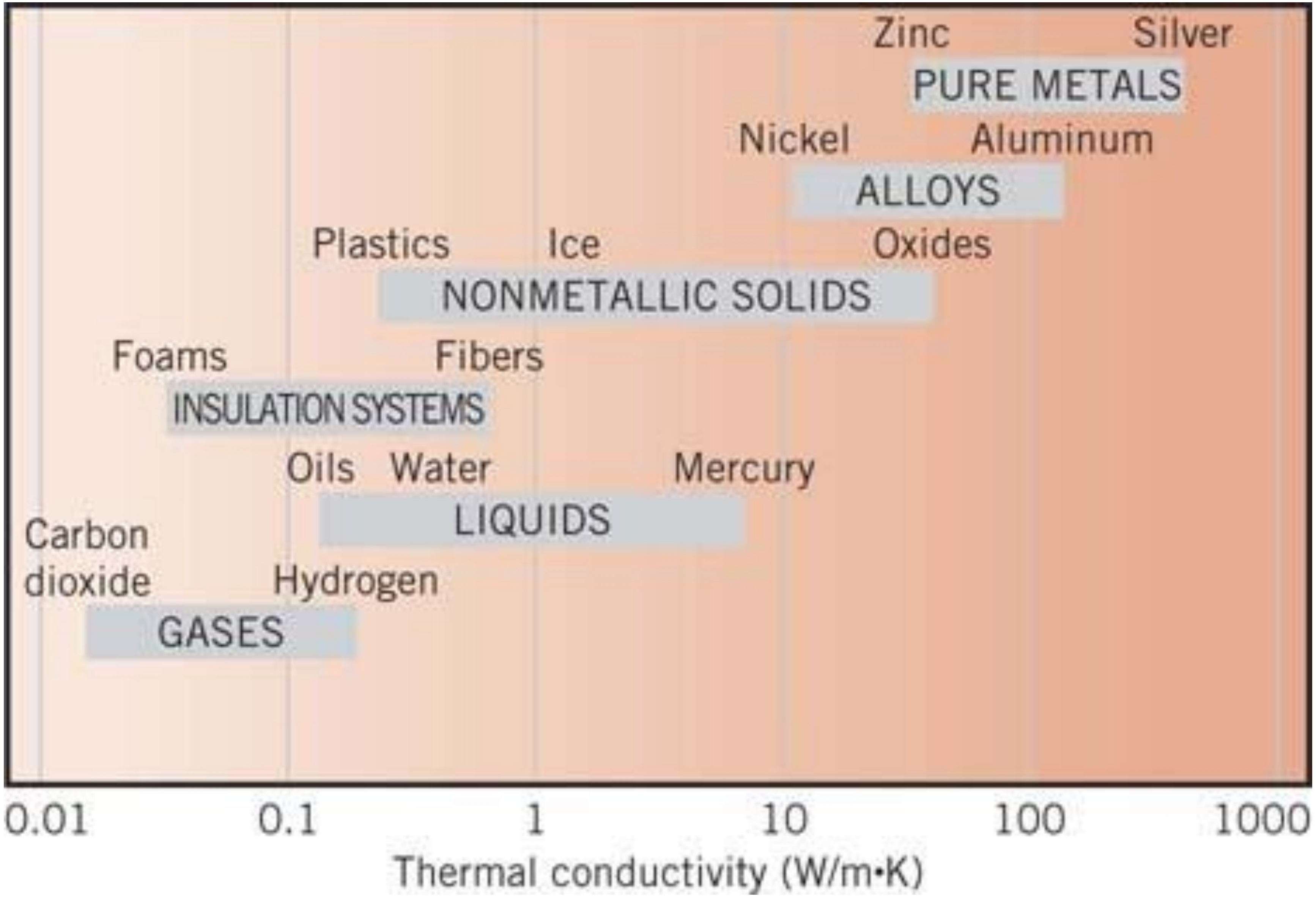
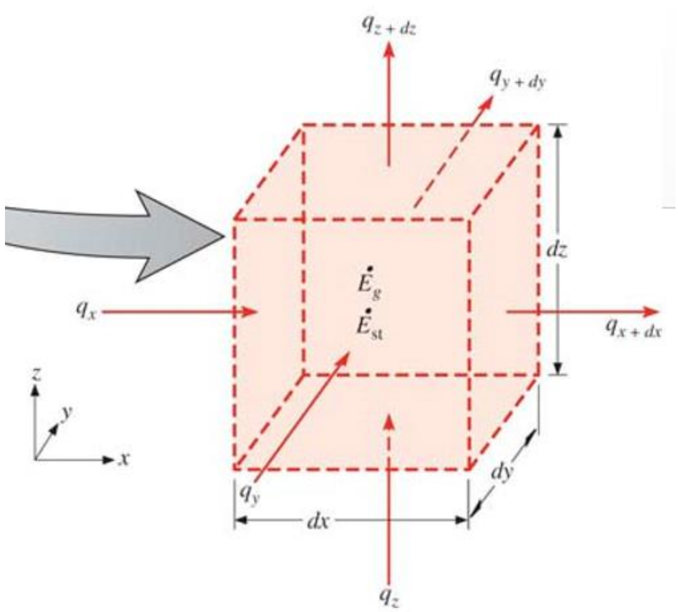
Varmetransmission
Kap. 2 Varmediffusionsligningen


