Design Point

Fixed Wing

Aerospace Design

TÉCNICO LISBOA

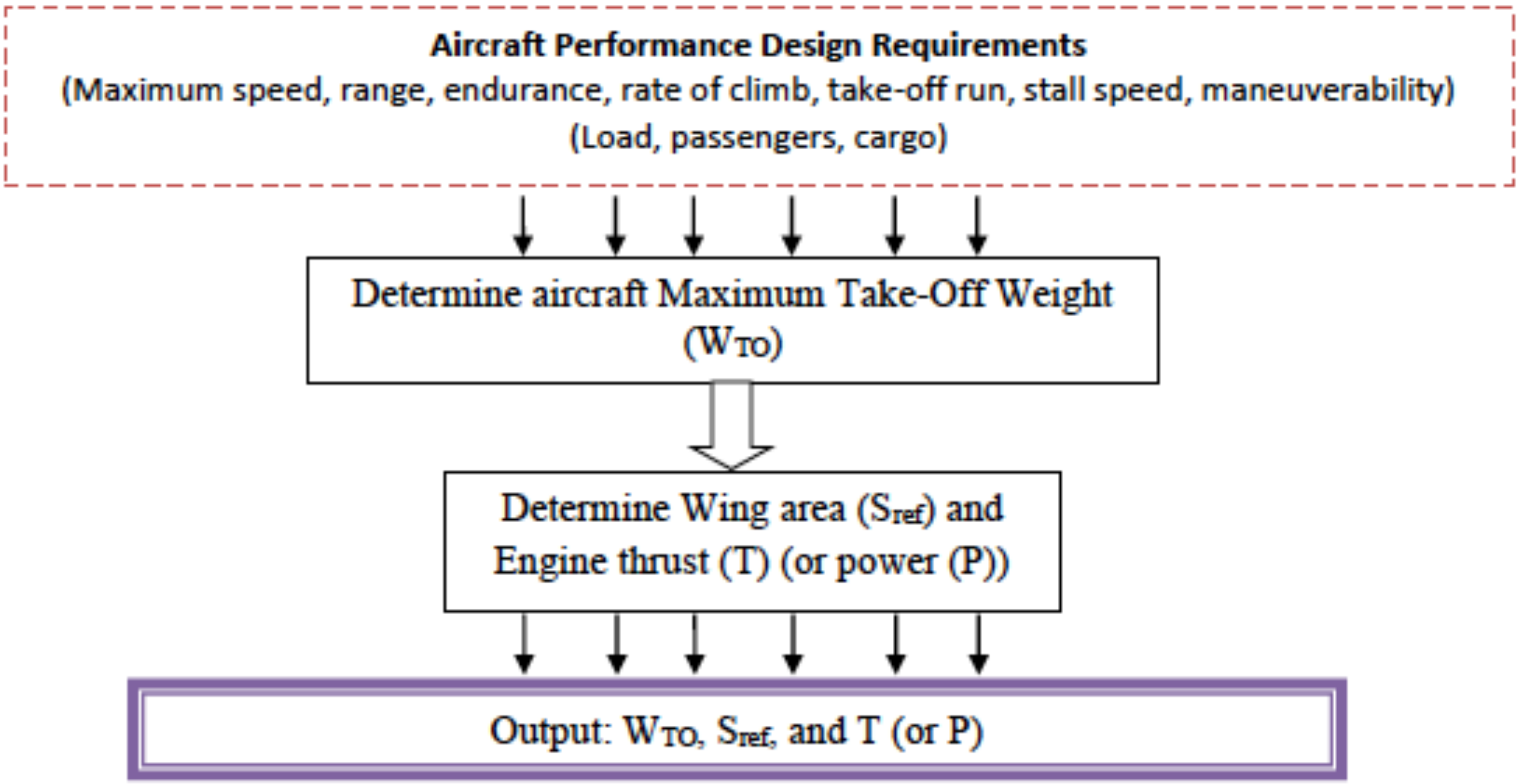
Initial Dimensions

- At this point an initial value for ***MTOW*** was already calculated and the ***fuel weight*** to perform the mission and comply with the requirements was obtained

- Now other important design variables and fractions are going to be estimated and calculated, which are the basis for any design

- The aircraft design can be done using an existing engine (called ***fixed engine*** approach) or a totally new engine design (***elastic engine***), in which the dimensions and performance of the engine are adapted to the design requirements

Preliminary Design Procedure



Thrust-to-Weight Ratio and Wing Loading

- Definition

- ***T/W*** is the ***Thrust-to-Weight Ratio*** (usually the MTOW of the aircraft is used)
- ***P/W*** is the ***Power-to-Weight Ratio***
- ***W/S*** is the ***Wing Loading*** or ***Weight-to-Wing Area Ratio*** (planform area, that is the reference area)

- ***T*** and ***P*** change with ***velocity*** and ***altitude***
- ***W*** changes over time (due to fuel consumption or expendable payload)

Thrust-to-Weight Ratio and Wing Loading

- So the previous ratios change depending on the mission type and within the same mission, depending on the mission phase
- Example:
  - Passenger aircraft designed for cruise with high *W/S*, and flaps for other phases to increase *S*
  - Fighter with low *W/S* for excess lift generation for high *g* manoeuvring

- The objective of this design phase is to obtain the combinations of ***T/W*** (or ***P/W***) and ***W/S*** so that the designed aircraft satisfies all the mission phases and requirements

- Unlike the first step in preliminary design phase at which the main reference was statistics, this phase is solely depending upon the aircraft performance requirements and employs flight mechanics theories. Hence, the technique is an analytical approach and the results are highly reliable without inaccuracy.

