# Heat transfer
# Kap. 3.1-3.5 One-Dimensional Steady-state conduction

Thermal diffusivity [m²/s]: $$\alpha = \frac{k}{\rho c_p}$$

$$\frac{\partial\left(k\,\frac{\partial T}{\partial x}\right)}{\partial x} + \frac{\partial\left(k\,\frac{\partial T}{\partial y}\right)}{\partial y} + \frac{\partial\left(k\,\frac{\partial T}{\partial z}\right)}{\partial z} + \dot{q} = \rho c_p \frac{\partial T}{\partial t}$$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = \frac{\rho c_p}{k}\frac{\partial T}{\partial t} \quad = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$ k constant and one-dimensional

$$\frac{d^2T}{dx^2} = 0$$ Steady-state and no heat generation

$$\frac{dT}{dx} = C_1$$ First integration

$$T(x) = C_1 x + C_2$$ Second integration: General solution

$$For\ x = 0 \quad \Rightarrow \quad C_2 = T_{s1}$$ BC 1

$$For\ x = L \quad \Rightarrow \quad T_{s2} = C_1 L + T_{s1} \Rightarrow \quad C_1 = \frac{T_{s2} - T_{s1}}{L}$$ BC 2

$$T(x) = \frac{T_{s2} - T_{s1}}{L}x + T_{s1}$$ Straight line (Particular solution)

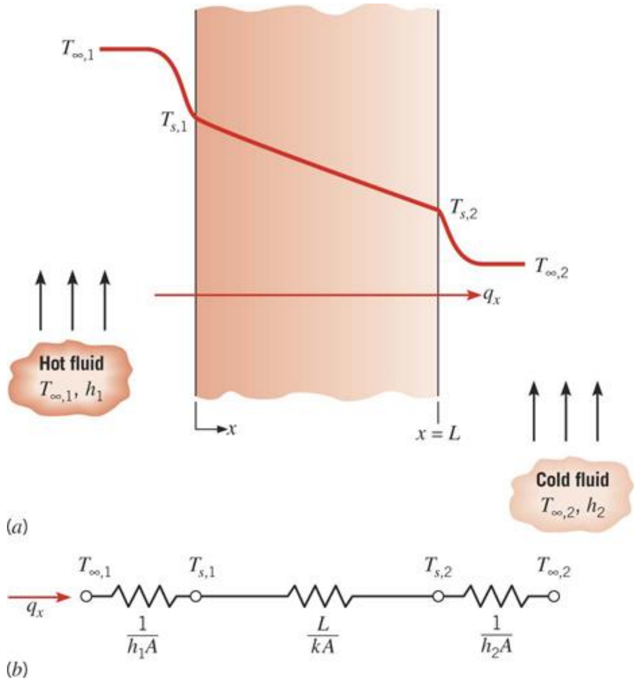
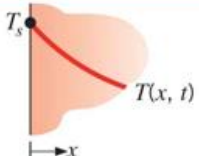



**TABLE 2.2** Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature

$$T(0, t) = T_s \qquad (2.31)$$



Temperature distribution: $$T(x) = \frac{T_{s2} - T_{s1}}{L} x + T_{s1}$$

$$\frac{dT}{dx} = \frac{T_{s2} - T_{s1}}{L}$$

Fourier: $$q_x = -kA \frac{dT}{dx}$$

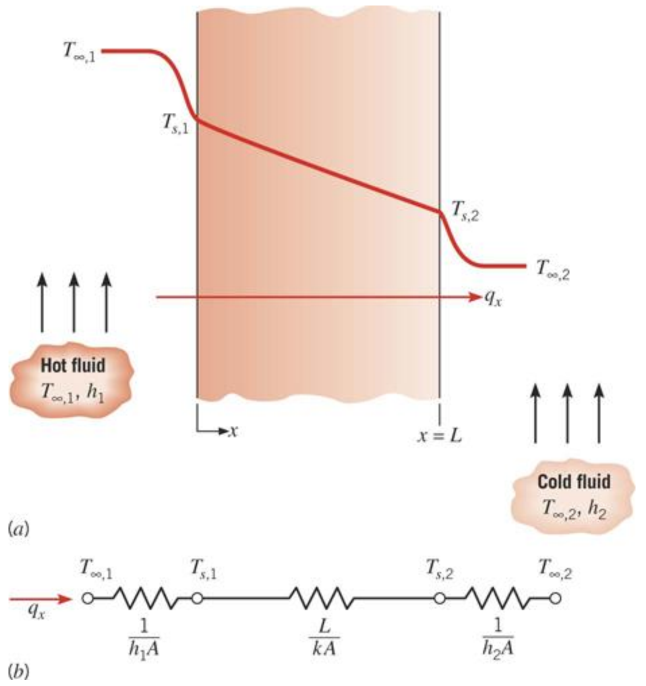
Heat transfer: $$q_x = \frac{kA}{L}(T_{s1} - T_{s2})$$

