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Changing Time Scale for Short-term Forecasting in Financial Markets

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ABSTRACT

A forecasting model based on high-frequency market makers' quotes of financial instruments is presented. The statistical behaviour of these time series leads to discussion of the appropriate time scale for forecasting. We introduce variable time scales in a general way and define the new concept of intrinsic time. The latter reflects better the actual trading activity. Changing time scale means forecasting in two steps, first an intrinsic time forecast against physical time, then a price forecast against intrinsic time. The forecasting model consists, for both steps, of a linear combination of non-linear price-based indicators. The indicator weights are continuously re-optimized through a modified linear regression on a moving sample of past prices. The out-of-sample performance of this algorithm is reported on a set of important FX rates and interest rates over many years. It is remarkably consistent. Results for short horizons as well as techniques to measure this performance are discussed.

KEY WORDS forecasting; time scales; high-frequency data; intrinsic time

INTRODUCTION

The subject of forecasting in the foreign exchange market is highly controversial. The majority of researchers in the field do not believe in the possibility of forecasting price movements. Their scepticism has been reinforced by many findings, in particular by Meese and Rogoff's (1983) work. However, the controversy continues: in a single issue of the *Journal of International Money and Finance* one can find three consecutive papers on this subject, each coming to a different conclusion. In the first case (Finn, 1986), the author reports that the models she studied failed to outperform the random walk, while in the following paper (Somanath, 1986) one reads '... the performance of structural models [in] the post Meese and Rogoff period is sufficient to establish the dominance of structural models over the random walk'. In the third (Hakkio, 1986), the author concludes '... although the hypothesis that the exchange rate follows a random walk cannot be rejected, not much weight should be put on this conclusion'.

The least one can say is that the subject is far from exhausted and the debate is still open. Selecting a few other examples from the abundant literature, we find results in the direction of

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non-forecastability reported in Diebold and Nason (1990) but successful forecasting models reported in Wolff (1988), Schinasi and Swamy (1989) and Swamy and Schinasi (1989). Looking at the long-term picture, we see an evolution of the subject. Twenty years ago the consensus was that no forecast was at all possible (see, for instance, Fama's (1970) famous paper on market efficiency. Now, with the development of non-linear models such as ARCH (Autoregressive Conditional Heteroscedasticity) processes, the consensus has shifted to acknowledging the predictability of volatility (see, for instance, Diebold, 1988; Taylor, 1986; Baillie and McMahon, 1989; Bollerslev, Chou and Kroner, 1992). Clearly, we have our own opinion on this problem, but the purpose of this paper is not to prove a theory; it is more modestly to explain our work on this question and to discuss some of the results we have obtained.

There are basically two approaches to the problem of forecasting exchange rates. The first builds upon common macro-economic models to test with varying stringencies market efficiency (see, Fama, 1970, 1991) or to explore purchasing power parity, the modelling of risk premia, and so on. (For a good review of these different models see, for instance, Baillie and McMahon, 1989, or MacDonald and Taylor, 1992). Following Meese and Rogoff (1983), who performed the first serious out-of-sample tests of these models, we term them structural models.

The second approach, often called the time series model, bases forecasts on a variety of prefiltering techniques and manipulations of the time series itself. This type of model can be univariate; that is, with only one time series to forecast itself, or multivariate, with a number of time series used for forecasting. (The classic book by Granger and Newbold (1977) is a good introduction to these types of models. There are many other books treating these methods, but we would only like to mention here that some new developments for non-linear models are presented in Priestley, 1989).

We take this second approach, applying univariate time series analysis in the sense that only past prices are used to compute forecasts. Two reasons explain our choice of this approach. First, the absence of any theory for the short-term movements of FX rates makes structural models irrelevant for these horizons. Second, the availability of high-density data leads us to believe that we can capture many of the market effects that are relevant to the short-term movements.

The Olsen & Associates forecasting models run in real-time and comprise part of the Olsen & Associates Information System (OIS). They take into account every price change in the market to provide Foreign Exchange (FX) Rate forecasts for most of the US dollar (USD, Standard abbreviations of the International Organization for Standardization (ISO, code 4217)) rates and for many cross rates from a few hours to a few weeks. These models are based on extensive statistical studies of our database of intra-day market maker quotes that we have collected since the beginning of 1986 from information vendors such as Reuters, Knight Ridder or Telerate (Müller *et al.*, 1990; Dacorogna *et al.*, 1993). The importance of intra-day prices lies both in the enhanced statistical significance gained from the extremely large number of observations and in the increased ability to analyse finer details of the behaviour of different market participants.

In the next section of this paper we show some statistical properties of FX-rates in the intra-day region. These properties are important in understanding why certain choices have been made when building our models. Moreover, they show that far more subtle forces are at work than could be described by an efficient market theory that results in a pure random walk model for the price movements.

In the third section we discuss the use of an optimal time scale in computing our forecasting model and show that one can account for some of the effects described in the second section through time transformations. In the fourth section, the forecasting model itself is presented. The fifth section is devoted to discussing the forecasting model results for FX rates and interbank interest rates and we draw our conclusions in the sixth section.

Because the model contains successive layers, we summarize in an appendix the different steps needed to obtain a forecast from this approach.

SOME STATISTICAL PROPERTIES OF FX- RATE TIME SERIES

A leptokurtic and non-stable price change distribution

In two recent papers (Müller *et al.*, 1990; Dacorogna *et al.*, 1993) we reported a set of empirical results that we would like to summarize here. We shall use the same notation as in these two papers. In particular, we denote x as the logarithmic middle price between bid and ask and thus all results we present here are computed for the logarithm of FX rates. The first property is that the shape of the price change distribution is *not stable* under time aggregation, as shown in Figure 1. Three distributions are shown for the USD/DEM rate for 30 minutes, 1 day and 1 week time intervals over a period of six years from 5 May 1986 to 5 May 1992. The cumulative frequency of price changes is plotted on the scale of the cumulative Gaussian probability function. The abscissa is in units of the mean absolute value of each time interval. Gaussian distributions would have the form of a straight line on such a scale. Our analysis shows, however, that the shorter the horizon, the more leptokurtic is the distribution. Only for weekly intervals does the distribution look almost Gaussian. Let us mention, without entering a detailed discussion, that unstable distributions of FX rate changes were also found by other authors studying daily data. McFarland, Petit and Sung (1982) and Boothe and Glassman (1987) suggested that these distributions are formed by reactions to different information flows.

Since Mandelbrot (1963) first worked on cotton prices it has become largely accepted that speculative prices produce *leptokurtic distributions* of price changes. This is also true for FX rates and, as reported above, we found this to be the case for a large range of time intervals. There is even a concern that the variance of the process may be infinite. In fact, our recent studies of the tails of the distribution (Pictet *et al.*, 1992) show that, although the second moment of the distribution exists (i.e. the variance is finite) for all time intervals including the intra-day region, the fourth moment does not converge for short-term intervals. Such behaviour implies difficulties when using squared deviations in least square fitting methods and also when measuring forecasting errors. These problems are treated later in this paper.

A scaling law for absolute price changes

We have shown empirical evidence of a *scaling law* that is followed by the absolute price changes (Müller *et al.*, 1990). The mean absolute change of the logarithmic middle price over a time interval is related to the size of this interval, Δt:

$$\overline{|x(t) - x(t - \Delta t)|} \equiv \overline{|\Delta x|} = \left(\frac{\Delta t}{\Delta T}\right)^D \tag{1}$$

where the bar over $|\Delta x|$ indicates the average over a long sample interval (six years in our case) and ΔT is an empirical time constant depending on the FX rate. The drift exponent $D = 1/E$ is about 0.59 for the major FX rates we have studied, whereas a pure Gaussian random walk model would imply $D = 0.5$ with the exponent $E = 2.0$. A similar law has been found for the root mean square (RMS) of the price change and for the interquartile ranges but with different

