



The profitability of low-volatility

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ABSTRACT

Low-risk stocks exhibit higher returns than predicted by established asset pricing models, but this anomaly seems to be explained by the new Fama-French five-factor model, which includes a **profitability factor**. We argue that this conclusion is premature given the lack of empirical evidence for a positive relation between risk and return. We find that exposure to market beta in the cross-section is not rewarded with a positive premium, regardless of whether we control for the new factors in the five-factor model. We also observe stronger mispricing for volatility than for beta, which suggests that the low-volatility anomaly is the dominant phenomenon. We conclude that the low-risk anomaly is not explained by the five-factor model.

1. Introduction

Vast empirical evidence shows that the unconditional Capital Asset Pricing Model fails to explain cross-sectional differences in average stock returns. The early tests of the model already indicated that the relation between beta and return is flatter than predicted; see, for instance, Black et al. (1972), Fama and MacBeth (1973), and Haugen and Heins (1975). Two decades later, Fama and French (1992) conclude that, if one controls for size effects, market beta is unpriced in the cross-section of stock returns, implying that firms with higher market sensitivity are not rewarded with higher average returns. Closely related to the low-beta anomaly is the low-volatility effect of Blitz and van Vliet (2007) and Blitz et al. (2013), who document that the relation between past stock volatilities and subsequent stock returns is not merely flat, but even negative in all major stock markets over recent decades. The low-volatility effect is also related to studies which report superior risk-adjusted returns for minimum-variance portfolios, such as Haugen and Baker (1991) and Clarke et al. (2010), and to the work of Ang et al. (2006, 2009), who find a similar anomaly for very short-term idiosyncratic volatility. More recent studies such as Baker et al. (2011), Baker and Haugen (2012), and Frazzini and Pedersen (2014) confirm the low-volatility and/or low-beta effects.

Various studies show that the three- and four-factor models fail to explain the low-risk anomaly. For instance, Blitz (2016) finds that the Fama and French (1993) three-factor model is unable to explain anomalously high returns of low-volatility stocks, and Frazzini and Pedersen (2014) report that the low-beta anomaly is not subsumed by the three- and four-factor models. However, Novy-Marx (2014) argues that the low-beta and low-volatility anomalies are explained by a three-factor model augmented with a profitability factor. Fama and French (2016) also find that their (2015) five-factor model, which adds profitability and investment factors to their original three-factor model, is able to explain returns on beta-sorted portfolios. Both papers use time-series regressions to come to these conclusions. This means that they first create beta- or volatility-sorted portfolios, and next regress the resulting time series of portfolio returns on the time series of the factors that comprise their proposed asset pricing models. The

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absence of economically large and statistically significant alphas in these regressions is interpreted as evidence that the low-beta and low-volatility anomalies are explained.

This paper does not question the empirical results of Fama and French (2016) and Novy-Marx (2014), but argues that direct evidence for a linear, positive relation between market beta and returns, which is assumed in their models, is still lacking. If the Fama and French (2015) and Novy-Marx (2014) asset pricing models were correct, it should be possible to construct portfolios which show that the positive, linear relation between beta and returns holds in practice, provided one controls appropriately for the other factors in their models. This can be tested by the use of Fama-MacBeth (1973) regressions, as the estimated coefficients in these regressions can be interpreted as returns on portfolios which have unit exposure (ex-ante) to factors, controlling for exposures (ex-ante) to all other factors included in the regression. Fama (2015) also argues for considering not just one, but multiple asset pricing tests, including Fama-MacBeth (1973) cross-section regressions. However, the rejections of the low-beta anomaly by Novy-Marx (2014) and Fama and French (2016) are solely supported by time-series spanning tests.

Using Fama-MacBeth regressions we test whether the factors in the five-factor model are rewarded with significant premia, and find that all factors are, except market beta. In other words, a unit exposure to market beta in the cross-section does not result in significantly higher returns, regardless of whether one controls for the additional factors proposed by Fama and French (2015). At the same time, the constant in the regressions, which ought to be zero according to this asset pricing model (if returns in excess of the risk-free return are used), is large and significant. Taken together, these results imply that the relation between market beta and return in the cross-section is flat instead of positive, which is consistent with the asset pricing models of Blitz (2014) and Clarke et al. (2014). Simply put, we are unable to construct high-beta portfolios with high returns and low-beta portfolios with low returns by controlling for factors such as profitability, while it should be possible to do so if the low-beta anomaly is fully explained by such factors.