Resistance $$R \equiv \frac{\Delta T}{q}$$

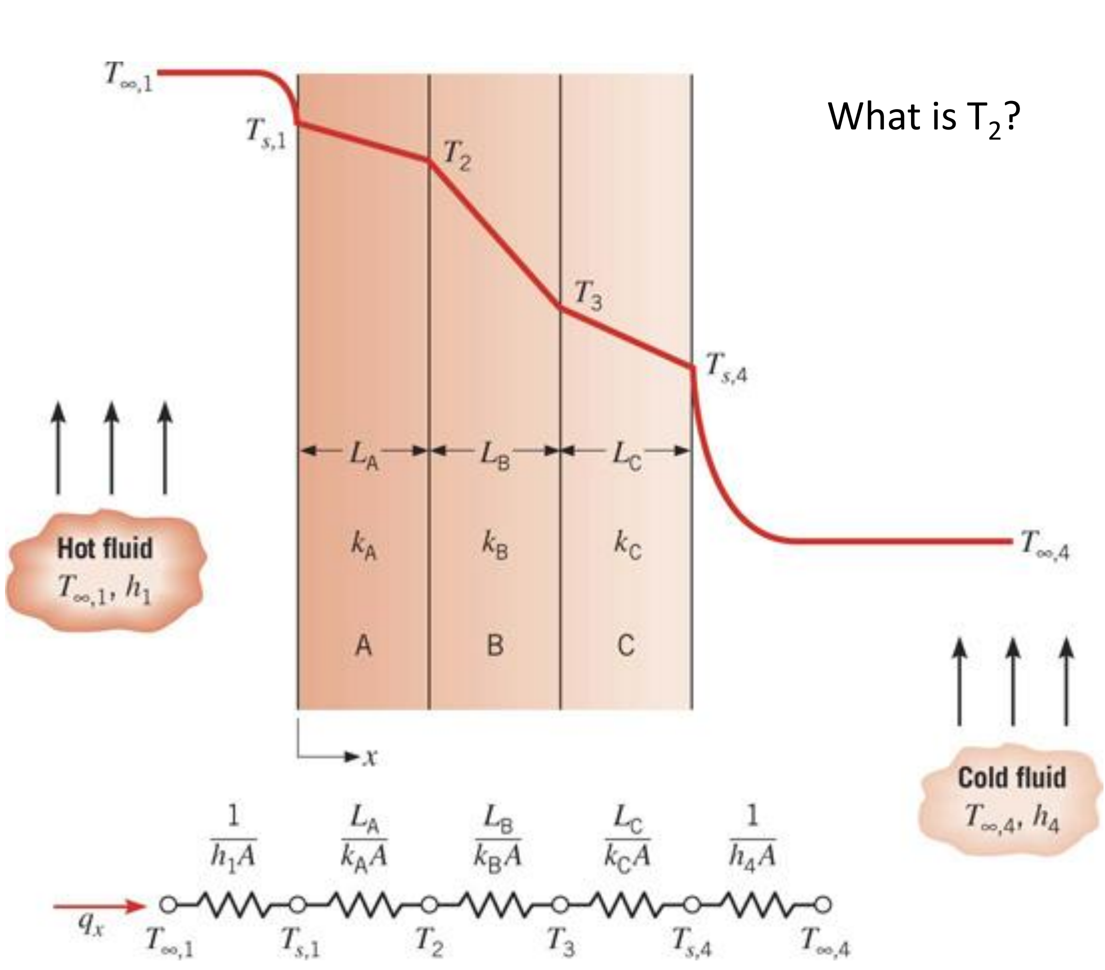
$R_{cond} \equiv ?$ $$\frac{T_{s1} - T_{s2}}{q_x} = \frac{L}{kA}$$

