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Profiling high-frequency equity price movements in directional changes

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Market prices are traditionally sampled in fixed time intervals to form time series. Directional change (DC) is an alternative approach to record price movements. Instead of sampling at fixed intervals, DC is data driven: price changes dictate when a price is recorded. DC provides us with a complementary way to extract information from data. It allows us to observe features that may not be recognized in time series. The argument is that time series and DC-based analysis complement each other. With data sampled at irregular time intervals in DC, however, some of the time series indicators cannot be used in DC-based analysis. For example, returns must be time adjusted and volatility must be amended accordingly. A major objective of this paper is to introduce indicators for profiling markets under DC. We analyse empirical high-frequency data on major equities traded on the UK stock market, and through DC profiling extract information ccomplementary to features observed through time series profiling.

Keywords: Directional change; Time series; Profiling; Return; Volatility; Events

1. Introduction

Market dynamics is traditionally captured through time series, and the observer decides how often data are sampled. High-frequency data however arrive in irregular intervals, and to summarize them in time series, first a representative price is selected for each of the equal intervals of the lower-frequency time series. Alternatively, Guillaume *et al.* (1997) introduced the concept of 'directional changes' (DC) as a new approach to sample data for analysis. This concept is formally defined in Tsang (2010). In DC, sample points are data driven—the observer lets the data determine when to sample the market. The observer decides the threshold of price changes that he/she considers significant, which could be 5, 1 or 0.5%. The idea is that different observers should see the market in the scale that he/she finds useful—an idea inspired by Mandelbrot and Hudson (2004). The market is next partitioned into alternating uptrends and downtrends. A change from a downtrend (uptrend) to an uptrend (downtrend) is recorded when the market price changes direction by the predefined threshold.

With most research based on time series, why should DC be introduced as a new way to analyse data? We argue that the DC approach provides an insight into data, which may not be revealed by time series. In time series analysis, the researcher determines how often data are sampled—in other words, determines the time scale of the *x*-axis and observes price changes on the *y*-axis. In DC-based analysis, the researcher determines how large a change is considered significant—in other words, determines the price scale of the *y*-axis and let the data tell when to record prices. With data sampled at irregular time intervals, most statistical analyses are not applicable to DC-based sampling. This paper defines and introduces indicators for DC-based sampling. As a first step of a new approach, we limit our study to descriptive statistics in this paper. Time series-based and DC-based analyses look at data from two different angles, and provide different perspectives of the same data. That is, the motivation behind this work. The analogy is: when using two eyes, we get a stereo image of an object. We are able to see features that we could not see with one eye.

One of the most significant insights into data under DC-based sampling is Glattfelder *et al.* (2011). They empirically

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discovered 12 scaling laws related to foreign exchange data series across thirteen currency exchange rates, based on the DC theory. Kablan and Ng (2011) developed a new method for capturing volatility using the DC-event approach. Aloud *et al.* (2012a) pointed out that the length of the price-curve coastline defined by DC events is longer than the coastline of price changes based on time series. Bisig *et al.* (2012) proposed useful measures of price movements under DC. Qi (2012) studied value-at-risk under the DC framework. Masry *et al.* (2013) and Masry (2013) studied trading patterns in FX markets. All these advances provide insights into data that are not observed in time series analysis.

A major objective of this paper is to introduce new indicators to support DC-based analysis: researchers have developed useful indicators in time series analysis—e.g. return and volatility. With data sampled at irregular intervals, time series indicators are not applicable to DC-based summaries. Returns must be time adjusted and volatility must be adjusted accordingly. This paper proposes new indicators for profiling markets under DC. The empirical analysis demonstrates how DC information is extracted. We demonstrate that information extracted through DC-based analysis is complementary to information extracted from time series.

The remainder of the paper is organized as follows. Section 2 explains the concept of DC and its component events. Section 3 introduces indicators for extracting information from data under the DC framework. Section 4 compares time series analysis and DC-based analysis to show the different perspectives they provide. Section 5 offers empirical profiling of high-frequency price movements under DC, for UK-traded blue-chip equities representing four major industries. It demonstrates how DC-based profiles help understanding market dynamics. The paper is concluded in Section 6.

