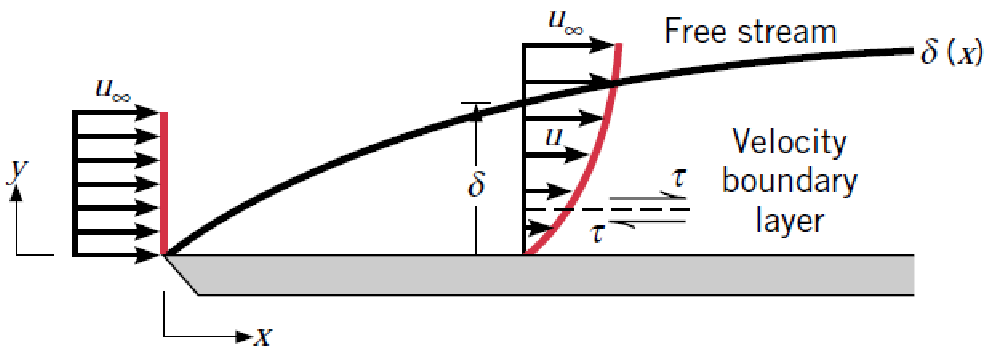
Heat transfer
Chap. 6 Introduction to convection

Boundary Layers: Physical Features

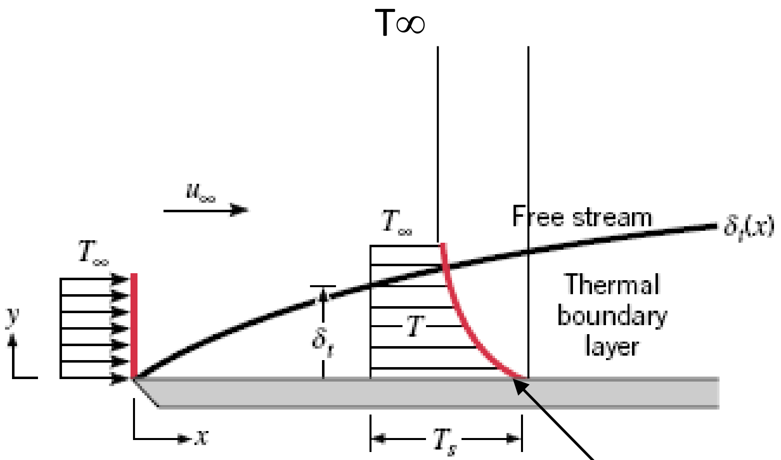
- Velocity Boundary Layer
  - A consequence of viscous effects associated with relative motion between a fluid and a surface.
  - A region of the flow characterized by shear stresses and velocity gradients.
  - A region between the surface and the free stream whose thickness δ increases in the flow direction.



$$\delta \rightarrow \frac{u(y)}{u_\infty} = 0.99$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} \qquad [\mathrm{N/m^2}]$$

- Thermal Boundary Layer
  - A consequence of heat transfer between the surface and fluid.
  - A region of the flow characterized by temperature gradients and heat fluxes.
  - A region between the surface and the free stream whose thickness δ_t increases in the flow direction.



$$\frac{T_s - T(\delta_t)}{T_s - T_\infty} \rightarrow 0.99 \Rightarrow \delta_t$$

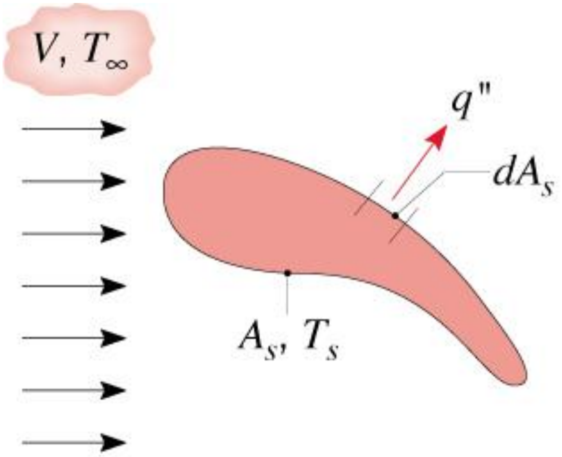
$$q_s'' = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad [\mathrm{W/m^2}]$$