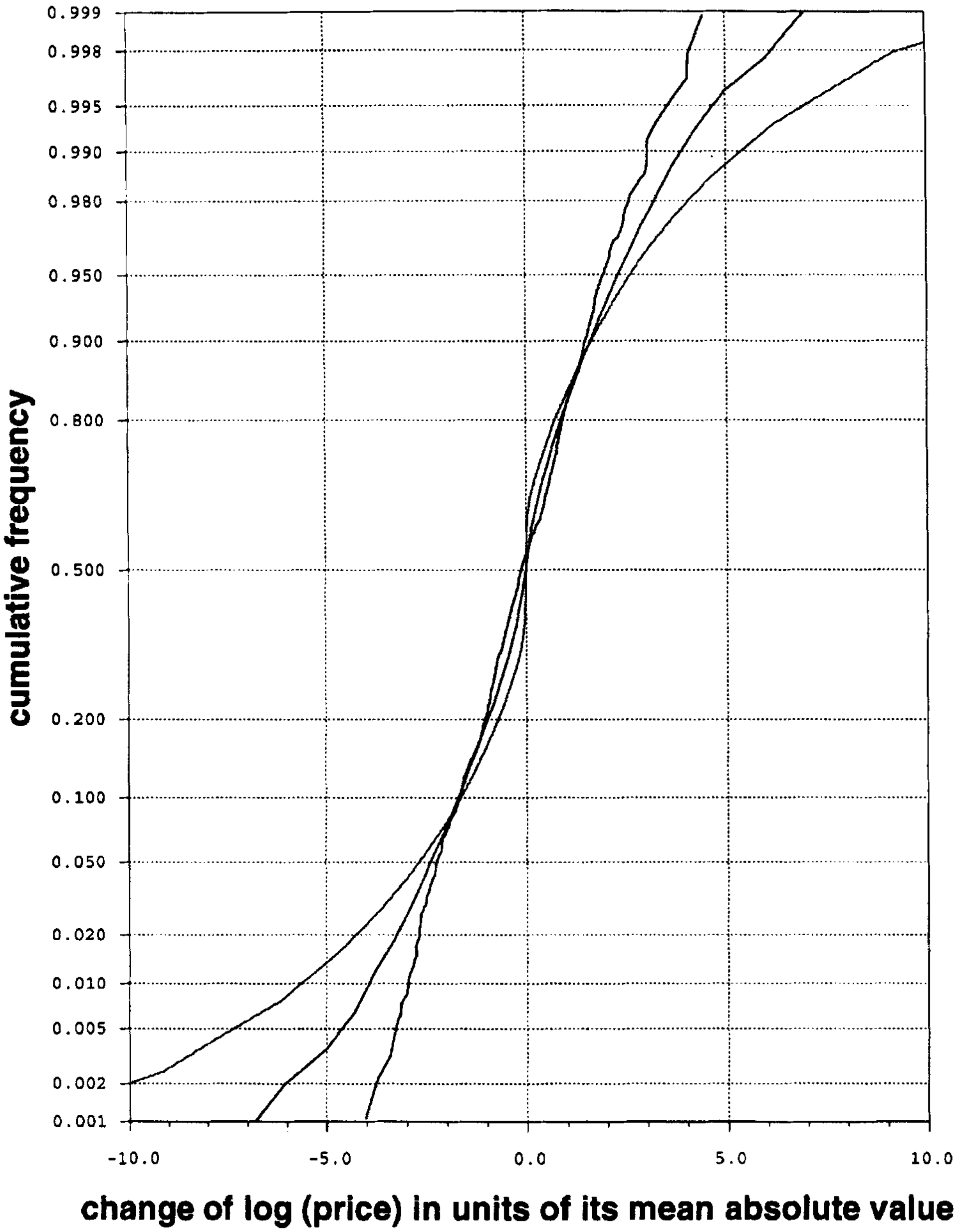

Figure 1. Cumulated distributions of USD/DEM rate for 30-minute intervals (the most leptokurtic one), for 1-day intervals and for 1-week intervals (the closest to a straight line). For 30-minute intervals $\overline{|\Delta x|} = 0.45\ 10^{-3}$. For 1-day intervals $\overline{|\Delta x|} = 4.14\ 10^{-3}$. For 1-week intervals $\overline{|\Delta x|} = 12.4\ 10^{-3}$

exponents: an exponent of ≈ 0.52 for the RMS and of ≈ 0.7 for the interquartile ranges. These distinct differences for the exponent can only be explained by varying distribution forms for the different time intervals.

The scaling law expressed in equation (1) holds for all time series studied and for a wide variety of time intervals ranging from 10 minutes to more than a year. An example of this law for the USD/DEM is shown in Figure 2 on a double logarithmic scale that makes this relationship appear linear.

Seasonal and conditional heteroscedasticity

The behaviour of a time series is called *seasonal* if it shows a periodic structure in addition to less regular movements. In Müller *et al.* (1990), we demonstrated daily and weekly *seasonal*

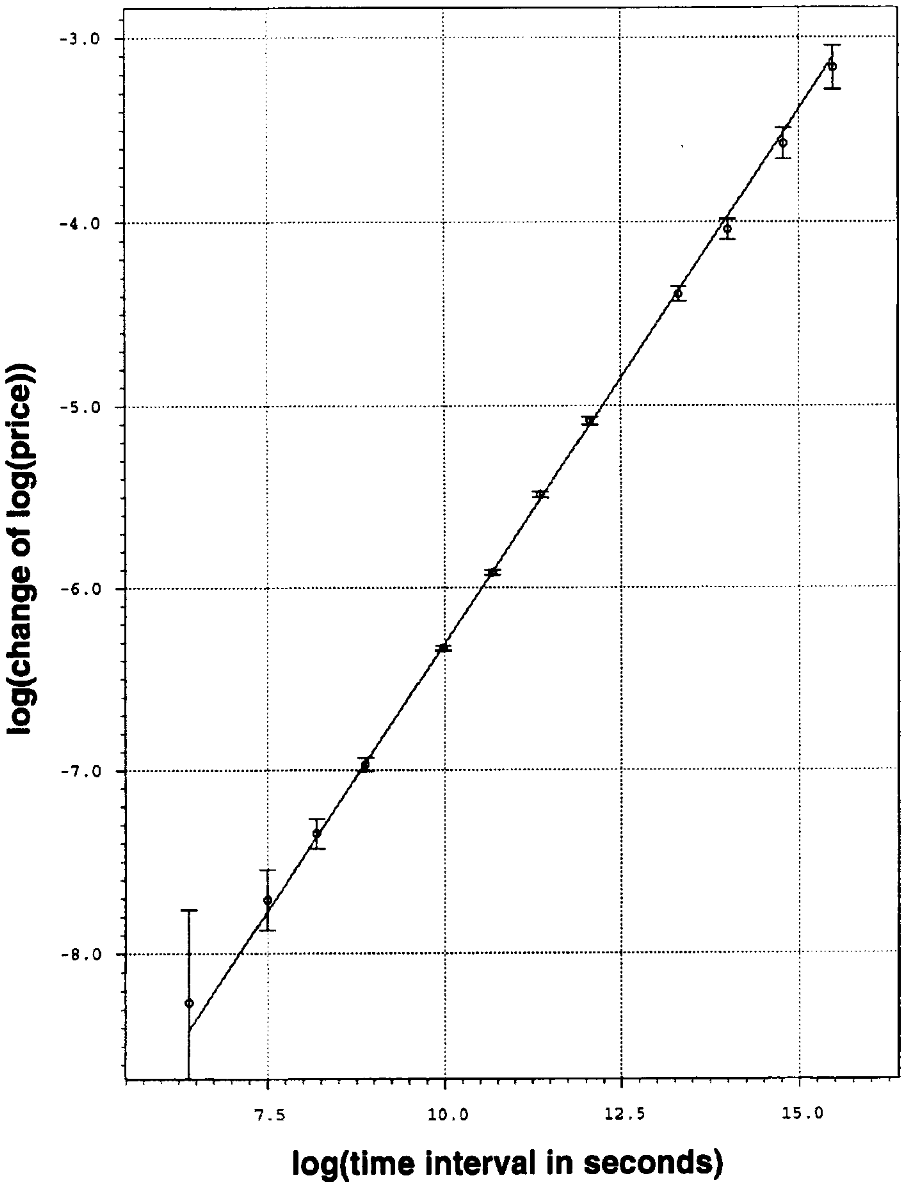

Figure 2. Scaling law for the USD/DEM. The scaling law is computed over a sample of six years for intervals ranging from 10 minutes to 2 mean months. The slope of the straight line is in this representation the drift exponent. Its value is 0.585 here

heteroscedasticity, a seasonal behaviour of FX price *volatility* rather than of FX prices themselves. This seasonality has been found in a study with intra-daily and intra-weekly sampling as well as in an autocorrelation analysis where autocorrelation coefficients are significantly higher for time lags that are integer multiples of the seasonal period (daily, weekly) than for other lags. With the intra-week analysis we show that mean absolute price

