Notes for EBU problem

Gautam

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1 EBU problem - 2D

1.1 Original Problem from the classical structure

We are solving for $u' = (E'z, H'x, H'y)$ on an $nx * ny$ grid with $nx = ny = n$, where $dim(E'z) = n * n$, $dim(H'x) = n * (n-1)$ and $dim(H'y) = (n-1) * n$. The PDE is given as

$$\frac{du'}{dt} = A'u, \tag{1}$$

where

$$A' = \begin{bmatrix} 0_{N\times N} & A'E_{N\times M} \\ A'H_{M\times N} & 0_{M\times M} \end{bmatrix} \tag{2}$$

with $N = n * n$ and $M = 2 * n * (n-1)$. The matrices $A'E$ and $A'H$ have two parts to themselves.

$$\begin{aligned} A'E &= A'E^1_{N*(M/2)}, A'E^2_{N*(M/2)} \quad \text{appended rowwise,} \\ A'H &= A'H^1_{(M/2)*N}, A'E^2_{(M/2)*N} \quad \text{appended columnwise.} \end{aligned} \tag{3}$$

where $A'E^1(A'H^1)$ and $A'E^2(A'H^2)$ are the 2D partial derivatives respecting PMC(PEC) boundary conditions with respect to y and x, respectively.

1.2 Restating the problem

The main pde to be implemented

To make the problem suitable to Hamiltonian Simulation. We redefined the matrices by padding 'zeroes' to each $A'E^1, A'E^2, A'H^1, A'H^2$ and A' and obtain new matrices AE^1, AE^2, AH^1, AH^2 and A. The new matrices have the dimensions

$$\begin{aligned} AE &= AE^1_{N*N}, AE^2_{N*N} \quad \text{appended rowwise,} \\ AH &= AH^1_{N*N}, AE^2_{N*N} \quad \text{appended columnwise,} \end{aligned}$$

$$A = \begin{bmatrix} 0_{N\times N} & 0_{N\times N} & AE^1_{N\times N} & AE^2_{N\times N} \\ 0_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ AH^1_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N} \\ AH^2_{N\times N} & 0_{N\times N} & 0_{N\times N} & 0_{N\times N}. \end{bmatrix} \tag{4}$$

The correcponding combined vector is defined as $u = [Ez_{n*n}, 0_{n*n}, Hx_{n*n}, Hy_{n*n}]$. To get the same vectors as the original problem the mapping is given by

$$\begin{aligned} &E'z = Ez \\ &H'x = Hx[0 : n*n - n] \\ &H'y = Hy[mask], \quad \text{mask} = k \in 1, 2, \ldots, n^2, \text{such that} \quad k \bmod n \neq 0. \end{aligned}$$

Therefore we will have a new equation given as

$$\frac{du}{dt} = Au. \tag{5}$$

1.3 Encoding A with smaller matrices

Let us define the following matrices $D^b_{PEC} = D^b$ and $D^f_{PMC} = D^f$ that are the discretized backward and forward derivatives with PMC and PEC boundaries respectively.

$$D^b = \begin{cases} D^b[i,i] = 1, 1 \leq i \leq n-2, \\ D^b[0,0] = 2, \\ D^b[i+1,i] = -1, 0 \leq i \leq n-3 \\ D^b[n-1,n-2] = -2, \end{cases}$$
$$D^f = \begin{cases} D^f[i,i] = -1, 1 \leq i \leq n-2, \\ D^f[i-1,i] = 1, 0 \leq i \leq n-1 \end{cases} \tag{6}$$

Next, the following matrices will be useful in encoding the full A matrix

$$\sigma_{00} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \sigma_{01} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \sigma_{10} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \sigma_{11} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}. \tag{7}$$

Using the matrices defined in Eq. (6) and Eq. (7), we can express the components of A as

$$\begin{aligned} AE^1 &= -D^b \otimes I^n, \\ AE^2 &= I^n \otimes D^b, \\ AH^1 &= -D^f \otimes I^n, \\ AH^2 &= I^n \otimes D^f. \end{aligned} \tag{8}$$

Thus, we can decompose A in the following form

$$A = \sigma_{01} \otimes \sigma_{00} \otimes AE^1 + \sigma_{01} \otimes \sigma_{01} \otimes AE^2 + \sigma_{10} \otimes \sigma_{00} \otimes AH^1 + \sigma_{10} \otimes \sigma_{10} \otimes AH^2, \tag{9}$$

where it is obvious to see that the actions of the individual components are as given below

$$(\sigma_{01} \otimes \sigma_{01} \otimes AE^1)u = -\frac{dHx}{dy} \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$(\sigma_{01} \otimes \sigma_{00} \otimes AE^2)u = \frac{dHy}{dx} \otimes \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
$$(\sigma_{10} \otimes \sigma_{00} \otimes AH^1)u = -\frac{dEz}{dy} \otimes \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$
$$(\sigma_{10} \otimes \sigma_{10} \otimes AH^2)u = \frac{dEz}{dx} \otimes \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{10}$$