

One Dimensional Random Variable



DATE / /

Mo Tu We Th Fr Sa Su

A random variable is a function
to that assigns a real number
 $x(s) = x$ to every element $s \in S$,
corresponding to random experiment.

Range space (R_x): Is the set of all
possible values of x .

Eg: Suppose we toss a pair of coins
then $S = \{HH, TH, HT, TT\}$,
we define: $x \rightarrow \text{no. of head appear}$

$$x(HH) = 2$$

$$x(HT) = x(TH) = 1$$

$$x(TT) = 0$$

$$\therefore \text{Range} \} = \{0, 1, 2\}$$

Now Range space, $R_x = \{0, 1, 2\}$.

Two types of RV:-

① Discrete RV

② Continuous RV

If x is a RV which takes finite
number or countably infinite
number of values; then it is called
discrete RV.

→ Probability mass func. (pmf) / Probability func. :-

If (X) is discrete RV which can
take the values x_1, x_2, \dots, x_n such
that $P(X=x_i) = P(x_i) = p_i$ then p_i
is called pmf, provided the following
condⁿ satisfy,

$$\textcircled{1} \quad p_i \geq 0$$

$$\textcircled{2} \quad \sum_{i=1}^n p_i = 1$$

★ @ continuous RV :-

If x is a RV which takes all the values in an interval, then x is called CRV.

→ Prob. density func. (pdf) :-

If x is CRV, say it is a func. $f(x)$. It is called pdf provided $f(x)$ satisfies the following cond:-

$$\textcircled{1} \quad f(x) > 0 \text{ for } x \in R_x$$

$$\textcircled{2} \quad \int_{-\infty}^{\infty} f(x) dx = 1 \quad \leftarrow x \text{ is right}$$

(3) for any a, b

$$P(a \leq x \leq b) = \int_a^b f(x) dx.$$

★ ~~Cumulative Distribution Function~~

If x is RV (discrete or continuous) then $P(x \leq x_0)$ is called cdf.

and is denoted by

$$(i) \text{ discrete: } F(x) = \sum_{x_j \leq x} P(x_j) = \sum_{x_j \leq x} p(x_j)$$

$$(ii) \text{ continuous: } F(x) = \int_{-\infty}^x f(x) dx$$

$$\text{Ansatz: } F(x) = P(-\infty < x \leq x_0).$$

* Properties :-

1. $f(x)$ is mon-decreasing fm. of x .
 $x_1 < x_2 \Rightarrow f(x_1) > f(x_2)$

2. $F(-\infty) = 0$, $F(\infty) = 1$

3. $P(a < x < b) = \int_a^b f(x) dx$

$$= \int_{-\infty}^{b'} f(x) dx - \int_{-\infty}^a f(x) dx$$

$$= F(b) - F(a)$$

4. $f(x) = \frac{d}{dx} F(x) \quad (x > 0)$

5. A RV x has prob. func. $P(x=k) = c_k$

$\sum P(x=k) = 1$. Then find

- i) Value of c ($x > 0$)
- ii) prob. if ($x \geq 5$)
- iii) $P(x < 1/2)$

since $\sum_{k=0}^{\infty} P(x=k) = 1 \Rightarrow \sum_{k=0}^{\infty} c_k = 1$

$$\frac{c}{0!} + \frac{c}{1!} + \frac{c}{2!} + \dots = 1 \quad (i)$$

$$c \left(1 + \frac{1}{2} + \frac{1}{4} + \dots \right) = 1 \quad (i)$$

$$c \left(\frac{1}{1 - \frac{1}{2}} \right) = 1$$

$$2c = 1 \Rightarrow c = 1/2 \quad (ii)$$

$$\therefore P(\alpha = k) = \frac{C}{\alpha^k} = \frac{1}{\alpha^{k+1}}$$

$$\text{ii) } P(\alpha > 5) = \frac{1}{\alpha^6} + \frac{1}{\alpha^7} + \dots$$

$$= \frac{\frac{1}{\alpha^6}}{1 - \frac{1}{\alpha}} \left(1 + \frac{1}{\alpha} + \dots \right)$$

$$= \frac{1}{\alpha^6} \left(\frac{1}{1 - \frac{1}{\alpha}} \right)$$

$$= \frac{1}{\alpha^6} \left(\frac{\alpha}{\alpha - 1} \right)$$

$$= \frac{1}{\alpha^5} = \frac{1}{32}$$

$$\text{iii) } P(\alpha < 2) = P(\alpha \leq 0) = 0.5$$

Q. If α be a R.V with pdt ~~st~~:

$$f(x) = \begin{cases} Kx^3, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$$

find value of (i) K

$$(i) P(Y_4 \leq x \leq 3/4)$$

$$(ii) P(\alpha > Y_2)$$

$$(iii) P(\alpha < 0.8)$$

$$\text{(i) } \int_{-\infty}^{\infty} Kx^3 dx \quad \int Kx^3 dx = 1 \Rightarrow$$

$$\frac{K}{4} \left[x^4 \right]_0^1 = 1$$

$$\frac{K}{4} (1) = 1 \Rightarrow K = 4$$

$$f(x) = \begin{cases} 4x^3 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

$$P\left(\frac{1}{4} < x < \frac{3}{4}\right)$$

$$= \int_{y_4}^{y_4} 4x^3 dx$$

$$= [ax^4]_{y_4}^{y_4}$$

$$= \frac{5}{16}$$

$$(iii) P(x > \frac{1}{2}) = \int_{y_2}^{\infty} 4x^3 dx$$

$$= \int_{y_2}^{\infty} 4x^3 dx + \int_{0}^{\infty} 4x^3 dx$$

$$= [ax^4]_{y_2}^1$$

$$= 1 - (\frac{1}{2})^4$$

$$= 1 - \frac{1}{16} = \frac{15}{16}$$

$$(iv) P(x < 0.8) = \int_{-\infty}^0 4x^3 dx + \int_0^{0.8} 4x^3 dx$$

$$= [ax^4]_0^{0.8} =$$

$$= \frac{256}{1685}$$

Q. Suppose that a random variable X with pdf $f(x) = \begin{cases} 8x & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$

find

- (i) cdf
(ii) $F(0.5)$

(i) $F(x) = \int_{-\infty}^x f(x) dx$

for $x < 0$ $F(x) = \int_{-\infty}^x f(x) dx = \int_{-\infty}^0 0 dx = 0$.

for $0 < x < 1$

$F(x) = \int_{-\infty}^0 f(x) dx + \int_0^x f(x) dx$

$= 0 + \int_0^x 8x dx$

$= [4x^2]_0^x$

$\Rightarrow \frac{x^2}{4} = 1 - 1 = 0$

for $x \geq 1$

$F(x) = \int_{-\infty}^0 f(x) dx + \int_0^1 f(x) dx + \int_1^x f(x) dx$

$= \int_{-\infty}^0 0 dx + \int_0^1 8x dx + \int_1^x 0 dx$

$= 0 + \int_0^1 8x dx + \int_1^x 0 dx$

$= 1$

$$F(x) = \begin{cases} 0 & x < 0 \\ x^2 & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$$

$$\frac{d(x^2)}{dx} = 2x \rightarrow \text{pdf}$$

$$F(0.5) = \int_{-\infty}^{0.5} 2x \, dx$$

$$= \int_{-\infty}^{0.5} 0 \, dx + \int_0^{0.5} 2x \, dx$$

$$= [x^2]_0^{0.5} = \frac{1}{4}$$

$$= 0.25$$

Q. A coin is known to come up heads as often as tails. The coin is tossed 3 times. Let x denotes no. of heads that appear. Write the prob. dist. of x and also eff.

$$P(H) = \frac{3}{4}, P(T) = \frac{1}{4}$$

$S = \{HHH, HHT, HTH, HTT, TTT, THT, TTH, THT\}$

X : no. of heads appear

$$Rx = \{0, 1, 2, 3\}$$

$$P(X=0) = P(TTT) = \left(\frac{1}{4}\right)^3$$

x	0	1	2	3
$P(x)$	$\frac{1}{64}$	$\frac{3}{64}$	$\frac{27}{64}$	$\frac{27}{64}$

$$\begin{aligned}
 P(X=1) &= P(HTT) + P(THT) + P(TTH) \\
 &= 3 \times \frac{3}{4} \times \left(\frac{1}{4}\right)^2 \\
 &= \frac{3}{64}
 \end{aligned}$$

$$\begin{aligned}
 P(X=2) &= P(HHT) + P(HTH) + P(THH) \\
 &= 3 \times \left(\frac{3}{4}\right)^2 \times \frac{1}{4} \\
 &= \frac{27}{64}
 \end{aligned}$$

$$\begin{aligned}
 P(X=3) &= P(HHH) \\
 &= \left(\frac{3}{4}\right)^3 \\
 &= \frac{27}{64}
 \end{aligned}$$

$$\begin{aligned}
 f(x) &= \sum_{x_j \leq x} P(x=x_j) \\
 P(x) &= \begin{cases} 0 & x < 0 \\ \frac{27}{64}, & 0 \leq x < 1 \\ \frac{27}{64} + \frac{27}{64} = \frac{54}{64}, & 1 \leq x < 2 \\ \frac{27}{64} + \frac{27}{64} + \frac{27}{64} = \frac{81}{64}, & 2 \leq x < 3 \\ \frac{81}{64} + \frac{27}{64} = 11, & x \geq 3 \end{cases}
 \end{aligned}$$

Mean

Difficulty :-

$$E(x) (= u) = \sum x_i p(x_i)$$

If $g(x)$ is a func. of $E(x)$ then
 $E(g(x)) = \sum g(x_i) p(x_i)$

My notes

Dis continuous :-

$$E(x) = \int_{-\infty}^{\infty} x f(x) dx.$$

If $u(x)$ is func. of x then

then $E(u(x)) = \int_{-\infty}^{\infty} u(x) f(x) dx.$

$$E(u(x)) = \int_{-\infty}^{\infty} u(x) f(x) dx.$$

Variance

Variance is denoted by

 $V(u)$ or σ^2 and also, (\bar{x}) and defined as $V(x) = E(x - E(x))^2 = E(x^2) - (E(x))^2$

→ the square root of $V(x)$ is called standard deviation of x .

Properties

$$\rightarrow E(c) = c \quad (c \text{ const.}) \quad (\text{iii})$$

$$\rightarrow E(ax+b) = a E(x) + b$$

$$\rightarrow E(E(x)) = E(x)$$

$$\rightarrow V(c) = 0$$

$$\rightarrow V(cx) = c^2 V(x)$$

PROOF:

$$V(x) = E(x^2) - (E(x))^2$$

$$= c^2 E(x^2) - (c E(x))^2$$

$$V(cx) = E(c^2 x^2) - (E(cx))^2$$

$$= c^2 E(x^2) - (c E(x))^2$$

$$= c^2 V(x)$$

10

Q. A RV x has following prob. distribution.

X	-2	-1	0	1	2	3
P(x)	0.1	K	0.2	0.2	0.8	0.8K

find

(i) K

(ii) $P(-2 < x < 2)$

(iii) edf of x

(iv) mean and variance of x

(v) mode & median

$$1. \sum P(x) = 1$$

$$0.6 + 6K = 1$$

$$6K = 0.4$$

$$K = 1/15$$

30

$$(ii) P(-2 < x < 2) = K + 0.2 + 0.8K = 3K + 0.2$$

$$= 3 \times \frac{1}{15} + 0.2$$

$$= \frac{3}{15} + 0.2$$

$$= 1/5 + 1/5$$

34

(iii) $F(x) = \begin{cases} 0 & x < -2 \\ 0.1 & -2 \leq x < -1 \\ Y_1 & -1 \leq x < 0 \\ 0.8 & 0 \leq x < 1 \\ Y_2 & 1 \leq x < 2 \\ 1.4 & 2 \leq x < 3 \\ 1.0 & x \geq 3 \end{cases}$

(iv) $u = \sum x_i P(x_i)$
 $= \frac{16}{15}$

Ans $E(x^2) = \sum x_i^2 P(x_i)$

$E(x^2) = \sum x_i^2 P(x_i)$
 $= 8.6 = 18$

$V(x) = E(x^2) - (E(x))^2$

$V(x) = \frac{554}{1000} = 0.554$

(v) mode - ~~the~~ value of x for which $P(x)$ is max.