Zinc
Silver
PURE METALS
Nickel
Aluminum
ALLOYS
Plastics
Ice
Oxides
NONMETALLIC SOLIDS
Foams
Fibers
INSULATION SYSTEMS
Oils
Water
Mercury
LIQUIDS
Carbon dioxide
Hydrogen
GASES
0.01
0.1
1
10
100
1000
Thermal conductivity (W/m•K)

Heat diffusion equation



$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_{gen} = \frac{dE_{st}}{dt}$$

Taylor $$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$\boldsymbol{q_x} + q_y + q_z - \left[\boldsymbol{q_x} + \boldsymbol{\frac{\partial q_x}{\partial x} dx} + q_y + \frac{\partial q_y}{\partial y} dy + q_z + \frac{\partial q_z}{\partial z} dz\right] + \dot{q} dxdydz = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

$$-\boldsymbol{\frac{\partial q_x}{\partial x} dx} - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz + \dot{q} dxdydz = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

$$-dy\, dz \frac{\partial \left(-k\, \frac{\partial T}{\partial x}\right)}{\partial x} dx - \cdots \ldots \ldots \ldots + \dot{q} dxdydz = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

Fourier

$$q_x = -kA \frac{\partial T}{\partial x}$$

$$q_x = -k\, dy\, dz \frac{\partial T}{\partial x}$$

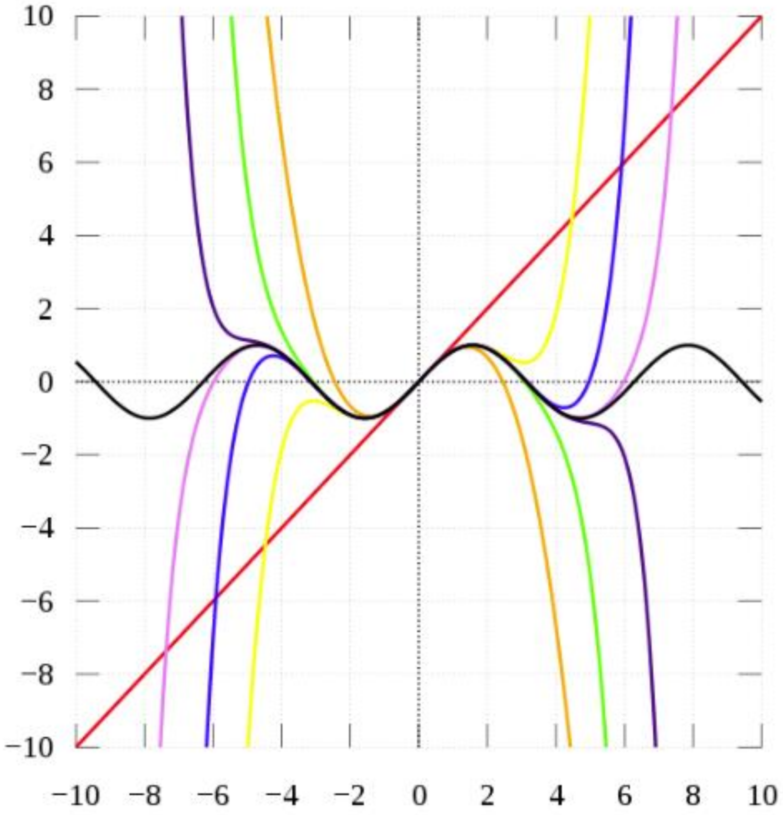
$$\frac{\partial \left(k\, \frac{\partial T}{\partial x}\right)}{\partial x} + \frac{\partial \left(k\, \frac{\partial T}{\partial y}\right)}{\partial y} + \frac{\partial \left(k\, \frac{\partial T}{\partial z}\right)}{\partial z} + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

Heat diffusion equation

Taylor expansion

$$f(x) = f(a) + \frac{f'(a)}{1!}(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \cdots,$$

$$\left(q + \frac{\partial q}{\partial x}\Delta x\right)$$



Sin(x) around a=0

$$\frac{\partial\left(k\,\frac{\partial T}{\partial x}\right)}{\partial x}+\frac{\partial\left(k\,\frac{\partial T}{\partial y}\right)}{\partial y}+\frac{\partial\left(k\,\frac{\partial T}{\partial z}\right)}{\partial z}+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

$k\ constant$

$$\underbrace{\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}}_{\text{Net transfer of thermal energy into the CV}}+\underbrace{\frac{\dot{q}}{k}}_{\text{Energy generation}}=\underbrace{\frac{\rho c_p}{k}\frac{\partial T}{\partial t}}_{\text{Energy storage change}}$$