$R_{conv} \equiv ?$ $$\frac{1}{hA}$$

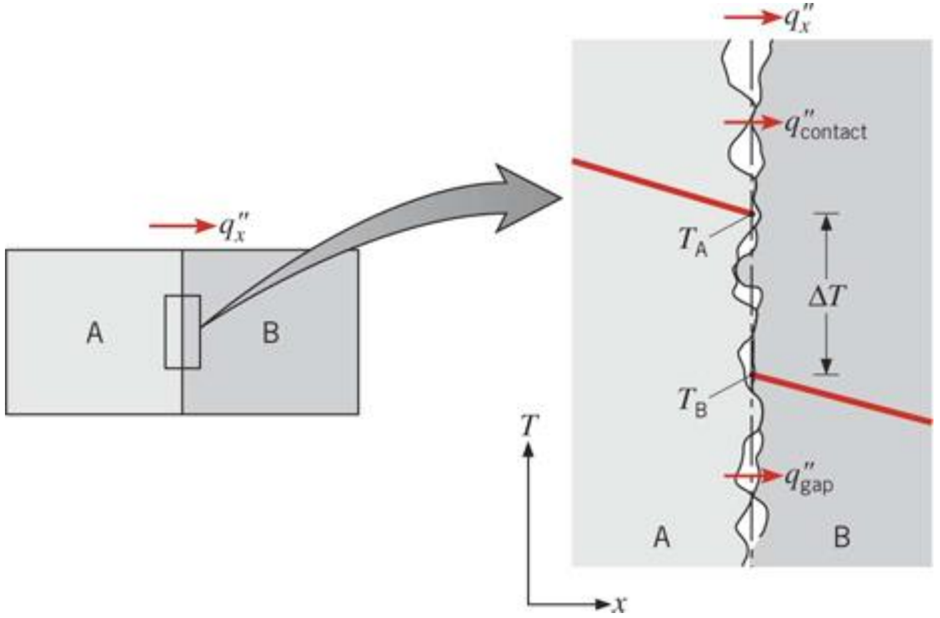
$$R_{rad} = \frac{1}{h_r A} \qquad h_r = \epsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2)$$