$\therefore \text{mode} = 2$

median - $\frac{x_1 + x_2}{2}$ where x_1 is largest value of x such that $P(x \leq x_1) \leq 0.5$
 $x_2 \rightarrow$ smallest value of x such that $P(x \leq x_2) \geq 0.5$

Q. RV X has pdf given by $f(x) = \begin{cases} k(1-x^2), & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

find:

(i) k

(ii) $P(0.1 < x < 0.2)$

(iii) mean and variance

(iv) mode, median

i) $\int_{-\infty}^{\infty} f(x) dx = 1$

$$\int_0^1 k(1-x^2) dx = 1$$

$$\frac{2}{3}k = 1$$

$$k = \frac{3}{2}$$

(ii) $P(0.1 < x < 0.2)$

$$\int_{0.1}^{0.2} \frac{3}{2}(1-x^2) dx = 0.1465$$

$$\int_{0.1}^{0.2} \frac{3}{2}(1-x^2) dx = \frac{22.293}{2000} = 0.1465$$

(iii) $E(X) = \int_{-\infty}^{\infty} x f(x) dx$

$$= \int_0^1 \frac{3x}{2}(1-x^2) dx.$$

Integrate by parts, $x^2 - 2x^3 + x^4$

$$= 0.375$$

$$30 \cdot 0.375 = 1.125$$

$$\text{Median} = \frac{3}{8}$$

Interpretation:

$$P(0.375 < X < 0.625) ?$$

$$E(x^2) = \int_0^1 3x^2(1-x^2)dx = 0.2 = 21$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \frac{19}{300} = 0.059375$$

$$(iv) \text{ if } \frac{d^2f(a)}{da^2} = 0.$$

$$\frac{d}{dx} \left(\frac{3}{2}(1-x^2) \right) = 0$$

$$f'(a) = -3a = 0$$

$$a = 0.$$

$$f''(a) = -3 < 0$$

$x = 0$ is mode.

$$\text{medium, } m = \int_{-\infty}^m f(a) da = \frac{1}{2}, \text{ or } \int_m^\infty f(a) da = \frac{1}{2}$$

$$\int_{-\infty}^m f(a) da = \frac{1}{2}.$$

$$\int_0^m \frac{3}{2}(1-x^2) dx = \frac{1}{2}$$

$$\frac{3}{2} \left[x - \frac{x^3}{3} \right]_0^m = \frac{1}{2}$$

$$m - \frac{m^3}{3} = \frac{1}{2} \quad \therefore m = 0.847.$$

$$3m - m^3 = 1$$

$$m^3 - 3m + 1 = 0$$

$$m = 1.582, -1.87, 0.847$$

Q. A CRV X has pdf $f(x) = Kx^2e^{-x}$, find mean and standard deviation also find $P(|x| \leq 1)$.

$$E(x) = \int_0^\infty x f(x) dx$$

$$\int_0^\infty Kx^2 e^{-\alpha x} dx = 1$$

$$K \int_0^\infty x^2 e^{-\alpha x} dx = 1$$

$$K \Gamma(3) = 1$$

$$K \Gamma(2) = 1$$

$$K = 1/2.$$

$$F(x) = \int_0^\infty x^2 e^{-\alpha x} dx$$

$$= \frac{1}{2} \int_0^\infty x^3 e^{-\alpha x} dx$$

$$= \frac{1}{2} \Gamma(4)$$

$$= \frac{1}{2} (3!) = \frac{6}{2} = 3.$$

$$E(x^2) = \int_0^\infty x^2 x^2 e^{-\alpha x} dx$$

$$= \frac{1}{2} \int_0^\infty x^4 e^{-\alpha x} dx$$

$$= \frac{1}{2} \Gamma(5) := \frac{1}{2} \cdot 4! = \frac{24}{2} = 12.$$

$$\text{Sumda} = \text{UfVda} - \text{fU}$$

$$SV = V_1$$

$$\text{DATA} \quad V_1 = V_2$$

Mo Tu We Th Fr Sa Su

$$\text{Std dev.} = \sqrt{f_{12}} = \sqrt{3.464} =$$

$$V(x) = E(x^2) - (E(x))^2$$

$$21 = 12 - (3)^2 = (2-3)^2$$

$$22 = 12 - 9 =$$

$$= 3.$$

$$1 = 9 - (2)^2 = (2-3)^2$$

$$\sigma = \sqrt{3}$$

$$P(|x| \leq 1) = P(-1 \leq x \leq 1),$$

$$221' 221'' 221''' | (x)$$

$$8840.0 - 8810.0 = 30 \rightarrow 1$$

$$\int_{-1}^1 f(x) dx = \int_{-1}^1 x^2 dx + \int_{-1}^1 x dx$$

$$1 = \frac{1}{2} \int_0^1 x^2 e^{-x} dx.$$

$$= \frac{1}{2} \times 0.803 (e^{-1})$$

- Q. A box contains 12 items of which 4 are defective. A sample of 8 items is selected from the box. Let x denotes the no. of defective items in the sample. Find the prob. distribution of x . Also determine mean, standard dev., mode & median.
- $x: \text{no. of dif. items}$

$$30. f_x = \begin{cases} 0, 1, 2, 3, 4 \\ f_p \end{cases} \quad P(X=0) = \frac{8C_8}{12C_3} = \frac{14}{55}$$

~~$$P(X=1) = \frac{4C_4}{12C_3} = \frac{1}{55} \quad P(X=2) = \frac{10C_3}{12C_3} = \frac{6}{11}$$~~

$$P(X=1) = \frac{^4C_1 \times {}^8C_2}{{}^{12}C_3} = \frac{28}{55}$$

$$P(X=2) = \frac{^4C_2 \times {}^8C_1}{{}^{12}C_3} = \frac{12}{55}$$

$$P(X=3) = \frac{^4C_3}{{}^{12}C_3} = \frac{1}{55}$$

X	0	1	2	3
P(X)	14/55	28/55	12/55	1/55
	0.2545	0.5	0.2182	0.0182
Mean = E(X) =	0.14/55	42/55	54/55	1

15

$$E(X^2) = 0.8 \sum x^2 p_x =$$

$$= \frac{17}{11}$$

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$$\text{range} = E(X^2) - (E(X))^2 = 16/11 - 17/11^2 = 8/11$$

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$$\text{mode} = 1 \Rightarrow \text{maximum frequency}$$

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$$\text{median} = \frac{x_1 + x_2}{2} = \frac{0+1}{2} = \frac{1}{2}$$

34

Given: a. CRV X has pdf $f(x) = \begin{cases} 6x(1-x), & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$

find mean, mode & median

$$f(x) =$$

$$F(x) = \int_{-\infty}^x x + \alpha dx$$

$$= \int_0^1 6x^2(1-x) dx.$$

$$= \frac{1}{2}.$$

~~$$E(x^2) = \int_0^1 x^2 f(x) dx$$~~

~~$$= \int_0^1 6x^3(1-x) dx$$~~

$$= \frac{8}{15}$$

~~$f'(x)$~~

$$f'(x) = \frac{d}{dx}(6x - 6x^2)$$

$$\Rightarrow 6 - 12x = 0$$

$$\Rightarrow x = \frac{1}{2}$$

$$f''(x) = -12x.$$

$$\text{mode} = x = \frac{1}{2} \cdot m$$

$$\text{median}, m = \int_{-\infty}^m 6x(1-x) dx = \frac{1}{2}$$

$$= 6 \int_0^m x - x^2 dx = \frac{1}{2} \Rightarrow \frac{x^2}{2} - \frac{x^3}{3} \Big|_0^m = \frac{1}{2}$$

$$\frac{9}{3} - \frac{x^2}{2} + \frac{1}{12} = 0$$

$$m = 1.8660, -0.8660, 0.5$$

H.W.

Let X be MGRV with pdf $f(x) = y_0 e^{-x}$,
 $-\infty < x < \infty$

- (i) P.T. $y_0 = 1$: Use mean, median, mode
(ii) variance.

5

$$f(x) = \int y_0 e^{-x} dx, \quad 0 < x < \infty$$

$$y_0 e^{-x}, \quad \text{for } x < 0$$

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$$\text{nb } (x+1)^{\frac{1}{2}} \text{ is } \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right.$$

$$\frac{1}{0} = \infty$$

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$$(x+1)^{\frac{1}{2}} \text{ is } \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right.$$

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$$(x+1)^{\frac{1}{2}} \text{ is } \left\{ \begin{array}{l} 1 \\ 0 \end{array} \right.$$

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$$0 = 0^{\frac{1}{2}} = 1 - e^{-1}$$

$$0^{\frac{1}{2}} = 1 - e^{-1}$$

$$1 - e^{-1} = 0.632$$

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$$= \text{nb } (x+1)^{\frac{1}{2}}$$

* Markov's Inequality

If x is a RV assuming non-negative values. Then $P(x \geq a) \leq \frac{E(x)}{a}$, $a > 0$

* Chebychev's Inequality

Let x be a RV with mean $E(x) = \mu$ and $SD = \sigma$ then for any $k > 0$ we have

$$P(\mu - k\sigma \leq x \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$$

$$\text{put } k = 1 \Rightarrow P(|x - \mu| \leq \sigma) \geq 1 - \frac{1}{1^2} = 0$$

$$= P(|x - \mu| \leq \sigma) \geq 1 - \frac{1}{1^2} = 0$$

Now we want to find σ such that $P(|x - \mu| \leq \sigma) \geq 0.99$

Use Chebychev's inequality to find

- (i) $P(|x - \mu| \leq 3) \geq 0.99$
- (ii) $P(-2 \leq x \leq 8) \geq 0.99$ $E(x) = 3$
- (iii) $P(|x - \mu| \geq 3) \leq 0.01$ $E(x^2) = 13$

Now we want to find σ such that

$$\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ &= 13 - 9 \end{aligned}$$

$$\sigma^2 = V(x) = 4 \quad \sigma = 2$$

$$\sigma = 2\sigma = 2$$

$$(13) \geq (2\sigma)^2 \geq 4$$

$$(i) \quad P(|x - \mu| \leq 3\sigma) \geq 1 - \frac{1}{9}$$

$$0.99 \geq (1 - \frac{1}{9}) \geq 0.99$$

$$3\sigma = 3$$

$$\sigma = 1$$

$$\sigma = 1.5$$

$$P(|x - \mu| \geq 3\sigma) \geq 1 - \frac{4}{9} = 5/9$$

$$(LÜ) P(\mu - K\sigma \leq x \leq \mu + K\sigma) \geq 1 - \frac{1}{K^2}$$

$$\mu = 50 \text{ Einf } 2.5$$

$$3 = K\sigma \Rightarrow K = 1 - 2 = 1.5$$

$$K = 5$$

$$K = 5/2$$

$$\therefore P(-2 < x < 8) \geq 1 - \frac{1}{K^2} = 1 - \frac{1}{25} = 0.96$$

$$1 - 1 < (x - \mu) > \sigma \Rightarrow 0.96 = 0.96$$

Q. Suppose that it is known that the no. of items produced by a factory during a week is a RV with mean 50.

(1) What can be said about the prob. that this week's production will

be exceeded by 75 or more?

