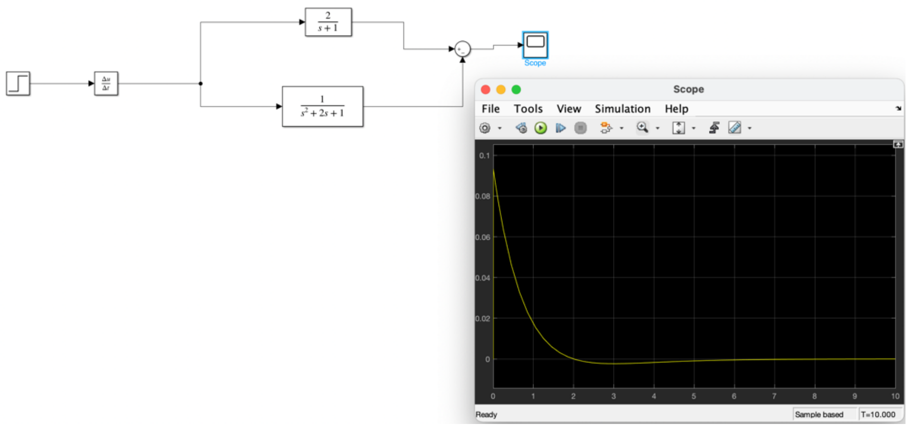
$$\frac{Y(s)}{X(s)} = \frac{Bs+1}{s^2+Bs+1} = \frac{Bs+1}{\left(s - \frac{-B+\sqrt{B^2-4}}{2}\right)\left(s - \frac{-B-\sqrt{B^2-4}}{2}\right)}$$

If we want the poles to be real, $\sqrt{B^2-4}$ *must be real. The smallest positive B to make the term real is 2.*

$$\Longrightarrow \frac{Y(s)}{X(s)} = \frac{2s+1}{(s+1)^2} = \frac{2}{s+1} - \frac{1}{(s+1)^2} \Longrightarrow Y(s) = X(s)\left(\frac{2}{s+1} - \frac{1}{(s+1)^2}\right);\ X(s) = \mathcal{L}\{\delta(t)\} = 1$$

$$\Longrightarrow Y(s) = \left(\frac{2}{s+1} - \frac{1}{(s+1)^2}\right) \xrightarrow{\mathcal{L}^{-1}} 2e^{-t}u(t) - te^{-t}u(t)$$



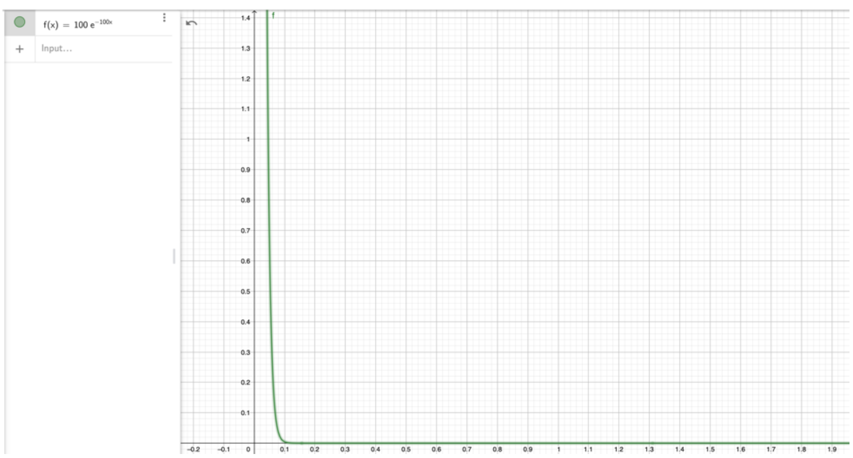


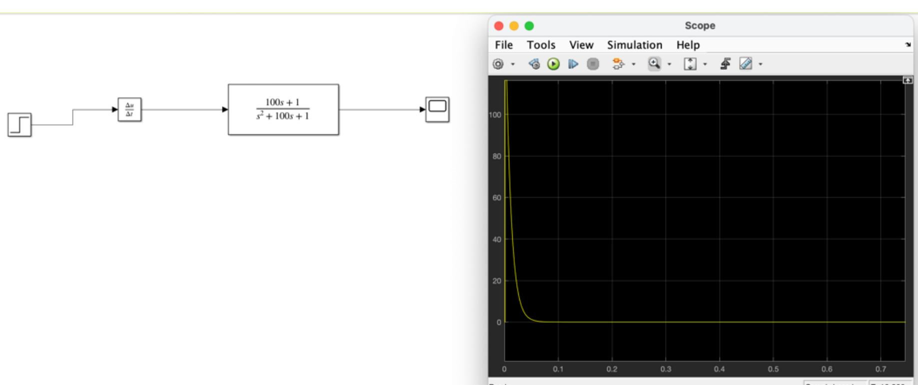
As we can see ,results are the same.

