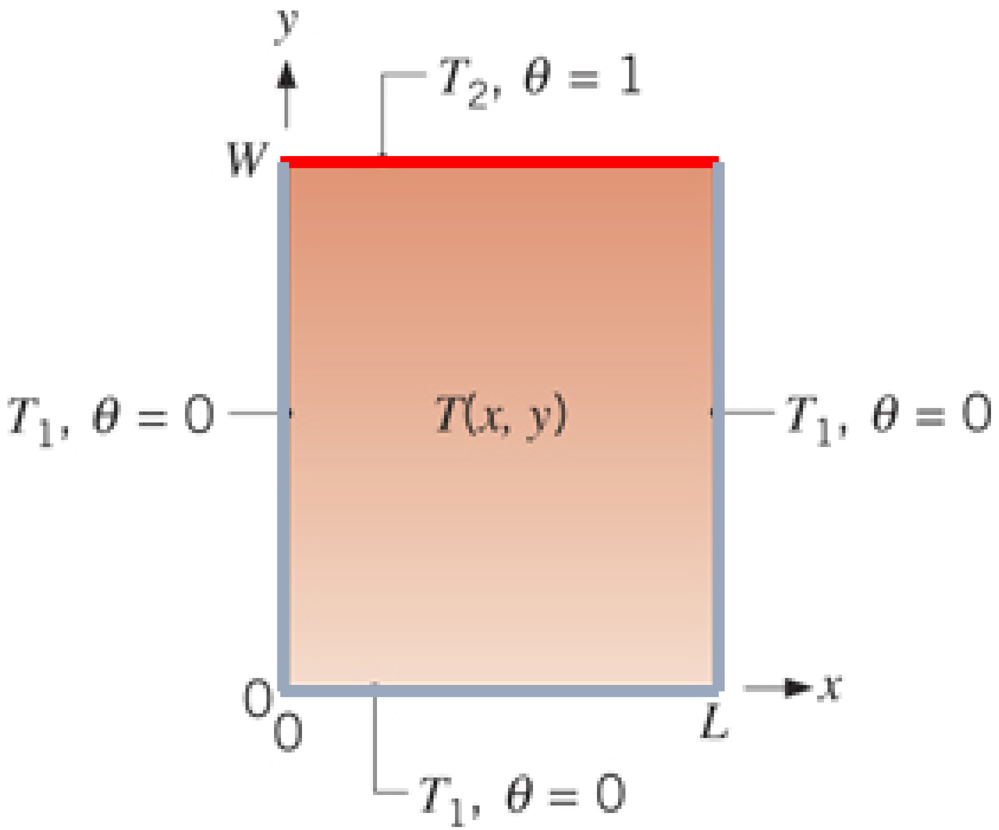
Heat transfer

Chap. 4 two-dimensional conduction



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial x^2} = 0$$

$$\theta(x,y) = \frac{T(x,y) - T_1}{T_2 - T_1}$$

Dimensionless

$$\frac{T(x,y) - T_1}{T_2 - T_1} = \theta(x,y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\frac{n\pi x}{L}\frac{\sinh n\pi y/L}{\sinh n\pi W/L}$$

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial x^2} = 0$$

$$q''_x = -k\frac{\partial T}{\partial x}$$

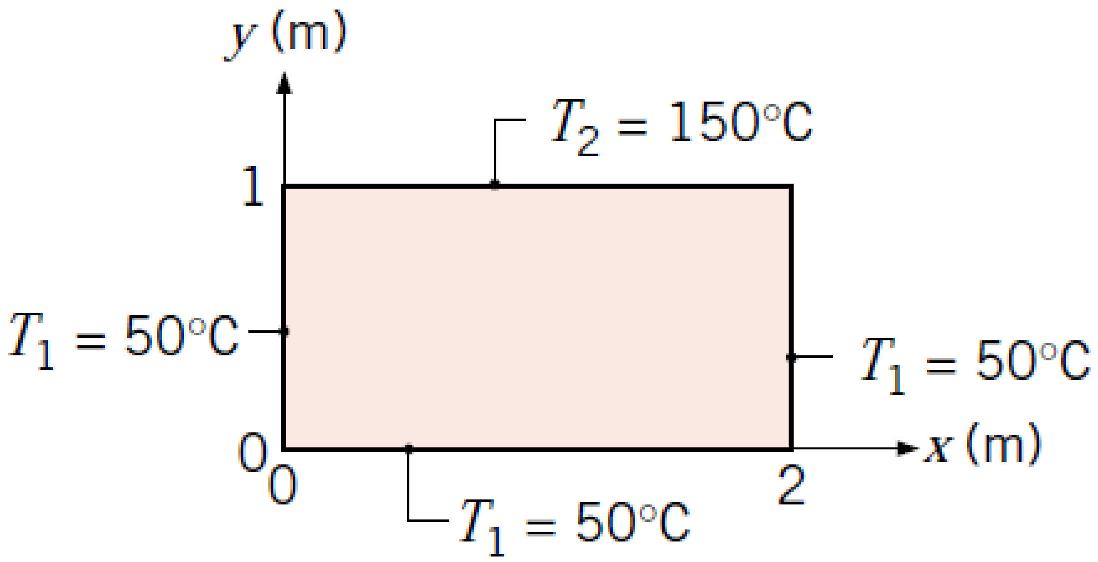
$$q''_y = -k\frac{\partial T}{\partial y}$$