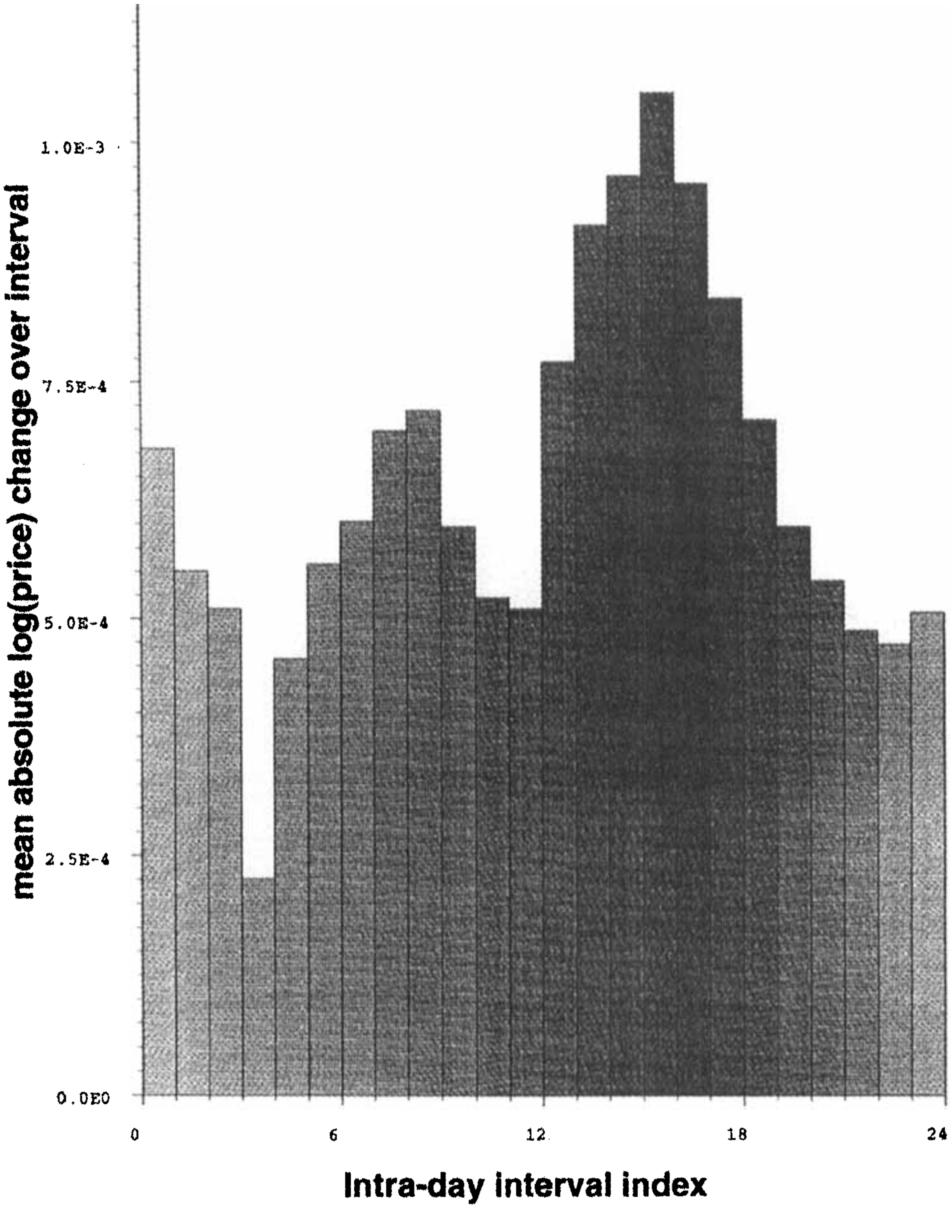

Figure 3. Intra-day seasonal volatility. The USD/DEM average hourly absolute price changes over a six-year period. The hourly grid is in Greenwich Mean Time (GMT). No adjustment is made for daylight-saving time

changes are much higher over working days than over Saturdays and Sundays when the market actors are hardly present. The corresponding intra-day analysis in the same paper shows that the mean absolute hourly price changes have distinct seasonal patterns. These patterns are clearly correlated with the *changing presence* of the main market places of the worldwide FX market. The lowest market presence outside the weekend happens during the lunch hour in Japan (noon break in Japan, night in America and Europe); it is at this time that the minimum of the mean absolute hourly price changes is found. Conversely, the maximum is found during the overlap of the business hours of the two main markets: Europe and America, (Dacorogna *et al.*, 1993). Figure 3 illustrates this seasonality by showing the average intra-day volatility patterns on an hourly grid. The daily maximum average volatility is roughly four times higher than the minimum.

Another very interesting property has been discovered in asset prices in recent years: *conditional heteroscedasticity*, which has been found in both the stock market and the FX market. The volatility of price changes is clustered in periods of high volatility and periods of low volatility. In examining daily or weekly data, many others have reported such behaviour for FX rates: (Diebold, 1988; Hsieh, 1988; Engle and Bollerslev, 1986; Baillie and Bollerslev, 1989). The most popular model for the clustering of volatility, the ARCH process, was first proposed by Engle (1982) in a study of UK inflation. Later Bollerslev (1986) proposed a generalized version, the GARCH (Generalized ARCH) process, that has been used extensively in modelling FX rates. For a good review of the literature on the subject see Bollerslev, Chou and Kroner, (1992). In fact, volume 52 of the *Journal of Econometrics* is dedicated to this subject, testifying to the current high interest in this model.

Many empirical studies also find that conditional heteroscedastic effects tend to weaken with less frequently sampled data. For example, Baillie and Bollerslev (1989) found that ARCH effects decrease as the sample interval lengthens from daily and become insignificant for data sampled monthly. This is in accordance with the asymptotic behaviour derived by Diebold (1988): under aggregation the distribution of an ARCH process converges towards a Gaussian random walk distribution. All these findings corroborate our findings of varying price distributions under aggregation as shown in Figure 1.

At the same time, our work confirms the existence of conditional heteroscedasticity in the FX market and extends it also to the intra-day data. However, our very short-term analysis (Dacorogna *et al.*, 1993) shows, in addition, an unusually long memory for the price changes over short intervals. There is still significant autocorrelation in the absolute price changes after a lag of 700 for 20-minute intervals. Stated otherwise, a 20-minute price change is significantly correlated with another 20-minute price change that occurred ten days ago! This long memory associated with a converging second moment (Pictet *et al.*, 1992) cannot be explained by simple ARCH or GARCH models. It is one of the reasons why we were compelled to develop a different type of model for forecasting volatility. Since we started this work, new attempts have been made to model this long memory effect: (Baillie, Bollerslev and Mikkelsen, 1993; Ding, Granger and Engle, 1993; Müller *et al.*, 1995; Baillie, 1995).

OPTIMAL TIME SCALE FOR FORECASTING FX-RATES

Traditional forecasting models in economics that are based on time series analysis concentrate on discovering a process that generates equally spaced values (see, for instance, Granger and Newbold, 1977; Taylor 1986). Little attention is given to the analysis of the underlying time scale. The usual assumption is an equally spaced (homogeneous) time series with elements separated by constant intervals of physical time. This assumption is, however, wrong, for in most actual 'daily' time series Saturdays and Sundays are skipped together with business holidays. In this case, the underlying time scale is no longer physical time but a new time scale which we call the *business time scale*. Such an observation seems to be trivial. However, the fact that time scales vary and are not identical to physical time is fundamental for almost all time series concerning human activities. A good example is given by the intra-day price change statistics presented in Figure 3. Since there are clear identifiable patterns related to FX market activity, defining an appropriate time scale that models current activity is essential for optimal short-term forecasting models. The different statistical properties we present here call for a better treatment of time than simply using physical time.

Changing the time scale can be also viewed as a way of introducing some of the 'fundamentals' that are missing in usual time series analysis. These fundamental economic variables certainly have an influence on price movements, but their individual and combined effects are difficult to isolate and thus to replicate. Their impact can be better seen, however, if one abstracts slightly and views the price movements as indelible 'footprints' left by these variables. In this sense we consider the FX markets to be highly efficient in reflecting the price information available in the market. A new time scale may therefore be used to capture the tracks left by economic factors.

Applying a new time scale to a process is formally equivalent to reformulating the generation process as a subordinated process, with the view that the subordinated process is easier to handle than the original one. For instance, a generation process of price changes with strong intra-day and intra-week volatility patterns cannot be stationary. Our model for the seasonal volatility fluctuations introduces a new *time scale* which, once used as the *directing process* $\vartheta(t)$ of the *subordinated* price generating process $x(t) = x^*[\vartheta(t)]$, makes the process x^* non-seasonal and, in the majority of cases, stationary.

Although a subordinated process is not the only possible way to treat the observed seasonality, other standard techniques of deseasonalization do not apply as the *volatility* itself is seasonal, not the raw time series.

Though the approach we propose here is not common among time series analysts, there have been a few tentative moves in this direction, and a variety of alternative time scales have been proposed, in different contexts, for treating the generation process of time series. In the early 1960s, Allais proposed the concept of *psychological time* to formulate the quantity theory of money and obtained some success in using it for modelling the movement of interest rates in Great Britain from 1815 to 1913 (Allais, 1974). Mandelbrot and Taylor (1967) suggested cumulating *transaction volume* to obtain a new time scale which they call the *transaction clock*. Clark (1973) proposed a subordinated stochastic process model that fitted cotton futures price data better than the stable family proposed by Mandelbrot. Stock (1988) studied post-war US GNP and interest rates and proposed a new time scale to model the conditional heteroscedasticity exhibited by these time series.