2. Directional changes

2.1. DC events

DC is an alternative way to summarize price changes (Guillaume *et al.* 1997). The basic idea is to partition the market into alternating Uptrends and Downtrends. An uptrend terminates when a Downturn DC Event takes place. Similarly, a downtrend terminates when an Upturn DC Event takes place. A Downturn (Upturn) DC Event is an event at which price drops (rises) by a threshold from its highest (lowest) price in the previous trend. Here, the threshold is a percentage that the observer considers significant. One observer may consider 0.05% a significant change, while another observer may consider 5% is significant. Observers who use different thresholds will observe different DC events and trends.

As a Downturn DC Event defines the beginning of a new downtrend, at the end of the Downturn DC Event, price would have dropped by the specified threshold from the highest price in the last (as well as current) trend. That highest point (or the lowest point in the case of upturn DC) is called an *Extreme Point*. It must be emphasized that the extreme point was only confirmed to be the extreme point in hindsight, when DC is confirmed (i.e. when price has changed by the threshold or more from the extreme point).

A downtrend continues until the next upturn DC event is observed, which defines the lowest price in the current downtrend and start the next uptrend. We refer to the price change from the end of the Downturn DC Event to the lowest price in the current trend an *Overshoot Event*. In other words, each trend comprises a DC Event and an Overshoot Event, as shown in figure 1. A formal definition of DC Events and Overshoot Events can be found in (Tsang 2010).

During a downtrend, the lowest price within the current trend, P_l, is continuously updated to the minimum of P_t (the current market price) and P_l (the last low price). Similarly, during an uptrend, the highest price within the trend, P_h, is continuously updated to the maximum of P_t (the current market price) and P_h (the Last High price) (Tsang 2010). At the beginning of the sequence, when we do not know whether we are in an uptrend or downtrend, the last high price P_h and last low price P_l are set to the initial market price at the beginning of the summarized period (Tsang 2010).

2.2. A more formal definition

In this subsection, we shall provide a formal definition of the above. A downturn DC event is an event when the absolute price change between the current market price P_t and the last high price P_h is lower than a fixed threshold (a percentage) θ:

$$P_t \leq P_h \times (1 - \theta) \tag{1}$$

A downturn DC event starts a downtrend. A downtrend is terminated by an upturn DC event, which is an event when the absolute price change between the current market price P_t and the last low price P_l is higher than a fixed threshold θ:

$$P_t \geq P_l \times (1 + \theta) \tag{2}$$

The starting point of an upturn DC event is an upturn point where the price last troughed at P_l. The end of an upturn DC event is an upturn confirmation DC point where the price has risen from the last upturn point by the threshold θ.

A downturn DC event is followed by a downward overshoot event that is ended by the next upturn DC event, which is itself followed by an upward overshoot event that is ended by the next DC downturn event (Tsang 2010) (see figure 1). The overshoot event (OS) represents the price movement beyond the DC event.

Under the DC framework, price movement is summarized in a four-event cycle:

$$\begin{aligned} &\ldots \rightarrow DownturnDCEvent \rightarrow \\ &DownwardOvershootEvent \rightarrow \\ &UpturnDCEvent \rightarrow \\ &UpwardOvershootEvent \rightarrow \\ &DownturnDCEvent \rightarrow \ldots \end{aligned}$$

Figure 1. Directional Changes (DCs), an example: DCs observed in EUR/USD with a threshold of 3%, three directional change events (red) and two overshoot events (green) are shown here; The extreme points and directional change confirmation points are shown in little circles and squares (solid black) respectively. (This figure is re-used with the permission of the author of [Tsang 2010].)

A *total price movement* (TM) is constituted by a downturn event and a subsequent downward overshoot event, or an upturn event and a subsequent upward overshoot event (Glattfelder *et al.* 2011), as illustrated in figure 1.