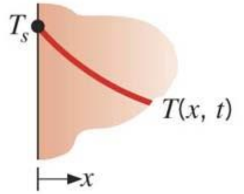
$$\nabla^2 T+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$Heat\ diffusion\ coefficient:\alpha=\frac{k}{\rho c_p}\quad\left[\frac{m^2}{s}\right]$$

TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

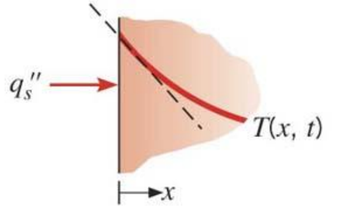
$$T(0, t) = T_s \qquad (2.31)$$



2. Constant surface heat flux

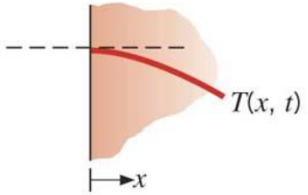
(a) Finite heat flux

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = q_s'' \qquad (2.32)$$



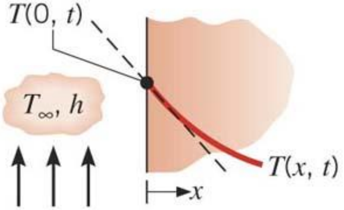
(b) Adiabatic or insulated surface

$$\frac{\partial T}{\partial x}\bigg|_{x=0} = 0 \qquad (2.33)$$



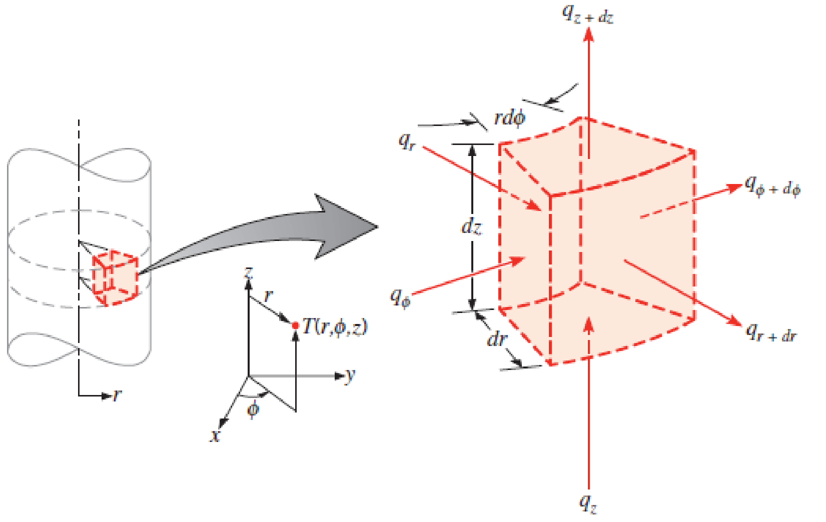
3. Convection surface condition

$$-k\frac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0, t)] \qquad (2.34)$$



- Cylindrical Coordinates:

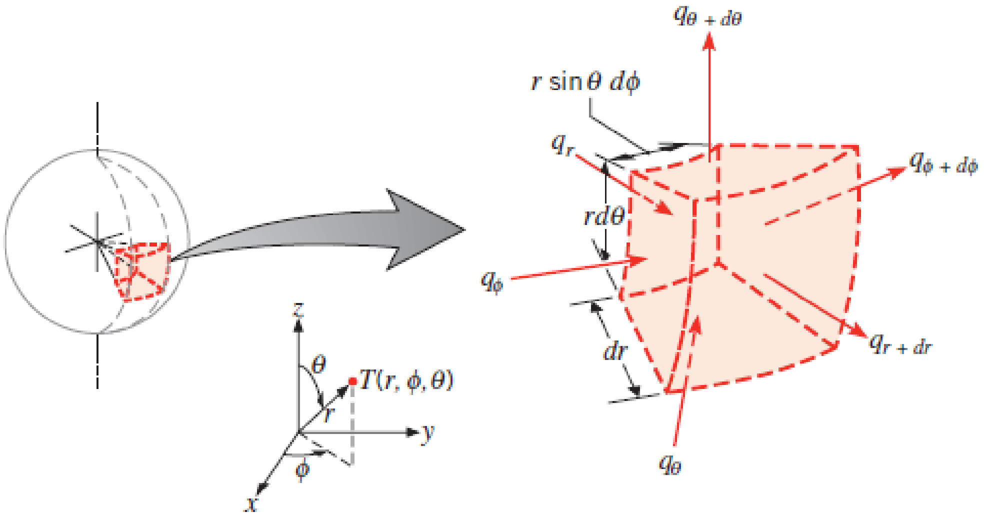
$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \tag{2.26}$$



- Spherical Coordinates:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

(2.29)



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