Assumptions:
- One-dimensional
- Steady-state
- No heat generation
- Constant k



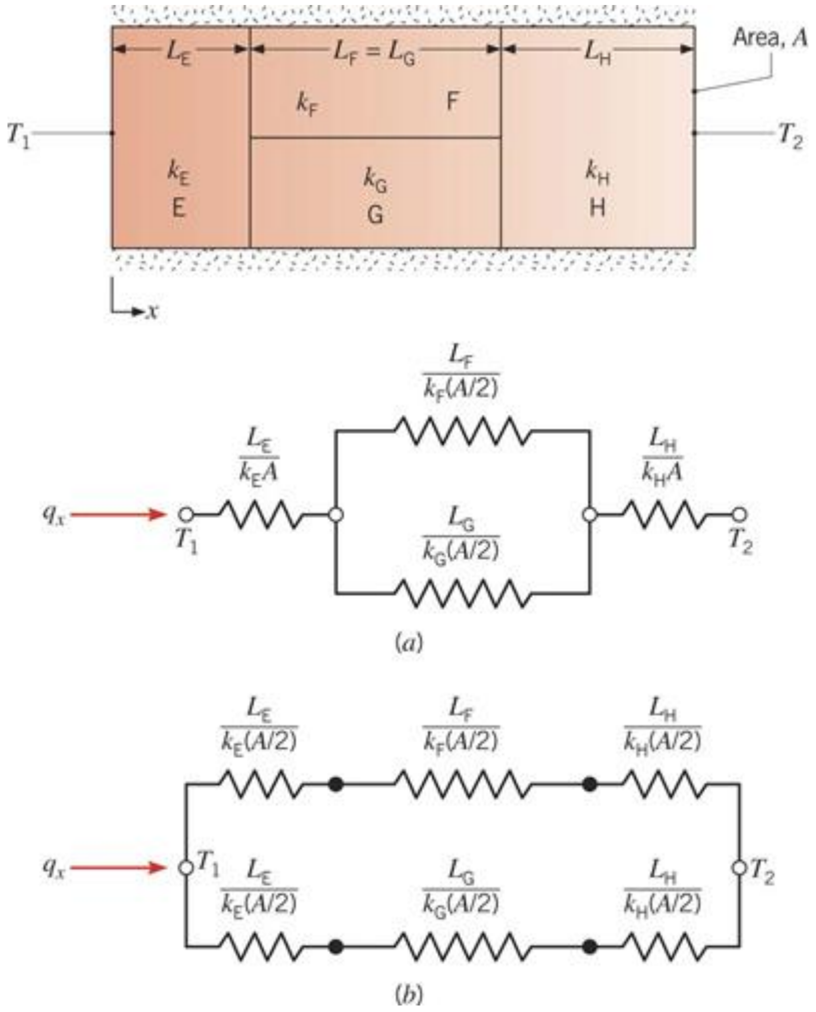

What is $T_2$?



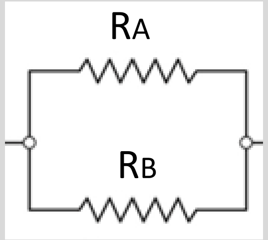

$$R''_c = \frac{T_A - T_B}{q''_x}$$

$$q \equiv \frac{T_{\infty 1} - T_{\infty 4}}{R_{tot}}$$

$$R_c = \frac{R''_c}{A} = \frac{T_A - T_B}{q''_x A}$$




Parallel resistances:




$$R_{AB} = \left(\frac{1}{R_A} + \frac{1}{R_B}\right)^{-1}$$

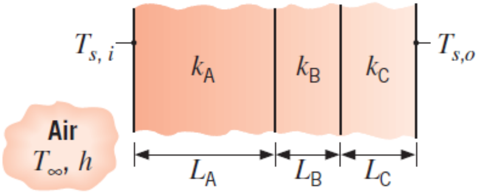
**TABLE 3.3** One-dimensional, steady-state solutions to the heat equation with no generation

| | Plane Wall | Cylindrical Wall[a] | Spherical Wall[a] |
|---|---|---|---|
| Heat equation | $\frac{d^2T}{dx^2} = 0$ | $\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$ | $\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$ |
| Temperature distribution | $T_{s,1} - \Delta T\frac{x}{L}$ | $T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$ | $T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$ |
| Heat flux ($q''$) | $k\frac{\Delta T}{L}$ | $\frac{k\,\Delta T}{r\ln(r_2/r_1)}$ | $\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$ |
| Heat rate ($q$) | $kA\frac{\Delta T}{L}$ | $\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$ | $\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$ |
| Thermal resistance ($R_{t,\text{cond}}$) | $\frac{L}{kA}$ | $\frac{\ln(r_2/r_1)}{2\pi Lk}$ | $\frac{(1/r_1)-(1/r_2)}{4\pi k}$ |