(2) If the variance of a week's production is known to be 25, then what can be said about the prob. that this week's production will be fewer than 60.

$$P = 8.1$$

$$E(x) = \mu = 50 \quad \sigma = 5 \quad \sigma^2 = 25$$

$$\sigma = \sqrt{\sigma^2} = 5$$

$$P(x > 75) \leq F(x)$$

$$1 - 1 < (x - \mu) > \sigma \Rightarrow 0.96 = 0.96$$

$$P(x > 75) \leq 0.04$$

$$75$$

$$P(x > 75) \leq 0.04$$

$$3$$

$$P(40 \leq x \leq 60) \geq 1 - \frac{1}{K^2}$$

$$\mu - K\sigma = 40$$
$$50 - K(5) \geq 40$$

$$P(40 \leq x \leq 60) \geq 1 - \frac{1}{25}$$

$$10 = SK$$

$$P(40 \leq x \leq 60) \geq 1 - \frac{1}{4}$$

$$10 = K = 2.$$

$$P(40 \leq x \leq 60) \geq \frac{3}{4}$$

28 a. $x \sim N(50, 25)$

29 a. $x \sim N(50, 25)$

30 a. $x \sim N(50, 25)$

b. $x \sim N(50, 25)$

under 30 we would see 10% to 4.

a pos weight building in in

15 b. $x \sim N(50, 25)$ only 10% in
VR community

20 c. $x \sim N(50, 25)$

$$25 = (x - 50, 25 - 25)$$

d. $x \sim N(50, 25)$

$$10 = (x - 50, 25 - 25)$$

25 a. $x \sim N(50, 25)$

b. $x \sim N(50, 25)$

25

26 a. $x \sim N(50, 25)$

for a VR community 90% in what

for that better (50 + 25) = 75

30

and requires to count below

2-D Random Variable

Let S be the sample space assoc. with a RE. Let $x = X(S) = x$, $y = Y(S) = y$ be a func. each assigning a real number to each outcome $s \in S$ then (x, y) is called 2-D RV.

* If possible values of (x, y) are finite or countably infinite then (x, y) is called 2-D discrete RV.

* If (x, y) can assume all the values in a specified region say R in xy plane, then (x, y) is called continuous 2-D RV.

* Joint Pmf.

$$P(x = x_i, y = y_j) = p_{ij}$$

$$\bullet p_{ij} \geq 0$$

$$\bullet \sum_j \sum_i p_{ij} = 1$$

If (x, y) is 2D discrete RV then p_{ij} is called joint pmf if

* Joint Pdt.

If (x, y) is 2D continuous RV, if a func. $f(x, y)$ called joint pdt provided $f(x, y)$ satisfies the following cond-n.

$$\textcircled{1} \quad f(x, y) \geq 0 \quad \forall (x, y) \in R$$

$$\textcircled{2} \quad \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy = 1$$

$$\textcircled{3} \quad P(a < x < b, c < y < d) = \int_a^b \int_c^d f(x, y) dx dy$$

* cdf

$$\textcircled{4} \quad \text{if } (x, y) \text{ in DRV, } F(x, y) = \sum_{y_j \leq y} \sum_{x_i \leq x} P(x_i, y_j)$$

$$\textcircled{5} \quad \text{if } (x, y) \text{ in CRV, } F(x, y) = \int_{-\infty}^x \int_{-\infty}^y f(x, y) dx dy.$$

$$\frac{\partial^2}{\partial x \partial y} F(x, y) = f(x, y)$$

* Properties

$$\rightarrow F(-\infty, y) = F(x, -\infty) = 0$$

$$\rightarrow F(\infty, \infty) = 1$$

$$\rightarrow P(a < x < b, c < y < d) = F(b, y) - F(a, y)$$

$$\rightarrow P(x \leq a, c < y < d) = F(a, d) - F(a, c)$$

$$\rightarrow P(a < x < b, c < y < d) = F(b, d) - F(b, c) - F(a, d) + F(a, c).$$

* Marginal Prob. Point

x	y₁	y₂	...	y_m	(x, y)
a₁	p₁₁	p₁₂	...	p_{1m}	f(x₁)
a₂	p₂₁	p₂₂	...	p_{2m}	f(x₂)
:	:	:			:
a_n	p_{n1}	p_{n2}	...	p_{nm}	f(x_n)
	g(y₁)	g(y₂)	...	g(y_m)	1

* Marginal Pmf

Here x, y are DRV

If we define as follows,

$p_{marg}(x_i)$

Marginal pmf of x :

$$p(x_i) = \sum_{j=1}^n p_{ij}$$

Marginal pmf of y :

$$q(y_j) = \sum_{i=1}^n p_{ij}$$

* Marginal pdf.

Let x, y be CRV then,

marginal pdf of x :

$$g(x) = \int f(x, y) dy$$

20

marginal pdf of y :

$$(y, 0) \rightarrow (y, \infty) : \int_0^\infty f(x, y) dx, x \geq 0 \Rightarrow g(y) = \int_0^\infty f(x, y) dx$$

* Condⁿ Prob. Distribution

Let (x, y) be DRV, point p_{ij} a

$p(x_i)$ & $q(y_j)$ marginal pmf of x & y .

$$P(x=x_i \cap y=y_j) = p_{ij}$$

$$\therefore P(y=y_j) = P(y=y_j)$$

34

$$P(Y_i = y_i | \alpha = x_i) = \frac{P_{ij}}{P(x_i)} = P(\alpha = x_i).$$

* Cond^n P_{dy}

If (x, y) are CRV

pdf $f(x, y) \propto g(x) u(y)$ marginal
pdf of x, y resp.

Cond^n pdt of x , $g(x|y) = \frac{f(x, y)}{g(y)}$.

" " y , $u(y|x) = \frac{f(x, y)}{g(x)}$, $g(x) > 0$

Q: Find

$$P(\alpha \leq 1), P(y \leq 3), P(\alpha \geq 1, y \leq 3)$$

$$P(\alpha \leq 1 | y \leq 3)$$

$$P(y \leq 3 | x \leq 1)$$

$$P(\alpha + y \leq 4)$$

if you find the following prob. distribution.

$x \setminus y$	1	2	3	4	5	6	Rowsum	Colsum
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{3}{32}$	$\frac{4}{32}$	1	1
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{5}{16}$	$\frac{1}{2}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	0	$\frac{2}{64}$	$\frac{1}{8}$	$\frac{1}{8}$
Colsum							1	0

$$\begin{aligned} i) P(\alpha \leq 1) &= P(\alpha = 0) + P(\alpha = 1) \\ &= P(x=0, y=y_i) + P(x=1, y=y_i) \\ &= \frac{7}{16} \end{aligned}$$

$$P(Y \leq 3) = P(Y=1) + P(Y=2) + P(Y=3)$$

$$\frac{8}{32} + \frac{8}{32} + \frac{11}{64}$$

$$= \frac{23}{64}$$

$$64$$

$$P(X \leq 1, Y \leq 3) = \frac{9}{32}$$

$$\text{Ansatz } P(X \leq 1 | Y \leq 3) = \frac{P(X \leq 1, Y \leq 3)}{P(Y \leq 3)}$$

$$= \frac{9/32}{23/64} = \frac{18}{23}$$

$$\text{CSE } P(Y \leq 3 | X \leq 1) \approx P(X \leq 1, Y \leq 3) = 9/32$$

$$(18/23) \times 18/32 = 18/32$$

$$20 \quad x \quad y$$

$$\text{mit } P(X+Y \leq 4) = \frac{3}{32} + \frac{2}{16} + \frac{1}{8} + \frac{2}{32}$$

$$0 \quad 2$$

$$0 \quad 3$$

$$0 \quad 4$$

$$= 18$$

$$32$$

25

$$1 \quad 1$$

$$1 \quad 2$$

$$1 \quad 3$$

$$2 \quad 1$$

$$2 \quad 2$$

30

$$(1-2)^2 + (0-1)^2 + (1-2)^2 + (1-2)^2 + (0-1)^2 + (1-2)^2 + (1-2)^2 + (1-2)^2 + (1-2)^2 + (1-2)^2$$

Q. $\forall (x, y)$ in Δ CRV

$$f(x, y) = \begin{cases} x^2 + \frac{xy}{3}, & 0 \leq x \leq 1, 0 \leq y \leq 2 \\ 0, & \text{otherwise} \end{cases}$$

i) marginal pdf of x & y

$$\begin{aligned} \text{(ii)} \cdot P(x > y_2) & \quad \text{(iii)} P(y < 1) \\ \text{(iv)} \cdot P((x+y) \approx 1) & \quad \text{(v)} P(x > y_2 | y > 1) \end{aligned}$$

$$\text{i) } g(x) = \int_{-\infty}^{+\infty} f(x, y) dy.$$

$$= \int_0^2 \left(x^2 + \frac{xy}{3} \right) dy$$

$$= x^2 [y]_0^2 + \frac{x}{3} [y^2]_0^2$$

$$= x^2 + \frac{x}{6} (4)$$

$$= x^2 + \frac{x}{3}, \quad 0 \leq x \leq 1$$

marginal pdf of y

$$h(y) = \int_{-\infty}^{+\infty} f(x, y) dx$$

$$= \int_0^1 \left(x^2 + \frac{xy}{3} \right) dx$$

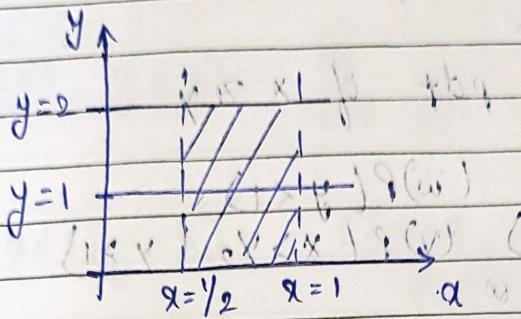
$$= [x^3]_0^1 + \frac{y}{6} [x^2]_0^1$$

$$= \frac{1}{3} + \frac{y^3}{6}, \quad 0 \leq y \leq 2$$

$$y_2 > 2 = 1$$

$$(ii) P(x > y_2) = \int_0^2 \int_{y_2}^{x+1} P(x > y_2)$$

$$P(a \leq x \leq b, c \leq y \leq d) = \int_a^b \int_c^d f(x, y) dy dx$$



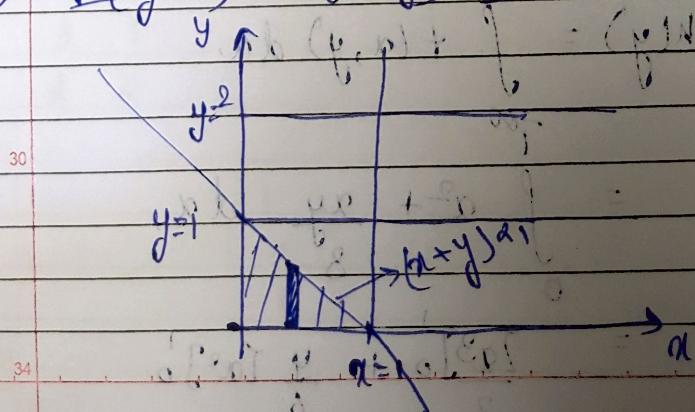
$$P(x > y_2) = P(y_2 < x < 1, 0 \leq y \leq 2)$$