Viscous sublayer
Low Reynolds number

$$q_{conv}'' = h(T_s - T_\infty)$$

$$h \equiv \frac{-k_f \partial T / \partial y|_{y=0}}{T_s - T_\infty} \qquad [\mathrm{W/m^2 \cdot K}]$$

Distinction between Local and Average Heat Transfer Coefficients



- Local Heat Flux and Coefficient:

$$q_s'' = h\left(T_s - T_\infty\right)$$

$$q = \int_{A_s} q'' dA_s = \left(T_s - T_\infty\right)\int_{A_s} h dA_s$$

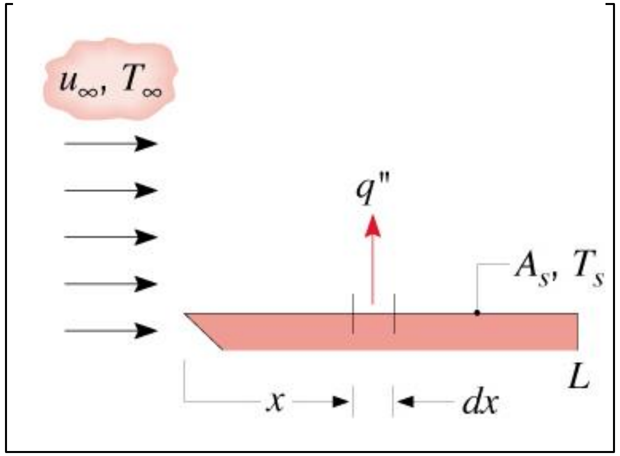
- Average Heat Flux and Coefficient for a Uniform Surface Temperature:

$$q = \bar{h}A_s\left(T_s - T_\infty\right) = \left(T_s - T_\infty\right)\int_{A_s} h dA_s$$

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s$$

- For a flat plate in parallel flow:

