Hennessy-Milner Logic

without recursion

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HML Syntax

BNF grammar

x ::= logical variable

a ::= action, a $\in$ *Act*

$\phi, \psi ::= tt | ff | \phi \wedge \psi | \phi \vee \psi | \langle a \rangle \phi | [a] \phi$

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Syntax sugar

$\langle \{a, b\} \rangle \phi ::= (\langle a \rangle \phi) \vee (\langle b \rangle \phi)$

$[\{a, b\}] \phi ::= ([a] \phi) \wedge ([b] \phi)$

.

Example

$\langle a \rangle tt \wedge [a] \langle b \rangle tt$

HML Semantics

???

$[\cdot]$

???

$[[\phi]] ::= \{p \in Proc | p \models \phi\}$

$(\mathrm{p} \models \phi) \equiv (p \in [[\phi]])$

HML Semantics

$[[\cdot]]$

$[[tt]] ::= Proc$
$[[ff]] ::= \varnothing$
$[[\phi \wedge \psi]] ::= [[\phi]] \cap [[\psi]]$
$[[\phi \vee \psi]] ::= [[\phi]] \cup [[\psi]]$
$[[\langle a \rangle \phi]] ::= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge (q \models \phi))\}$
$[[[a]\phi]] = \{p \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (q \models \phi))\}$

HML Semantics

$[[tt]] ::= Proc$

???

.

$[[ff]] ::= \varnothing$

???

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$[[\phi \vee \psi]] ::= [[\phi]] \cup [[\psi]]$

???

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$[[\phi \vee \psi]] ::= [[\phi]] \cup [[\psi]]$

???

HML Semantics

$[[\langle a\rangle\phi]] ::= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge (q \models \phi))\}$

???

HML Semantics

$[[[a]\phi]] = \{p \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (q \models \phi))\}$

???

Practice

Question: p $\models \langle a \rangle ff$ − what it means?

.

$[[\langle a \rangle ff]] =$
$= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge (q \in [[ff]]))\} =$
$= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge (q \in \varnothing))\} =$
$= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge false)\} =$
$= \{p \in Proc | \exists q \in Proc.false\} =$
$= \{p \in Proc | false\} =$
$= \varnothing$

.

Answer: ???

Practice

Question: p $\models [a]\,ff$ – what it means?

.

$[[[a]ff]] =$

$= \{p \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (q \in [[ff]]))\} =$

$= \{p \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow false)\} =$

$= \{p \in Proc | \nexists q \in Proc.(p \xrightarrow{a} q)\}$

.

Answer: ???

Practice

Question: p $\models [a]tt$ – what it means?

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$$
\begin{aligned}
&[[[a]tt]] = \\
&= \{p \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (q \in [[tt]]))\} = \\
&= \{p \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow true)\} = \\
&= \{p \in Proc | \forall q \in Proc.true\} = \\
&= \{p \in Proc | true\} = \\
&= \text{Proc}
\end{aligned}
$$

.

Answer: ???

Practice

Question: p $\models \langle a \rangle tt$ – what it means?

.

$$
\begin{aligned}
&[[\langle a \rangle tt]] = \\
&= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge (q \in [[tt]]))\} = \\
&= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge (q \in Proc))\} = \\
&= \{p \in Proc | \exists q \in Proc.((p \xrightarrow{a} q) \wedge true)\} = \\
&= \{p \in Proc | \exists q \in Proc.(p \xrightarrow{a} q)\}
\end{aligned}
$$

.

Answer: ???

.

!!!

$\langle a \rangle tt \equiv \exists(\xrightarrow{a})$

Practice

Question: p $\models [a]\langle b\rangle tt$ – what it means?

.

$[[[a]\langle b\rangle tt]] =$

$= \{\mathrm{p} \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (q \in [[\langle b\rangle tt]]))\} =$

$= \{\mathrm{p} \in Prc | \forall q \in Prc.((p \xrightarrow{a} q) \Rightarrow (q \in \{q \in Prc | \exists r \in Prc.(q \xrightarrow{b} r)\}))\} =$

$= \{\mathrm{p} \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (\exists r \in Proc.(q \xrightarrow{b} r))) =$

$= \{\ \mathrm{p} \mid \forall q.(p \xrightarrow{a} q) \Rightarrow \exists r.(q \xrightarrow{b} r)\}$

.

Answer: ???

.

!!!

$[a]\,\langle b\rangle tt \equiv (\exists(\xrightarrow{a}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b}))$

Practice

Question: p $\models [a][b]\langle c\rangle tt$ – what it means?

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$[[[a][b]\langle c\rangle tt]] =$
$= \{\text{p} \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (q \in [[[b]\langle c\rangle tt]]))\} =$
$= \{\text{p} \in Proc | \forall q \in Proc.((p \xrightarrow{a} q) \Rightarrow (\forall r \in Proc.((q \xrightarrow{b} r) \Rightarrow (r \in [[\langle c\rangle tt]]))\}))\} =$
$= \{\text{p} — \forall q.(p \xrightarrow{a} q) \Rightarrow \forall r.(q \xrightarrow{b} r) \Rightarrow \exists s.(p \xrightarrow{c} s)\}$

.

Answer: ???

.

!!!

$[a]\,[b]\,\langle c\rangle tt \equiv (\exists(\xrightarrow{a} \cdot \xrightarrow{b}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c}))$

Practice

Question: ??? $\cdot \xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c} \cdot$

Answer: $\phi ::= (\langle a \rangle tt) \wedge ([a]\langle b \rangle tt) \wedge ([a][b]\langle c \rangle tt)$

Explanation:

$\langle a \rangle tt \equiv \exists(\xrightarrow{a})$

$[a]\,\langle b \rangle tt \equiv (\exists(\xrightarrow{a}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b}))$

$[a]\,[b]\,\langle c \rangle tt \equiv (\exists(\xrightarrow{a} \cdot \xrightarrow{b}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c}))$

.

$(\ \langle a \rangle tt) \wedge ([a]\langle b \rangle tt) \wedge ([a][b]\langle c \rangle tt) \equiv$

$\equiv \exists(\xrightarrow{a}) \wedge (\exists(\xrightarrow{a}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b})) \wedge (\exists(\xrightarrow{a} \cdot \xrightarrow{b}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c})) =$

$= (\exists(\xrightarrow{a} \cdot \xrightarrow{b})) \wedge (\exists(\xrightarrow{a} \cdot \xrightarrow{b}) \Rightarrow \exists(\xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c}))$

$= \exists(\xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c})$

!!!

$\exists(\xrightarrow{a} \cdot \xrightarrow{b} \cdot \xrightarrow{c}) \equiv ((\langle a \rangle tt) \wedge ([a]\langle b \rangle tt) \wedge ([a][b]\langle c \rangle tt)$