$$\Rightarrow \quad q = Sk\Delta T_{1-2}$$

S shape factor [m]

Eksempel

- A two-dimensional rectangular plate is subjected to
- prescribed boundary conditions. Using the results of
- the exact solution for the heat equation presented in
- Section 4.2, calculate the temperature at the midpoint
- (1, 0.5) by considering the first five nonzero terms of
- the infinite series that must be evaluated.



Two-dimensional
steady-state conduction

Shape factor

$$q = Sk\Delta T_{1-2}$$

$$Resistance \;\; R_{t,cond} = \frac{\Delta T_{1-2}}{q} = \frac{1}{Sk}$$

TABLE 4.1 **Conduction shape factors and dimensionless conduction heat rates for selected systems.**

(*a*) Shape factors [$q = Sk(T_1 - T_2)$] S

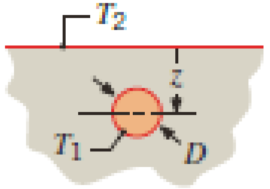
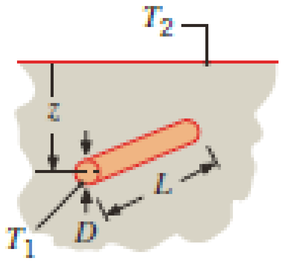
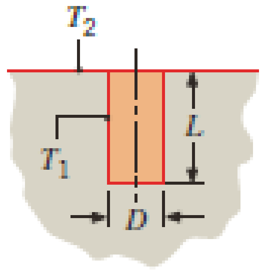
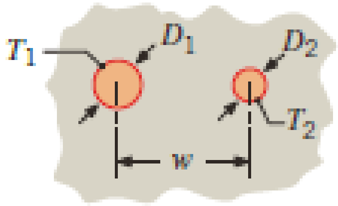
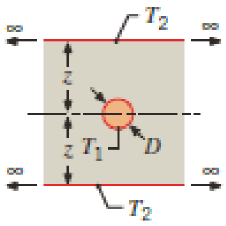
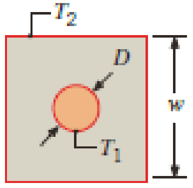
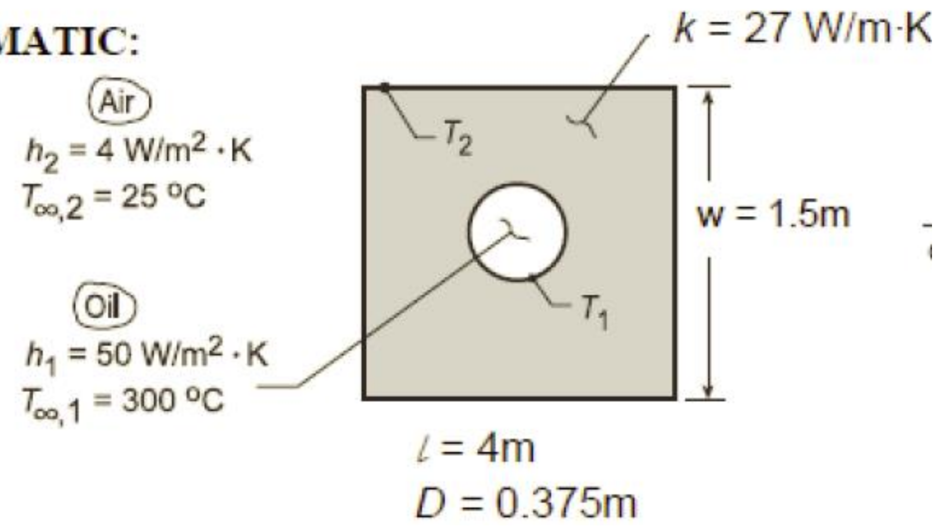
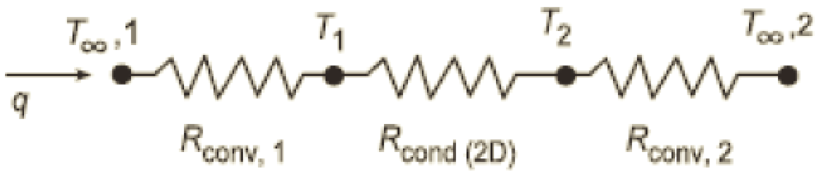
System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$
		$L \gg D$ $z > 3D/2$	$\dfrac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$

