Predicate Logic

1-order Logic

Golovach Ivan

RChain Coop

July, 2018

BNF grammar

x ::= logical variable

$\phi, \psi ::= tt | ff | \neg\phi | \phi \wedge \psi | x | \forall x.\phi | \exists x.\phi$

Predicate Logic ::= Propositional Logic $+ \forall x.\phi + \exists x.\phi$

.

Example

Term / formulae

$\forall x.\exists y.((x \wedge \neg y) \Rightarrow (x \vee z))$

$\forall x.\phi / \exists x.\phi$ – "*binders*"
Remove variable name from free variables set.

BNF grammar

$\mathrm{FN}(\forall x.\phi) ::= FN(\phi) \setminus \{x\}$
$\mathrm{FN}(\exists x.\phi) ::= FN(\phi) \setminus \{x\}$

.

.

Example

Closed Term / formulae
$\forall x.\exists y.(x \wedge \neg y)$

What is "value" of this formulae?

Reduction rules: $\forall$

$\forall x.tt \rightarrow tt$

$\forall x.ff \rightarrow ff$

.

$\forall x.\neg\phi \rightarrow \neg(\exists x.\phi)$

$\forall x.(\phi \wedge \psi) \rightarrow (\forall x.\phi) \wedge (\forall x.\psi)$

$\forall x.(\phi \vee \psi) \rightarrow ???$

.

$\forall x.x \rightarrow ff$

$\forall x.y \rightarrow ???y$

Reduction rules: $\exists$

$\exists x.tt \rightarrow tt$

$\exists x.ff \rightarrow ff$

.

$\exists x.\neg\phi \rightarrow \neg(\forall x.\phi)$

$\exists x.(\phi \wedge \psi) \rightarrow ???$

$\exists x.(\phi \vee \psi) \rightarrow (\exists x.\phi) \vee (\forall x.\psi)$

.

$\exists x.x \rightarrow tt$

$\exists x.y \rightarrow ???$

Predicate Logic

Reduction Rules:

???

Predicate Logic

Set comprehension notation

$\forall x \in \mathbb{N}.(x + 1 \in \mathbb{N}) = true$
$\forall x \in \mathbb{N}.(x - 1 \in \mathbb{N}) = false$
$\exists x \in \mathbb{N}.(x + 1 \in \mathbb{N}) = true$
$\exists x \in \mathbb{N}.(x - 1 \in \mathbb{N}) = true$

.

$\{\mathrm{x} \in \mathbb{N} | x < 5\} = \{1, 2, 3, 4\}$

.

$\{\mathrm{x} \in \mathbb{N} | \exists y \in \mathbb{N}.(\exists z \in \mathbb{N}.(x > y + z))\} = \{3, 4, 5, ...\} = \mathbb{N} \setminus \{1, 2\}$