An intra-day business time scale: the ϑ-scale

We attribute the observed seasonal heteroscedasticity to the changing presence of traders in the FX markets and we introduce a new time scale, the ϑ-scale. In this time scale, price changes have a non-seasonal volatility. We call its derivative against physical time the *activity*, $a(t)$. This new variable measures, for each time t, the active presence of traders in the FX market through the price changes they induce. The activity of the worldwide FX market is then decomposed into three submarkets, $a_k(t)$, geographically centred in East Asia, Europe and North America. The model parameters can be computed by fitting them to the average hourly

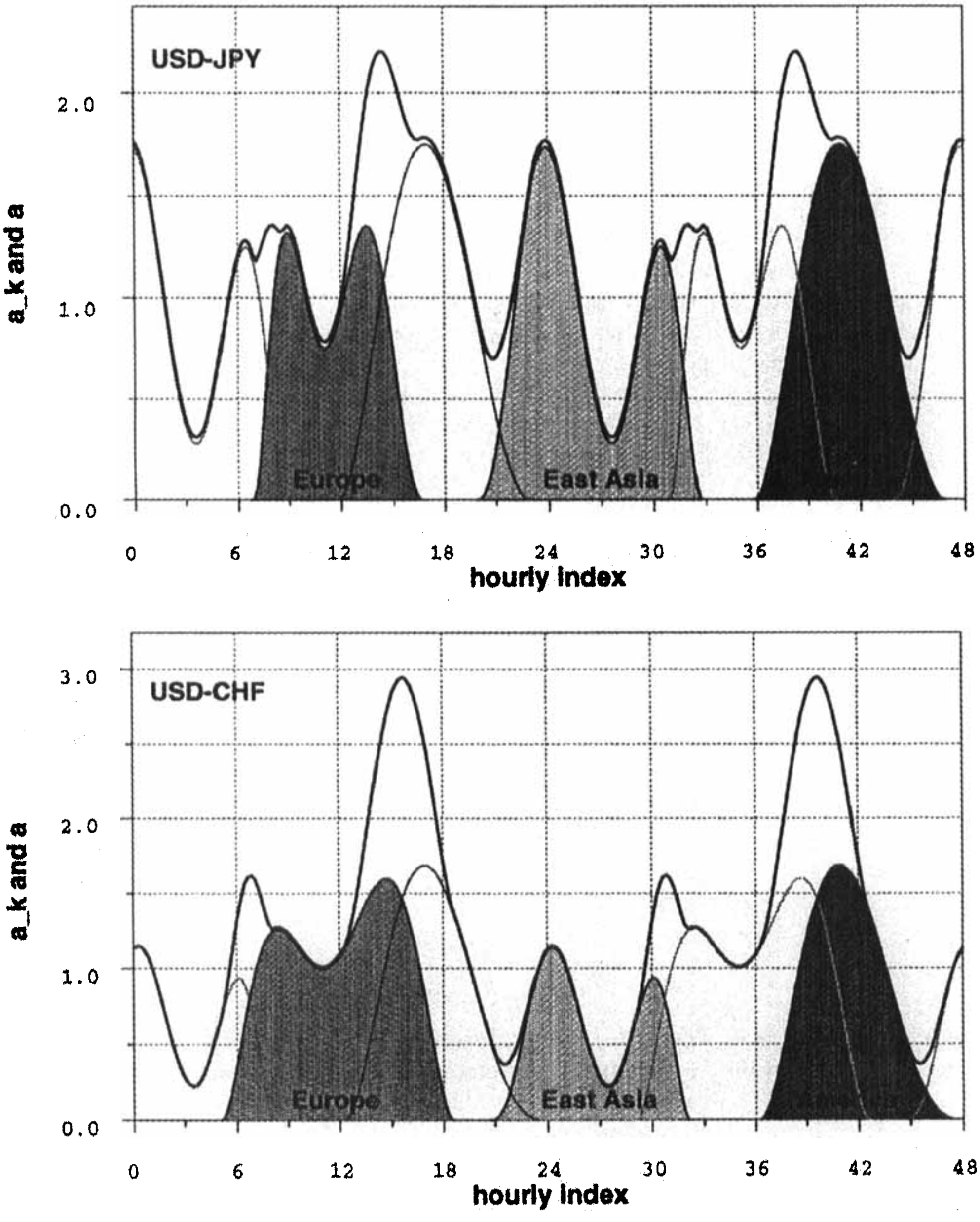

Figure 4. The activity model. The activity model for USD-JPY and USD-CHF decomposed into the three different continental markets over a period of 48 hours during normal business days. The thick line represents the total activity $a(t)$. It is the sum of a_0, the basic activity, and the three market activities $a_k(t)$. We have chosen to display the activity over 48 hours so the East Asian market can be seen without discontinuity

volatility over every 168 hours of the week (more information about the method is available in Dacorogna *et al.*, 1993). Examples of this activity model are shown in Figure 4.

The activity model is used to construct the ϑ-scale as the time integral of worldwide activity.

$$\vartheta(t) \equiv a_0(t - t_0) + \sum_{k=1}^{3} \int_{t_0}^{t} a_k(t')\, \mathrm{d}t' \tag{2}$$

where a_0 is a basic activity and a_k is the activity of a particular market out of three generic markets (East Asia, Europe and America). The a_k are polynomial functions of the time in a business day that present a relative minimum (lunch break) between two maxima, for East Asia and Europe, and only a maximum for America (no lunch break), as shown on Figure 4. The time scale $\vartheta(t)$ is a strictly monotonic function of physical time t. Any time interval from t_1 to $t_2(>t_1)$ corresponds to a ϑ-time interval of the positive size $\vartheta_2 - \vartheta_1$. Because of the regular weekly pattern of a (the activity is fitted to weekly seasonal volatility patterns), ϑ is predictable and may also be computed according to equation (2) in the future. The activity variable is normalized in such a way that ϑ-time can be measured in the *same units* as physical time (e.g. hours, days, weeks); one full week in ϑ-time corresponds to one week in physical time.

This scale, when used for the analysis of time series with high-frequency data, yields results that are no longer overshadowed by seasonality (see Dacorogna *et al.*, 1992). We also construct our forecasting model on this time scale to remove the seasonality of volatility from the computation of the model indicators.

Intrinsic time: an answer to the clustering of volatility

Our goal behind this new change of time scale is twofold: a treatment of the conditional heteroscedasticity present in the FX data and a more dynamical memory of our forecasting model. This is the second layer of our forecasting model on top of the business time ϑ-scale.

As mentioned above, some researchers have followed a similar approach. To treat conditional heteroscedasticity, Stock (1988) specifically uses two types of time deformation; one based on the time series itself and one based on business cycle variables. In the same paper he shows how this way of solving the problem can be compared to ARCH models. In our approach, we also use the time series itself to construct a time deformation. It is based on the scaling law defined in equation (1) and on the price volatility:

$$\tau(t_c) \equiv \tau(t_{c-1}) + k \frac{\vartheta(t_c) - \vartheta(t_{c-1})}{\Delta\vartheta_r} v_r^E \Delta T \tag{3}$$

where t_c is the current time. The factor k is calibrated in a way that τ-time flows neither slower nor faster than physical time or ϑ-time in the long-term average. The last two factors together are an inverse scaling law applied to a variable v_r, which is the recent volatility (not annualized); the exponent E is the inverse of the drift exponent D in equation (1); $\Delta\vartheta_r$ is a range parameter, the ϑ-time interval size of the price changes considered for computing the recent volatility $v_{\Delta\vartheta}$. In the implementation of Dacorogna *et al.* (1992), volatility is defined quite simply as an absolute price change:

$$v_r = |\, x(\vartheta(t_c)) - x(\vartheta(t_c) - \Delta\vartheta_r)\, | \tag{4}$$

where $\Delta\vartheta = 1$ hour is chosen to reflect short-term volatility. For a complete discussion of the parameters included in volatility measurements, see Guillaume *et al.* (1994).

The underlying time scale of the τ definition is always ϑ-time rather than physical time, thus analysing the time series in its deseasonalized form. The results for interest rates presented later in this paper are actually computed with a forecasting model that uses a different definition of v_r based on moving averages of the absolute price changes.

If there is no new price, the τ-time does not flow. Only if a new price arrives does the τ-time increase by an amount related to the size of the price movement. If the price movement is large, the time interval will appear longer than the physical time interval and so the 'memory' of the system, with fixed range in the τ-time scale, will actually be shortened when regarded in physical time. Intuitively, this seems very natural behaviour if many events take place in a given period of time: older events tend to be forgotten more rapidly than if only one event happens in the same time period.

This second new time scale, the τ-scale, does not directly use the physical time t, and does not need to have fundamental information about the behaviour of the series. The only information needed to define the scale are the values of the time series themselves. Thus we have chosen to call this time scale *intrinsic time*. The consequence of using such a scale is to expand periods of high volatility and contract those of low volatility, thus better capturing the relative importance of events to the market. Any moving average based on the intrinsic time τ dynamically adapts its range to market events. Therefore a forecasting model based on the τ-scale has a *dynamic memory* of the price history.

There is, however, a problem when using such a time scale. The intrinsic time τ is only known for the past, in contrast to the business time scale ϑ which is known also for the future since it is based on average behaviour. Thus a forecasting model for the price actually needs to be composed of *two* forecasting models: one for the intrinsic time and one for the price. The first forecast only requires the forecasting of an amplitude and not a direction since time cannot flow backwards.

THE O&A FORECASTING MODELS FOR THE FX-RATES

In the second section, we saw that the price generation process is far from being linear and in the previous section we tried to capture as much of these non-linearities as possible through successive changes of the underlying time scales. After these transformations, there are still many non-linearities left. Other researchers also found that there was remaining heteroscedasticity in the error terms after using ARCH models (Bollerslev, Chou and Kroner, 1992). Nevertheless, we chose a linear form for the forecasting model, the reason being the simplicity of the optimization procedure of such models. The idea is, then, to model the remaining non-linearities through introducing non-linear indicators and continuous optimization.

From this section onwards we assume that the forecasting model is constructed on the business time scale ϑ and all the following equations are written in terms of this new scale. The relation to the physical time scale is given by equation (2).

