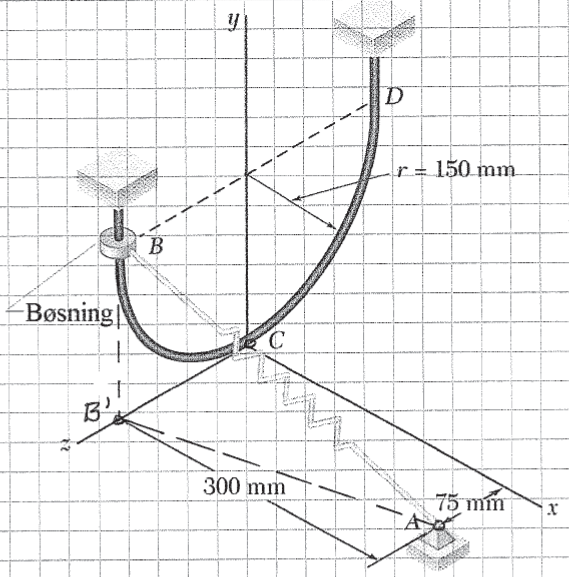
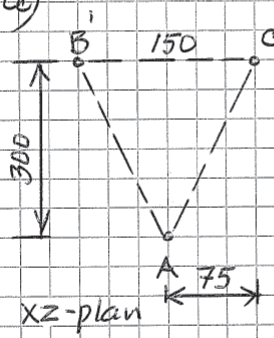
M3DYN1- Reeksamen august 2020 - Løsningsforslag

Opgave 1



a)



$L_{AC} = \sqrt{300^2 + 75^2} = 309{,}23 \text{ mm} = AB'$

$L_{AB} = \sqrt{AB'^2 + B'B^2} = \sqrt{309{,}23^2 + 150^2} = 343{,}69 \text{ mm}$

b) $T_B + \bar{V}_B + U'_{BC} = T_C + \bar{V}_C$

$T_B = 0$ (hvile)

$\bar{V}_B = \frac{1}{2} k x_B^2 = \frac{1}{2} \cdot 320 \cdot 0{,}14369^2 = 3{,}303 \text{ J}$

$x_B = L_{AB} - L_0 = 343{,}69 - 200 = 143{,}69 \text{ mm}$

$U'_{BC} = 0$ (ingen ydre kræfter)

$T_C = \frac{1}{2} m v_C^2 = \frac{1}{2} \cdot 0{,}5 \cdot v_C^2 = 0{,}25 v_C^2$

$$mgh_C = 0{,}5 \cdot 9{,}81 \cdot (-0{,}15) = -0{,}736 \text{ J}$$

$$\tfrac{1}{2} k x_C^2 = \tfrac{1}{2} \cdot 320 \cdot 0{,}10923^2 = 1{,}909 \text{ J}$$

$$x_C = L_{AC} - L_0 = 309{,}23 - 200 = 109{,}23 \text{ mm}$$

$$V_C = mgh_C + \tfrac{1}{2} k x_C^2 = -0{,}736 + 1{,}909 = 1{,}173 \text{ J}$$

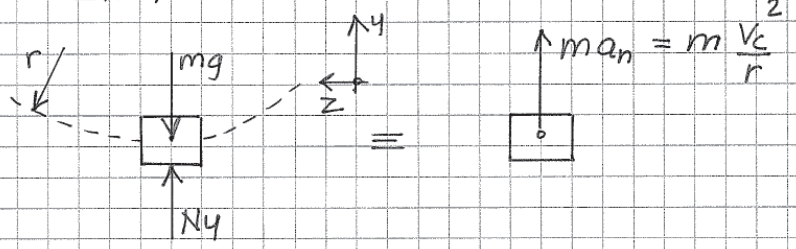
$$T_B + V_B + U_{BC} = T_C + V_C$$

$$0 + 3{,}303 + 0 = 0{,}25\, v_C^2 + 1{,}173$$

$$\rightarrow \quad v_C = 2{,}92 \text{ m/s}$$

c)

yz-plan



FBD KD

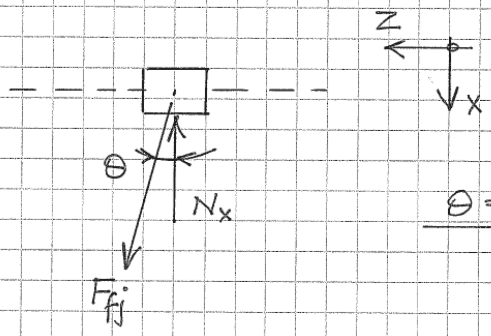
$$\uparrow \Sigma F_y = ma_n = m\frac{v_C^2}{r}$$