We also find more pronounced mispricing for volatility than for beta. This suggests that the low-volatility anomaly is stronger than the low-beta anomaly, and, given that the two are closely related, that the low-volatility anomaly is the dominant phenomenon. These results are consistent with the earlier findings of Blitz and van Vliet (2007), who find higher alpha spreads for volatility-sorted portfolios than for beta-sorted portfolios. Lastly, we show that our results are robust to the choice of profitability measure, and also, using two distinct methodologies, to the well-known errors-in-variables problem.

2. Data

We consider all common stocks (share codes 10 and 11) in the CRSP database traded on NYSE, AMEX, and NASDAQ exchanges, except those with share price below 1 dollar. Following Frazzini and Pedersen (2014), we estimate stock and market return volatilities over the past year (minimum 120 days) and correlations with the market portfolio over the preceding five year period (minimum three years). For volatilities we use log returns ($\ln(1+r)$), whereas for correlations we use the average of past three log returns to control for non-synchronous trading. If daily data are not available, we use past twelve monthly returns to calculate volatilities and sixty (minimum 36) for correlations. These estimates are used to calculate market betas, which we shrink towards one using the commonly employed shrinkage factor of $1/3$, as proposed originally by Blume (1971, 1975).

For the calculation of size, value and momentum characteristics we follow the standard Fama-French methodology. The market capitalization (ME) of a stock is its price times the number of shares outstanding, and size is defined as the natural logarithm of market capitalization at the end of the previous month. The balance sheet and income statement information stems from Compustat North America annual files. Book value is the book value of shareholders' equity, plus balance sheet deferred taxes and investment tax credit, if available, minus the book value of preferred stock (calculated using the redemption, liquidation, or par value, in that order). If available, we use shareholders' equity from either Compustat or Moody's Industrial manuals, otherwise, we measure stockholders' equity as the book value of common equity plus the par value of preferred stock, or the book value of assets minus total liabilities. The valuation of a stock is defined as its book-to-market ratio, i.e. BE/ME , calculated as the book value of common equity at the previous calendar year's fiscal year-end divided by the market value of equity at the end of the previous calendar year, updated at the end of June each year. The momentum of a stock is defined as its total return over the preceding twelve months excluding the most recent month.

For profitability, Fama and French (2015, 2016) use an operating profitability ratio which is defined for all stocks, while Novy-Marx (2014) uses a gross profitability ratio which is undefined for financials. As their results suggest that for explaining the alpha of low-risk strategies it does not matter which definition is used, we use the Fama and French (2015, 2016) measure as our base-case definition of profitability. This means that we calculate operating profitability as annual revenues minus cost of goods sold, interest expense, and selling, general, and administrative expenses divided by book equity. In the penultimate section of the paper, we challenge the robustness of our results to the choice of profitability measure. Next to Novy-Marx (2014) gross profitability we also consider cash-based operating profitability, based on Ball et al. (2016), and return on equity based on Hou et al. (2015). Gross profitability is defined as annual revenues minus cost of goods sold divided by total assets. Cash-based operating profitability is defined as revenues minus costs of goods sold minus reported sales, general, and administrative expenses minus change in account receivables, inventory, and prepaid expenses, plus change in deferred revenue, trade account payable, and accrued expenses. All changes are calculated on the year-to-year bases.

Return on equity (ROE) is defined as income before extraordinary items divided by book equity, but we deviate slightly from the definition of Hou et al. (2015) by using annual instead of quarterly earnings data. In this way we ensure consistency with the frequency of the other accounting factors that we consider. Another advantage of annual ROE is that it is available for the full span of our sample, while the quarterly measure of Hou et al. (2015) is only available from 1972. Moreover, Novy-Marx (2015) analyzes the

profitability measure of [Hou et al. \(2015\)](#) and concludes that their ROE factor is a convoluted proxy for profitability, as it derives a significant portion of its pricing power from past earnings surprises that it incorporates through the use of quarterly earnings data. This enables their ROE factor to price momentum portfolios. He goes on to argue that gross profitability is a superior measure of firm's profitability once this is accounted for. This also supports the use of annual data.

Investment (asset growth) is the percentage change in firms' total assets from year t-2 to t-1. Stocks for which one or more data items are missing are dropped from the sample for that month. The start date of the sample period is July 1963, as this is the earliest date for which the new Fama-French factors are available, and the sample period ends in December 2015. The average number of stocks per month is 2972.