3. Useful indicators in DC summarizing

DC is a new way of summarizing price changes. In order to analyse price dynamics, we need to extract useful information from DC summaries. In Tsang *et al*. 2015, we proposed indicators that could be useful for extracting information. We further develop and define a set of indicators here—they will be described next in sections 3.1 to 3.5. These indicators provide us with a vocabulary to describe profiles of price movements in markets under the DC framework.

3.1. Number of DC events

N_{DC} is the total number of DC events that happened over the profiled period. It measures the frequency, or volatility of DC events. Based on the same threshold, a time period that has higher N_{DC} value is more volatile than another time period of the same length. By recording the N_{DC} within the profiled period, DC provides us an alternative way to measure the volatility of market price movements.

3.2. Time for completion of a trend

DC is defined based on events, so it uses intrinsic, as opposed to physical, time (Glattfelder *et al.* 2011). However, that does not mean that it ignores physical time. The amount of physical time that a trend takes to complete is a significant piece of information. We define an indicator T as the physical time that it takes between the extreme points that begin and end a trend (figure 2).

3.3. Total price movements value at extreme points

Total price movements value at extreme points ($\mathrm{TMV_{EXT}}$) measure the price distance between the extreme points that begin and end a trend, normalized by θ, which is the threshold used for generating the DC summary. It measures the maximum possible profit for each trend. $\mathrm{TMV_{EXT}}$ is defined by:

$$TMV_{\mathrm{EXT}_i} = \frac{P_{\mathrm{EXT}\_i+1} - P_{\mathrm{EXT}\_i}}{P_{\mathrm{EXT}_i} \times \theta} \tag{3}$$

Here P_{EXT_i} represents the price at the ith DC extreme point, P_{EXT_i+1} represents the price at the $(i+1)$-th DC extreme point, θ is the threshold used (figure 2).

Figure 2. Example: T, P_{EXT} and θ in EUR/USD, θ = 3%, P_{EXT} is the price at directional change extreme point (solid black squares), T is the time that it takes between two consecutive directional change extreme points. (This figure is re-used with the permission of the author of [Tsang 2010].)

3.4. Time-independent coastline

Since $\mathrm{TMV}_{\mathrm{EXT}}$ represents the maximum possible profit of each TM event, we define the length of the price-curve coastline under DC (θ) as the sum of all absolute value of $\mathrm{TMV}_{\mathrm{EXT}}$ over the profiling period:

$$C_{\mathrm{DC}} = \sum_{i=1}^{N(\theta)} |\mathrm{TMV}_{\mathrm{EXT}_i}| \tag{4}$$

Here, θ is the threshold (in %), N (θ) is the total number of DC events over the profiling period under θ and $\mathrm{TMV}_{\mathrm{EXT}_i}$ is the Total Price Movements Value at each DC extreme point.

The calculation of C_{DC} only pays attention to price changes; time is ignored. It shows us the maximum possible profits available from the profiled period.

3.5. Time-adjusted return of DC

We define time-adjusted return of DC (R_{DC}) to measure the return in each upturn or downturn event, i.e. the ratio between each TMV and time interval (T). We define R_{DC} as:

$$R_{\mathrm{DC}} = \frac{|\mathrm{TMV}_{\mathrm{EXT}}| \times \theta}{T} \tag{5}$$

Here, $\mathrm{TMV}_{\mathrm{EXT}}$ is total price movements value at extreme points and T is the time interval between each EXT, θ is the threshold used. R_{DC} measures the percentage of price rising/dropping per time unit.

One could define a coastline based on time-adjusted returns R_{DC}. For example, one could take the accumulative (unsigned) returns to represent coastline. However, its equivalence in time series is unfamiliar to researchers. Therefore, while it is potentially useful, we leave this option open at this stage.c

4. Contrast between time series and DC-based analyses

In this section, we shall explain that DC adds a different angle to looking at price dynamics. It complements the angle seen from time series.