A linear combination of non-linear indicators

The model equations are based on *indicators*. These are computed with the help of moving averages. Indicators for market prices come conceptually from simple trading systems used in practice by market participants (see, for instance, Dunis and Feeny, 1989 or Murphy, 1986). Those trading systems yield *buy* and *sell* signals by evaluating an indicator function: the crossing of a certain threshold by the indicator on the positive side is regarded as a *buy* signal, on the negative side as a *sell* signal. An indicator is thus used as a *predictor* of a variable or its change; for instance, a price change.

Finding an ideal indicator, if it exists at all, would be enough to make a good price forecast. We, however, have no ideal indicators. Therefore we need to combine different ones appropriately to optimize their respective influence. Partly to avoid the severe problems of non-linear optimization, the forecasting models are based on a *linear combination* of price indicators z_x that can be treated by multiple linear regression. For a fixed forecasting horizon $\Delta\vartheta_f$ (corresponding to a Δt_f in physical time), the price forecast $\tilde{x}_f$ is computed according to

$$\tilde{x}_f = x_c + \sum_{j=1}^{n_x} c_{x,j}(\Delta\vartheta_f) z_{x,j}(\Delta\tilde{\tau}_f, \tau_c) \tag{5}$$

where x_c is the current price and n_x is the number of indicators used in the model (13 per horizon). All the indicators are computed in the intrinsic time scale (τ-scale) but note that the coefficients $c_{x,j}(\Delta\vartheta_f)$ are results of a multiple linear regression to be described below. This regression is done in the ϑ-scale to facilitate the computation of the model and it is clearly a first approximation to a full intrinsic time model that would use coefficients also computed in the τ-scale.

One element in equation (5) is not yet defined: $\Delta\tilde{\tau}_f$, the forecasting horizon expressed in intrinsic time. This quantity must be computed from its own forecasting model which is very similar to that in equation (5). The forecasting horizon, $\Delta\tilde{\tau}_f$, can be written as an intrinsic time forecast following

$$\Delta\tilde{\tau}_f \equiv \tilde{\tau}_f - \tau_c = \sum_{j=1}^{n_\tau} c_{\tau,j}(\Delta\vartheta_f) z_{\tau,j}(\Delta\vartheta_f, \vartheta_c) \tag{6}$$

where the forecasting model is computed in the ϑ-scale. The coefficients $c_{\tau,j}(\Delta\vartheta_f)$ are again the results of a multiple linear regression, n_τ is the number of indicators used in the τ model (five per horizon), and $z_{\tau,j}(\Delta\vartheta_f, \vartheta_c)$ are the intrinsic time indicators.

In contrast, to most traditional forecasting models, this one does not rely on a fixed basic time interval but is designed using a concept of continuous time. In fact, the time when a price is recorded in our database depends on the market itself; this means an unequally spaced time series. Moreover, the use of the τ-scale implies that our forecasting models must be computed simultaneously over several fixed time horizons $\Delta\vartheta_f$.

Given a forecasting horizon in physical time Δ_{tf} and the price history until x_c, one can compute $\Delta\vartheta_f$ with equation (2) and τ_c with equation (3). Assuming that we have computed a sufficiently large set of indicators, $z_{\tau,j}(\Delta\vartheta_f, \vartheta_c)$ and $z_{x,j}(\Delta\tilde{\tau}_f, \tau_c)$, and coefficients, $c_{\tau,j}(\Delta\vartheta_f)$ and $c_{x,j}(\Delta\vartheta_f)$, the price forecast can be computed by choosing the appropriate $\Delta\tilde{\tau}_f$ with equation (6) and inserting it in equation (5). In the next two sub sections we shall define the indicators and show how to compute the coefficients $c_{x,j}$.

Moving averages, momenta and indicators

The indicators used in equations (5) and (6) are based on momenta. Following standard time series techniques, we define *momentum* as a variable that measures the movements of a time series. Such a variable is based on moving averages. We choose to use *exponential* moving averages because they may be conveniently expressed in terms of recursion formulae.

Although moving averages are well defined for homogeneous time series (the reader can refer to Granger and Newbold, 1977, for instance), their formulation is much less developed for the case of continuous time series in which we are interested. That is why we present here the more general case. For continuous time series, the exponential moving average (EMA) at time τ_c is an integral,

$$\mathrm{EMA}_x(\Delta\tau_r, \tau_c) \equiv \frac{1}{\Delta\tau_r}\int_{-\infty}^{\tau_c} \mathrm{e}^{-(\tau_c-\tau)/\Delta\tau_r} x(\tau)\,\mathrm{d}\tau \tag{7}$$

where the time scale τ may be any scale, not only the intrinsic time and $\Delta\tau_r$ is the EMA *range* or the typical depth of the relevant past.

The recursion formulae for different types of interpolation can easily be derived. Here we only give the result of this derivation:

$$\mathrm{EMA}_x(\Delta\tau_r, \tau_c) = \mu\mathrm{EMA}_x(\Delta\tau_r, \tau_{c-1}) + (1-\mu)x_c + (\mu - \nu)\Delta x_c \tag{8}$$

where Δx_c is defined as $x_c - x_{c-1}$. For a time series representing a function with *all types* of interpolation, we obtain

$$\mu = \mathrm{e}^{-\alpha} = \bar{\mathrm{e}}^{\Delta\tau_c/\Delta\tau_r} \tag{9}$$

where the time interval $\Delta\tau_c$ is defined as $\tau_c - \tau_{c-1}$.

The variable ν depends on the interpolation method. Here are results for the most frequently used methods:

$$\nu = \begin{cases} \dfrac{1-\mathrm{e}^{-\alpha}}{\alpha} = \dfrac{1-\mu}{\alpha} & \text{for the linear interpolation} \\ 1 & \text{for taking the preceeding value in the series} \\ \dfrac{\mathrm{r}}{\mathrm{r}+1} = \mu & \text{for an homogeneous time series of interval } r \end{cases} \tag{10}$$

Together with the recursions, one needs to specify the initial value to be able to compute the EMA. There is usually no information before the first series element x_1, which makes it the natural choice for this initialization:

$$\mathrm{EMA}_x(\Delta\tau_r, \tau_1) = x_1 \tag{11}$$

The error made by this initialization declines by the factor $\mathrm{e}^{-(\tau_c - \tau_1)/\Delta\tau_r}([r/(r+1)]^{c-1}$ for a homogeneous time series). That is why part of the data must be reserved to build up the EMAs before optimizing the forecasting models themselves.

The EMAs alone are insufficient to study movements in the time series. We introduce here the momentum:

$$m_x(\Delta\tau_r, \tau_c) \equiv x_c - EMA_x(\Delta\tau_r, \tau_c) \tag{12}$$

which compares the most recent price to its own moving average. A momentum can be computed by a recursion formula that is easily derived from equation (8). It is also possible to define the *first* and the *second* momenta. The first momentum, $m_x^{(1)}$, is the difference of two exponential moving averages (or momenta) with *different ranges*. It can be considered as a sort of first derivative of $x(T)$. The second momentum, $m_x^{(2)}$, is the linear combination of three exponential moving averages (or momenta) with different ranges; it indicates the overall *curvature* of the series for a certain depth of the past.

In the previous sub section we have introduced the concept of indicators. Here we want to define those that are used in our forecasting models. Momenta of any kind can be considered as indicators and are sometimes used as such in technical analysis trading systems. The variety of indicators proposed and used by market participants is extensive (Murphy, 1986; Dunis and Feeny, 1989). We limit ourselves to describing those that are used in constructing our own forecasting model. They model some very primitive trading systems and are non-linear functions of different types of momenta. Following equation (5), let us call an indicator of price changes of range $\Delta\tau_r$, $z_x(\Delta\tau_r, \tau_c)$. It is defined as,

$$z_x(\Delta\tau_r, \tau_c) = \left[\frac{m_x^{(o)}(\Delta\tau_r, \tau_c)}{\sqrt{1 + (m_x^{(o)}(\Delta\tau_r, \tau_c)/m_{\max})^2}} \right]^p \tag{13}$$

where $m_x^{(o)}(\Delta\tau_r, \tau_c)$ are normalized momenta of order o of price changes, the power p is the accentuator of the indicator movements and $m_{\max}$ is related to the maximum value the indicator can take:

$$\lim_{m_x^{(o)} \to \infty} = (m_{\max})^p \tag{14}$$

In the case of price indicators, the power p must be an odd number to keep the sign of the moving average. The shape of the non-linear function is shown in Figure 5 for different powers p and for a $m_{\max}$ of one. This shape shows how the indicator plays the role of a primitive trading system. If the momentum has a high positive or negative value the indicator z_x saturates. It is as though the indicator is fully exposed in a *long* or *short* position. The power p both plays the role of a threshold (no threshold if $p = 1$) and influences how the model approaches its full long or short position. The $(m_{\max})^p$ value plays the role of the *quantity* of capital invested and also influences the shape of the indicator function. This analogy should, however, not be pushed too far. We do not try to model the traders' behaviour specifically.