TABLE 4.1 *Continued*

System	Schematic	Restrictions	Shape Factor
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\dfrac{2\pi L}{\ln(8z/\pi D)}$
Case 6 Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L \gg w$	$\dfrac{2\pi L}{\ln(1.08\, w/D)}$

SCHEMATIC:





$$q = \frac{T_{\infty,1} - T_{\infty,2}}{R_{conv,1} + R_{cond(2D)} + R_{conv,2}}$$

$$S = \frac{2\pi L}{\ln\left(\frac{1.08w}{D}\right)} = 17.176\ \text{m}$$

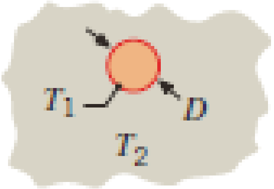
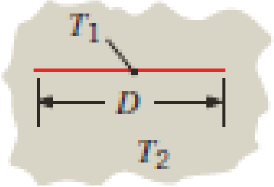
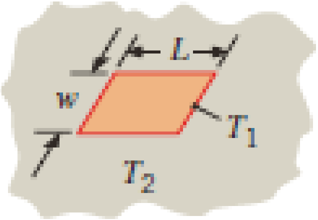
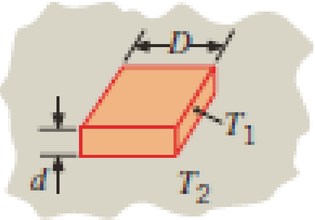
$$R_{2D} = \frac{1}{Sk} = 0.00216\,\frac{K}{W}$$

Eksempel

A hole of diameter D =0.375 m is drilled through the center of a solid block of square cross section with w = 1,5 m on a side. The hole is drilled along the length, l = 4 m, of the block, which has a thermal conductivity of k = 27 W/m . K. The outer surfaces are exposed to ambient air, with $T_{,2}$ = 25°C and $h2$ = 4 W/m^2 .K, while hot oil flowing through the hole is characterized by $T_{,1}$ = 300°C and $h1$ =50 W/m^2 .K.

Determine the corresponding heat rate and surface temperatures.

(b) Dimensionless conduction heat rate $[q = q^*_{ss}\, kA_s(T_1 - T_2)/L_c;\ L_c = (A_s/4\pi)^{1/2}]$

System	Schematic	Active Area, A_s	q^*_{ss}
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2		πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2		$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14 Infinitely thin rectangle of length L, width w, and temperature T_1 in an infinite medium of temperature T_2		$2wL$	0.932
Case 15 Cuboid shape of height d with a square footprint of width D and temperature T_1 in an infinite medium of temperature T_2		$2D^2 + 4Dd$	d/D — q^*_{ss} 0.1 — 0.943 1.0 — 0.956 2.0 — 0.961 10 — 1.111

Infinite medium

Uniform surface temperature

$$L_c = \sqrt{\frac{A_s}{4\pi}}$$

$$q^*_{ss} = \frac{qL_c}{kA_s(T_1 - T_2)}$$

Opgave 4.8

Radioactive wastes are temporarily stored in a spherical container, the center of which is buried a distance of 10 m below the earth's surface. The outside diameter of the container is 2 m, and 500 W of heat are released as a result of the radioactive decay process. If the soil surface temperature is 20°C, what is the outside surface temperature of the container under steady-state conditions? On a sketch of the soil–container system drawn to scale, show representative isotherms and heat flow lines in the soil.

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