$$N_y - mg = m\frac{v_C^2}{r} \rightarrow N_y = m\frac{v_C^2}{r} + mg$$

$$N_y = 0{,}5 \cdot \frac{2{,}92^2}{0{,}15} + 0{,}5 \cdot 9{,}81$$

$$N_y = 33{,}33 \text{ N}$$

XZ-plan



$$\Theta = \tan^{-1}\frac{75}{300} = 14{,}04^\circ$$

$$\uparrow \Sigma F_x = 0$$

$$N_x - F_{fj}\cdot\cos\Theta = 0 \rightarrow N_x = F_{fj}\cos\Theta$$

$$F_{fj} = k\cdot x_c = 320\cdot 0{,}10923 = 34{,}95\ N$$

$$N_x = 34{,}95\cdot\cos 14{,}04^\circ = 33{,}91\ N$$

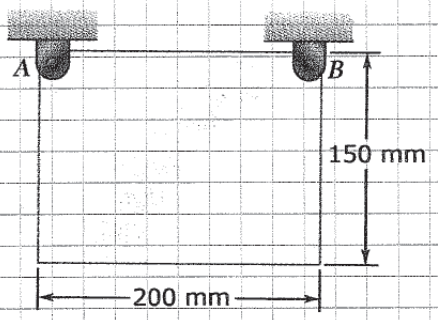
$$F = \sqrt{N_y^2 + N_x^2} = \sqrt{33{,}33^2 + 33{,}91^2} = 47{,}55\ N$$

Alternativ med vektornotation:

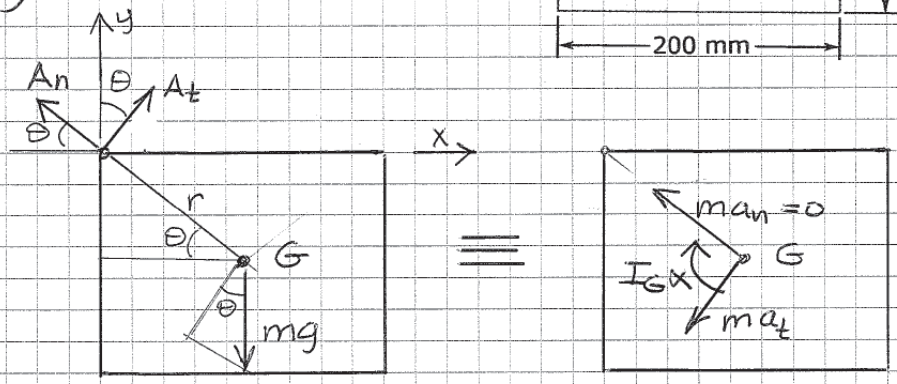
$$\bar{F} = -33{,}91\,\bar{i} + 33{,}33\,\bar{j}\ [N]$$

Opgave 2

$\theta = \tan^{-1}\frac{75}{100} = 36{,}87^\circ$



a)



FBD KD

b) $I_G = \frac{1}{12} m(b^2 + l^2)$ (0/4 s. 691)

$I_G = \frac{1}{12} \cdot 10 \,(0{,}15^2 + 0{,}2^2) = 0{,}052\ [kgm^2]$

c) $I_A = I_G + m r^2$ $r = 0{,}125[m]$ (3-4-5)

$I_A = 0{,}052 + 10 \cdot 0{,}125^2 = 0{,}208\ [kg\,m^2]$

d)

① $\curvearrowleft \Sigma F_t = m a_t$

② $\nwarrow \Sigma F_n = m a_n = 0 \quad (w = 0)$

③ $\circlearrowleft \Sigma M_G = I_G \alpha$

④ $\circlearrowleft \Sigma M_A = I_A \alpha$

④ $mg \cdot 0,1 = I_A \alpha$

$$\alpha = \frac{mg \cdot 0,1}{I_A} = \frac{10 \cdot 9,81 \cdot 0,1}{0,208} = 47,16 \ s^{-2} \ ck$$

$$\vec{\alpha} = -47,16 \vec{k}$$

e)

① $-A_t + mg\cos\theta = m a_t = m r \alpha$

$A_t = mg\cos\theta - mr\alpha$

$A_t = 10 \cdot 9,81 \cos 36,87^\circ - 10 \cdot 0,125 \cdot 47,16$

$A_t = 19,53 \ N$

② $A_n - mg\sin\theta = 0$

$A_n = mg\sin\theta = 10 \cdot 9,81 \sin 36,87^\circ$

$A_n = 58,86 \ N$

$F_{Ai}\vec{i} = A_t \sin\theta - A_n \cos\theta = 19,53 \sin 36,87^\circ - 58,86 \cos 36,87^\circ$

