Propositional Logic

0-order Logic

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BNF grammar

$\phi, \psi ::= tt | ff | \neg\phi | \phi \wedge \psi$

Syntax sugar

$\phi \vee \psi ::= \neg(\neg\phi \wedge \neg\psi)$
$\phi \Rightarrow \psi ::= \neg(\phi \wedge \neg\psi)$

Example

Term / formulae
$(\mathrm{tt} \wedge ff) \Rightarrow \neg(ff \vee tt)$

Reduction rules

$\neg tt \to ff$

$\neg ff \to tt$

tt $\wedge tt \to tt$

tt $\wedge ff \to ff$

ff $\wedge tt \to ff$

ff $\wedge ff \to ff$

Example

$(\text{tt} \wedge ff) \Rightarrow (ff \vee tt) \to$

ff $\Rightarrow (\neg(\neg ff \wedge \neg tt)) \to$

ff $\Rightarrow (\neg(tt \wedge ff)) \to$

ff $\Rightarrow \neg ff \to$

ff $\Rightarrow tt \to$

tt

Theorem

Confluence: closed term reduction is confluent.

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Def: Term / formulae meaning

Every closed term means its reduction.

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Def: Term equality $\equiv$

Terms are equal $(\equiv) if.have.the.same.meaning(reduction)$.

??? de morgan laws

$\neg\neg\phi \equiv \phi(doublenegation)$

$\neg\phi \wedge \phi \equiv ff(???)$

$\neg\phi \vee \phi \equiv tt(???)$

.

Commutativity and associativity of disjunction

$\phi \wedge \psi \equiv \psi \wedge \phi$

$(\phi \wedge \psi) \wedge \omega \equiv \phi \wedge (\psi \wedge \omega)$

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Commutativity and associativity of conjunction

$\phi \vee \psi \equiv \psi \vee \phi$

$(\phi \vee \psi) \vee \omega \equiv \phi \vee (\psi \vee \omega)$

.

Distributivity

$(\phi \wedge \psi) \vee \omega \equiv (\phi \vee \psi) \wedge (\phi \vee \omega)$

$(\phi \vee \psi) \wedge \omega \equiv (\phi \wedge \psi) \vee (\phi \wedge \omega)$

$\mathrm{P} \Rightarrow Q, P \rightarrow Q$

.....or

$\mathrm{P} \Rightarrow Q, P \vdash Q$

.....or

$\mathrm{P} \Rightarrow Q$

P

Q

Example

$\mathrm{Today.is.Tuesday} \Rightarrow John.will.go.to.work$

Today.is.Tuesday

John.will.go.to.work

$((\mathrm{P} \Rightarrow Q) \wedge P) \Rightarrow Q$

$(\neg(P \wedge \neg Q) \wedge P) \Rightarrow Q$

$((\neg P \vee \neg\neg Q) \wedge P) \Rightarrow Q$

$((\neg P \vee Q) \wedge P) \Rightarrow Q$

$((\neg P \wedge P) \vee (Q \wedge P)) \Rightarrow Q$

$(\mathrm{ff} \vee (Q \wedge P)) \Rightarrow Q$

$(\mathrm{Q} \wedge P) \Rightarrow Q$

???

BNF grammar

x ::= logical variable

$\phi, \psi ::= tt | ff | \neg\phi | \phi \wedge \psi | x$

.

.

Example

Term / formulae

$\phi(x, y) = (x \wedge ff) \Rightarrow \neg(x \vee y)$

$\mathrm{FN}(\cdot) - Free.Names.calculation$

$\mathrm{FN(tt)}$: term $\rightarrow$ *variableset*

$\mathrm{FN(tt)} ::= \varnothing$

$\mathrm{FN(ff)} ::= \varnothing$

$\mathrm{FN(x)} ::= \{ \mathrm{x} \}$

$\mathrm{FN}(\neg\phi) ::= FN(\phi)$

$\mathrm{FN}(\phi \wedge \psi) ::= FN(\phi) \cup FN(\psi)$

Example

$\mathrm{FN}((\mathrm{x} \wedge ff) \Rightarrow \neg(x \vee y)) =$

$\mathrm{FN}(\mathrm{x} \wedge ff) \cup FN(\neg(x \vee y)) =$

$\mathrm{FN(x)} \cup FN(ff) \cup FN(x) \cup FN(y) =$

$\{\mathrm{x}\} \cup \varnothing \cup \{x\} \cup \{y\} =$

$\{\mathrm{x}, \mathrm{y}\}$