The main difference time series and DC is that the former fixes time, and focuses on changes in prices, whereas the latter fixes the threshold on price changes, and focuses on *when* to sample. As a result, they observe the same data from different angles.

4.1. Return

Unlike time series analysis, which observes price changes in fixed periods, DC lets price changes decide when to sample prices. In DC, return is calculated for each trend. As each trend may take different amount of time to complete they need to be time-adjusted before they can be compared or aggregated. Time-adjusted returns in DC capture returns of DC events.

Time-adjusted returns in the trends should not be directly compared with returns in time series. This is because DC fixes the threshold, and focus on when to take sample. Therefore, by stipulation, returns in each trend would be at least as big as the threshold. So time-adjusted returns observed in DC tend to be bigger than returns observed in time series. Direct comparison between the two returns observed could therefore be misleading. They should be seen as two different angles in looking at price changes.

4.2. Coastlines

The coastline measures the potential profits that one could make in a market over a period. In time series, the coastline is measured by the accumulation of returns over each period. Its equivalence in DC could be measured by C_{DC} (maximum possible returns over the profiled period).

By definition, DC captures extreme points in trends. Therefore, DC coastlines are at least as long as time series coastlines, often longer (Aloud *et al.* 2012b). That means there is more potential to gain from forecasting and trading in DC than in time series.

4.3. Volatility

In time series, volatility is often measured by variance on returns. DC provides different perspectives on volatility:

(1) T: The time that it takes to complete a trend is one way to look at volatility. Everything being equal, the longer a market remains in the same trend, the less volatile one may consider the market to be.
(2) N_{DC}: If two profiles cover two time periods of the same length, then the number of DCs is a measure of volatility. In that case, N_{DC} is inversely proportional to T. A higher N_{DC} value suggests a higher frequency of DCs in a market, which suggests higher volatility.
(3) $\mathrm{TMV}_{\mathrm{EXT}}$: While the number of DCs measures the frequency of DCs, $\mathrm{TMV}_{\mathrm{EXT}}$ measures the magnitude of price change in each trend. Everything being equal, the greater the magnitude of price changes per trend, the more volatile one may consider the market to be.

Thus, T and N_{DC} reflect the frequency of DCs and $\mathrm{TMV}_{\mathrm{EXT}}$ reflects the magnitude of the trends. Together, they provide additional angles to look at volatility, which are not captured by variance in returns in time series.

4.4. Statistical observations: power laws

The introduction of the overshoot concept allows (Glattfelder *et al.* 2011) to discover 12 power laws in foreign exchange price movements. They found that on average, if one observes a DC with threshold θ, then the scale of the overshoot is also θ. This means on average, the price change in a DC event equals the price change in an overshoot event. They also found that, on average, the time that it takes to complete an overshoot is twice the time that it takes to confirm a DC. In other words, the average time that DC takes is one-third of T in each trend, and the average time that overshoot takes is two-third of T (where T is the time that the whole trend takes to complete). These regularities were only observable when one adopts the notion of DC and overshoots.

4.5. Summary on contrast between time series and DC

DC is still in its infancy. It is still limited in what we can use DC indicators to profile market dynamics. But useful information can be gained from the research so far. This has been explained in the above subsections. Here is a summary.

The returns that time series look at are returns over fixed period of time, while the returns that DC looks at (R_{DC}) are returns over DC events. Given the same number of data points, DC coastlines are often longer than time series coastlines for the same period, because by definition, DC captures the extreme points (Aloud *et al.* 2012b).

The three indicators (N_{DC}, $\mathrm{TMV}_{\mathrm{EXT}}$ and T) introduced in DC provide three additional measures of volatility. The introduction of overshoot enabled (Glattfelder *et al.* 2011) to observe power laws in the foreign exchange market. Table 1 summarizes the indicators discussed so far.

5. Profiling high-frequency price movements in equity markets

In this section, we explain how DC profiling could help us to observe price movements in four companies. These four companies were chosen to represent four sectors in the FTSE 100 Index, which are shown in table 2.