3. Main results

According to the asset pricing models of [Fama and French \(2015\)](#) and [Novy-Marx \(2014\)](#), expected stock returns should be linearly proportional to beta, as in the CAPM, after accounting for the other priced factors in their respective models. A popular way to analyze the significance of a factor while controlling for other factors is to conduct double or triple sorts. However, that approach becomes practically infeasible when one wants to control for more than three factors. This paper uses firm-level [Fama-MacBeth \(1973\)](#) regressions instead. Every month, we run cross-section regressions of stock returns in excess of the risk-free return on a set of characteristics. The crucial realization here is that the coefficients estimated in such a regression can be interpreted as returns on portfolios with a unit exposure (ex-ante) to a factor in the cross-section, while controlling for all other factors included in the regression. The next step consists of calculating the average premium associated with each factor over all months and the corresponding t-statistics (using Newey-West corrected standard errors), and verifying whether these levels are consistent with the predictions of the asset pricing models under investigation. We include momentum as one of the control factors in our analyses even though it is not part of the [Fama and French \(2015\)](#) five-factor model, because it is widely recognized as an important driver of stock returns in the cross-section. We note that our conclusions do not materially change if momentum is excluded from the analysis. Our results are also robust to using weighted least squares (WLS) instead of OLS regressions, using either market capitalization or price to weigh observations.

All explanatory variables are winsorized at the 1% and 99% levels in order to avoid a large potential influence of outliers, and cross-sectional z-score normalizations are applied to all variables, except market beta. As a result, the intercept of the regression can be interpreted as the expected excess return on a stock with average factor characteristics, adjusted for the part of the return that can be empirically attributed to its market beta. Based on the CAPM and its extensions, the intercept of the regressions should not be statistically different from zero, and the reward to a unit of beta exposure should be equal to the equity risk premium, which amounts to 0.50 percent per month over our sample period.

[Table 1](#) reports results for various Fama-MacBeth regressions. Regression I tests the CAPM, as next to the intercept it only includes market beta. Every month, we run a cross-section regression of stock returns in excess of the risk-free rate on their market betas. This procedure yields a time-series of estimated coefficients, for which we report the averages and the corresponding t-statistics. If the CAPM holds and beta explains the cross-sectional differences in average stock returns, the intercept should not be

Table 1
Base-case Fama-MacBeth results.

	I	II	III
Intercept	1.03 (5.69)	0.85 (4.83)	0.74 (4.35)
Beta	-0.20 (-0.71)	-0.01 (-0.02)	0.11 (0.40)
Size		-0.16 (-1.80)	-0.22 (-2.68)
Value		0.18 (3.89)	0.15 (3.48)
Momentum		0.34 (6.52)	0.32 (6.42)
Profitability			0.21 (6.07)
Investment			-0.20 (-9.91)
R-square (%)	2.60	5.42	5.77

This table reports the results of [Fama-MacBeth \(1973\)](#) regressions. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1963 to December 2015 with share price above \$1. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t, value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in t-1 and market cap at the end of December of t-1; momentum is the total stock return from t-12 to t-2; profitability is the ratio of operating profits and book equity at the fiscal year ending in t-1, and investment is growth of total assets for the fiscal year ending in t-1. All variables are winsorized at 1% and 99%, and we normalize all variables except beta. Reported are the average coefficients and t-statistics calculated using Newey-West corrected standard errors.

statistically different from zero, while the slope, i.e. the reward to beta exposure, should be positive, matching the equity risk premium over our sample period. The results in the table show that the reverse is true in reality: the intercept is large and highly statistically significant, while the return associated with beta exposure is indistinguishable from zero. This implies a flat, instead of a positive, relation between risk and return in the cross-section.

In regression II, the size, value, and momentum characteristics are added to the regression. The sign and significance of the estimated premia for these factors is fully consistent with the existing literature; however, they do not help to save the market beta. Controlling for the influence of size, value and momentum, the average return to a unit of beta exposure in the cross-section remains indistinguishable from zero, while the intercept remains positive and significant. The interpretation of these results is that the base-case expected return is positive and the same for every stock, regardless of whether it co-varies strongly with the market or not, and that only the size, value and momentum characteristics of a stock add or detract from that base-case expected return. These results are consistent with the asset pricing models proposed by Blitz (2014) and Clarke et al. (2014), in which market beta also does not carry a premium in the long run, but is only included to help explain the short-term cross-sectional variation in stock returns.

Regression III shows the results when adding the two new variables of Fama and French (2015), profitability and investment. In this regression, market beta should clearly show up as a priced factor, if it were true that with the new factors the low-beta anomaly is resolved. Consistent with the predictions of the five-factor model, both new factors are rewarded with significant premia. However, the table also shows that, again, the return to beta is insignificant, while the intercept of the regressions remains positive and significant. In other words, contrary to the predictions of the asset pricing models of Fama and French (2015) and Novy-Marx (2014), we are unable to find clear evidence of a positive relation between beta and return when controlling for the factors they argue explain the anomaly.