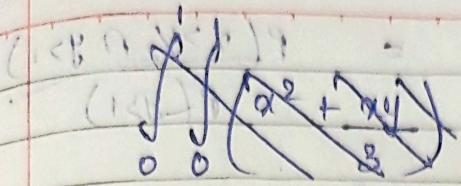
$$= \int_0^1 \int_0^{2-x} (x^2 + \frac{xy}{3}) dy dx$$

$$= \int_0^1 \int_0^{2-x} (x^2 + 4x) dx$$

$$= \frac{5}{6}$$

$$(iii) P(y > 1) \rightarrow P(x+y > 1)$$



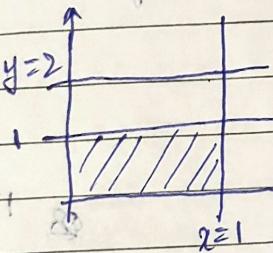


$$\int_0^1 \int_0^{1-x} \left(x^2 + \frac{xy}{3} \right) dy dx.$$

$$\int_0^1 x^2(1-x) + \frac{x(1-x)^2}{6} dx.$$

$$f(x) = \frac{1}{72} \left\{ x^6 + \frac{x^6}{6} \right\},$$

$$P(Y \leq 1) = \int_0^1 \int_{y=0}^x \left(x^2 + \frac{xy}{3} \right) dy dx$$

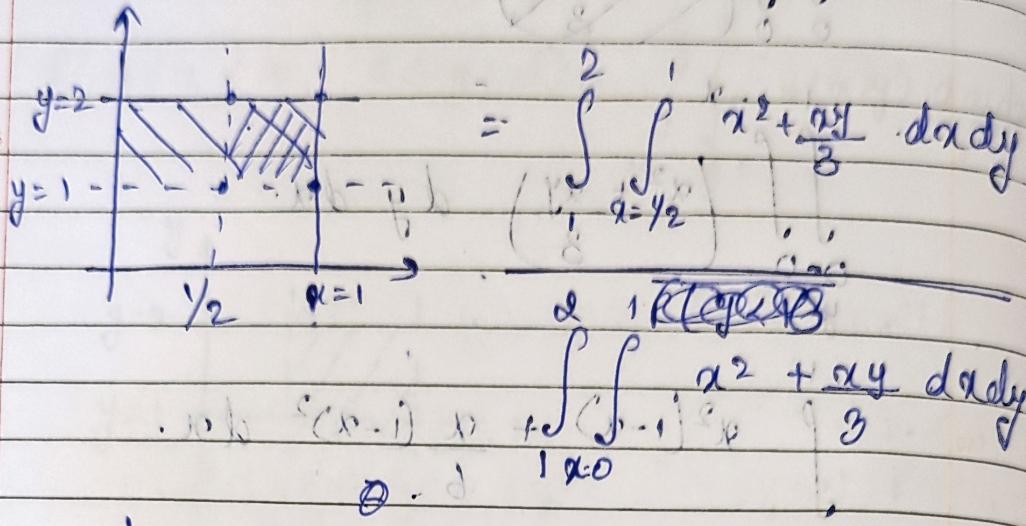


$$P(Y \leq 1) = \int_0^1 \int_{y=0}^x \left(\frac{x^3}{3} \right)_0^1 + \frac{y}{6} [x^2]_0^1 dy$$

$$= \int_0^1 \frac{1}{3} + \frac{y}{6} dy$$

$$= \frac{5}{12}$$

$$P(X > \frac{1}{2}, Y > 1) = P(X > \frac{1}{2} \cap Y > 1)$$



$$\int_1^2 \left[\frac{x^3}{3} \right]_{y_2}^1 + \frac{y}{6} \left[x^2 \right]_{y_2}^1 dy$$

$$\int_0^1 \left[\frac{x^3}{3} \right]_0^1 + \frac{y}{6} \left[x^2 \right]_0^1 dy$$

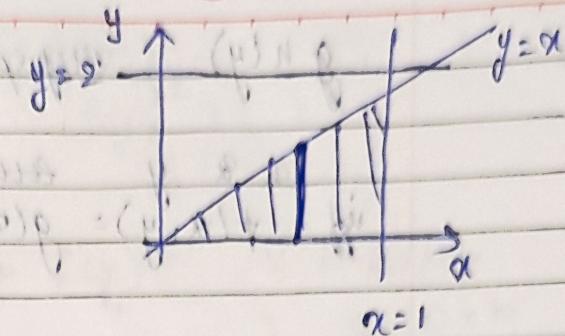
$$\int_1^2 \left[\frac{7}{8}y + \frac{3y}{24} \right]_1^2 dy$$

$$\int_1^2 \left[\frac{7}{8}y + \frac{1}{6}y \right] dy$$

$$= \frac{23}{28}$$

$P(Y \leq x)$

$$\int_{x=0}^{\infty} \int_{y=0}^x \left(x^2 + \frac{xy}{3} \right) dy dx$$



$$\int_0^1 \left[x^3 + \frac{x^2}{6} [y^2]_0^x \right] dx$$

$$(10) \int_0^1 \left[x^3 + \frac{x^5}{6} \right] dx$$

$$(11) = (10) + \int_0^1 x^5 dx$$

$$(12) \int_0^1 x^5 dx = (10) 9$$

$$\frac{7}{6} (x^6)_0^1 = \frac{7}{6}$$

Final answer $\sqrt{6}/6$ or $(\sqrt{6}/6)^{1/2}$

This is the required answer

\star Independent RV (Ans)

Let (x, y) be a-D DRN with joint pmf $p(x_i, y_j)$ and marginal pdfs of x and y , $p(x_i)$ & $q(y_j)$ resp. Then we say that x & y are independent iff $p(x_i, y_j) = p(x_i) \cdot p(y_j)$.

\star (Ans) \Rightarrow (Ans) way

Let x, y be a-D continuous RV with joint pdf $f(x, y)$ & marginal pdf of x & y , $f_x(x)$ &

$g(x, y)$ is exp. then

for x, y and all independent RV iff $f(x, y) = g(x) \cdot h(y)$, $\forall x, y$.

Let (x, y) be 2D DRV & joint pmf $p(x_i, y_j)$. If $p(x_i)$ & $q(y_j)$, then Expected value

$$E(x) = \sum_i x_i p(x_i)$$

$$E(y) = \sum_j y_j q(y_j) = p(y_j)$$

$$E(xy) = \sum_{i,j} x_i y_j p_{ij}$$

Let (x, y) be 2D cRV with joint pdf & marginal pdf's $g(x)$ & $h(y)$

$$E(x) = \int x g(x) dx.$$

20

using VCF

$$\int_{-\infty}^{\infty} x f(x, y) dy$$

$$= \int_{-\infty}^{\infty} x \int_{-\infty}^{\infty} f(x, y) dy dx$$

$$E(y) = \int_{-\infty}^{\infty} y h(y) dy = \int_{-\infty}^{\infty} y f(x, y) dx$$

30

$$E(xy) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x y f(x, y) dx dy$$

34

$$V(x) = E(x^2) - (E(x))^2$$

$$V(y) = E(y^2) - (E(y))^2$$

$$E(x+y) = E(x) + E(y), \quad ; \quad E(ax+by) = aE(x) + b$$

$E(xy) = E(x)E(y)$ if x & y independent

$$V(x+y) = V(x) + V(y)$$

* Covariance of $(x,y) = \text{cov}(x,y)$

$$\text{cov}(x,y) = E(xy) - E(x)E(y)$$

if x & y are independent, $\text{cov}(x,y) = 0$

* Co-eff of correlation

$$\rho = \frac{\text{cov}(x,y)}{\sqrt{V(x)V(y)}} = \frac{E(xy) - E(x)E(y)}{\sqrt{V(x)V(y)}}$$

$$\rho = \frac{\text{cov}(x,y)}{\sqrt{V(x)V(y)}} = -1 \leq \rho \leq 1$$

$$\rho = \frac{(1)(2) + (3)(4)}{\sqrt{V(x)V(y)}} = \frac{1}{\sqrt{V(x)V(y)}}$$

- Q. The marginal pdf of x & y
 find (i) $E(x)$ & $E(y)$
 (ii) $\text{cov}(x, y)$
 (iii) $\text{var}_x, \text{var}_y$ if $f(x, y) = (x, y)$,
 (iv) P

Joint probability mass function $f(x, y) = (x, y)$

	x	y		Row sum (prob)
10	-4	-4	$(-4, -4)$	
1	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{2}$
5	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

$$\text{col sum } q(y) = \frac{3}{8} = \left(\frac{8}{8}\right) \text{ (all rows)}$$

$$15. (x) = (y) = (x, y) = (y, x) = 0$$

marginal pmf of x marginal pmf of y

$$0 = (y, x) = 0$$

x	1	5	y	-4	2	7
$p(x)$	$\frac{1}{2}$	$\frac{1}{2}$	$q(y)$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{4}$

$$i) E(x) = \sum x_i p(x_i)$$

$$(y) = (x) = i \cdot (y) = y = 9$$

(y) (x)

$$= 1\left(\frac{1}{2}\right) + 5\left(\frac{1}{2}\right)$$

$$= \frac{1+5}{2} = 3$$

$$j) E(y) = \sum y_i q(y_i)$$

$$= -4\left(\frac{3}{8}\right) + 2\left(\frac{3}{8}\right) + \frac{7}{4}$$

$$= \frac{7}{4} - \frac{6}{8} = 1$$

6 - 12

34

$$E(xy) = \sum_{i,j} a_{ij} y_j p_{ij}$$

$$= \left(1 \times -4 \times \frac{1}{8} \right) + \left(1 \times 2 \times \frac{1}{4} \right) + \left(1 \times 7 \times \frac{1}{8} \right)$$

$$+ \left(5 \times -4 \times \frac{1}{4} \right) + \left(5 \times 2 \times \frac{1}{8} \right) + \left(5 \times 7 \times \frac{1}{8} \right)$$

$$= \frac{3}{2} - \frac{(1)(1)(1)}{16}$$

$$(ii) \text{ cov}(x,y) = E(xy) - E(x)E(y)$$

$$= \frac{3}{2} - 3 \times 1$$

$$= -\frac{3}{2}$$

~~$E(x^2) = \sum x_i^2 p(x_i)$~~

$$E(x^2) = \sum x_i^2 p(x_i)$$

$$= 1^2 \times \frac{1}{2} + 5^2 \times \frac{1}{4}$$

$$\sum 18$$

$$E(y^2) = \sum y_i^2 q(y_i) = (-4)^2 \times \frac{3}{8} + (2^2 \times \frac{3}{4})$$

$$= 16 \times \frac{3}{8} + (4^2 \times \frac{1}{4})$$

$$18 \times \frac{3}{8} + (4^2 \times \frac{1}{4}) = \frac{79}{4}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= 13 - 3^2$$

$$= 4$$

$$V(y) = E(y^2) - (E(y))^2$$

$$= \frac{79}{4} - 1$$

$$= \frac{75}{4}$$

$$\rho = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y} = \frac{-3/2}{\sqrt{4} \sqrt{75/4}} = -\frac{3}{2\sqrt{75}}$$

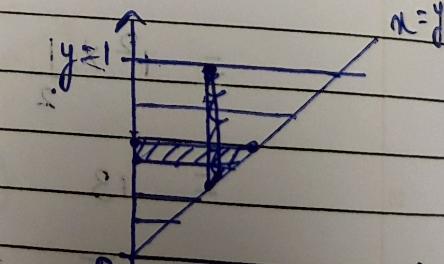
$$\rho = \frac{E(XY) - (E(X)E(Y))}{\sqrt{E(X^2) - (E(X))^2} \sqrt{E(Y^2) - (E(Y))^2}}$$