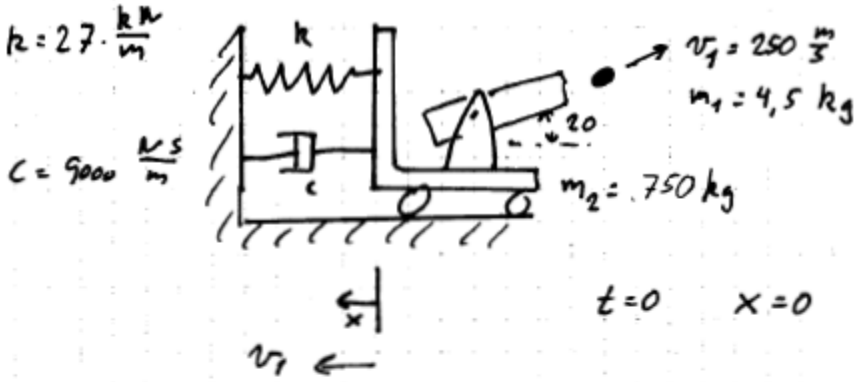
$F_{Ai} = -35,37 \vec{i} \ [N]$

$F_{Aj} = A_t \cos\theta + A_n \sin\theta$

$F_{Aj} = 19,53 \cos 36,87^\circ + 58,86 \sin 36,87^\circ$

$F_{Aj} = 50,94 \vec{j} \ [N]$

Opgave 3



①

$t=0 \quad x=0 \quad \dot{x} = v_2$

Kanonens hastighed i x-aksens retning lige efter affyring

$$\Delta G_x = 0 \Rightarrow m_1 \cdot v_1 \cdot \cos 20 = m_2 \cdot v_2$$

$$\Downarrow$$

$$v_2 = \frac{m_1}{m_2} \cdot v_1 \cdot \cos(20) = \frac{4,5}{750} \cdot 250 \cdot \cos(20)$$

$$v_2 = 1,41 \ \frac{m}{s} = \dot{x}_0$$

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{27 \cdot 10^3}{750}} = 6 \ \frac{rad}{s}$$

$$\zeta = \frac{c}{2 \cdot m \cdot \omega_n} = \frac{9000}{2 \cdot 750 \cdot 6} = 1 \Rightarrow \text{kritisk dæmpning}$$

$$x = (A_1 + A_2 t) \cdot e^{-\omega_n \cdot t}$$

$$\dot{x} = A_2 \cdot e^{-\omega_n \cdot t} + (A_1 + A_2 \cdot t) \cdot (-\omega_n) \cdot e^{-\omega_n \cdot t}$$

$$\dot{x} = (A_2 - A_1 \cdot \omega_n - A_2 \cdot \omega_n \cdot t) \, e^{-\omega_n \cdot t}$$

Bestemmelse af konstanter

$t = 0 \Rightarrow x = 0 \quad \wedge \quad \dot{x} = v_2 = 1{,}41 \frac{m}{s}$

$$x = (A_1 + A_2 \cdot t) e^{-\omega_n \cdot t}$$
$$\Downarrow$$
$$0 = (A_1 + A_2 \cdot 0) e^{-6 \cdot 0}$$
$$\Downarrow$$
$$A_1 = 0$$

$$\dot{x} = (A_2 - A_1 \cdot \omega_n - A_2 \cdot \omega_n \cdot t) e^{-\omega_n \cdot t}$$
$$\Downarrow$$
$$1{,}41 = (A_2 - 0 \cdot 6 - A_2 \cdot 6 \cdot 0) \cdot e^{-6 \cdot 0}$$
$$\Downarrow$$
$$A_2 = 1{,}41$$

Når $x = x_{max}$ gælder at $\dot{x}(t) = 0$

$$0 = (1{,}41 - 0 \cdot 6 - 1{,}41 \cdot 6 \cdot t) \cdot e^{-6 \cdot t}$$
$$\Downarrow$$
$$0 = 1{,}41 - 8{,}46 \cdot t$$
$$\Downarrow$$
$$t = \frac{1}{6} s = 0{,}1667 s$$

$$x(t = \tfrac{1}{6} s) = (0 + 1{,}41 \cdot \tfrac{1}{6}) \cdot e^{-6 \cdot \frac{1}{6}}$$

$$= 0{,}0865 m = \underline{\underline{86{,}5 mm}}$$