In the analysis above, each stock's covariation with the market portfolio, i.e. its market beta, was estimated in a univariate framework, that is, our estimate of the market beta is closely related to the slope estimate from a simple regression of stock returns on the market portfolio (with the difference that we estimate correlations and volatilities over different time periods). An alternative to this would be to estimate the sensitivity to the market portfolio using the multivariate regression, that is, accounting for the effects of the other four factors in the five-factor model. We estimate this beta over past 60 months (minimum 24), shrink it towards one using the same shrinkage parameter as before (1/3), and run the Fama-MacBeth regressions using this measure instead of the univariate one. These results can be found in Table A1 in the Appendix A. Note that the sample starts in July 1968, as the Fama-French (2015) factor returns are available from July 1963. Our conclusions remain unchanged: the estimated market premium remains insignificant indicating that high beta stocks do not earn higher returns than their low beta counterparts.

We further examine the robustness of our results by splitting the universe at two size breakpoints and repeat the analysis within these groups. Following academic convention, we define large-caps as stocks with market capitalization above NYSE median, and small-caps as those below this threshold. We also examine whether this relationship holds amongst micro-caps, defined as stock with market capitalization below the 20th percentile of NYSE listed stocks. These results are reported in Table A2 in the Appendix A. Once again, the sample starts in July 1968 to ensure significant coverage on all variables, especially amongst micro-caps (our conclusions do not change if we start in 1963). We observe that the results are robust across all size groups.

4. Significance tests

Our results unambiguously indicate that none of the examined models manages to establish a positive relationship between market beta and average returns; however, they do not say anything about the magnitude of the deviation between the expected (theory-based) and realized premium for market beta. In order to determine the significance of our results, we conduct two follow-up tests. For the first test, we note that under the null hypothesis that there is a linear relationship between market beta and stock returns, a unit of beta exposure in the cross-section should be rewarded with the market risk premium. We test whether this premise is supported by the data by regressing the time series of the estimated market premiums (i.e. coefficients for market beta) on realized

Table 2

Regression of time-series of estimated returns to beta in the cross-section on the market risk premium.

	I	II	III
Alpha	-0.74 (-3.86)	-0.53 (-2.68)	-0.40 (-2.11)
Slope	1.09 (25.34)	1.05 (23.70)	1.04 (24.22)

This table reports results of time-series regressions of the estimated returns to market beta in the cross-section on the realized market premium (Mkt-Rf). To estimate returns to market beta in the cross-section, we use Fama MacBeth (1973) regressions. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1963 to December 2015 with share price above \$1. For specification I, we use market beta as the explanatory variable in Fama-MacBeth regressions; in specification II, we use beta, size, value and momentum; in specification III we use beta, size, value, momentum, profitability, and investment. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t, value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in t-1 and market cap at the end of December of t-1; momentum is the total stock return from t-12 to t-2; profitability is the ratio of operating profits and book equity at the fiscal year ending in t-1, and investment is growth of total assets for the fiscal year ending in t-1. All variables are winsorized at 1% and 99%, and we normalize all variables except beta.

market returns (Mkt-Rf). According to the examined linear pricing models, the estimated premium to beta in the cross section should be explained entirely by exposure to the market factor, so the intercept of this regression should be zero. If, instead, the intercept is positive (negative), then the return to a unit of beta is higher (lower) than predicted.

Table 2 shows that the ex-post estimated slope coefficients are close to one and highly significant, which is what we would expect for portfolios with unit beta exposure ex-ante; but, more importantly, the estimated coefficient on the constant term is significantly below zero in all three cases. In other words, we observed before that beta in the cross-section is not rewarded with a significantly positive return, and this additional test shows that this deviation from the models' prediction is statistically significant.

Our second test is also aimed at assessing the significance of the deviation from the predicted relation between market beta and return. We do so by re-running the Fama-MacBeth regressions, only this time using beta-adjusted returns on the left-hand side as the dependent variable. In other words, for each stock return in the regression we not only subtract the risk-free return, but also its beta times the market excess return in that period. The main difference with the previous test is that the beta adjustment executed in this way is applied ex-ante to each stock in the universe, while in the prior test, the adjustment was done using the time-series of ex-post estimated returns to market beta. If the linear relation between beta and return holds, using beta-adjusted returns on the left-hand side of the regressions should result in a zero coefficient for beta on the right-hand side of the regressions; if, instead, the coefficients are positive (negative), the return to a unit of beta is higher (lower) than predicted. The results are shown in the first three columns of Table 3. We find statistically significant, negative premiums for beta on the right-hand side of the regressions, contrary to the predictions of the CAPM and its extensions, which is another confirmation of the low-beta anomaly. We do note that the statistical significance weakens as we add more control factors, and even drops below the commonly used 5% threshold level in the regression which includes profitability and investment factors. This last bit of ambiguity disappears when we consider not only beta, but also volatility.