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

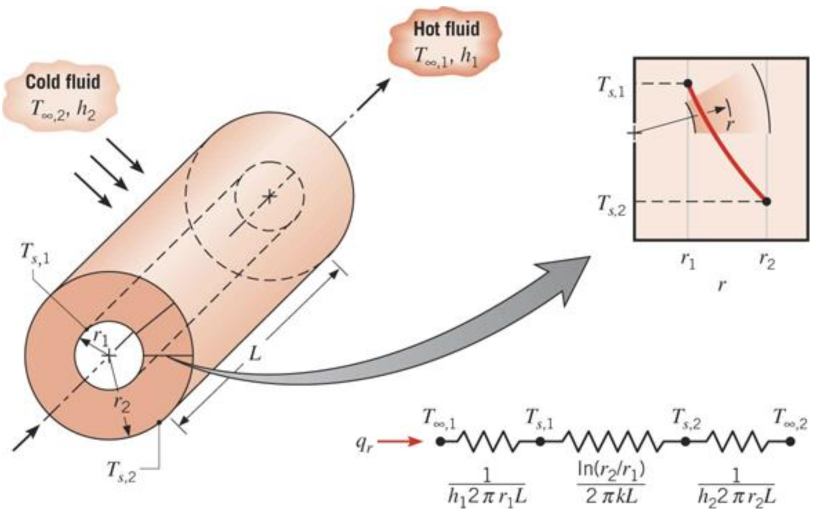
- Eksampel:opg 3.17

The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 25$ W/m · K and $k_C = 60$ W/m · K, and known thickness, $L_A = 0.40$ m and $L_C = 0.20$ m. The third material, B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.20$ m, but unknown thermal conductivity $k_B$.




Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 20$°C, an inner surface temperature of $T_{s,i} = 600$°C, and an oven air temperature of $T_\infty = 800$°C. The inside convection coefficient $h$ is known to be 25 W/m$^2$ · K. What is the value of $k_B$?

Cylinder $$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{dr}\right)+\frac{1}{r^2}\frac{\partial}{\partial\phi}\left(k\frac{\partial T}{d\varphi}\right)+\frac{\partial}{\partial \mathrm{z}}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{dt}$$




Assumptions:

- One-dimensional
- Steady-state
- No heat generation
- Constant k

Temperature distribution?
Heat transfer?

$$q_r=-kA_r\frac{dT}{dx}$$

$$q_r=-k\;2\pi rL\frac{dT}{dr}$$

$$T(r)=\frac{T_{s1}-T_{s2}}{\ln\left(\frac{r_1}{r_2}\right)}\ln\left(\frac{r}{r_2}\right)+T_{s2}$$

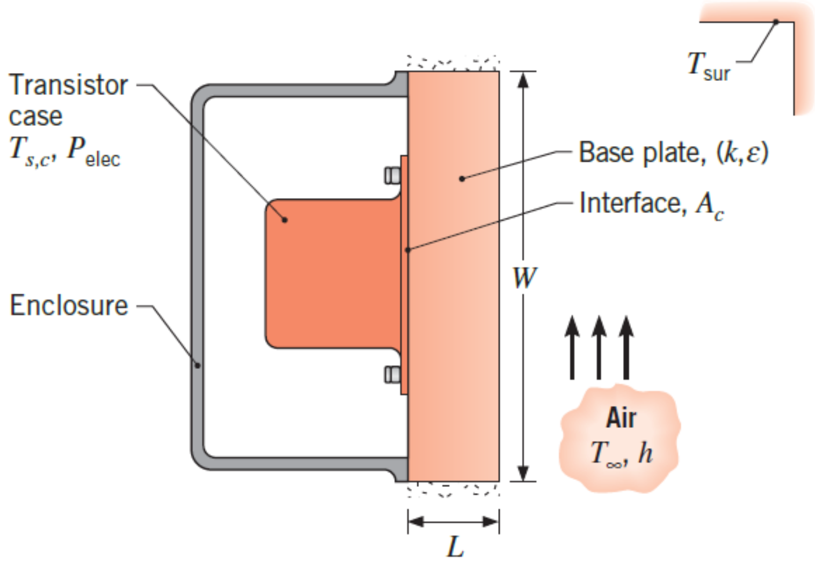
$$\frac{dT}{dr}=\frac{T_{s1}-T_{s2}}{\ln\left(\frac{r_2}{r_1}\right)r}$$