Q. Find coeff of correlation of x & y
for the following pdf.

$$f(x,y) = \begin{cases} 2, & (x,y) \in R \\ 0, & \text{otherwise} \end{cases}$$

Marginal pdf of x:

$$g(x) = \int_{-\infty}^{\infty} f(x,y) dy$$



$$\int_{-\infty}^{\infty} g(x) dx = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) dy dx = \int_{-\infty}^{\infty} \int_{x}^{1-x} 2 dy dx = 2 \int_{-\infty}^{\infty} (1-2x) dx = 2 \left[x - x^2 \right]_{-\infty}^{\infty} = 2$$

$$\therefore g(x) = 2(1-x), 0 \leq x \leq 1$$

$$1(y) = \int_{-\infty}^{\infty} f(\alpha|y) d\alpha = \int_{\alpha=0}^y d\alpha$$

$$\begin{aligned} E(x) &= \int_{x=0}^1 x g(x) dx \\ &= \int_0^1 (x^2 - 2x^3) dx \\ &= \left[\frac{x^3}{3} - \frac{2x^4}{4} \right]_0^1 = \left(\frac{1}{3} - \frac{2}{4} \right) = -\frac{1}{6} \\ &= \frac{1}{3} \end{aligned}$$

$$E(y) = \int_0^1 y w(y) dy = \left(\frac{1}{3} \right)^2 - \left(\frac{2}{3} \right)^2 = \frac{1}{9} - \frac{4}{9} = -\frac{3}{9} = -\frac{1}{3}$$

$$= \int_0^1 y (w(y)) dy = \left(\frac{1}{3} \right)^2 - \left(\frac{2}{3} \right)^2 = \frac{1}{9} - \frac{4}{9} = -\frac{3}{9} = -\frac{1}{3}$$

$$= \left[\frac{y^3}{3} \right]_0^1 - \left(\frac{2}{3} \right)^2 = \frac{1}{3} - \frac{4}{9} = -\frac{1}{3}$$

$$E(xy) = \iint_{0 \leq y \leq 1, 0 \leq x \leq y} xy \cdot L(x,y) dxdy$$

$$= \iint_{0 \leq y \leq 1, 0 \leq x \leq y} y^3 \cdot L(x,y) dxdy$$

$$= \int_0^1 y^3 dy = \frac{1}{4}$$

~~$V(x) = \frac{x}{\sqrt{3}}$~~
 ~~$V(y) = \sqrt{\frac{x}{3}}$~~

~~$P = E(xy) - E(x)E(y)$~~

$E(x^2) = \int_{x=0}^1 x^2 (2-x) dx.$

$= \frac{1}{6}$

$E(y^2) = \int_0^8 y^2 (8y) dy$

$= \frac{\alpha y^4}{4} \Big|_0^1$

$= \frac{1}{2}$

$V(x) = E(x^2) - [E(x)]^2$
 $= \frac{1}{6} - \frac{1}{3^2}$
 $= \frac{1}{18}$

$V(y) = E(y^2) - [E(y)]^2$
 $= \frac{1}{18}$

$Cov = E(xy) - E(x)E(y)$
 $= \frac{1}{9} - \frac{1}{3} \cdot \frac{2}{3}$
 $= \frac{1}{9} - \frac{2}{9}$
 $= -\frac{1}{9}$

$\therefore V_{xy} = \frac{1}{9}$

$$P = \frac{\sqrt{Y_{36}}}{\sqrt{Y_8 \cdot Y_8}} = \frac{Y_{36}}{Y_8} = \frac{1}{2}$$

Q. A fair coin is tossed 3 times. Let α : 0 or 1 according as H or T occurs on the first toss. y : no. of heads find the joint prob. distribution and $\text{cov}(x, y)$.

$$P(x=0, y=0) = 0$$

$$P(x=0, y=1) = P(HTT) = \frac{1}{8}$$

$$P(x=0, y=2) = P(HHT) \text{ or } P(HTH) = \frac{2}{8}$$

$$P(x=0, y=3) = P(HHH) = \frac{1}{8}$$

$$P(x=1, y=0) = P(TTT) = \frac{1}{8}$$

$$P(x=1, y=1) = P(THH \text{ or } THT) = \frac{2}{8}$$

$$P(x=1, y=2) = P(HTH) = \frac{1}{8}$$

$$P(x=1, y=3) = P(HHT) = \frac{1}{8}$$

$x \setminus y$	0	1	2	3	$P(x)$
0	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{3}{8}$	$\frac{1}{2}$
1	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0	$\frac{1}{2}$
$y \setminus x$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	1

$$E(x) = \sum x_i p(x_i) = 0 \cdot \frac{1}{2} + 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$E(y) = \sum y_j q(y_j) = 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

$$E(xy) = \sum x_i y_j p(x_i, y_j) = 0 \cdot 0 \cdot \frac{1}{8} + 0 \cdot 1 \cdot \frac{1}{8} + 1 \cdot 0 \cdot \frac{1}{8} + 1 \cdot 1 \cdot \frac{1}{8} + 2 \cdot 0 \cdot \frac{3}{8} + 2 \cdot 1 \cdot \frac{3}{8} = \frac{12}{8} = \frac{3}{2}$$

$$\text{cov}(x, y) = E(xy) - E(x)E(y) = \frac{3}{2} - \frac{1}{2} \cdot \frac{3}{2} = \frac{3}{4}$$

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$$

$$\Gamma(n) = 2 \int_0^{\pi/2} e^{-x^2/2} x^{n-1} dx$$

NOTE

Mo Tu We Th Fr Sa Su

$$E(Bay) := \sum_{i,j} x_i y_j P(x_i, y_j)$$

$$= 0 + 1 \cdot 0 \cdot 1/8 + 1 \cdot 1 \cdot 2/8 + 1 \cdot 2 \times 1/8 + 0$$

$$E(y) = \frac{1}{8} + 2 \cdot \frac{1}{8} + 3 \cdot \frac{1}{8} + 4 \cdot \frac{1}{8} = \frac{15}{8}$$

$$\text{var}(y) = E(y^2) - E(y)^2$$

$$\text{cov}(x, y) = E(xy) - E(x)E(y)$$

$$= 4^2 - 4 \cdot \frac{15}{8} = \frac{11}{2}$$

$$E(X) = (1+2+3+4)/4 = \frac{15}{8}$$

$$E(Y) = (1^2+2^2+3^2+4^2)/4 = \frac{30}{8}$$

$$E(XY) = (1 \cdot 1 + 2 \cdot 2 + 3 \cdot 3 + 4 \cdot 4)/4 = \frac{35}{8}$$

Q. joint PDF of X & Y = $f(x,y)$ is given by

$$f(x,y) = \begin{cases} Kxye^{-(x^2+y^2)}, & x>0, y>0 \\ 0 & \text{otherwise} \end{cases}$$

find value of K and $P(T < x \& Y < y)$

$$P(T < x \& Y < y) = \int_0^x \int_0^y Kxye^{-(x^2+y^2)} dy dx$$

$$1 = \int_0^\infty \int_0^\infty Kxye^{-(x^2+y^2)} dy dx = 1$$

$$\Rightarrow K \cdot \int_{y=0}^{\infty} \int_{x=0}^{\infty} xy e^{-y^2} dy dx = 1$$

$$\Rightarrow K \cdot \frac{1}{2} (\Gamma(1)) \times \frac{1}{2} (\Gamma(1)) = 1$$

$$\Rightarrow K = 1$$

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1} dx$$

$$\Gamma(n) = \alpha \int_0^\infty e^{-\alpha^2 x^2} dx$$

$$E(XY) := \sum x_i y_j P(x_i, y_j)$$

$$= 0 + 1 \cdot 0 \cdot \frac{1}{8} + 1 \cdot 1 \cdot \frac{2}{8} + \\ 1 \cdot 2 \cdot \frac{1}{8} + 0$$

$$\text{cov}(x,y) = E(xy) - E(x)E(y)$$

$$= Y_2 - \frac{1}{2}(3/2) \quad (0=2)$$

$$xy = (x+y) - \frac{1}{2}(y) \quad (0=0)$$

$$- xy = (1+1) - 5/2 \quad (4=5) \quad (0=0)$$

$$xy = 2(1+1) - (8+4) \quad (8=8) \quad (0=0)$$

$$xy = (1+4) - (0+1) \quad (5=5) \quad (1=1)$$

Q. joint pdf of RV (X, Y) is given by
 $f(x, y) = \begin{cases} Kxye^{-(x^2+y^2)}, & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

find value of K such that X and Y
 are independent.

$$\int \int_{-\infty}^{\infty} Kxye^{-(x^2+y^2)} dx dy = 1$$

$$1 = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Kxye^{-(x^2+y^2)} dx dy = (K\pi)^2 \quad \text{using } \int e^{-r^2} dr = \sqrt{\pi}$$

$$\Rightarrow K \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} yxe^{-y^2} dy dx = \int_{-\infty}^{\infty} x e^{-x^2} dx = 1$$

$$\Rightarrow K \cdot \frac{1}{2}(\Gamma(1)) \times \frac{1}{2}(\Gamma(1)) = 1$$

$$\Rightarrow K/4 = 1 \Rightarrow K = 4$$

g(x) =

$$\frac{2\pi^{-1/2}}{m=0}$$



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$$g(x) = \int_{y=0}^{\infty} + (x, y) dy$$

$$= \int_0^{\infty} 4 \alpha y e^{-(x^2+y^2)} dy$$

Integrate,

$$= \alpha x e^{-x^2} \int_0^{\infty} y e^{-y^2} dy$$

$$= \alpha x e^{-x^2} \Gamma(1)$$

$$= \alpha x e^{-x^2} (0 \leq y < \infty)$$

$$u(y) = \int_0^{\infty} 4 \alpha y e^{-(x^2+y^2)} dx$$

$$= 2 y e^{-y^2} \int_0^{\infty} x e^{-x^2} dx$$

$$= 2 y e^{-y^2} \Gamma(1)$$

$$= 2 y e^{-y^2} - 0 \leq y < \infty$$

$$g(x) \cdot u(y) = 2 x e^{-x^2} \cdot 2 y e^{-y^2}$$

$$= 4 \alpha y e^{-(x^2+y^2)}$$

$x \leq y$ θ

y-limit - vert
x-limit - horz.



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Q. Suppose that joint pdf of (x, y) is given by

$$f(x, y) = \begin{cases} e^{-y}, & x > 0, y > x \\ 0, & \text{elsewhere} \end{cases}$$

10 $P(x > 2, y < 4)$

~~0 to 00~~

$$g(x) = \int_{-\infty}^{\infty} f(x, y) dy$$

15 $y = \alpha x$

$$= \int_x^{\infty} e^{-y} dy$$

20

$$= [e^{-y}]_x^{\infty} = e^{-\alpha x}$$

25

$$w(y) = \int_{-\infty}^y g(x) dx = \int_0^y e^{-\alpha x} dx$$

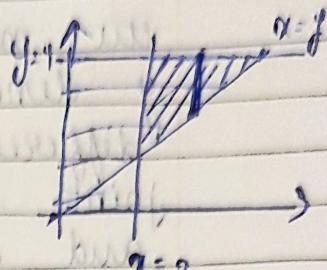
$$= ye^{-\alpha y} \quad y \geq 0$$

$$\therefore y \geq x$$

$$P(x > 2 | y < 4) = P(x > 2 \cap y < 4) / P(y < 4)$$

Num

$$\int_0^4 \int_{y/2}^{\infty} e^{-x} dy dx$$



$$= \int_0^4 -[e^{-y}]_x^4 dx$$

$$= \int_0^4 -e^{-4} + e^{-x} dx$$

$$= -e^{-4}(4) - [e^{-x}]_0^4$$

$$(iv) = 2 - 2e^{-4} - [e^{-4} - e^{-2}]$$

$$\frac{\partial}{\partial y} = 0 \quad 2e^{-4} - e^{-4} + e^{-2}$$

$$\frac{\partial}{\partial z} = 0 \quad e^{-2} - 3e^{-4}$$

$$\frac{\partial}{\partial x} = 0 \quad 4 - 4e^{-4}$$

$$P(y > 4) = \int_0^4 \int_{y/2}^{\infty} e^{-x} dy dx$$

$$= \int_0^4 -2e^{-4} + e^{-x} dx$$

$$= -4e^{-4} - [e^{-x}]_0^4$$

$$= -4e^{-4} + e^{-4} + 1$$

$$= 1 - 3e^{-4}$$

$$P(x > 2 | y < 4) = \frac{e^{-2} - 2e^{-4}}{1 - 3e^{-4}} = 0.0885$$

Q. A fair 6 sided die is rolled twice in succession. Let n denote the max of 2 outcomes and y denotes absolute value of the differences of 2 outcomes. Determine joint prob dist of n & y and find variance of n and variance of y .