The use of beta-adjusted returns on the left-hand side also allows us to let beta compete head on with volatility, by simply adding volatility to the set of explanatory variables on the right-hand side of the regressions. If any of the considered models hold, both beta and volatility should be unpriced on the right-hand side. These results are reported in the last three columns of Table 3. Controlling for only beta and volatility (specification IV), beta remains the dominant source of mispricing. Interestingly, this changes when we add other control factors. In these specifications (V and VI) we observe that the negative alpha shifts entirely from beta to volatility, and that the statistical significance of this negative alpha of volatility is much larger, in absolute terms, than the levels observed previously for beta. Consequently, the addition of volatility renders the estimated beta mispricing completely insignificant, suggesting that the low-volatility anomaly is the dominant phenomenon. This is consistent with the earlier findings of Blitz and van Vliet (2007), who come to the same conclusion based on the observation that alpha spreads are higher for volatility-sorted portfolios than for those sorted on beta.

Table 3
Fama-MacBeth results using beta-adjusted returns on the left-hand side.

	I	II	III	IV	V	VI
Intercept	1.04 (5.70)	0.86 (4.85)	0.74 (4.37)	1.07 (4.62)	0.32 (1.64)	0.30 (1.54)
Beta	-0.70 (-3.46)	-0.51 (-2.35)	-0.39 (-1.90)	-0.75 (-3.91)	0.06 (0.29)	0.08 (0.41)
Volatility				0.01 (0.07)	-0.32 (-4.44)	-0.27 (-4.04)
Size		-0.16 (-1.80)	-0.22 (-2.68)		-0.36 (-6.01)	-0.38 (-6.39)
Value		0.18 (3.88)	0.15 (3.47)		0.15 (3.34)	0.12 (2.85)
Momentum		0.34 (6.52)	0.32 (6.42)		0.36 (7.27)	0.34 (6.98)
Profitability			0.21 (6.07)			0.17 (5.73)
Investment			-0.20 (-9.89)			-0.20 (-9.69)
R-square (%)	1.36	4.25	4.61	3.14	4.76	5.05

This table reports the results of modified Fama-MacBeth (1973) regressions. Each month, we regress beta-adjusted stock returns on a set of characteristics and reported are the average coefficients and t-statistics calculated using Newey-West corrected standard errors. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1963 to December 2015 with share price above \$1. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t, value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in t-1 and market cap at the end of December of t-1; momentum is the total stock return from t-12 to t-2; profitability is the ratio of operating profits and book equity at the fiscal year ending in t-1, and investment is growth of total assets for the fiscal year ending in t-1. All variables are winsorized at 1% and 99%, and we normalize all variables except beta.

5. Robutness to choice of profitability measure

The Fama and French (2015) five-factor model is motivated by the valuation theory of asset pricing. One of the implications of this theory is that high profitability, for a given level of investment and book-to-market, implies high internal rate of return on expected dividends, and consequently, high expected returns. As their main measure of profitability, they opt for operating profitability, but this choice has been challenged in some studies. For instance, Ball et al. (2016) show that operating profitability is a poor measure of profitability as it includes accounting accruals, that have been shown to predict negative future returns. They propose a cash-based profitability measure that is purged of accruals, and find that it dominates the Fama-French variable. Hou et al. (2015) put forward the q-factor model motivated by the neoclassical q theory of investment. Their model consist of four factors: market, size, investment, and profitability, with their measure of profitability being return on equity. In the investment model, the value factor is not necessary to explain the cross-section of stock returns, whereas the dividend discount model directly implies its existence. Novy-Marx (2013), on the other hand, has his own measure of profitability, gross-profits-to-assets.

In this section, we run another robustness test using the profitability measures that have been proposed by Ball et al. (2016), Novy-Marx (2013, 2014), and Hou et al. (2015) together with the respective controls that they have in their models. As Ball et al. (2016) do not explicitly propose another asset pricing model, we use the Fama and French (2015) five-factor model and replace operating profitability with cash-based operating profitability. Novy-Marx considers models with and without the momentum factor, so we also include the two alternatives. The market, size, and investment (asset growth) characteristics of Fama and French (2015) and of Hou et al. (2015) are defined in the same way. As all three of the above mentioned papers exclude financial firms, we do so as well by removing all stocks with four-digit SIC codes starting with 6. Results are presented in Table 4. We also include the Fama and French (2015) model for comparison purposes, as the stock universe is restricted to non-financials.