Part5

$$\frac{Y(s)}{X(s)} = \frac{Bs+1}{s^2+Bs+1} \xrightarrow{B=100} \frac{Y(s)}{X(s)} \approx \frac{100s+1}{(s+100)(s+0.01)} = 100\frac{s+0.01}{(s+100)(s+0.01)} = \frac{100}{s+100} \Longrightarrow$$

$$Y(s) = X(s)\frac{100}{s+100};\ X(s) = \mathcal{L}\{\delta(t)\} = 1 \Longrightarrow Y(s) = \frac{100}{s+100} \xrightarrow{\mathcal{L}^{-1}} 100e^{-100t}u(t)$$





As we can see ,results are the same.

$\boldsymbol{Part6}$

*Based on the result of **part 3**, if there is no damper, when the car faces a bump, it will be contantly turbulent therefore this state is not ideal.*

*Based on the result of **part 5**, if B equals 100, meaning we have a strong damper. When the car faces a bump, it suppresses the input dramatically in a short amount of time. This state is not ideal as well because it could cause some damages.*

*In **part 4** as we can see, the damper suppresses the input with less intesity than part5, which causes less damages and also there will be no turbulence, therefore this state is the best choice.*

$\boldsymbol{Q_3: Part1}$

$$\mathcal{UL}\left\{\frac{dy}{dt}\right\} = sY(s) - y(0^-) = sY(s) - 1$$

$$\mathcal{UL}\left\{\frac{d^2y}{dt^2}\right\} = s\mathcal{UL}\left\{\frac{dy}{dt}\right\} - y'(0^-) = s(sY(s) - 1) - 1 = s^2Y(s) - s - 1$$

$$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y(t) = x(t) \xrightarrow{\mathcal{UL}} s^2Y(s) - s - 1 + 3(sY(s) - 1) + 2Y(s) = Y(s)(s^2 + 3s + 2) - s - 4 = X(s)$$

$$Y(s) = \frac{X(s)}{s^2 + 3s + 2} + \frac{s + 4}{s^2 + 3s + 2}\,(\mathbf{1})$$

$$x(t) = 5u(t) \xrightarrow{\mathcal{UL}} \frac{5}{s}\,(\mathbf{2})$$

$$(\mathbf{1}),(\mathbf{2}) \Longrightarrow Y(s) = \frac{\frac{5}{s}}{s^2 + 3s + 2} + \frac{s + 4}{s^2 + 3s + 2} = 5\left(\frac{1}{s} - \frac{1}{s+1}\right) - \frac{5}{2}\left(\frac{1}{s} - \frac{1}{s+2}\right) + \left(\frac{3}{s+1} - \frac{2}{s+2}\right)$$

$$= 5\left(\frac{1}{2s} - \frac{1}{s+1} + \frac{1}{2(s+2)}\right) + \left(\frac{3}{s+1} - \frac{2}{s+2}\right) \xrightarrow{\mathcal{UL}^{-1}} \frac{5}{2}u(t) - 5e^{-t}u(t) + \frac{5}{2}e^{-2t}u(t) + 3e^{-t}u(t) - 2e^{-2t}u(t)$$

$$= \frac{5}{2} - 2e^{-t}u(t) + \frac{1}{2}e^{-2t}u(t)$$

The green response, is input response, and the blue response, is response from initial condition.

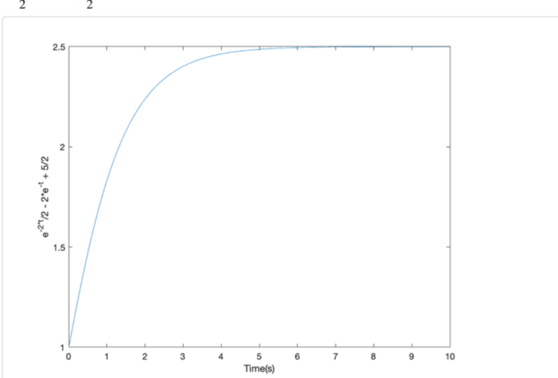
$\boldsymbol{Part2}$

ode(t) =

$$\frac{\partial^2}{\partial t^2}\,y(t) + 3\frac{\partial}{\partial t}\,y(t) + 2\,y(t) = 5$$

ySolv(t) =

$$\frac{e^{-2t}}{2} - 2\,e^{-t} + \frac{5}{2}$$



As we can see the results are the same