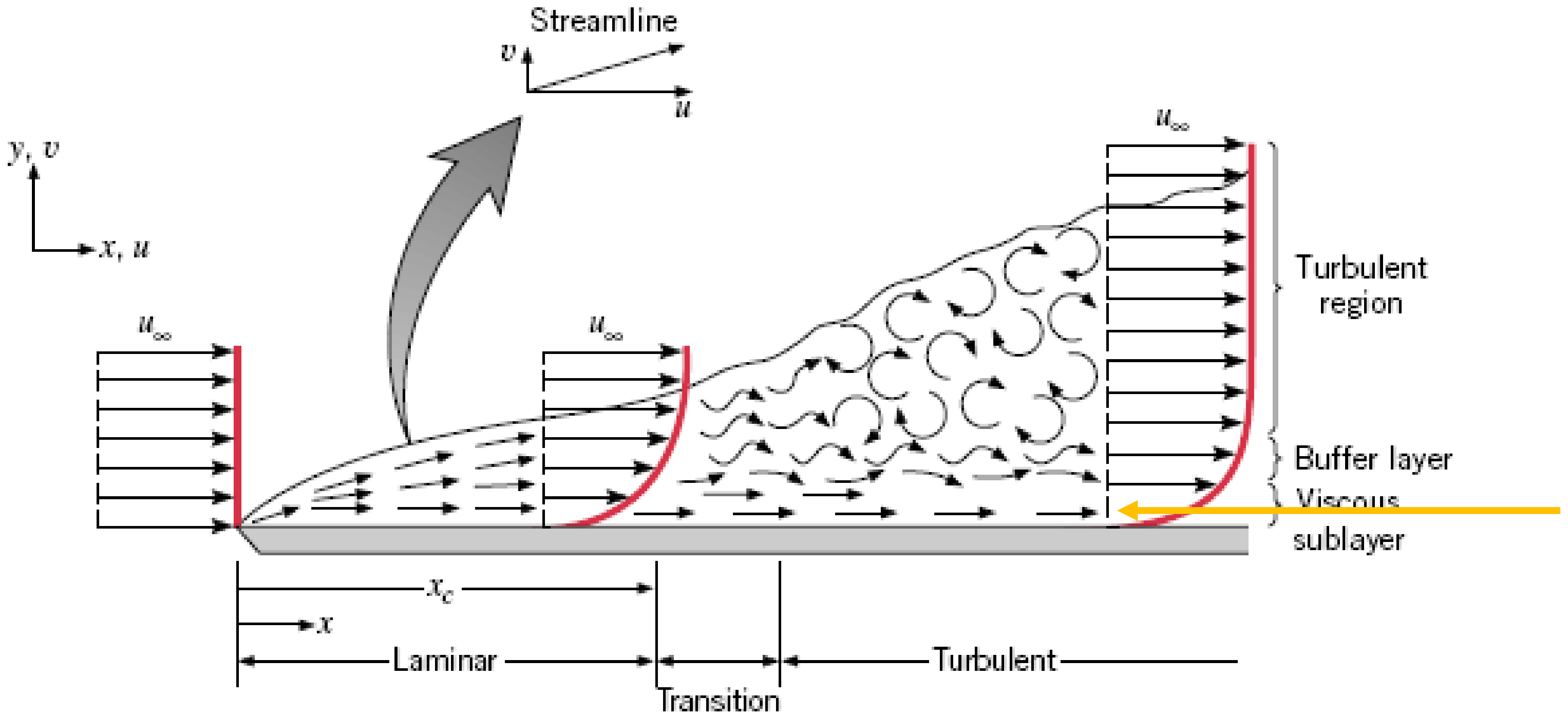
$$\bar{h} = \frac{1}{L}\int_o^L h dx$$



Eksempel 1

Water at a temperature of $T_\infty = 25°C$ flows over one of the surfaces of a steel wall (AISI 1010) whose temperature is $T_{s,1} = 40°C$. The wall is 0.35 m thick, and its other surface temperature is $T_{s,2} = 100°C$. For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.

Boundary Layer Transition



$$Re = \frac{Inertia\ forces}{Viscous\ forces}$$

- Transition criterion for a flat plate in parallel flow:

$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} \rightarrow \text{critical Reynolds number}$$

$x_c \rightarrow$ location at which transition to turbulence begins

$$10^5 \lesssim Re_{x,c} \lesssim 3 \times 10^6$$

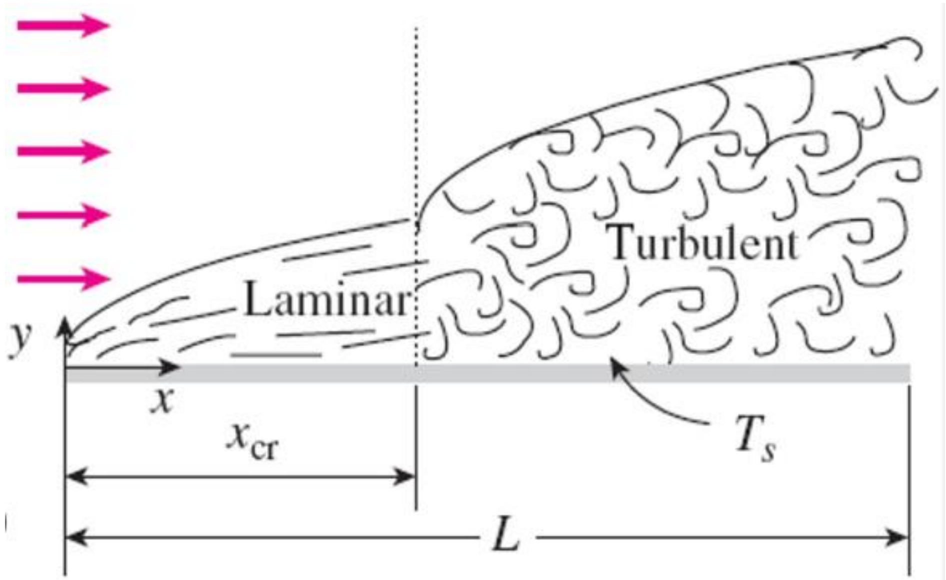
$Re_{x,c} = 5 \times 10^5$ is typically used.