In all model specifications, market beta remains unpriced in the cross-section of stock returns, while all other characteristics are economically and statistically significant. Consequently, all of these empirical models appear to be challenged by the same problem: the predicted positive, linear relationship between market beta and returns is not found in the cross-section.

Table 4
Robustness to choice of profitability measure.

	FF	BALL	HXZ	NM1	NM2
Intercept	0.76 (4.27)	0.80 (4.35)	0.82 (4.42)	0.85 (4.70)	0.87 (5.04)
Beta	0.12 (0.40)	0.07 (0.23)	0.05 (0.15)	0.02 (0.06)	0.00 (0.00)
Size	-0.19 (-2.27)	-0.20 (-2.23)	-0.21 (-2.58)	-0.12 (-1.28)	-0.15 (-1.71)
Value	0.16 (3.61)	0.19 (3.96)		0.23 (4.63)	0.23 (4.69)
Investment	-0.23 (-10.17)	-0.16 (-6.52)	-0.26 (-9.90)		
Momentum					0.31 (5.87)
Operating profitability	0.23 (5.80)				
Cash-based operating profitability		0.28 (10.67)			
Return on equity			0.18 (4.79)		
Gross profitability				0.18 (5.20)	0.16 (4.86)
R-square (%)	4.92	4.84	4.52	4.83	5.51

This table reports the results of Fama-MacBeth (1973) regressions. We include non-financial stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1963 to December 2015 with share price above \$1. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t, value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in t-1 and market cap at the end of December of t-1; momentum is the total stock return from t-12 to t-2; operating profitability is the ratio of operating profits and book equity at the fiscal year ending in t-1, cash-based operating profitability is operating profitability (of Fama-French) minus change in account receivables, inventory, and prepaid expenses, plus change in deferred revenue, account payable, and accrued expenses. All changes are calculated on the year-to-year bases. Return on equity as income before extraordinary items divided by book equity. Investment is growth of total assets for the fiscal year ending in t-1. All variables are winsorized at 1% and 99%, and we normalize all variables except beta. Reported are the average coefficients and t-statistics calculated using Newey-West corrected standard errors.

Table 5
Fama-MacBeth results with portfolio beta.

	I	II	III
Intercept	1.02 (5.70)	0.84 (4.87)	0.74 (4.38)
Beta	-0.19 (-0.70)	0.00 (-0.01)	0.11 (0.41)
Size		-0.16 (-1.80)	-0.21 (-2.68)
Value		0.18 (3.90)	0.15 (3.50)
Momentum		0.34 (6.51)	0.32 (6.41)
Profitability			0.21 (6.05)
Investment			-0.20 (-9.90)
R-square (%)	2.58	5.41	5.76

This table reports the results of [Fama-MacBeth \(1973\)](#) regressions. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1963 to December 2015 with share price above \$1. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. This beta is shrunk towards one using shrinkage parameter of 1/3. We form 50 portfolios by sorting stocks on their past market betas, calculate the beta of the overall portfolio, and assign this value to each stock in the corresponding portfolio. Size is the natural logarithm of firm's market capitalization at the end of month t , value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in $t-1$ and market cap at the end of December of $t-1$; momentum is the total stock return from $t-12$ to $t-2$; profitability is the ratio of operating profits and book equity at the fiscal year ending in $t-1$, and investment is growth of total assets for the fiscal year ending in $t-1$. All variables are winsorized at 1% and 99%, and we normalize all variables except beta. Reported are the average coefficients and t-statistics calculated using Newey-West corrected standard errors.

6. Robutness to measurment errors

Thus far, the analysis in this paper was conducted on the individual-firm level, which may raise the question of whether our results are affected by the errors-in-variables problem. If our independent variables are measured with systematic errors, this may lead to biases in our estimates and incorrect inferences. In order to explicitly address this problem, we test the robustness of our findings using two different methodologies.

We already showed that our results are robust to two different ways of estimating market beta; however, here we explicitly address the concern that our beta estimate is unpriced simply because it is riddled with estimation errors. To this end, we form 50 portfolios by sorting stocks on their past market betas, calculate the beta of the overall portfolio, and assign this value to each stock in the corresponding portfolio. This approach is similar to that used in [Fama and French \(1992\)](#) where they assign betas of size-beta portfolios to each stock in the portfolio. Assuming that the errors in estimation of single-stock betas are not perfectly correlated, this should lead to a reduction in estimation noise. Results are presented in [Table 5](#) and show that market beta remains unpriced in the cross-section in all three model specifications. We note that this conclusion does not depend on the number of portfolios used for stock grouping.