5.1. Profiling four blue-chip companies

We used tick transaction prices in two time periods, September 2014 and February 2015, to profile each of the four equities. Since the value of threshold will affect the results of profiles, we applied the same threshold (1%) to these four equities. In DC profiles, we calculated the DC indicators which we presented in Section 3. The profiles also included some simple statistical analysis of the indicators, such as the median, mean and standard value of the indicators. We developed a program based on Matlab platform for producing DC profiles. Full output of the program for the analysis below will be published online†.

†Full output reported in this paper will be accessible from http://www.bracil.net/finance/DirectionalChanges/

Table 1. Contrast between time series indicators and DC-based indicators.

	Time series indicators	Directional change indicators
Return: Different angles on returns	Returns measured in each (fixed) period	Percentage of price changes measured in each trend. Since they are sampled in irregular times, this percentage must be time adjusted for comparison
Coastlines: DC coastlines are often longer than time series coastlines (Aloud *et al.* 2012b)	Accumulation of Returns	C_{DC}: maximum possible returns over the profiled period
Volatility: Time series and DC provide different perspectives on volatility	Variance on Returns	N_{DC}: measures the frequency of DCs TMV_{EXT} measures the scale of price changes T measures the time that it takes to complete a trend
Statistical observations: Different observations made possible by different indicators	Many observations, such as fat tails and volatility clustering	Power law found on overshoot event, which is made possible by the introduction of overshoot value at extreme points (Glattfelder *et al.* 2011)

Table 2. Four companies and the four sectors that they represent.

Key	Company name	Sector
AZN	AstraZeneca PLC	Healthcare
BT	BT Group PLC	Technology
HSBA	HSBC Holdings PLC	Financial
MKS	Marks & Spencer Group PLC	Services

5.2. *Profiles of the companies*

In table 3, we included the values of six significant indicators. The final row of table 3 (C_{DC}) and chart 1 show the maximum potential profit for each company during the time periods.

The coastline of AZN is 171.84% in September 2014 and 304.69% in February 2015. Compared with other three companies in the same time period, AZN always has longest coastline in its DC profiles. That means one has the potential to generate more profit in AZN than in other three companies. Besides, among these four companies, only the coastline of MKS drops from September 2014 to February 2015—it drops from 132.38 to 75.27%. The other three companies' coastlines all rise substantially. For AZN, BT and HSBA, February 2015 presented traders with more profit potentials.

Volatility can be reflected by two indicators in DC profiles: the *frequency* of DCs and the *magnitude* of price changes in each trend. The frequency of DCs is reflected by the time that it takes to complete a trend, T. The magnitude of price changes is reflected by the total price movements in each trend, TMV.

T is the physical time that a trend takes to complete. Everything being equal, the longer time each trend takes to complete, the less volatile one may consider a market to be. In chart 2, we have reported the median T and its standard deviation.

Chart 2 shows that the median value of T for HSBA in February 2015 is 88.44 min, which is over four times of the median value of T for AZN in the same time period. HSBA has the second highest value of median T among the four companies during in February 2015. Considering market volatility in T, we conclude that HSBA in February 2015 has small risks in trading comparatively. As far as T is concerned, HSBA has the lowest volatility in September 2014, and MKS has the lowest volatility in February 2015 among the four companies studied.

T only tells half of the story about volatility. TMV_{EXT} tells the other. TMV_{EXT} measures the scale of price changes. Everything being equal, the bigger the scale of changes, the more volatile one may consider the market to be. In chart 3, we reported the median value of TMV_{EXT} as well as their standard deviations (the raw data are shown in table 3).

Chart 3 shows that all the median TMV_{EXT} values are comparable to each other in the range between 1.5 and 1.76, apart from MKS in September 2014, which has a value of 2.14. This suggests that, as far as TMV_{EXT} is

Table 3. Summarized DC Profiles with a threshold of 1% on AZN, BT, HSBA and MKS with second-by-second transaction prices, September 2014 and February 2015.