$$Rx: \{1, 2, 3, 4, 5, 6\}$$

$$Ry: \{0, 1, 2, 3, 4, 5\}$$

x	y	$P(x)$
1	0	1/36
1	1	2/36
1	2	2/36
1	3	2/36
1	4	2/36
1	5	2/36
2	0	2/36
2	1	2/36
2	2	2/36
2	3	2/36
2	4	2/36
2	5	2/36
3	0	2/36
3	1	2/36
3	2	2/36
3	3	2/36
3	4	2/36
3	5	2/36
4	0	2/36
4	1	2/36
4	2	2/36
4	3	2/36
4	4	2/36
4	5	2/36
5	0	2/36
5	1	2/36
5	2	2/36
5	3	2/36
5	4	2/36
5	5	2/36
6	0	2/36
6	1	2/36
6	2	2/36
6	3	2/36
6	4	2/36
6	5	2/36
$E(x)$	161/36	1

$$E(x) = \sum_{i=1}^6 x_i P(x_i)$$

$$\frac{161}{36}$$

$$E(y) = \frac{35}{18}$$

$$E(x^2) = \sum x^2 P(x)$$

$$= \frac{791}{86}$$

5

$$V(x) = 0.2 \times 0.8 + 0.1 \times 0.2 = 0.16 + 0.02 = 0.18$$

Two ways to do it:

(1) $E(x^2) - [E(x)]^2$ or $E(x^2) - E(x)^2$

- Q. find cov of x, y . Test (x, y) are independent. justify.

$$E(x^2) + E(y^2) - [E(x) + E(y)]^2 =$$

$E(x^2)$	0.1	0	$P(x)$
$E(y^2)$	0.2	0	0.2
0	0.1	0.2	0.1

$$E(x^2) + E(y^2) - [E(x) + E(y)]^2 =$$

$E(x^2)$	0.1	0.2	0.6	0.2	1
$E(y^2)$	0.2	0.6	0.2	1	

$$E(x^2) = 0.6$$

$$E(y^2) = E(y) + E(x^2) - E(x)^2 = 0.4$$

$$E(xy) = \sum x_i y_i p_{ij}$$

$$E(xy) = E(x)E(y) = 0.1 \times 0.2 = 0.02$$

$$E(x) = \sum x_i p(x_i) = 0.1 + 0.2 = 0.3$$

$$E(y) = \sum y_i p(y_i) = 0.2 + 0.6 = 0.8$$

$$E(xy) = E(x)E(y) = 0.3 \times 0.8 = 0.24$$

$$\text{cov} = E(xy) - E(x)E(y) = 0.24 - 0.24 = 0$$

34

$$P(x_i, y_j) = P(x_i) \cdot P(y_j)$$

$$i=j=1$$

$$P_{11} = 0$$

$$P(x_1) \cdot P(y_1) = 0.2 \times 0.2 = 0.04$$

∴ not independent

Q. If x & y are AD RV find $N(x+y)$
(same above que)

$$N(x+y) = E((x+y)^2) - [E(x+y)]^2$$

$$= E(x^2 + y^2 + 2xy) - [E(x) + E(y)]^2$$

$$= E(x^2) + E(y^2) + 2E(xy) - [E(x) + E(y)]^2$$

$$= E(x^2) + [E(y^2) + 2E(xy) - (E(x))^2 - (E(y))^2]$$

$$\begin{aligned} 25 \\ 1.0 &= E(x^2) - (E(x))^2 + E(y^2) - (E(y))^2 \\ &\quad - 2[E(xy) - E(x)E(y)]. \\ 3 &= 1.0 + (1.0) - 2[E(xy) - E(x)E(y)]. \end{aligned}$$

$$= N(x) + N(y) - 2\text{cov}(x, y)$$

$$30 \quad 0.0 = 0.0 + 0.0 -$$

$$\therefore \boxed{N(x) + N(y) - 2\text{cov}(x, y)}$$

$$34 \quad N(3x+4) = 8^2 N(x) + 0.0 = 64$$

Q. 2 independent RV x_1 & x_2 have means 5 & 4 and variances 10 and 9.
 Find the COV b/w $U = 3x_1 + 4x_2$
 $V = 3x_1 - x_2$ and also find coeff of correlation b/w U & V .

$$E(x_1) = 5 \quad E(x_2) = 4$$

$$V(x_1) = 10 \quad V(x_2) = 9$$

$$COV(U, V) = E(UV) - E(U)V - E(V)U$$

$$\begin{aligned} E(x_1 x_2) &= p E(x_1) E(x_2) \\ &= 5 \times 4 p = \\ &= 20 \end{aligned}$$

$$(UV) - (U)(V) = (U)(V)$$

~~$E(x_1^2) = E(x_1)^2 = 10$~~

$$\begin{aligned} E(x_1^2) - (E(x_1))^2 &= 10 \\ E(x_1^2) - 25 &= 10 = 9 \\ E(x_1^2) &= 35 \end{aligned}$$

$$V(x_1) = 10 \quad V(x_2) = 9$$

~~$E(x_2^2) - (E(x_2))^2 = 9$~~

~~$E(x_2^2) = 25$~~

~~$V(x_1 + x_2) = V(x_1) + V(x_2) = 19$~~

$$\begin{aligned} E(UV) &= E((3x_1 + 4x_2)(3x_1 - x_2)) \\ &= E(9x_1^2 + 9x_1 x_2 - 4x_2^2) \\ &= 9E(x_1^2) + 9E(x_1)E(x_2) - 4E(x_2^2) \\ &= 9 \times 35 + 9 \times 5 \times 4 - 4 \times 25 \\ &= 315 + 180 - 100 \end{aligned}$$

$$(UV) - (U)(V) = (UV)$$

$$\begin{aligned} (3+4) E(UV) - E(U)E(V) &= (3+4) [E(9x_1^2 + 4x_2^2)] - E(3x_1 - x_2) \\ &= [8E(x_1^2) + 4E(x_2^2)] [3E(x_1) - E(x_2)] \\ &= [3 \times 35 + 4 \times 25] (3 \times 5 - 4) \\ &= (15+16) (11) \\ &= 31 \times 11 = 341 \end{aligned}$$

$$\text{Cov} = \frac{\text{Cov}(U, V)}{\sqrt{V(U) N(V)}}$$

$$N(U) = N(3x_1 + 4x_2)$$

$$= 89 N(x_1) + 16 N(x_2)$$

$$= 9 \times 10 + 16 \times 9$$

$$= 90 + 144$$

$$= 234$$

$$N(V) = N(3x_1 - x_2)$$

$$= 9 N(x_1) + N(x_2)$$

$$= 9 \times 10 + 9 = 99$$

$$= 99$$

$$\text{Cov}(U, V) = E(UV) - E(U)E(V)$$

$$= 0.54$$

$$= ((10)(3)) - (9)(1)$$

$$\rho = \frac{0.54}{\sqrt{234} \times \sqrt{99}} = 0.08547$$

$$= \frac{0.54}{\sqrt{234} \times \sqrt{99}}$$

Q. x, y, z are uncorrelated RV with \bar{x} mean and standard deviation s_x , \bar{y} mean and standard deviation s_y , \bar{z} mean and standard deviation s_z . If $U = x+y$ & $V = y+z$ find corr of corr b/w U & V .

$$(E(UV) - E(U)E(V)) = (E(UV))$$

$$(E(UV)) = E(E(y)) = E(z) = 0$$

$$\text{Var}(x) = s_x^2 = 16, \text{Var}(y) = 16, \text{Var}(z) = 7$$

$$\therefore \text{Var}(U) = \text{Var}(x+y) = 16 + 16 = 32, \text{Var}(V) = \text{Var}(y+z) = 16 + 7 = 23$$

$$E(UV) = E(UV) = E(U)V + U E(V)$$

$$= E((x+y)(y+z)) = E(x+y)E(y+z)$$

$$= E(xy + xz + y^2 + zy) = [E(x)E(y) + E(x)E(z) + E(y)E(z)]$$

$$= 2 \times 8 \times (10 + 10 + 7) = 2 \times 8 \times 27 = 432$$

$$= 1440$$

$$= 1440$$

$\therefore x, y, z$ are uncorrelated ($P=0$) $\text{cov}(x, y) = \text{cov}(y, z) = \text{cov}(x, z) = 0$.

$$\begin{aligned} & \text{E}[E(xy) - E(x)E(y)] + [E(xz) - E(x)E(z)] + \\ & [E(yz) - E(y)E(z)] + [E(y^2) - (E(y))^2] \\ & = 0 + 0 + 0 + V(y) \\ & = 144 \end{aligned}$$

$$V(x+y+z) = V(x+y) V(y+z)$$

$$\begin{aligned} & = [V(x) + V(y)] [V(y) + V(z)] \\ & = 8400 \quad 32617 \end{aligned}$$

$$\rho = \frac{144}{\sqrt{32617}} = 0.7973$$

* Uniformly distributed = RV.

Suppose x is a continuous 1D RV. It is said to be uniformly dist. in an interval $[a, b]$ if pdf of x is

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{otherwise} \end{cases}$$

Q If x is uniformly distributed RV in interval $-2 \leq x \leq 2$ then find

$$= \left(\frac{1}{2} \cdot 1 \right) \cdot P(\alpha < 1) + \left(1 - P(1 \leq \alpha < 1) \right) \cdot P(\alpha > 1)$$

$$\begin{aligned} & \cdot P(\alpha < 1) + P(\alpha > 1) \cdot \left\{ \frac{1}{2} + \frac{1}{4} \left(\frac{1}{2} \cdot \frac{1}{2} \right) \cdot (y+2) \right\} \quad y < 2 \\ & \left\{ \frac{1}{2} + \frac{1}{4} \left(\frac{1}{2} \cdot \frac{1}{2} \right) \cdot (y+2) \right\} + 0 \cdot \left(1 - P(1 \leq \alpha < 1) \right) \end{aligned}$$

$$y = f(\alpha)$$

$$(y) = g + g + g$$

$$(g+g) + (g+g) + (g+g) = y = \frac{1}{4}y + 1$$

$$\left[(g+g) + (g+g) + (g+g) \right] = y = \frac{1}{4}y + 1$$

$$P(\alpha < 1) = \int_{-2}^1 f(\alpha) d\alpha$$

$$\begin{aligned} & \int_{-2}^1 0 d\alpha = 0 \\ & = \int_{-2}^1 \frac{1}{4}y d\alpha \end{aligned}$$

$$V_1 = \lim_{n \rightarrow \infty} \frac{3}{4} \cdot n$$

$$\begin{aligned} & P(1 \leq \alpha < 1) \rightarrow P(1 \leq \alpha < 1) \\ & P(1 \leq \alpha < 1) = 1 - P(\alpha < 1) \end{aligned}$$

$$\begin{aligned} & \text{Gesuchte Wahrscheinlichkeit} \\ & = 1 - \int_{-2}^{3/2} \frac{1}{4}y d\alpha \\ & = 1 - \left[\frac{1}{4} \cdot \frac{1}{2}y^2 \right]_{-2}^{3/2} \end{aligned}$$

$$= 1 - \frac{1}{4} \left(\frac{3}{2}^2 - (-2)^2 \right)$$

$$= 1 - \frac{1}{4} \left(\frac{9}{4} - 4 \right)$$

$$= \frac{3}{4}$$

$$\text{वर्णन } \text{ अवकलन } = \int x f(x) dx$$

$$\text{वर्णन } \text{ अवकलन } = \int_a^b x \frac{1}{b-a} = \frac{1}{b-a} \cdot \frac{x^2}{2} \Big|_a^b$$

$$= \frac{b^2 - a^2}{2(b-a)}$$

$$\text{यदि अवकलन की विधि } = \frac{b+a}{2}$$

$$\begin{aligned} F(x^2) &= \int_a^b x^2 f(x) dx \\ &= \int_a^b \frac{x^2}{b-a} dx \end{aligned}$$

$$\text{अवकलन } = (y, x) [x^3]_a^b \text{ तो, बहुपद } 3x^2$$

$$\text{वर्णन } \text{ अवकलन } = x^3(b-a)$$

$$\text{प्रतिकर्षा } \text{ अवकलन } = \frac{b^3 - a^3}{3(b-a)}$$

$$= \frac{(b-a)(b^2 + a^2 + ab)}{3(b-a)}$$

$$= \frac{1}{3} (b^2 + a^2 + ab)$$

$$N(x) = b(x^2) - (R(x))^2$$

$$= \frac{b^2 + ab + a^2}{3} - \frac{(a+b)^2}{4}$$

$$= 4(b^2 + ab + a^2) - 3(a^2 + b^2 + 2ab)$$

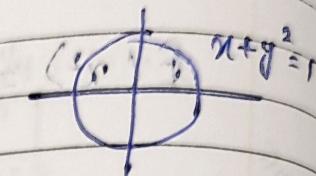
$$= \frac{(a-b)^2}{12}$$

Q. Suppose a r.v. continuous is uniformly distributed over the region R then its joint pdf of $(x, y) \in R$

$$f(x, y) = \begin{cases} \frac{1}{\text{Area of region } R}, & \text{if } (x, y) \in R \\ 0, & \text{otherwise} \end{cases}$$