$$q_r=-k\;2\pi L\frac{T_{s1}-T_{s2}}{\ln\left(\frac{r_2}{r_1}\right)}$$

Independent of r

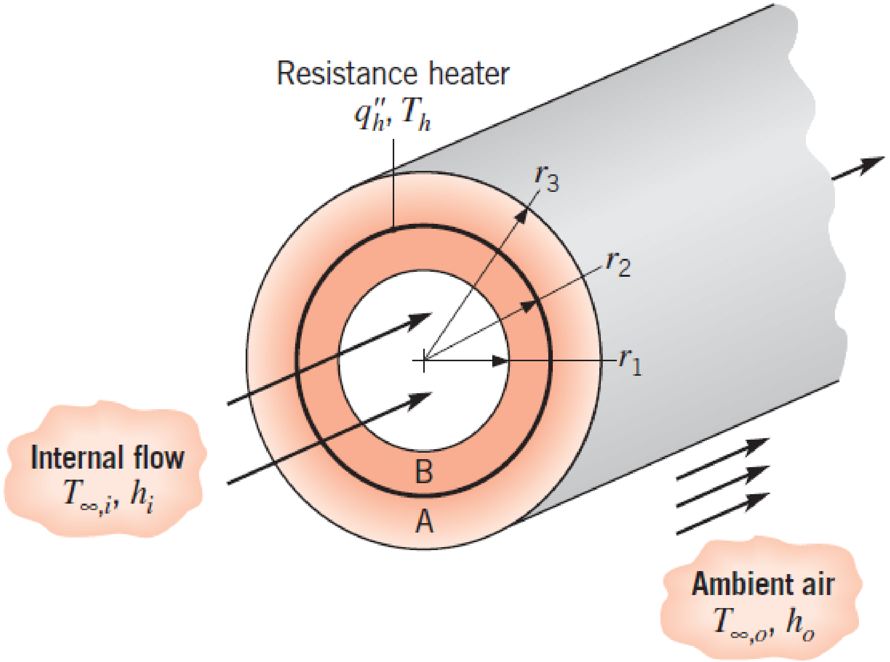
# 3.28

**3.26** Consider a power transistor encapsulated in an aluminum case that is attached at its base to a square aluminum plate of thermal conductivity $k = 240\ \text{W/m} \cdot \text{K}$, thickness $L = 8\ \text{mm}$, and width $W = 24\ \text{mm}$. The case is joined to the plate by screws that maintain a contact pressure of 1 bar, and the back surface of the plate transfers heat by natural convection and radiation to ambient air and large surroundings at $T_\infty = T_{sur} = 30°\text{C}$. The surface has an emissivity of $\varepsilon = 0.9$, and the convection coefficient is $h = 8\ \text{W/m}^2 \cdot \text{K}$. The case is completely enclosed such that heat transfer may be assumed to occur exclusively through the base plate.



3.50

**3.47** A composite cylindrical wall is composed of two materials of thermal conductivity $k_A$ and $k_B$, which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible.




Liquid pumped through the tube is at a temperature $T_{\infty,i}$ and provides a convection coefficient $h_i$ at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{\infty,o}$ and provides a convection coefficient of $h_o$. Under steady-state conditions, a uniform heat flux of $q''_h$ is dissipated by the heater.

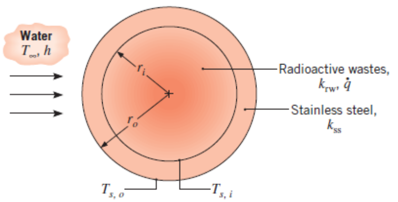
(a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.

(b) Obtain an expression that may be used to determine the heater temperature, $T_h$.

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, $q'_o/q'_i$. How might the variables of the problem be adjusted to minimize this ratio?

- 3.88

**3.86** Radioactive wastes ($k_{rw}$ = 20 W/m · K) are stored in a spherical, stainless steel ($k_{ss}$ = 15 W/m · K) container of inner and outer radii equal to $r_i$ = 0.5 m and $r_o$ = 0.6 m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m³, and the outer surface of the container is exposed to a water flow for which $h$ = 1000 W/m² · K and $T_\infty$ = 25°C.




(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of $r_i$, $T_{s,i}$, $k_{rw}$, and $\dot{q}$. Evaluate the temperature at $r$ = 0.

(d) A proposed extension of the foregoing design involves storing waste materials having the same thermal conductivity but twice the heat generation ($\dot{q} = 2 \times 10^5$ W/m³) in a stainless steel container of equivalent inner radius ($r_i$ = 0.5 m). Safety considerations dictate that the maximum system temperature not exceed 475°C and that the container wall thickness be no less than $t$ = 0.04 m and preferably at or close to the original design ($t$ = 0.1 m). Assess the effect of varying the outside convection coefficient to a maximum achievable value of $h$ = 5000 W/m² · K (by increasing the water velocity) and the container wall thickness. Is the proposed extension feasible? If so, recommend suitable operating and design conditions for $h$ and $t$, respectively.