The definition given in equation (13) can easily be extended to any time series. For instance, the same definition can be used to construct indicators for the intrinsic time in the ϑ-scale, $z_\tau(\Delta\vartheta_r, \vartheta_c)$, where the parameters are now defined as functions of τ_c computed using equation

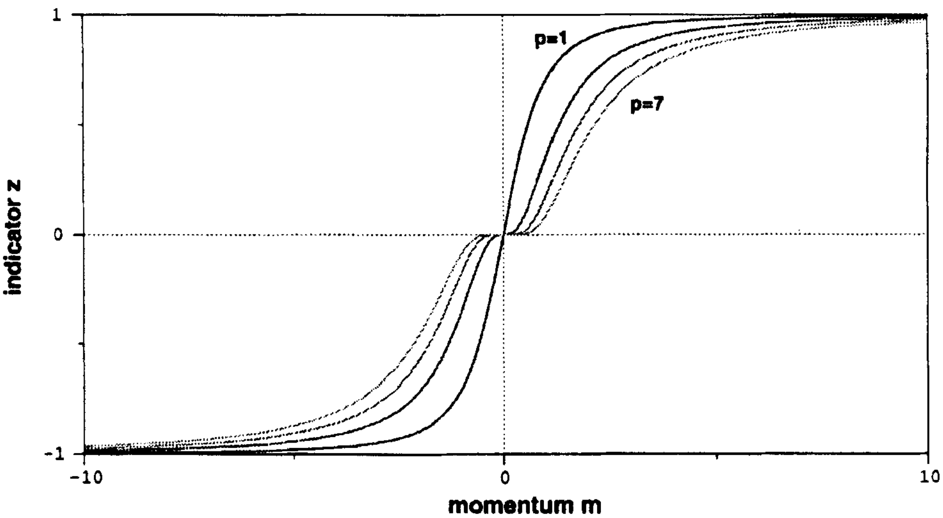

Figure 5. The non-linear function used for computing the indicator from the momentum

(3) on the ϑ-scale. The function $\tau(\vartheta)$ is a monotonic positive definite function so all its momenta are positive (time never flows backward). Thus the power of the function can also be even and it is only the upper-right quadrant of Figure 5 that is now relevant. The primitive trading system analogy does not work in this case and it is only by levelling off the indicator that we avoid overemphasizing the large movements.

In our implementation of this algorithm, the indicators are *continuously updated.* Every new price received from the market makers causes the model to recompute all its indicators for all horizons. It then updates the forecast again for each time horizon.

Continuous optimization of the coefficients

Not only are the indicators continuously updated but the coefficients are as well. Each coefficient $(c_{x,j}, c_{\tau,j})$ of the forecasting model is determined by fitting the model to a *chosen portion* of the price curve in the most recent past. We take this portion as a function of the forecasting horizon (a few months for hourly forecasts, up to a few years for three-month forecasts). The idea of taking horizon-dependent finite samples to optimize the models arises from the fact that there are different regimes in the market and that short-term horizons are particularly sensitive to them. Furthermore, short-term traders are not influenced much by a past older than three months. Using long samples to optimize short forecasting horizons will make the model less adaptive to regime changes.

Adaptation to long-term regime or structural changes is enabled by re-evaluation of the optimization as soon as enough new information becomes available. The optimization sample size is kept fixed (in ϑ-time), rolled forward, and then linear regression is re-applied to the new sample. This technique is similar to that presented in Schinasi and Swamy (1989) and Swamy and Schinasi (1989) except that we use a fixed sample size while they add the new data to their sample. The model is optimized through the usual generalized least-squares method except for two modifications.

Our forecasting models run in real time and the continuous re-optimization can generate instabilities (rapid jumps from positive to negative forecasts) when standard linear regression techniques are used. The instabilities originate from both the indicators and their coefficients in the linear combination (equations (5) and (6)). The indicators are moderately volatile: we avoid too volatile indicators by limiting the power of the exponents in the indicator construction to 3 for simple momenta and 7 for higher momenta. Moderately volatile indicators can cause instabilities only if their coefficients are large. The coefficients are less volatile than the indicators (because of the large optimization samples), but they may have high values if the regression by which they are optimized is near-singular because of high correlation between the indicators. Within a particular sample, the high positive and negative coefficients typical in the solution of a near-singular regression matrix would balance each other out. However, as soon as these coefficients are used with changing indicator values outside this sample, the equilibrium is lost and the high coefficients may boost the forecast signal. We have already eliminated one source of near-singularity by avoiding indicators that are too similar in the same forecast. However, this has proven insufficient in some cases.

The standard regression technique is applied under the assumption of precise regressors and a dependent variable with a Gaussian error. Our regressors (called indicators), however, originate from the same database as the dependent variable (the price change); thus, they are prone to database errors (missing data, badly filtered data, and so on) and to errors in the construction of the ϑ and τ time scales. Taking into account the regressor errors allows solution of the problem of near-singularities in a natural way. Instead of considering the jth regressor $z_{j,i}$ at the ith observation (where we have dropped the variable index and the horizon for ease of notation), we consider the *imprecise* regressors $\hat{z}_{j,i} = z_{j,i} + \varepsilon_{j,i}$, where $\varepsilon_{j,i}$ is a random error with a variance ρ^2 times that of z_j. We call the small parameter ρ the typical relative error of the indicators and we assume it is roughly the same for all indicators of the type we have defined earlier. Without going into the details of the calculation (the interested reader may obtain the full derivation upon request), such a change modifies the final version of the system of n_τ, or n_x equations. The kth equation can be written as:

$$\sum_{j=1}^{n} c_j(1 + \rho^2\delta_{jk}) \sum_{i=1}^{N} w_i \hat{z}_{j,i} \hat{z}_{k,i} = \sum_{i=1}^{N} w_i Y_i z_{k,i} \tag{15}$$

where N is the number of observations used in the regression, n is the number of indicators (n_x in equation (5) and n_τ in equation (6)), w_i is a weighting function depending on the type of moving averages used (here it is an exponential) and δ_{jk} is the usual Kronecker symbol: 1 for $j=k$ and 0 for $j \neq k$. The quantity Y_i is the usual response term of the regression: $x(\vartheta_i + \Delta\vartheta_f) - x(\vartheta_i)$. There is only one addition to the original regression: the diagonal elements of the system matrix are multiplied by a constant factor $1 + \rho^2$, slightly greater than 1.

The effect of increasing the diagonal values of the original matrix by ρ^2 is to guarantee a minimum regularity of the modified matrix even if the original is near-singular or even singular. The variable ρ^2 can be interpreted as the parameter of this minimum regularity. This desired effect is also accompanied by a slight decrease of the absolute values of the coefficients c_j, because the right-hand side of the equation system remains unaffected by the modification. The decreases are insignificant, the only exceptions being for coefficients inflated by near-singularity in the original regression. There, the absolute values decrease substantially, which is what we want in any case.

The other departure from the usual regression technique is a modification of the regression response Y_i necessitated by the leptokurtic behaviour of price changes. The forecast signals are much less leptokurtic than the price changes, hence the optimization is dominated by exceptionally large real price movements rather than the 'normal' price movements. This is also accentuated by the fact that it is squared price changes that enter into the computation of the least squares. Furthermore, the users of our forecasting models are more interested in the correct direction of the forecast than in the absolute size of a price change forecast. A pure linear regression is thus inappropriate.

The minimization of the sum of squared deviations, however, has an important advantage: it can be reached by solving a system of linear equations. Theoretically, though, the minimum sum of squares could be replaced by any utility function. Our problem is thus to find, within the framework of the regression technique, a more appropriate optimization (or utility) function. The best way to achieve this goal is through a *mapping function* of the price changes: the forecast should fit the mapped price changes $\hat{Y}_i$ rather than the *real* price changes Y_i.

A suitable mapping function makes the mapped price changes less leptokurtic than the original ones. The rest of the regression problem remains unchanged. The desired effects can be obtained with an underproportional mapping function having the following properties:

- Small price changes should be amplified when considered in the regression, in order to establish a sufficient penalty against forecasts of the wrong direction.
- Large price changes should be reduced when considered in the regression, so the distribution function of mapped price changes is no longer leptokurtic.
- The mapping effects should decrease with the increasing time horizon size.

The choice of such a mapping function M is arbitrary provided it has the above properties. The function used in our model is

$$\hat{Y}_i = M(Y_i) = \frac{AY_i}{[Y_i^2 + B]^{\alpha}} \tag{16}$$

with the parameters A, B, and α depending on the time horizon $\Delta\vartheta_f$. The same parameters are used for all different FX rates. They have been calibrated by trial and error in order to keep the full sample variance of the mapped price changes at the same level as that of the original price changes. The parameter or must satisfy the condition $0 \leq \alpha < 0.5$ because the mapping function must be an underproportional bijection.

DISCUSSION OF THE FORECASTING RESULTS

Two questions are relevant for testing any forecasting model of foreign exchange rates,

(1) What data should be used for testing?
(2) What is a good measure of forecasting accuracy?

Since the classic paper of Meese and Rogoff (1983), researchers are aware of the need for out-of-sample tests to truly check forecast validity. Because of the statistical nature of FX rates, there would be little significance in the forecast accuracy measured on the same data that were used for optimizing the models. The real test comes when the model is run on data that were not used in constructing the model. In our case, the model being run in real-time, we have a continuous out-of-sample test. Besides the question of in- and out-of-sample, there is a question as to what constitutes the relevant quantity for measuring the accuracy of forecasting methods. Makridakis, Wheelwright and McGee (1983) review the main measures. We limit ourselves here to presenting the reasons why we chose a certain type of measure and how we compute the uncertainty of these measures.