Second, instead of using individual-firm data, we form sets of portfolios and conduct our analysis on the portfolio level. We have two requirements for stock grouping: (i) the resulting portfolios should exhibit significant variation in average (expected) returns; (ii) they should exhibit significant variation in characteristics. In the asset pricing literature, studies often use the 5×5 size-book-to-market sorted portfolios; however, not only do we consider these portfolios, but also 5×5 portfolios sorted on size and each of the other control variables included in our study. That is, each month we form 25 size-beta, 25 size-value, 25 size-momentum, 25 size-profitability, and 25 size-investment sorted portfolios, and calculate the corresponding portfolio characteristics. Stocks are equal-weighted and the investment universe and sample period are the same as in our base case. For each of the portfolio sets, we run Fama-MacBeth cross-section regressions just as we do for single stocks. Results are reported in [Table 6](#).

Our conclusions do not change regardless of which set of test portfolios is used: beta is unpriced in the cross-section of stock returns. All other characteristics have the correct sign, although they are not always significant. As a possible reason, we contend that some of our double sorts don't produce enough variation in expected returns (in the case of size-beta), or not enough dispersion in certain portfolio characteristics. Nevertheless, regardless of how we group stocks, our conclusion do not change. In unreported test, we also use industry-sorted portfolios, as suggested by [Lewellen et al. \(2010\)](#), as well portfolios sorted on value and profitability, the two factors that arguably explain the low-risk anomaly, and also find our conclusions unchanged.

Table 6
Fama MacBeth results on portfolio-level.

	Size-beta	Size-value	Size-momentum	Size-profitability	Size-investment
Intercept	0.93 (4.31)	0.86 (2.58)	1.12 (3.59)	0.64 (1.65)	0.57 (1.53)
Beta	-0.12 (-0.43)	-0.10 (-0.25)	-0.30 (-0.75)	0.13 (0.27)	0.24 (0.55)
Size	-0.16 (-1.94)	-0.24 (-3.08)	-0.18 (-2.06)	-0.22 (-2.41)	-0.15 (-1.88)
Value	0.03 (0.44)	0.18 (2.75)	0.14 (2.17)	0.13 (2.45)	0.07 (1.36)
Momentum	0.12 (2.21)	0.07 (1.62)	0.34 (6.49)	0.06 (1.44)	0.05 (1.27)
Profitability	0.05 (0.76)	0.21 (3.77)	0.17 (2.88)	0.18 (3.28)	-0.00 (-0.07)
Investment	-0.10 (-3.57)	-0.09 (-2.83)	-0.06 (-2.10)	-0.04 (-1.60)	-0.15 (-4.85)
R-square (%)	60.12	54.94	58.28	53.89	55.01

This table reports the results of [Fama-MacBeth \(1973\)](#) regressions conducted on the portfolio level. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1963 to December 2015 with share price above \$1, and non-missing values for considered characteristics. Each month we form 25 size-beta, 25 size-value, 25 size-momentum, 25 size-profitability, and 25 size-investment sorted portfolios, and calculate the corresponding portfolio characteristics. Portfolio returns are equal-weighted and the investment universe and sample period are the same as in our base case. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t , value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in $t-1$ and market cap at the end of December of $t-1$; momentum is the total stock return from $t-12$ to $t-2$; profitability is the ratio of operating profits and book equity at the fiscal year ending in $t-1$, and investment is growth of total assets for the fiscal year ending in $t-1$. All variables are winsorized at 1% and 99%, and we normalize all variables except beta. Reported are the average coefficients and t -statistics.

7. Conclusion

We find that exposure to market beta in the cross-section is not rewarded with significantly higher returns, regardless of whether one controls for the additional factors proposed by [Fama and French \(2015, 2016\)](#). At the same time, the constant in the regressions, which ought to be zero according to their asset pricing model, is large and significant. Taken together these results imply that the relation between risk and return in the cross-section is flat instead of positive. We also find that the mispricing is even more pronounced for volatility than for beta. This suggests that the low-volatility anomaly is stronger than the low-beta anomaly, and, given that the two are closely related, that the low-volatility anomaly is the dominant one. We challenge the robustness of our results using different profitability measures that have been discussed in the literature and find that none of them is able to establish a positive relationship between market beta and stock returns. We also find that our results are robust to using characteristics-sorted portfolios instead of individual stocks, which addresses the concern that our findings might be affected by the errors-in-variables problem.