Ex: area if region is bounded by

$$x^2 + y^2 = 1$$



$$f(x) = \begin{cases} \frac{1}{\pi}, & (x, y) \in R \\ 0, & \text{otherwise} \end{cases}$$

Q. Find joint pdf of (x, y) which is uniformly distributed over a region R where $R: x=y, y=x^2$. Also find marginal pdf of x and y and variance of y .

$$\text{area} = \int \int dy dx.$$

$$= \int (x - x^2) dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3} \right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

$$= \frac{1}{6} - \frac{1}{3} = \frac{1}{6}$$

$$= \frac{1}{6} \cdot 2 = \frac{1}{3}$$

$$= \frac{1}{3}$$

$$\text{Given } f(x,y) = \frac{1}{x+y} = 6$$

$$g(x) = \int_{y=0}^{\infty} f(x,y) dy \\ = 6(x - x^2) \quad 0 < x < 1$$

$$h(y) = \int_{x=y}^{x=\sqrt{y}} f(x,y) dx \\ = 6[\sqrt{y} - y] \quad 0 < y < 1$$

$$E(y) = \int_0^1 y h(y) dy$$

$$= \int_0^1 y 6(\sqrt{y} - y) dy$$

$$= 0.4 \quad = 12/5$$

$$E(y^2) = \int_0^1 \int_0^x y^2 6 dy dx \quad = 3/14$$

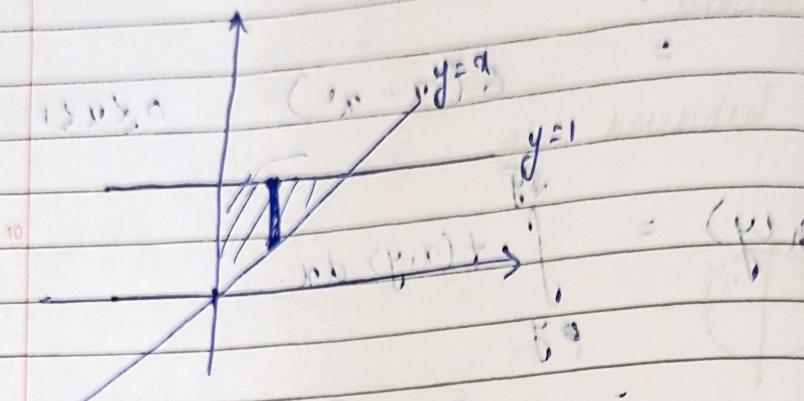
~~$$= \int_0^1 6(x - x^2) dx \quad V(y) = 3/14 - (12/5)^2 \\ = 19/350$$~~

$$= 6 \left[\frac{1}{2}x^2 - \frac{1}{3}x^3 \right]$$

$$= \frac{1}{1}$$

~~$$(34) V(y) = E(y^2) - (E(y))^2 \\ = \frac{1}{1} - \frac{(12/5)^2}{1} = 21/25$$~~

Q. Suppose that (X, Y) is uniformly dist. in the triangular region $R = \{(x, y) | 0 \leq y \leq x\}$ and joint pdf. $f(x, y)$.



$$\begin{aligned}
 \text{Area} &= \int_0^1 \int_{y=x}^{y=1} dy dx \\
 &= \int_0^1 (1-x) dx \\
 &= \left[x - \frac{x^2}{2} \right]_0^1 \\
 &= 1 - \frac{1}{2} = \frac{1}{2}.
 \end{aligned}$$

$$\begin{aligned}
 P(X > Y) &= \int_0^1 \int_{y=x}^{y=1} f(x, y) dy dx \\
 &= \int_0^1 \int_x^1 2 dx dy
 \end{aligned}$$

$$\begin{aligned}
 P(X > Y) &= \int_0^1 \int_x^1 2(1-x) dy dx \\
 &= \int_0^1 2(1-x) dx
 \end{aligned}$$

$$\begin{aligned}
 P(X > Y) &= 2 \left[\left(\frac{1}{2}x \right) + \frac{1}{2}(1-x)^2 \right]_0^1 = 2 \left(1 - \frac{3}{4} \right) = \frac{1}{2}.
 \end{aligned}$$

8. If RV K is uniformly distributed over $[0, 5]$, what is the prob that products of 5 equalities are equal such that $4xK + (K+2) = 0$

$$P(K) = \begin{cases} \frac{1}{5-0} = \frac{1}{5} & 0 < K \leq 5 \\ 0 & \text{otherwise} \end{cases}$$

$$4x^2 + 4xK + (K+2) = 0$$

$$4x^2 + 4xK + K^2 - K - 2 = 0$$

$$16K^2 - 16K - 2(16) \geq 0$$

$$K^2 - K - 2 \geq 0$$

$$K^2 - K - 2 \geq 0$$

$$(K^2 - K + K - 2) \geq 0$$

$$K(K-1) + (K-2) \geq 0$$

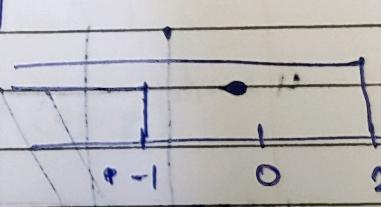
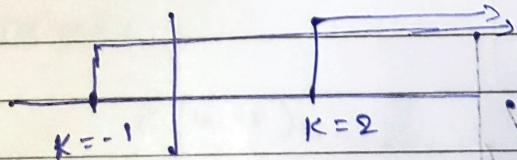
$$(K+1) \geq 0 \quad \text{and} \quad (K-2) \geq 0$$

$$K \geq -1$$

$$K \geq 2$$

$$(K+1) < 0 \quad \text{and} \quad K-2 < 0$$

$$K < -1$$



$$\therefore K \geq 2$$

$$K < -1$$

$$5$$

$$3$$

 \therefore not

possible

$$P(K \geq 2) = \int_{-1}^{5} \frac{1}{5} dK$$

$$= \frac{3}{5}$$

Q. Suppose that the dimensions of a rectangular metal plate may be considered to be independent continuous RV

$$x: g(x) = \begin{cases} -x+1 & 1 \leq x \leq 2 \\ -x+3 & 2 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

$$y: h(y) = \begin{cases} \frac{1}{2} & 2 \leq y \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

Find PDF of dimension of plate, $A = xy$

$$0 \leq A \leq 4 - 1 = 3$$

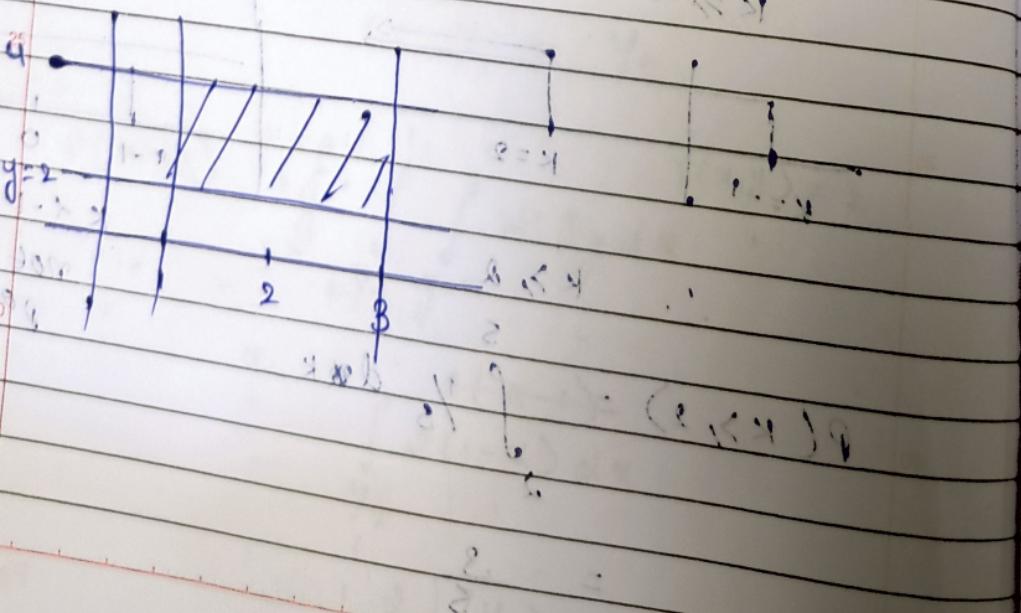
$$0 \leq x - y = 3$$

$$f(x,y) = \begin{cases} \frac{1}{2}(3-x) & 2 \leq x \leq 3, 2 \leq y \leq 4 \\ 0 & \text{elsewhere} \end{cases}$$

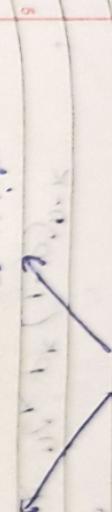
$$0 \leq 3-x \leq 1 \quad 0 \leq 1 \leq 1$$

$$0 \leq x \leq 3$$

$$1 \leq y \leq 4$$



Distributions.



discrete

continuous

Binomial

Gamma

Poisson

Exponential

Chi-square

Normal distribution

* Binomial Distribution

Event consists of 'n' repeated trials.

→ Event = trials : result may be classified

as success or failure.

→ Prob. of success will denoted by p and failure $1-p = q$.

→ Repeat trials are independent.

→ Here, $(q^n) + x$ is no. of success in 'n' repeated trials.

Then let X be a BRV on n repeated trials

$$\rho(X=k) = \sum_{m=0}^n {}^n C_k p^k (1-p)^{n-k}$$

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$$\rho(X=k) = \begin{cases} 0 & k < 0 \\ {}^n C_k p^k (1-p)^{n-k} & k \geq 0 \end{cases}$$

30

$$\sum_{k=0}^n \rho(X=k) = (1-p)^n + {}^n C_1 p(1-p)^{n-1} + \dots + p^n$$

34

$$f(x) = \sum a_k x^k$$

$$= \sum_{k=0}^n k p \lambda x^k$$

$$= \sum_{k=0}^n k n c_k p^k (1-p)^{n-k}$$

$$\text{Ansatz: } f(x) = \sum_{k=0}^n \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^n k n c_k (k-1)! (n-k)! p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^n n! (n-1)! (n-2)! \dots (n-k)! p^k (1-p)^{n-k}$$

$$= \sum_{k=1}^n n! (n-1)! (n-2)! \dots (n-k)! p^{k-1} (1-p)^{n-k}$$

$$= np \left[p + (1-p) \right]^{n-1} = np$$

$$= np$$

$$E(np) = np$$

$$b) E(x^2) = \sum_{k=0}^n k^2 P(x=k)$$

$$= 1 \cdot \sum_{k=1}^n (k^2 - k + k) P(x=k)$$

$$= \sum_{k=0}^n (k^2 - k) P(x=k) + \sum_{k=1}^n k P(x=k)$$

$$= (m+1)(m+2) \dots (m+k) \frac{m!}{(m-k)!} \frac{p^k}{(1-p)^{m-k}} mp$$

$$= \sum_{k=0}^{m-1} k(m-1)(m-2)! \frac{m!}{(m-k-2)!} \frac{p^k}{(1-p)^{m-k-2}} c_k$$