Appropriate measures of forecasting accuracy

Most standard measures rely on the Mean Square Error (MSE) and the Mean Absolute Error (MAE) for each time horizon. These errors are then compared to similar ones produced by a naive forecasting model serving as a benchmark. One naive model may be the random walk forecast where expected returns are zero and the best forecast for the future is the current price. These accuracy measures are, however, all parametric in the sense that they rely on the desirable properties of means and variances which occur when the underlying distributions are normal. The selection of the random walk model to derive the benchmark MSE or MAE is inherently inappropriate. It is in effect comparing the price volatility (MSE or MAE) with the forecasting error. There is no reason to expect that the heteroscedasticity and the leptokurticity of price changes would not affect the MSE or MAE for a particular horizon. Thus the significance of their comparison with the forecast MSE or MAE is unclear, for it might only reflect the properties of price volatility.

Such considerations have led us to formulate non-parametric methods of analysing forecast accuracy. These are generally 'distribution free' measures in that they do not assume a normally distributed population, and so can be used when this assumption is not valid. One measure which has this desirable property is the percentage of forecasts in the right direction. To a trader, for instance, it is more important to correctly forecast the direction (up or down) of any trend than its magnitude. We term this measure the *direction quality*, also known as the sign test:

$$Q_D(\Delta t_f) \equiv \frac{N(\{\tilde{x}_f \mid (\tilde{x}_f - x_c)(x_f - x_c) > 0\})}{N(\{\tilde{x}_f \mid (\tilde{x}_f - x_c)(x_f - x_c) \neq 0\})} \tag{17}$$

where N is a function that gives the number of elements of a particular set of variables $\{x\}$, and x_c and $\tilde{x}_f$ have the same definition as in equation (5). We give the forecasting horizon in physical time Δt_f because the quality must be measured in the time scale in which people look at the forecasts. It should be clarified here that this definition does not test the cases where either the forecast is the same as the current price or when the price at time $\vartheta_c + \Delta\vartheta_f$ is the same as the current one. To illustrate this problem, let us note that the random walk forecast cannot be measured by this definition. Other definitions could be used, like counting the case when the direction is zero as half right and half wrong. Excluding the cases where one of the two variables is zero would be a problem if this occurs very often. Our results show that it occurs only seldom: for the real signal $(x_f - x_c)$, a few per cent of the observations at the very short horizons, and almost never for the forecast signal $(\tilde{x}_f - x_c)$.

Unlike more conventional forecasts—for instance, a weather forecast—a FX rate forecast is still interesting, from a practical point of view, even if its direction quality is only significantly above 50%. No trader expects to be right all the time. In practice we assume that a Q_D significantly higher than 50% means that the forecasting model is better than the random walk. The problem lies in defining the word 'significant'. As much as one would like to be independent of the random walk assumption, we are still forced to go back to it in one way or another, as here, when we want to define the significance level of the direction quality. As a first approximation, we define the significance level as the 95% confidence level of the random walk:

$$\sigma_{Q_D} \approx \frac{1.96}{2\sqrt{n}} \tag{18}$$

where n is the number of tests. The factor 2 comes from the assumption of an equal probability of having positive or negative signals. It is a similar problem to the one of tossing a fair coin.

Another measure we use in conjunction with the previous one is the *signal correlation* between the forecasting signal and the real price signal:

$$C(\Delta t_f) \equiv \frac{\sum_{i=1}^{n'} (x_{f,i} - x_{c,i})(\tilde{x}_{f,i} - x_{c,i})}{\sqrt{\sum_{i=1}^{n'} (x_{f,i} - x_{c,i})^2 \sum_{i=i'} (\tilde{x}_{f,i} - x_{c,i})^2}} \tag{19}$$

where n' is the number of possible measures in the full sample, n the number of full forecasting horizons in the full sample and $i' = n - n'$. Here again the forecasting horizon is given in physical time, Δt_f. We estimate the significance of this quantity using $1.96/\sqrt{n'}$.

Both the direction quality and the signal correlation unfortunately have a slight drawback: they do not give a measure of the effectiveness of the forecast amplitude. Nevertheless, we believe that they are superior to standard measures because of the non-normality of the price change distributions. The direction quality, which for all practical purposes is the most relevant indication of the forecast, and the signal correlation must be highly significant before we accept a model as being 'good'. Thus used in conjunction, these measures provide a powerful test of model quality.

Model optimization and testing

Optimization consists of two distinct but interrelated operations, corresponding to the two main types of parameters in the models. The linear model coefficients $c_{\tau,j}$ and $c_{x,j}$ are optimized through least squares (see above) and, under the control of this process, always fulfil the strict out-of-sample condition when applied to a forecast. On the other hand, the non-linear parameters of the indicators described above must be optimized by trial and error to meet the above criteria of direction quality and signal correlation. The data set used in selecting the best combination of indicators is termed the *in-sample* period for it is in which the model parameters are fully optimized.

In Table I we show how our sample is divided to satisfy the different requirements of model initialization, in-sample optimization and out-of-sample tests. The initialization period is needed for both initializing the different EMAs (see equation (11) and the comment afterwards) and computing the first set of linear coefficients $c_{\tau,j}$ and $c_{x,j}$. The period from 1 December 1986 to 1 September 1990 (46 months) is our in-sample period because it was used to design the model and its parameters. The results presented in the next section are computed over one specific period using our database of intra-day market makers' quotes: from 3 September 1990 to 2 September 1993 (36 months). This period is pure *post ex-ante* testing, that is, data from this period were not used at all for building the model. These 36 months constitute our out-of-sample test for the FX rates.

As a further evidence of the quality of this model, we tested it on the deposit rates used in the transactions between banks and collected from the same source, Reuters. The market for this type of interest rate is very similar to the FX market and is often composed of the same agents. The interest rate forecasting model uses exactly the same structure and is not optimized, so it is justified to treat the full sample from 5 January 1987 to 2 August 1993 as an out-of-sample test.

Effectiveness of O&A forecasting models on very short-term horizons

We present here results from the O&A forecasting model for very short horizons: 2, 4 and 8 hours for the FX rates and 12, 24 and 48 hours for the interest rates. This choice was made for two reasons. First, it is, to our knowledge, the first time that an attempt has been made to study models for such short horizons. Second, the statistical relevance of these results is high because of the very large number of observations that can be obtained from just a few years.

The quality measures are computed for each time horizon at an interval of 1/12 of the horizon. As mentioned above, the forecast accuracy is always measured in the physical time scale since it is in this scale that the different forecasts are useful. The number of relevant points in the statistical computation varies depending on the horizon and on the number of missing data. If there is no recent price quote at the time when the forecast must be tested we ignore the point. For the 2 hours horizon it varies from around 70,000 tests to 90,000, for 4 hours from 35,000 to 45,000, and for 8 hours 17,000 to 24,000. These very large numbers insure that our statistical results are highly significant even though the significance levels are

Table I. The sample division for model building and testing, in the case of the FX rates

From	To	Data types	Data size	Usage
1.6.73	1.2.86	Daily	152 months	Model initialization
1.2.86	1.12.86	Intra-day	10 months	Model initialization
1.12.86	1.9.90	Intra-day	46 months	In-sample period
3.9.90	2.9.93	Intra-day	36 months	Out-of-sample tests

computed from the number of independent observations (≈8000 for 2 hours, ≈4000 for 4 hours and ≈2000 for 8 hours). For the interest rates the horizons are longer because the variations are less strong in the short term. The number of independent observations is, in this case: ≈4000 for 12 hours, ≈2000 for 24 hours and ≈1000 for 48 hours because the sample extends over a longer period.

We currently have 56 currencies running on the OIS, but in order not to overwhelm this paper with numbers, we only show results for the five most important FX rates against the USD and five of the most traded cross rates. For the other currencies the results are very similar. The direction quality and the signal correlation are given in percentages for the out-of-sample testing period in Table II. The direction quality 95% significance limits according to equation (18) are 50.9%, 51.2%, and 51.7% for the 2-hour, 4-hour, and 8-hour forecasts respectively; the signal correlation 95% significance limits are 1.7%, 2.4%, and 3.4%. In all the cases, the direction quality is above 50% and the signal correlation is positive. The + sign indicates if *both measures* are above the 95% significance level. If at least one of them is below the 95% significance level, the result is marked by a − sign. Looking at these signs shows clearly that, except for USD-CHF and for GBP-USD, the quality achieved is highly significant. Out of 30 cases, only nine do not show significant quality for both measures. Decomposing the results per horizon, we have 70% of significant results for the 2-hour, 90% for the 4-hour and 50% for the 8-hour horizons.