Time	September 2014				February 2015			
Company	AZN	BT	HSBA	MKS	AZN	BT	HSBA	MKS
N_{DC}	82	37	43	57	138	126	81	46
Median TMV_{EXT}	1.74	1.53	1.66	2.14	1.76	1.67	1.74	1.5
Standard deviation of TMV_{EXT}	1.26	0.88	0.7	1.23	1.19	1.2	1.33	0.7
Median T (minutes)	79.41	115.56	145.2	78.79	19.7	32.46	88.44	112.75
Standard deviation of T	178.77	391.37	381.85	290.07	129.61	148.9	162.61	307.93
C_{DC} (%)	171.84	65.28	80.84	132.28	304.69	261.72	176.78	75.27

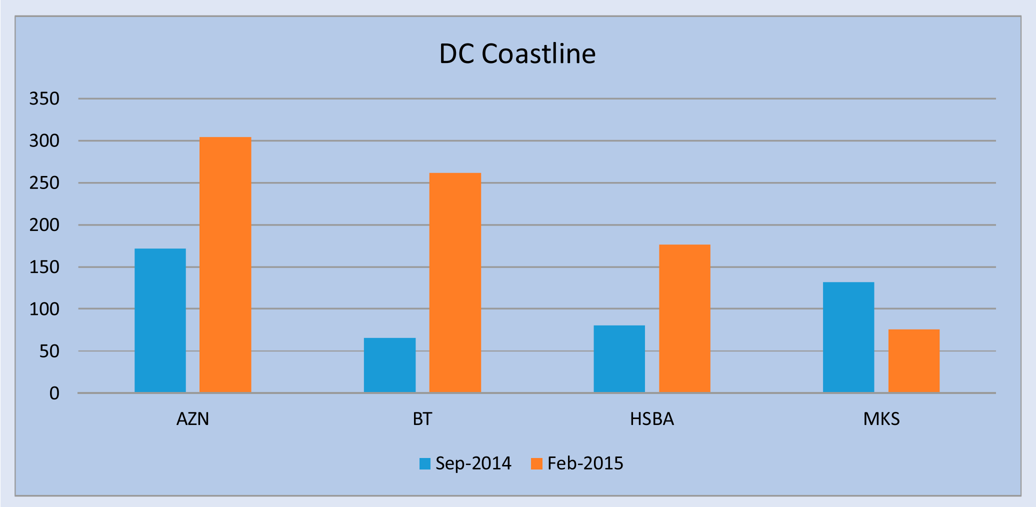

Chart 1. The coastline (C_{DC}) from DC profiles for AZN, BT, HSBA and MKS in two different time periods. The blue column is for September 2014 and the orange column is for February 2015.

