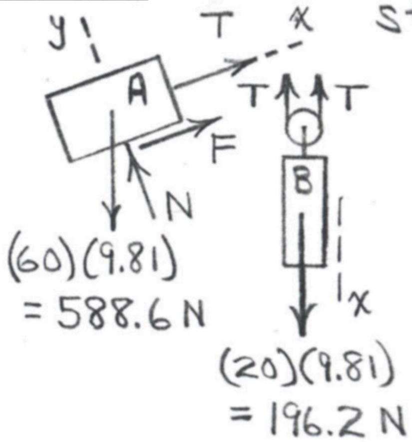
M3DYN1 F2020 - Løsningsforslag

Opgave 1

Check for motion by assuming static equilibrium.



B : $2T = 196.2$, $T = 98.1$ N

A : $\Sigma F_x = 0$: $98.1 - 588.6 \sin 30° + F = 0$, $F = 196.2$ N

$F_{max} = \mu_s N = (0.25)(588.6)\cos 30° = 127.4$ N

$F > F_{max} \Rightarrow$ motion ($\swarrow$)

From kinematics, $a_A = 2a_B = 2a$

A : $\Sigma F_x = ma_x$: $T + 0.2(588.6 \cos 30°) - 588.6 \sin 30° = 60(2a)$

B : $\Sigma F_x = ma_x$: $-2T + 196.2 = 20a$

Solution : $\underline{a = -0.725 \text{ m/s}^2}$, $\underline{T = 105.4 \text{ N}}$

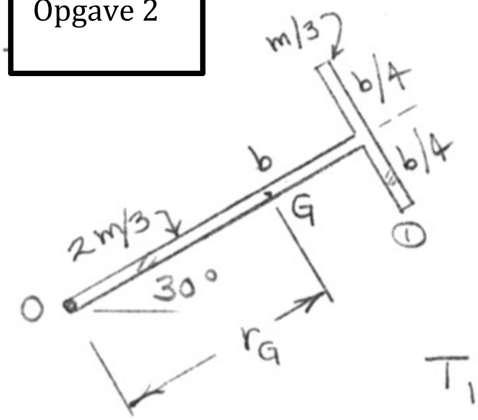
e)

$$v_A(t) = \int_0^t a_A \, dt = a_A t = -0{,}725 * 2 * 10 = -14{,}5 \frac{m}{s}$$

$$x_A(t) = \int_0^t v(t)dt = \int_0^t a_A t dt = a_A \frac{t^2}{2} = -0{,}725 * \frac{2 * 10^2}{2} = -72{,}5\, m$$

Legeme A har bevæget sig 72,5 m nedad

Opgave 2



$$r_G = \frac{\frac{2m}{3}\frac{b}{2} + \frac{m}{3}b}{m} = \frac{2b}{3}$$

$$I_O = \frac{1}{3}\frac{2m}{3}b^2 + \frac{m}{3}\left(\frac{1}{12}\left(\frac{b}{2}\right)^2 + b^2\right)$$

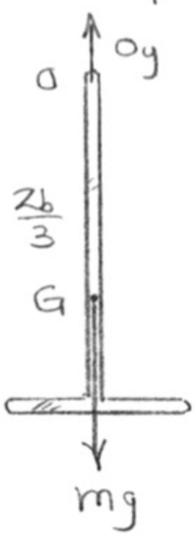
$$= \frac{9}{16}mb^2$$

$$T_1 + U_{1-2} = T_2 :$$

$$0 + mg\frac{2b}{3}\left(\sin 30° + 1\right) = \frac{1}{2}\left(\frac{9}{16}mb^2\right)\omega_2^2$$

$$\omega_2^2 = \frac{32}{9}\frac{g}{b}$$

FBD at position ②:



$$\Sigma F_y = ma_y :$$

$$O_y - mg = m\frac{2b}{3}\frac{32g}{9b}$$

$$\underline{O_y = \frac{91}{27}mg \quad \text{up}}$$

KD I pos 2, samme position, som vist ovenfor, da $\alpha = 0$ (ω er maks.) er også $a_{Gx} = r\alpha = 0$ og den eneste vektor, der skal påføres G, er ma_{Gy} opad, hvis også $I_G\alpha$ og ma_{Gx} påføres, skal der gøres rede for størrelserne af disse ud fra nævnte argumantation.

Opgave 3

a)

Masse - inertimoment om O)

$$I_0 = I + md^2 = \frac{1}{12}ml^2 + \left(\frac{l}{6}\right)^2 m = \underline{\frac{1}{9}ml^2}$$

Da der regnes fra ligevægt kan differentialligningen opskrives direkte (se bort fra statiske laster som sædvanligt, og antag små viinkeldrejninger):

$$\sum M_0 = I_o\alpha \Rightarrow -c\frac{2l}{3}\dot{\theta}\frac{2l}{3} - 2k\frac{l}{3}\theta\frac{l}{3} = \frac{1}{9}ml^2\ddot{\theta} \Rightarrow \ddot{\theta} + 4\frac{c}{m} + 2\frac{k}{m} = 0$$

2

$$\omega_n^2 = 2\frac{k}{m} \Rightarrow \underline{\underline{\omega_n^2 = \sqrt{\frac{2k}{m}}}} = \sqrt{\frac{2\cdot 30}{10}} = \underline{2,45 \ \frac{rad}{s}}$$

$$2\zeta\omega_n = 4\frac{c}{m} \Rightarrow \underline{\underline{\zeta = 2\frac{c}{m\omega_n}}}$$

b) $\zeta = 1 \Rightarrow$ kritisk dæmpet

$$1 = 2\frac{c}{m\omega_n} \Rightarrow c = \frac{m\omega_n}{2} = \frac{10\cdot 2,45}{2} = \underline{\underline{12,25}} \ N\cdot s/m$$

c) $\theta(t) = (A_1 + A_2 t)e^{-\omega_n t}$

$$\underline{= A_1 e^{-\omega_n t} + A_2 t e^{-\omega_n t}}$$

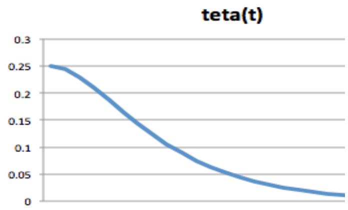
$$\underline{\dot{\theta}(t) = -\omega_n A_1 e^{-\omega_n t} + A_2 e^{-\omega_n t} - A_2 t \omega_n e^{-\omega_n t}}$$

Til tiden $t = 0 \Rightarrow \theta(0) = 0,25$ rad $\wedge$ $\dot{\theta}(0) = 0$

$$\theta(0) = \underline{A_1 = 0,25 \ rad}$$

$$\dot{\theta}(0) = -\omega_n A_1 + A_2 = 0 \Rightarrow A_2 = \omega_n A_1 = 2,45 \frac{rad}{s} \cdot 0,25 \ rad$$

$$\underline{= 0,61 \ \frac{rad^2}{s} \left(\frac{1}{s}\right)}$$



$$\underline{\underline{\theta(t) = (0,25 + 0,61t)e^{-2,45t}}} \ [rad]$$