The quality measures of the interest rate forecasts are presented in Table III. The direction quality significance limits according to equation (18) are 51.4%, 52.0%, and 52.8% for the 12-hour, 24-hour, and 48-hour forecasts respectively; the signal correlation significance limits are 2.8%, 4.0%, and 5.6%. This means that all the results in Table III indicate a significant success

Table II. Out-of-sample forecasting results for four USD rates, the gold price, and five cross rates for the period from 3 September 1990 to 2 September 1993 (%)

FX rate	Forecast horizon (hours)	Direction quality	Signal correl.	Sig.	FX rate	Forecast horizon (hours)	Direction quality	Signal correl.	Sig
USD-DEM	2	51.9	+2.2	+	DEM-JPY	2	52.1	+4.1	+
	4	51.9	+4.0	+		4	51.2	+3.5	+
	8	52.3	+3.2	−		8	51.4	+2.7	−
USD-JPY	2	52.5	+3.2	−	GBP-JPY	2	53.2	+5.3	+
	4	52.6	+4.8	+		4	53.1	+6.3	+
	8	51.8	+3.4	+		8	52.8	+7.4	+
GBP-USD	2	51.5	+1.4	−	GBP-DEM	2	54.6	+7.7	+
	4	51.6	+3.9	+		4	53.5	+4.2	+
	8	50.6	+3.4	−		8	53.2	+4.6	+
USD-CHF	2	51.8	+0.7	−	DEM-CHF	2	54.6	+6.1	+
	4	51.8	+1.5	−		4	54.0	+5.1	+
	8	51.6	+2.6	−		8	53.5	+6.9	+
XAU-USD	2	54.1	+3.0	+	JPY-CHF	2	52.4	+4.2	+
	4	53.1	+3.4	+		4	51.5	+4.4	+
	8	52.9	+2.6	−		8	51.4	+5.8	+

Notes:
All direction qualities are above 50%, all signal correlations above 0%, both significantly so in cases marked by +. In the cases marked by −, at least one of the two quality tests is insignificant.

Table III. Interest rate forecasting results for the period from the 5 January 1987 to 2 August 1993 (%)

Interest rate	Forecast horizon (hours)	Direction quality	Signal correlation	Interest rate	Forecast horizon (hours)	Direction quality	Signal correlation
USD–3m	12	60.4	+20.2	USD–6m	12	58.4	+9.6
	24	55.9	+11.2		24	54.2	+9.8
	48	55.1	+8.2		48	54.7	+10.0
DEM–3m	12	61.5	+21.2	DEM–6m	12	61.7	+21.4
	24	58.0	+17.6		24	57.2	+15.8
	48	53.0	+5.7		48	54.5	+15.2
JPY–3m	12	61.0	+20.6	JPY–6m	12	61.5	+20.1
	24	56.9	+14.6		24	56.6	+16.9
	48	56.5	+12.0		48	58.3	+18.8
GBP–3m	12	58.3	+11.1	GBP–6m	12	56.6	+14.6
	24	55.6	+8.1		24	54.9	+7.2
	48	53.8	+7.6		48	53.3	+6.1
CHF–3m	12	59.2	+14.5	CHF–6m	12	58.6	+19.2
	24	55.4	+15.4		24	55.7	+13.8
	48	56.2	+12.8		48	56.5	+10.9

Notes:
Different currencies, different maturities (3m = 3 months, 6m = 6 months), different forecast horizons (from 12 to 48 hours) (%).

of the model. The results are even better than those of the FX rate forecasts in Table II. The results of Table III have been computed out of sample. The FX model of Table II was applied to interest rates without any modification: no interest rate data were used to 'fit' the model.

CONCLUSION

In this paper, we have shown that with the help of very high-frequency data the statistical properties of FX rates can be better understood and that specifying forecasting models for very short-term horizons is possible. These models contain ingredients all designed to better capture the dynamics at work in the financial markets as illustrated by the success of the models when applied to interest rates. The most important model characteristics are:

- Univariate time series analysis but based on intra-day non-homogeneous data.
- Variable time scales to capture both seasonal heteroscedasticity (ϑ-scale) and autoregressive conditional heteroscedasticity (τ-scale).
- Linear combination of non-linear indicators.
- Multiple linear regression with two modifications to avoid instabilities and to correct for the leptokurtic behaviour of the price changes.
- Continuous optimization of the model coefficients in a finite size, forecasting horizon-dependent sample.

The forecast quality of these models is evaluated on a very large sample with two different measures that avoid statistical problems arising from the nature of the FX rate time series. The rigorous separation of in- and out-of-sample measures, the large number of observations, and the stringent significance levels mean that the statistical results of the forecast evaluation are convincing evidence that our models beat the random walk for most of the 10 studied currencies and for all of the interest rates for very short-term forecasting horizons. These results are also corroborated by those we obtain on the other currencies that are running in the OIS.

What are the consequences of such results for the economic theory of market efficiency? We believe that they point to the extension and improvement of methods and tools for defining and analysing market efficiency. The accepted theory was probably never conceived for such short horizons and, even more importantly, it takes an unrealistic view of market response to new information. Being developed only in a static framework, the theory assumes that economic actors integrate new price information *instantaneously*, and very little attention is paid to the time needed for a piece of information to be available to all market participants and to the diverse interpretation of that information. In the context of very short time horizons these factors play critical roles in market adjustments. It is reasonable to assume that the markets need a *finite time* to adjust to any information and that this time depends on the nature of the information.

We think that these adjustments can be modelled and hence that a certain predictability of price movements exists. Our forecasting models, while a positive step in this direction, are nevertheless only a first one and there is still room for improvements through a better understanding and definition of intrinsic time and through the search for better indicators.

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APPENDIX: THE FORECASTING MODEL AT A GLANCE

The forecasting model is built in a layered structure. In this appendix we want to show how, from a price collected on Reuters, we arrive at a prediction for a particular horizon. When a new price is collected in the database the process of obtaining a new price forecast is divided into two operations:

(1) The updating of the model with a new price, and
(2) The computation of a forecast for a particular point in time.

Our forecasting model does not have a preferred time horizon and is supposed to answer requests for horizons going from a few minutes up to few weeks. This is why the model maintains internally many horizons simultaneously. Moreover, the model is actually composed of two forecasting models, one for the intrinsic time τ and one for the price x. Thus, we need to maintain two sets of parameters, one for each model.

When a new price is collected in the database

- *Model update*: a new price is collected at time t_c. This implies that the internal parameters of the forecasting model are no longer up to date and need to be re-evaluated. The following are the different operations necessary to have an up-to-date model:

  (1) We update the current intrinsic time τ_c according to equation (3).

(2) The model maintains two sets of 41 momenta (equation (12)) of different horizons in a geometrical series separated by a factor 1.4. There are intrinsic time momenta, $m_\tau(\Delta\vartheta_r, \vartheta_c)$ (computed in the ϑ-scale), and logarithmic price, x, momenta, $m_x(\Delta\tau_r, \tau_c)$ computed in intrinsic time. The next step is to update both sets of momenta.

(3) The indicators $z_\tau(\Delta\theta_r, \theta_c)$ and $z_x(\Delta\tau_r, \tau_c)$ are simple non-linear functions, equation (13 of the momenta and are computed directly in the next step to evaluate the regression coefficients. The momenta needed for all indicator computations are obtained, for each indicator horizon, by interpolating the momenta with the two neighbouring horizon in the geometrical series of momenta.

(4) Two sets of 23 forecasting horizons for the intrinsic time forecast and for the price forecast are maintained in parallel. As for the momenta, the horizons are in a geometrical series separated by a factor 1.6. Both factors (1.4 and 1.6) are chosen as a compromise between the accuracy of the forecast and the memory size of the program. For each of the forecasting horizons, we compute all indicators $z_{\tau,j}$ and $z_{x,j}$. They are multiplied with the coefficients ($c_{\tau,j}$ and $c_{x,j}$), stored in the same sequence of 23 horizons, to compute a forecast according to equations (6) and (5). Every few prices, these coefficients are updated through a modified regression algorithm on a moving sample as described in section 4.3.

- *Computation of a price forecast*: Once all these steps are completed, we are ready to compute a forecast for a time point t_f in the future. The following shots how a price forecast is obtained.

(1) Transform $\Delta t_f = t_f - t_c$ in $\Delta\vartheta_f$ on the business time scale: $\Delta t_f \rightarrow \Delta\vartheta_f$. Most probably, $\Delta\vartheta_f$ will not coincide with one of the chosen forecasting horizons.

(2) Use $\Delta\vartheta_f$ to find the new $\Delta\tilde{\tau}_f$ with the intrinsic time forecast by linearly interpolating between the two neighbouring horizons that are maintained in the list: $\Delta\vartheta_j \rightarrow$ equation (6) $\rightarrow \Delta\tilde{\tau}_f$.

(3) Use $\Delta\tilde{\tau}_f$ to compute the price indicators $z_x(\Delta\tilde{\tau}_f, \tau_c)$ and $\Delta\vartheta_f$ to find the corresponding coefficients to compute the forecast for price change.

$$\left.\begin{array}{l}\Delta\tilde{\tau}_f \rightarrow z_x(\Delta\tilde{\tau}_f, \tau_c)\\ \Delta\vartheta_f \rightarrow c_{x,j}(\Delta\vartheta_f)\end{array}\right\} \rightarrow \tilde{x}_f = x_c + \sum_{j=1}^{m} c_{x,j}(\Delta\vartheta_f) z_{x,j}(\Delta\tilde{\tau}_f, \tau_c)$$

The final forecast will be obtained by linearly interpolating between the forecasts of the two neighbouring horizons that are maintained in the list.

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