Opgave 3

$m_1 := 300\ kg$

$k_1 := 900\ \frac{N}{m}$

$c_1 := \frac{2 \cdot 300\ kg \cdot 3\ \frac{rad}{s} \cdot 1.25}{2} = 1125\ N \cdot \frac{s}{m}$

Estimat for c for at få "pæne" overdæmpede resultater

$c_{eq} := 2 \cdot c_1 = \left(2.25 \cdot 10^3\right)\ N \cdot \frac{s}{m}$

$k_{eq} := 3 \cdot k_1 = \left(2.7 \cdot 10^3\right)\ \frac{N}{m}$

$\omega_{n\_eq} := \sqrt{\frac{k_{eq}}{m_1}} = 3\ \frac{rad}{s}$

$f_{nn\_Hz} := \frac{\omega_{n\_eq}}{2 \cdot \pi} = 0.477\ \frac{1}{s}$

$\zeta := \frac{c_{eq}}{2 \cdot m_1 \cdot \omega_{n\_eq}} = 1.25$

Overdæmpet systen

Foreskrift, plot og x_max

$\omega_n := 3 \qquad \zeta := 1.25$

$\lambda_1 := \omega_n \cdot \left(-\zeta + \sqrt{\zeta^2 - 1}\right) = -1.5$

$\lambda_2 := \omega_n \cdot \left(-\zeta - \sqrt{\zeta^2 - 1}\right) = -6$

$x(t) := A_1 \cdot exp\left(\lambda_1 \cdot t\right) + A_2 \cdot exp\left(\lambda_2 \cdot t\right)$

$v(t) := \frac{d}{dt} x(t) \rightarrow -1.5 \cdot A_1 \cdot e^{-1.5 \cdot t} - 6 \cdot A_2 \cdot e^{-6 \cdot t}$

$A_1 := x(0) = -1 \xrightarrow{solve,\ A_1} -A_2 - 1$

$A_2 := v(0) = 18 \xrightarrow{solve,\ A_2} -\left(0.25 \cdot A_1\right) - 3$

$$C_1 := -\left(-\frac{1}{4}\cdot C_1 - 3\right) - 1 = C_1 \xrightarrow{solve,\,C_1} \frac{8}{3}$$

$$C_2 := -\left(0.25\cdot C_1\right) - 3 = -3.667$$

$$x(t) := C_1\cdot\exp\left(\lambda_1\cdot t\right) + C_2\cdot\exp\left(\lambda_2\cdot t\right)$$

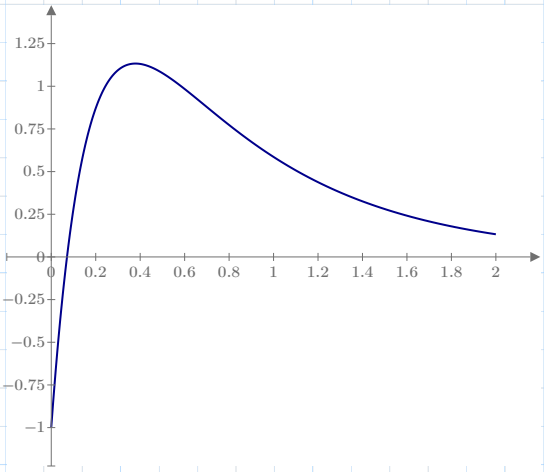
$$v_x(t) := \frac{\mathrm{d}}{\mathrm{d}t}x(t) \rightarrow 21.999999999999999\cdot e^{-6\cdot t} - 4.0\cdot e^{-1.5\cdot t}$$

$$t_{max} := v_x(t) = 0 \xrightarrow{solve,\,t} 0.37883290938631670871$$

$$x(0) = -1$$

$$v_x(0) = 18$$

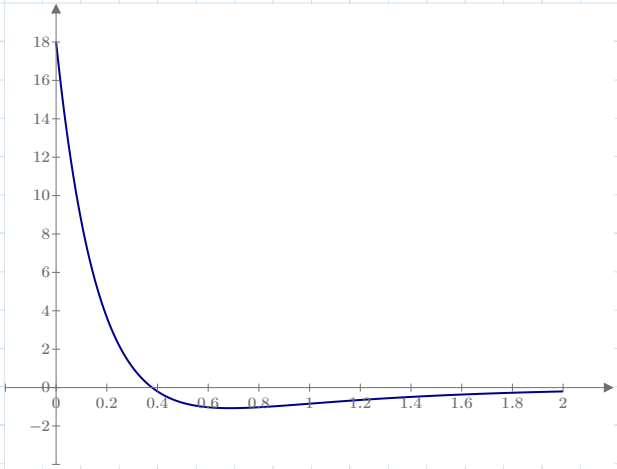
$$x_{max} := x\left(t_{max}\right) = 1.133$$



$x(t)$

t

Ekstra information



$v_x(t)$

t

$$q(t) := \frac{8}{3} \cdot e^{-1.5 \cdot t} - \frac{11}{3} \cdot e^{-6 \cdot t}$$

$$C_1 \cdot \lambda_1 \cdot e^{\lambda_1 \cdot t_{max}} + C_2 \cdot \lambda_2 \cdot e^{\lambda_2 \cdot t_{max}} = 4.441 \cdot 10^{-16}$$

$$\frac{\ln\left(\frac{11}{2}\right)}{\lambda_1 - \lambda_2} = 0.379$$

$$\lambda_1 - \lambda_2 = 4.5$$

$$\ln(11) = 2.398$$

$$\ln(2) = 0.693$$

$$\ln\left(\frac{11}{2}\right) = 1.705$$

max udsving

$$C_1 \cdot \exp\left(\lambda_1 \cdot \left(\frac{\ln\left(\frac{11}{2}\right)}{\lambda_1 - \lambda_2}\right)\right) + C_2 \cdot \exp\left(\lambda_2 \cdot \left(\frac{\ln\left(\frac{11}{2}\right)}{\lambda_1 - \lambda_2}\right)\right) = 1.133$$