74.319 Assignment 1: Formal Representation and Reasoning.

Due: October 16, 2006, by the start of class. All answers must be typeset using a text editor or word processing program; hand-written answers will not be accepted (nor will late assignments). Your assignment must be submitted in a folder, along with a signed honesty declaration. There are 52 marks in total. Each assignment will be worth proportionally the same value to your overall grade irrespective of the total marks on the assignment.

This assignment involves some exercises in representation – specifically, First Order Logic, Clause Form, Unification, and Resolution Refutation. The point of this assignment is to get you thinking about formal representation for problem solving and the processes involved in reasoning before we start using a computer to do this. Pay attention to the level at which you are representing facts – try to make it so that concepts are general and could be extended.

I realize that doing formal symbols in a typesetting program is tedious. To make life a little easier and to be able to use standard text files, we will use the notation **forall(X):wff(X)** for a universal quantifier, and **exists(X):wff(X)** for an existential quantifier. ~, ^, v (use the lowercase letter v) and -> can be used directly off the keyboard for not, and, or, and implication. You are certainly welcome to use appropriate symbols if you feel like prettying it up in a symbol font, but this is good enough.

Note that all of these proofs require resolution refutation: no marks are available for any other form of reasoning.

Question 1. 12 Marks.
Consider the following English statements, which form a logical system:
Convert the following to first order logic statements. Each numbered statement below should be a single FOL statement.

1) The Rock is human.
2) Seven of Nine is Borg.
3) If you are not Borg, then Resistance is Futile.
4) Someone can smell what The Rock is cooking.
5) Anytime someone fights someone else, if that first someone wins, then the other someone does not win.
6) The Rock fought Seven of Nine, and The Rock won.
7) If a human fights a Borg, and the Borg does not win, then the human is lucky.
8) Some Borg are lucky.
9) Someone fought the Law and the Law won.

Convert these first order logic statements (5 marks). Then convert them to clause form (4 marks - you must **SHOW YOUR WORK** if you want any part marks to be considered) and **using resolution refutation**, prove that **The Rock is Lucky** (3 marks). Each clause form statement should state the number of the original FOL statement that it came from,

and each should also be numbered so the marker can follow your reasoning in the resolution proof. Each new resolution should show the two parents clauses used to derive the resolution. You must also state the necessary substitutions for unification where unification is performed.

Question 2. 20 Marks.

Convert the following sentences into first order logic statements (7 marks - each numbered statement below should be a single FOL statement). Then put these in clause form (6 marks) You must **SHOW YOUR WORK** if you want any part marks to be considered. Each clause form statement should state the number of the original FOL statement that it came from, and each should also be numbered so the marker can follow your reasoning in the proofs that follow. Each new resolution should show the two parents clauses used to derive the resolution. You must also state the necessary substitutions for unification where unification is performed.

1)   Spiderman, The Green Goblin, Wolverine, and Mary Jane are persons.
2)   The Green Goblin has special powers and he also cheats.
3)   The Green Goblin is not a fan of Mary Jane.
4)   Spiderman is a mutant, and so is Wolverine.
5)   Wolverine is a fan of Captain Picard.
6)   All persons with special powers are mutants.
7)   All people who do not cheat are good.
8)   Some mutants are not X-Men.
9)   Mary Jane loves all good persons who are not X-Men.
10)   There is no such thing as an X-Man who is not a fan of Captain Picard, and there is no such thing as an X-Man who cheats.
11)   All X-Men are mutants.
12)   All mutants who are fans of Captain Picard are X-Men.
13)   Spiderman does not cheat and is not a fan of Captain Picard.

Now, using the above system, perform the following proofs **using resolution refutation** (show your work by stating the numbers of each pair of clauses you are resolving **and the necessary substitutions to unify these**, numbering each new deduction as we did in class):

i) Prove that Mary Jane loves Spiderman (5 marks).
ii) Prove that Wolverine does not cheat (2 marks).

Question 3: 20 Marks.
Make up **your own** logical system and a hypothesis to prove on a topic of your choice (and try to keep it reasonably family-suitable, ok?). Show the statements in English and first order logic, convert them to clause form and solve **using resolution refutation** as you did for the previous three questions. Your logical system's statements must use existential and universal quantifiers, and must have at least 8 statements, at least 4 of which should involve implication. Marks will be allocated in proportion to the intricacy

of your system and the extent of the work you've done in representing it. The system should reason about some set of coherent concepts as the questions above do, be logically correct, and have statements that involve some work (as opposed to just defining individual facts to fill up space). The system must also be entirely your own work, and thus it should also NOT essentially logically duplicate the above examples or the Marcus/Caesar example from class while just changing symbols/subject matter. What you ask to prove should also be possible given your system, and your proof should be done correctly as well.