Of course, the results in this paper represent just one attempt at obtaining a positive risk-return relation by controlling for the factors that supposedly explain the low-risk anomaly. The fact that this attempt is unsuccessful does not rule out that portfolios constructed in a different manner do exhibit a clear positive risk-return relation consistent with the predictions of the [Fama and French \(2015\)](#) and [Novy-Marx \(2014\)](#) models. For instance, the market betas or factor exposures used in this paper might not be appropriate, and it is possible that a different methodology would lead to different conclusions. But as long as the data indicates that portfolios with higher risk do not generate higher returns, it is premature to conclude that the low-risk anomaly has been resolved.

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Appendix A

See [Tables A1 and A2](#).

Table A1

Fama-MacBeth results with multivariate beta.

	I	II	III
Intercept	0.71 (3.14)	0.62 (2.89)	0.59 (2.71)
Beta	-0.02 (-0.15)	0.08 (0.79)	0.11 (1.15)
Size		-0.09 (-1.07)	-0.15 (-2.06)
Value		0.26 (4.09)	0.20 (3.64)
Momentum		0.35 (5.36)	0.33 (5.32)
Profitability			0.22 (4.67)
Investment			-0.23 (-9.77)
R-square (%)	0.58	3.58	4.05

This table reports the results of [Fama-MacBeth \(1973\)](#) regressions. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1968 to December 2015 with share price above \$1. Beta is the slope coefficient on the market factor estimated using multivariate regressions of stock excess returns on the five factor model from t-60 to t-1 (min t-24 to t-1). We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t, value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in t-1 and market cap at the end of December of t-1; momentum is the total stock return from t-12 to t-2; profitability is the ratio of operating profits and book equity at the fiscal year ending in t-1, and investment is growth of total assets for the fiscal year ending in t-1. All variables are winsorized at 1% and 99%, and we normalize all variables except beta. Reported are the average coefficients and t-statistics calculated using Newey-West corrected standard errors.

Table A2

Fama-MacBeth in different size market segments.

	Above NYSE median			Below NYSE median			Below NYSE 20 percentile		
	I	II	III	I	II	III	I	II	III
Intercept	0.96 (3.85)	1.04 (4.81)	0.91 (4.36)	1.15 (5.69)	0.92 (4.67)	0.80 (4.22)	1.17 (5.59)	0.83 (4.11)	0.73 (3.70)
Beta	-0.29 (-0.84)	-0.38 (-1.22)	-0.25 (-0.83)	-0.33 (-1.01)	-0.08 (-0.23)	0.05 (0.15)	-0.36 (-1.01)	0.02 (0.06)	0.14 (0.42)
Size		-0.07 (-1.67)	-0.08 (-2.07)		-0.08 (-1.03)	-0.13 (-1.83)		-0.16 (-2.29)	-0.20 (-3.07)
Value		0.08 (1.81)	0.13 (2.33)		0.23 (4.26)	0.16 (3.38)		0.24 (4.31)	0.16 (3.12)
Momentum		0.21 (3.18)	0.20 (3.24)		0.36 (6.31)	0.34 (6.21)		0.41 (7.66)	0.39 (7.52)
Profitability			0.13 (3.51)			0.24 (6.13)			0.26 (5.61)
Investment			-0.10 (-4.37)			-0.25 (-10.85)			-0.27 (-10.37)
R-square (%)	5.06	8.81	9.44	2.28	4.25	4.57	2.07	3.62	3.94

This table reports the results of [Fama-MacBeth \(1973\)](#) regressions in three size market segments: Large, Small and Micro caps. We include all common stocks traded on NYSE, AMEX and NASDAQ exchanges from July 1968 to December 2015 with share price above \$1. Large stocks are those with market capitalization above median capitalization of NYSE listed stocks; Small stocks are those below this threshold, and Micro stocks are those with capitalization below 20th percentile of capitalization of NYSE stocks. Beta is estimated using past five years of daily stock returns – correlations and volatilities are separately estimated over the five (min three) and one (min 120 days) year windows, respectively, and beta is calculated as the ratio of volatilities multiplied by correlation; if daily data is not available, we use past sixty (min 36) and twelve monthly returns to calculate correlations and volatilities, respectively. We shrink beta towards one using shrinkage parameter of 1/3. Size is the natural logarithm of firm's market capitalization at the end of month t, value is the natural logarithm of the ratio of firms book equity for the fiscal year ending in t-1 and market cap at the end of December of t-1; momentum is the total stock return from t-12 to t-2; profitability is the ratio of operating profits and book equity at the fiscal year ending in t-1, and investment is growth of total assets for the fiscal year ending in t-1. All variables are winsorized at 1% and 99%, and we normalize all variables except beta. Reported are the average coefficients and t-statistics calculated using Newey-West corrected standard errors.

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