Nusselt's Number:

$Local$ $$Nu_x = \frac{h_x x}{k_f}$$

$Average$ $$\overline{Nu}_L = \frac{\bar{h}L}{k_f}$$

Eksempel 2



The Boundary Layer Equations

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

***x*-momentum:**

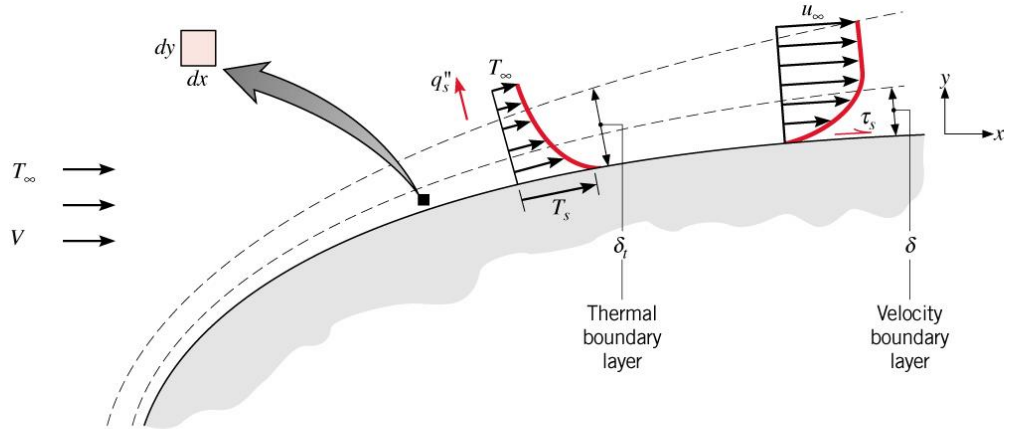
$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P'}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right)$$

***y*-momentum:**

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P'}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\right)$$

***z*-momentum:**

$$u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P'}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$



- Consider concurrent velocity and thermal boundary layer development for steady, two-dimensional, incompressible flow with constant fluid properties (μ, c_p, k) and negligible body forces.
- Apply conservation of mass, Newton's 2nd Law of Motion and conservation of energy to a differential control volume and invoke the boundary layer approximations.

Velocity Boundary Layer:

$$\frac{\partial^2 u}{\partial x^2} << \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial p}{\partial x} \approx \frac{dp_\infty}{dx}$$

Thermal Boundary Layer:

$$\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$$

- Conservation of Mass:

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0 \qquad (6.27)$$

In the context of flow through a differential control volume, what is the physical significance of the foregoing terms, if each is multiplied by the mass density of the fluid?

- Newton's Second Law of Motion:

x-direction :

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (6.28)$$

What is the physical significance of each term in the foregoing equation?

Why can we express the pressure gradient as dp_∞/dx instead of $\partial p / \partial x$?

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$

Boundary Layer Equations (cont.)

- Conservation of Energy:

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (6.29)$$

Boundary Layer Similarity

- As applied to the boundary layers, the principle of similarity is based on determining similarity parameters that facilitate application of results obtained for a surface experiencing one set of conditions to geometrically similar surfaces experiencing different conditions. (Recall how introduction of the similarity parameters *Bi* and *Fo* permitted generalization of results for transient, one-dimensional conduction).