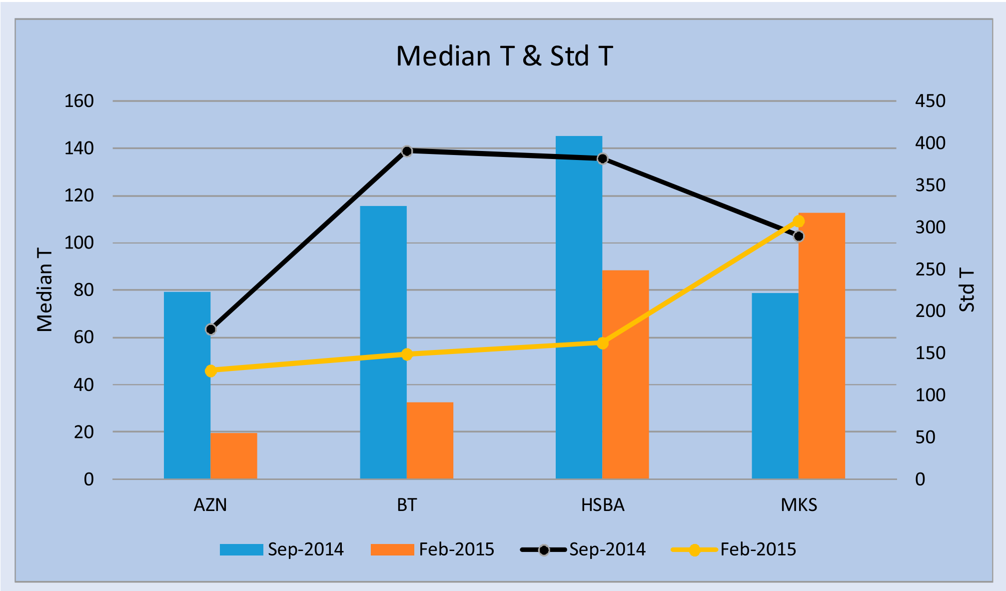

Chart 2. The median and standard deviation of T from DC profiles for AZN, BT, HSBA and MKS in two different time periods. Blue and orange column represent median T. Black and yellow line stand for standard deviation of T.

concerned, the profiled periods (apart from MKS in September 2014) have similar risks. That means we can rely on the other indicator, T, to measure volatility of the profiled periods.

What we have shown in this Section is that the coastline, frequency of DCs and magnitude of DCs enrich our analysis in studying return and risk in markets. As mentioned above, the new indicators introduced allow researchers to statistically observe the market, such as the power law (Glattfelder *et al*. 2011).

5.3. *Contrast between time series and DC-based analyses*

Starting with the same data, one can study price changes with both time series and DC. They extract different information from the same data—in this case, tick data. Therefore, what they observe should be consistent with each other. However, they provide us with different perspectives. In this section, we use AZN February 2015 data to illustrate our point.

We used a threshold of 1% to generate a DC profile from the tick-to-tick data for this period. This gives us 138 trends. We sample 138 points at fixed intervals within this period to form the time series. Then, we extract the information from both time series and DC-based analyses. In table 4, we summarize some of the indicators which could be extracted by both approaches.

As explained in Section 4.1, 'returns' from time series and 'time-adjusted returns' from DC-based analysis generate are not directly comparable with each other because data are sampled in irregular intervals in DC profiling. Table 4

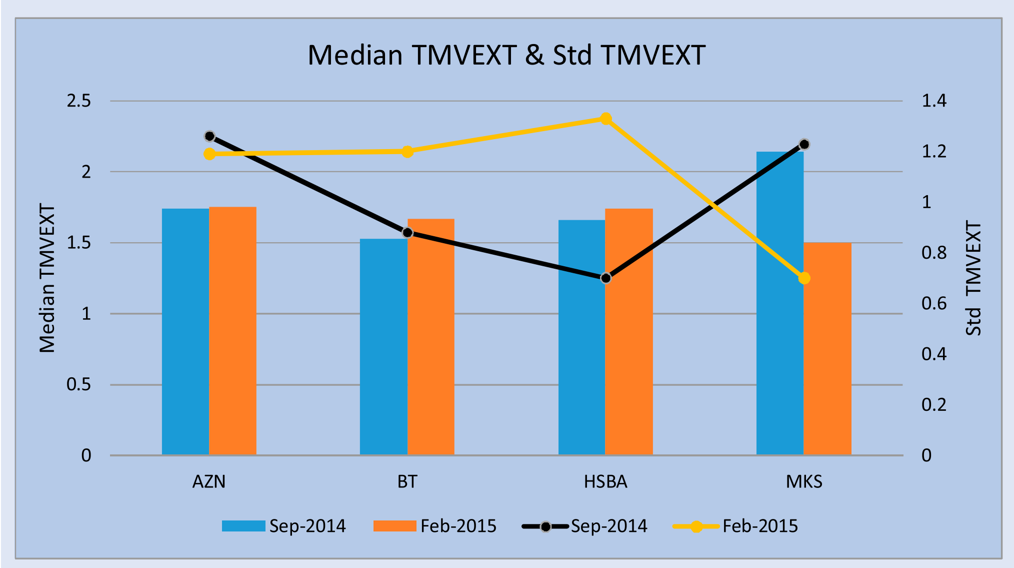

Chart 3. The median TMV_{EXT} and standard deviation of TMV_{EXT} from DC profiles for AZN, BT, HSBA and MKS in two different time periods. Blue and orange column represent median TMV_{EXT}. Black and yellow line stand for standard deviation of TMV_{EXT}.