10

=

$$= \sum_{k=0}^{m-1} (m-1)(m-2)! \frac{(m-1)p^2}{(1-p)^2} \frac{p^{k-2}}{(1-p)^{m-k-2}}$$

$$k=2 \quad (k-2)! (m-2)-(k-2))!$$

$$(1-p)^{(m-2)-(k-2)} + mp$$

15

$$(1-p)^{m-2} + (1-p)^{m-2} p^2 \sum_{k=0}^{m-2} \binom{m-2}{k} p^k (1-p)^{m-2-k} + mp$$

$$= (mp^2 + m^2 p^2) + ((p + (1-p))^2 - 1) mp^2 + mp$$

20

$$= (mp^2 + m^2 p^2) + ((p + (1-p))^2 - 1) mp^2 + mp$$

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$$N(x) = E(x^2) - (E(x))^2$$

$$= \frac{\partial^2 p}{\partial p^2} - mp^2 + mp - (mp)^2$$

$$(c=0) q + (p=0) q + (q=0) q = (c-q) q$$

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$$= (mp^2 + (1-p)^2) - (mp)^2$$

$$= mp^2 (1-p)^2$$

35

8. 6 fair coins are tossed
find prob of getting

- ① Exactly 3 head
- ② atmost 3 heads
- ③ at least 3 heads
- ④ at least 1 head

$$\text{Ans: } \text{no. of heads} = 6 \\ p = \gamma_2 = \frac{1}{2}$$

$$\textcircled{1} \quad P(X=3) = {}^6C_3 (\gamma_2)^3 (1-\gamma_2)^{6-3}$$

$$= {}^6C_3 (\gamma_2)^3 (1-\gamma_2)^3$$

$$= {}^6C_3 \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right)^3$$

$$\textcircled{2} \quad P(X \leq 3) = P(X=0) + P(X=1) + P(X=2) \\ + P(X=3)$$

$$= {}^6C_0 (\gamma_2)^0 (1-\gamma_2)^6 + {}^6C_1 (\gamma_2)^1 (1-\gamma_2)^5 +$$

$$+ {}^6C_2 (\gamma_2)^2 (1-\gamma_2)^4 + {}^6C_3 (\gamma_2)^3 (1-\gamma_2)^3 +$$

$$= \frac{21}{32}$$

$$\textcircled{3} \quad P(X \geq 3) = P(X=3) + P(X=4) + P(X=5) +$$

$$= {}^6C_3 (\gamma_2)^3 (1-\gamma_2)^3 + {}^6C_4 (\gamma_2)^4 (1-\gamma_2)^2 + {}^6C_5 (\gamma_2)^5 (1-\gamma_2) +$$

$$= \sum_{k=3}^6 {}^6C_k (\gamma_2)^k (1-\gamma_2)^{6-k}$$



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$$\begin{aligned} P(X \geq 1) &= 1 - P(X < 1) \\ &= 1 - P(X=0) \\ &= 1 - 6(0.4)^6(0.6)^6 \end{aligned}$$

ANSWER
 63. $\frac{1}{2}(1 - 6(0.4)^6(0.6)^6)$

84.

Q. Determine the prob. of getting 29 exactly twice with 8 throws with a pair of fair dice.

$$(6,3), (3,6), (4,5), (5,4)$$

$$P(\text{getting } a) = \frac{4}{36} = \frac{1}{9} = P_{\text{exactly}}(29) = \frac{8}{9}$$

$$\Rightarrow P(X=2) = {}^8C_2(1/9)^2(8/9)^6$$

Q. 20. Out of 800 families with 5 children each having money, we find expect to have

$$(a) 3 boys \quad (b) 4 boys$$

(b) 5 girls

Assume equal prob. for boy and girl

X: no. of boys

$$n=5$$

$$p = \frac{1}{2}$$

ANSWER

$$\therefore P(X=3) = {}^5C_3(1/2)^3(1/2)^2$$

$$= \frac{5}{16}$$

∴ Out of 800 families having 3 boys

$$= \frac{800 \times 5/16}{800} = \frac{5}{16}$$

Page 850

Note

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$$b) P(x=0) = {}^5C_0 (\frac{1}{2})^0 (\frac{1}{2})^5 = \frac{1}{32}$$

$$\text{no. of faulty} = \frac{800 \times 1}{1000} = \frac{8}{10} = 0.8$$

Q. In sampling from a large number of parts manufactured by machine, the mean no. of defective units in a sample of 10 is 1. Out of 1000 such samples, how many would be expected to contain at least 8 defective parts.

$$X: \text{no. of defective parts}$$

$$\text{Given } P(x \geq 8) = 1 - P(x \leq 7)$$

$$P(x \geq 8) = ?$$

$$P(x \geq 8) = 1 - \sum_{k=0}^{7} P(X=k) = 1 - \sum_{k=0}^{7} {}^10C_k (0.1)^k (0.9)^{10-k}$$

$$= 1 - 10C_0 (0.1)^0 (0.9)^{10} - 10C_1 (0.1)^1 (0.9)^{9} - \dots - 10C_7 (0.1)^7 (0.9)^{3}$$

Q. 10

If x is binomially distributed RV with expected value μ and variance σ^2 , find distribution of

Page

10. Binomial Distribution

$$f(x) = \alpha^x e^{-\alpha} \quad V(x) = 4/3$$

x	0	1	2	3	4	5	6	7	8
$P(x)$	$\frac{e^{-\alpha}}{1}$	$\frac{\alpha}{2!} e^{-\alpha}$	$\frac{\alpha^2}{2!} e^{-\alpha}$	$\frac{\alpha^3}{3!} e^{-\alpha}$	$\frac{\alpha^4}{4!} e^{-\alpha}$	$\frac{\alpha^5}{5!} e^{-\alpha}$	$\frac{\alpha^6}{6!} e^{-\alpha}$	$\frac{\alpha^7}{7!} e^{-\alpha}$	$\frac{\alpha^8}{8!} e^{-\alpha}$

$$\frac{64}{243}, \quad (1-\alpha), \quad 1/3, \quad P_1 = 1/3, \quad Q_1 = 2/3,$$

$$n = 3 \times 2 \\ = 6$$

* 10 Poisson Distribution

A. RV \bar{x} is said to be taking all values $0, 1, 2, \dots$ is said to be Poisson RV with parameter $\alpha > 0$ if its PMF is given by:

$$P(x=k) = \frac{e^{-\alpha} \alpha^k}{k!}, \quad k=0, 1, 2, \dots$$

$$\sum_{k=0}^{\infty} P(\bar{x}=k) = \sum_{k=0}^{\infty} \frac{e^{-\alpha} \alpha^k}{k!} = e^{-\alpha} \sum_{k=0}^{\infty} \frac{\alpha^k}{k!} = e^{-\alpha} (e^\alpha) = 1$$

$$E(\bar{x}) = \sum_{k=0}^{\infty} k P(\bar{x}) = \sum_{k=0}^{\infty} k (P(\bar{x}=k))$$

$$= \sum_{k=0}^{\infty} k \frac{\alpha^k}{k!(k-1)!} e^{-\alpha}$$

$$= e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^k}{(k-1)!} e^{-\alpha}$$

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$$\therefore E(\bar{x}) = \alpha$$

$$= \alpha e^{-\alpha} \sum_{k=1}^{\infty} \frac{\alpha^{k-1}}{(k-1)!}$$

34

$$= \alpha e^{-\alpha} e^\alpha$$

Page

$$E(\alpha^k) = \sum_{k=0}^{\infty} k^k p_k P(X=k), \quad (1)$$

$$= \sum_{k=0}^{\infty} k^k e^{-\alpha} \alpha^k \frac{e^{-\alpha} \alpha^k}{k!} = e^{-\alpha} \alpha^k \sum_{k=0}^{\infty} \frac{e^{-\alpha} \alpha^k}{k!} = e^{-\alpha} \alpha^k \cdot 1 = e^{-\alpha} \alpha^k$$

$$= e^{-\alpha} \sum_{k=0}^{\infty} (k+1)^k \frac{\alpha^k}{k!}, \quad (2)$$

$$\text{Let } E(X^2 - \alpha \sum_{k=0}^{\infty} (k+1)^k \frac{\alpha^k}{k!}) = \sum_{k=0}^{\infty} \frac{\alpha^{k+2}}{k+1} (k+1)^k \frac{\alpha^k}{k!}$$

$$= e^{-\alpha} \left[\sum_{k=2}^{\infty} \frac{\alpha^2 \alpha^{k-2}}{k-1} \frac{(k+1)^k \alpha^k}{k!} (k-2)! \right]$$

$$= e^{-\alpha} \alpha^2 e^{\alpha} + e^{-\alpha} \alpha e^{\alpha}$$

$$= \alpha^2 + \alpha$$

$$E(X^2) = E(X^2) - (E(X))^2 + (E(X))^2 = (\alpha + \alpha^2) - \alpha^2 + \alpha^2 = \alpha^2 + 2\alpha$$

$$V(X) = E(X^2) - (E(X))^2$$

$$= \alpha^2 \alpha^2 + \alpha - \alpha^2$$

$$(15) = \alpha^4 + \alpha - \alpha^2 = (\alpha^2 + 1)(\alpha^2 - 1) = (\alpha + 1)(\alpha - 1)$$

$$\therefore V(X) = \alpha^2 - 1$$

$$\alpha \rightarrow 10, \quad 0 < \alpha$$

$$10 \rightarrow \alpha$$

$$0 < \alpha < 10$$

$$10 > \alpha > 0 \quad \therefore \quad 0 < \alpha < 10$$

8. Let X be binomially distributed RV with parameter n & p . Suppose $n \rightarrow \infty$ & $p \rightarrow 0$ such that $p \rightarrow \alpha$ under these cond'n we have prob. of $P(X=k) = e^{-\alpha} \alpha^k / k!$, $k=0,1,2,\dots$

$$P(X=k) = {}^n C_k p^k (1-p)^{n-k}$$

$$= \frac{n!}{k!(n-k)!} p^k (1-p)^{n-k}$$

$$= \frac{n(n-1)}{k!} (n-2) \dots (n-(k-1)) \cancel{(n-k)!}$$

$$= \frac{n^k}{k!} (1-p/n)(1-p/n)\dots (1-\frac{k-1}{n}) p^k (1-p)^{n-k}$$

20

$$P(X=k) = \lim_{n \rightarrow \infty} \frac{n^k p^k}{k!} \left(1 - \frac{p}{n}\right) \left(1 - \frac{p}{n}\right) \dots$$

$$\left(1 - \frac{k-1}{n}\right) \left(1 - \frac{\alpha}{n}\right)^{n-k}$$

25

$$= \frac{(\alpha)^k}{k!} \times \lim_{n \rightarrow \infty} \frac{\left(1 - \frac{\alpha}{n}\right)^n}{\left(1 - \frac{\alpha}{n}\right)^k}$$

$$= \frac{\alpha^k}{k!} \left[\left(1 - \frac{\alpha}{n}\right)^{-n/\alpha} \right]^{-\alpha} \xrightarrow[n \rightarrow \infty]{\lim \left(1 + \frac{1}{n}\right)^n} e^{-\alpha}$$

30

$$= \frac{\alpha^k}{k!} e^{-\alpha}$$

Gamma (in another way)

Q. The no. of road acc. for a day in certain city is distributed as gamma variable with 'average' of 6 and variance 18. find the prob. that there will be more than 8 accidents (ii) b/w 5 and accidents on a particular day.

$$E(X) = \frac{r}{\alpha} = 6$$

$$V(X) = \frac{r}{\alpha^2} = 18$$

$$\frac{r}{\alpha} = \frac{6}{\alpha} \Rightarrow \alpha = 1/3$$

$$\text{Since } \frac{r}{\alpha} = 6 \Rightarrow r = 6 \times \frac{1}{3} = 2.$$

$$f(x) = \begin{cases} \frac{\alpha^r}{r!} (x^\alpha)^{r-1} e^{-\alpha x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{\gamma_3}{\Gamma(2)} (\gamma_3 x)^{2-1} e^{-\gamma_3 x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$= \begin{cases} \gamma_3 x e^{-\gamma_3 x}, & x > 0 \\ 0, & \text{otherwise} \end{cases}$$