- Dependent boundary layer variables of interest are:

$$\tau_s \text{ and } q'' \text{ (or } h\text{)}$$

- For a prescribed geometry, the corresponding independent variables are:

Geometrical: Size (L), Location (x,y)

Hydrodynamic: Velocity (V)

Fluid Properties:

Hydrodynamic: ρ, μ

Thermal: c_p, k

Hence,

$$u = f\left(x, y, L, V, \rho, \mu\right)$$

$$\tau_s = f\left(x, L, V, \rho, \mu\right)$$

and

$$T = f\left(x, y, L, V, \rho, \mu, c_p, k, T_s, T_\infty\right)$$

$$h = f\left(x, L, V, \rho, \mu, c_p, k, T_s, T_\infty\right)$$

- Key similarity parameters may be inferred by non-dimensionalizing the momentum and energy equations.

- Recast the boundary layer equations by introducing dimensionless forms of the independent and dependent variables.

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{V} \qquad \upsilon^* \equiv \frac{\upsilon}{V}$$

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

- Neglecting viscous dissipation, the following normalized forms of the *x*-momentum and energy equations are obtained:

$$u^* \frac{\partial u^*}{\partial x^*} + \upsilon^* \frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \qquad (6.35)$$

$$u^* \frac{\partial T^*}{\partial x^*} + \upsilon^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}} \qquad (6.36)$$

$Re_L \equiv \dfrac{\rho V L}{\mu} = \dfrac{VL}{\nu} \rightarrow$ the Reynolds Number

$Pr \equiv \dfrac{c_p \mu}{k} = \dfrac{\nu}{\alpha} \rightarrow$ the Prandtl Number

- For a prescribed geometry,

$$u^* = f\left(x^*, y^*, Re_L\right)$$

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \left(\frac{\mu V}{L}\right) \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless shear stress, or local friction coefficient, is then

$$C_f \equiv \frac{\tau_s}{\rho V^2 / 2} = \frac{2}{Re_L} \frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} \tag{6.45}$$

$$\frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, Re_L\right)$$

$$C_f = \frac{2}{Re_L} f\left(x^*, Re_L\right) \tag{6.46}$$

What is the functional dependence of the average friction coefficient?

- For a prescribed geometry,

$$T^* = f\left(x^*, y^*, Re_L, Pr\right)$$

$$h = \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty} = -\frac{k_f}{L}\frac{\left(T_\infty - T_s\right)}{\left(T_s - T_\infty\right)}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless local convection coefficient is then

$$Nu \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = f\left(x^*, Re_L, Pr\right) \qquad (6.48;\ 6.49)$$

$Nu \rightarrow$ local Nusselt number

What is the functional dependence of the average Nusselt number?

How does the Nusselt number differ from the Biot number?

The Reynolds Analogy

- Equivalence of dimensionless momentum and energy equations for negligible pressure gradient ($dp^*/dx^* \sim 0$) and $Pr \sim 1$:

$$\underbrace{u^* \frac{\partial u^*}{\partial x^*} + \upsilon^* \frac{\partial u^*}{\partial y^*}}_{\text{Advection terms}} = \underbrace{\frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}}}_{\text{Diffusion}}$$

$$\overbrace{u^* \frac{\partial T^*}{\partial x^*} + \upsilon^* \frac{\partial T^*}{\partial y^*}}^{\text{Advection terms}} = \overbrace{\frac{1}{Re} \frac{\partial^2 T^*}{\partial y^{*2}}}^{\text{Diffusion}}$$

- Hence, for equivalent boundary conditions, the solutions are of the same form:

$$u^* \equiv \frac{u}{V}$$

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

$$u^* = T^*$$

$$\left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=0} = \left. \frac{\partial T^*}{\partial y^*} \right|_{y^*=0}$$

$$C_f \frac{Re}{2} = Nu \qquad (6.66)$$

or, with the Stanton number defined as,

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{Re\,Pr}$$

With $Pr = 1$, the Reynolds analogy, which relates important parameters of the velocity and thermal boundary layers, is

$$\frac{C_f}{2} = St \qquad (6.69)$$

- Modified Reynolds (Chilton-Colburn) Analogy:
  - An empirical result that extends applicability of the Reynolds analogy:

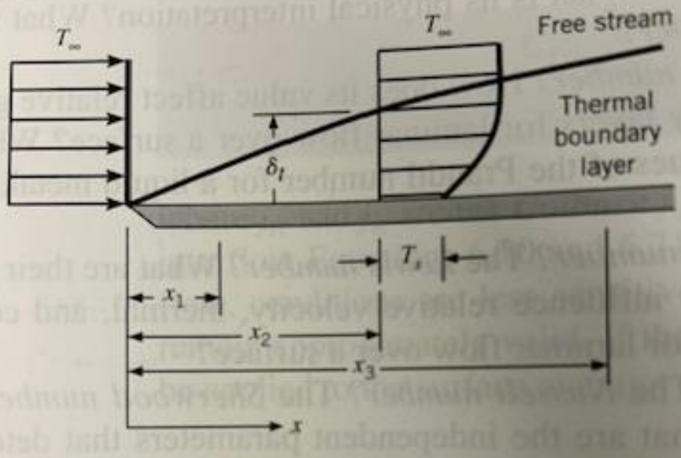
$$\frac{C_f}{2} = St\,Pr^{2/3} \equiv j_H \qquad 0.6 < Pr < 60 \qquad (6.70)$$

Colburn j factor for heat transfer

  - Applicable to laminar flow if $dp^*/dx^* \sim 0$.
  - Generally applicable to turbulent flow without restriction on dp^*/dx^*.

Opgave 6.1

6.1 The temperature distribution within a laminar thermal boundary layer associated with flow over an isothermal flat plate is shown in the sketch. The temperature distribution shown is located at $x = x_2$.



(a) Is the plate being heated or cooled by the fluid?

(b) Carefully sketch the temperature distributions at $x = x_1$ and $x = x_3$. Based on your sketch, at which of the three x-locations is the local heat flux largest? At which location is the local heat flux smallest?

(c) As the free stream velocity increases, the velocity and thermal boundary layers both become thinner. Carefully sketch the temperature distributions at $x = x_2$ for (i) a low free stream velocity and (ii) a high free stream velocity. Based on your sketch, which velocity condition will induce the larger local convective heat flux?

Opgave 6.16

Consider airflow over a flat plate of length $L = 1$ m under conditions for which transition occurs at $x_c = 0.5$ m based on the critical Reynolds number, $Re_{x,c} = 5 \times 10^5$.

(a) Evaluating the thermophysical properties of air at 350 K, determine the air velocity.

(b) In the laminar and turbulent regions, the local convection coefficients are, respectively,

$$h_{lam}(x) = C_{lam}\, x^{-0.5} \quad \text{and} \quad h_{turb} = C_{turb}\, x^{-0.2}$$

where, at $T = 350$ K, $C_{lam} = 8.845\ \text{W/m}^{3/2} \cdot \text{K}$, $C_{turb} = 49.75\ \text{W/m}^{1.8} \cdot \text{K}$, and x has units of m. Develop an expression for the average convection coefficient, $\bar{h}_{lam}(x)$, as a function of distance from the leading edge, x, for the laminar region, $0 \leq x \leq x_c$.

(c) Develop an expression for the average convection coefficient, $\bar{h}_{turb}(x)$, as a function of distance from the leading edge, x, for the turbulent region, $x_c \leq x \leq L$.