Table 4. Contrast between time series indicators and DC-based indicators (with threshold = 1%) in AZN, February 2015; DC was summarized with a threshold of 1%, which resulted in 138 trends observed; to enable fair comparison, 138 data points were sampled from the high-frequency data in the same period to form the time series.

	Time series indicators	Directional change indicators
Return: Time series and DC provide different angles on returns	The mean and median absolute returns were 0.36 and 0.27%, respectively	The mean and median absolute percentage price differences from beginning to end of trend were 2.22 and 1.76%, respectively
Coastlines: It measures the maximum possible profit available	Coastline as measured by Sum of absolute returns is 49%	Coastline as measured by C_{DC} is 305%
Volatility: Time series and DC provide different perspectives on volatility	Volatility as measured by standard deviation of returns was 0.51%	Volatility as measured by median time (T) to complete a trend was 19.7 min (N_{DC} was 138); median return (TMVEXT) was 1.76%

shows that mean return for time series was 0.36%, while mean time-adjusted return for DC profiling is 2.22%. The substantial difference is in fact partly explained by the way that DC samples its data. DC-based analysis used a threshold of 1% to generate results in table 4. Therefore, every trend would see at least a 1% change (the rest is overshoot). The time-adjusted return in DC profiles tells us the magnitude of overshoot.

Coastline is a useful indicator in DC profiles. By stipulation, DC captures extreme points in trends. Therefore, DC coastlines should be at least as long as time series coastlines; in fact, they are often much longer (Aloud *et al.* 2012b). As table 4 shows, coastline in DC is 305% in AZN, February 2015, which is over six times bigger than the coastline in time series analysis in the same time period (49%). So, everything being equal, there is more potential to gain from forecasting and trading in DC than in time series. This motivates us to develop DC indicators to forecast DCs points.

In Section 4.3, we explained that there are two ways to measure volatility in DC profiling. Table 4 shows that the standard deviation of returns for time series was 0.51%. Volatility in DC profiling is measured by two dimensions: (a) the frequency of DCs is measured by median time to complete a trend (T), which was 19.7 min. The smaller this number, the more frequent that direction changes; (b) the magnitude of price changes in the trends is measured by median return (TMV_{EXT}), for which 1.76% was recorded. All three numbers, 0.51% (for time series), 19.7 min and 1.76% (for DC profiling), are useful for assessing the volatility of the profiled period. None of them can be replaced by the others.

6. Conclusion and further research

In this paper, we have introduced DC as an alternative way to summarize and profile price changes. DC is different from traditional time series, and therefore provides an additional angle for capturing and analysing market changes. It complements traditional time series analysis. We have introduced a set of indicators for capturing and extracting information from data. For example, the coastline captures the potential profit of trading within a time period under a selected threshold. DC profiling also adds new ways to look at volatility: N_{DC}, TMV_{EXT} and T. The former two measure the frequency of DCs and the latter the magnitude of overshoots. Such new measures (though not in the vocabulary introduced in this paper) enabled (Glattfelder *et al.* 2011) to discover power laws in market data, which cannot be discovered otherwise. We have demonstrated how DC profiles could be used to summarize price changes in high-frequency data in equity markets. Through these indicators, we can discover useful information that cannot be captured in time series analysis.

With the indicators presented in this paper, we aim to initiate further research under the DC framework. One direction is to define new indicators for DC profiles. More indicators will help us capture more useful information about data. Another direction is to combine DC analysis with traditional time series analysis to explore synergies.

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