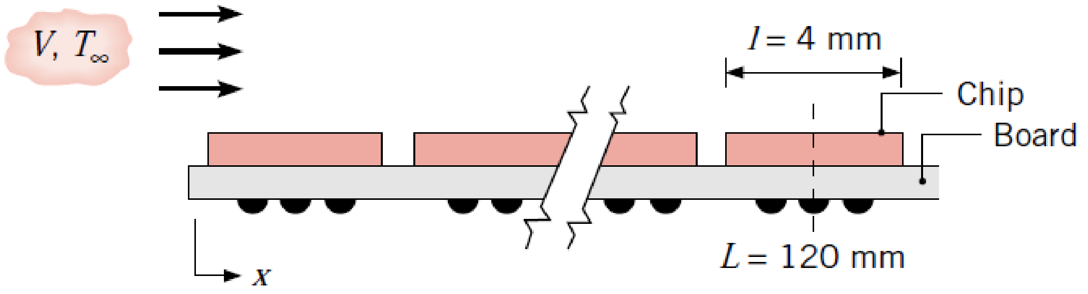
(d) On the same coordinates, plot the local and average convection coefficients, h_x and $\bar{h}_x$, respectively, as a function of x for $0 \leq x \leq L$.

Opgave 6.33

Forced air at $T_\infty = 25°\text{C}$ and $V = 10$ m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm by 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$Nu_x = 0.04 Re_x^{0.85}\, Pr^{1/3}$$

$$Nu_x = 0.04 Re_x^{0.85}\, Pr^{1/3}$$



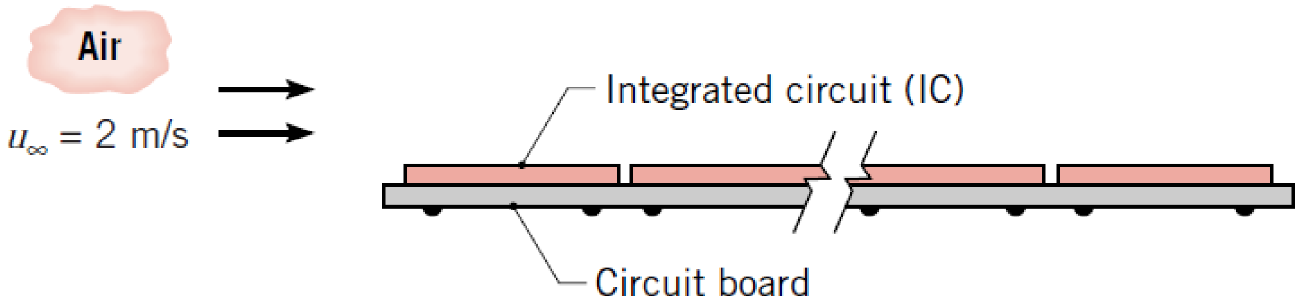
Estimate the surface temperature of the chip if it is dissipating 30 mW.

Opgave 6.42

As a means of preventing ice formation on the wings of a small, private aircraft, it is proposed that electric resistance heating elements be installed within the wings. To determine representative power requirements, consider nominal flight conditions for which the plane moves at 100 m/s in air that is at a temperature of $-23°C$ and has properties of $k = 0.022\ \mathrm{W/m \cdot K}$, $Pr = 0.72$, and $\nu = 16.3 \times 10^{-6}\ \mathrm{m^2/s}$. If the characteristic length of the airfoil is $L = 2$ m and wind tunnel measurements indicate an average friction coefficient of $\overline{C}_f = 0.0025$ for the nominal conditions, what is the average heat flux needed to maintain a surface temperature of $T_s = 5°C$?

Opgave 6.43

6.39 A circuit board with a dense distribution of integrated circuits (ICs) and dimensions of 120 mm by 120 mm on a side is cooled by the parallel flow of atmospheric air with a velocity of 2 m/s.



From wind tunnel tests under the same flow conditions, the average frictional shear stress on the upper surface is determined to be 0.0625 N/m^2. What is the allowable power dissipation from the upper surface of the board if the average surface temperature of the ICs must not exceed the ambient air temperature by more than 25°C? Evaluate the thermophysical properties of air at 300 K.

AARHUS
UNIVERSITET