- The aircraft performance requirements that are utilized to size the aircraft in this step are:

  - Stall speed (V_s)
  - Maximum speed (V_{max})
  - Maximum rate of climb (ROC_{max})
  - Take-off run (STO)
  - Ceiling (h_c)
  - Turn requirements (turn radius and turn rate)

Thrust-to-Weight Ratio

- Historical *T/W*

Aircraft Type	Typical T/W [N/N]
Fighter jet (air to air combat)	0.80-1.30
Fighter jet (high speed intercept)	0.55-0.80
Fighter jet (close air support)	0.40-0.60
Training jet	0.40
STOL (short take-off and landing)	0.40-0.60
Military transport/bomber	0.25
Civil aircraft (long range)	0.20-0.35
Civil aircraft (short to intermediate range)	0.30-0.45

$T/W_0 = a\,M_{max}^{C}$	a	C
Trainer jet	0.488	0.728
Fighter jet (air to air combat)	0.648	0.594
Fighter jet (general)	0.514	0.141
Military transport/bomber	0.244	0.341
Civil aircraft	0.267	0.363

- ***Low T/W*** means ***efficiency*** in cruise
- ***High T/W*** are required for maneuvering but means ***inefficiency*** in cruise

Power-to-Weight Ratio

- Historical *P/W*

Aircraft Type	Typical P/W [W/N]
Motorized glider	7
Homebuilt	13
General aviation (single engine)	12
General aviation (multi engine)	30
Agriculture aircraft	15
Twin turboprop	33
Hydroplane	16

$P/W_0 = a\,V_{max}{}^C$ [W/N] **with** V_{max} [km/h]	a	C
Motorized glider	7.24	0.00
Homebuilt (metal/wood)	0.61	0.57
Homebuilt (composite)	0.51	0.57
General aviation (single engine)	3.67	0.22
General aviation (twin engine)	4.89	0.32
Agriculture aircraft	1.02	0.50
Twin turboprop	1.63	0.50
Hydroplane	4.38	0.23

- An equivalent T/W for propellered aircraft can be expressed as

$$\frac{T}{W} = \frac{\eta_p}{V}\frac{P}{W}$$

Thrust-to-Weight Ratio

- Thrust Matching

- For aircraft designed primarily for efficiency during cruise, ***thrust matching*** is a better estimate for the required ***T/W***
- In level steady state flight, the thrust must equal the drag

$$\left(\frac{T}{W}\right)_{\text{cruise}} = \frac{1}{\left(L/D\right)_{\text{cruise}}}$$

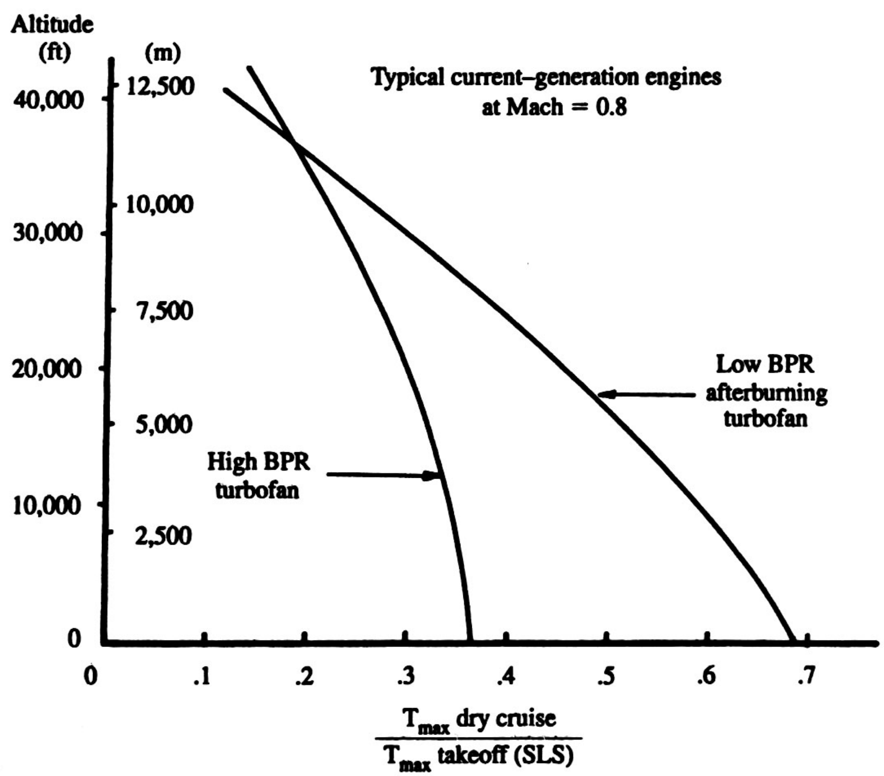
- The take-off *T/W* required for cruise matching can be approximated using

$$\left(\frac{T}{W}\right)_{\text{take-off}} = \left(\frac{T}{W}\right)_{\text{cruise}} \left(\frac{W_{\text{cruise}}}{W_{\text{take-off}}}\right) \left(\frac{T_{\text{take-off}}}{T_{\text{cruise}}}\right)$$

Thrust-to-Weight Ratio

- Thrust Matching

- The thrust ratio between take-off and cruise conditions should be obtained from actual engine data if possible



Thrust lapse at cruise

- Historical *W/S*

Aircraft Type	Typical W/S [N/m^2] at take-off
Motorized glider	295
Homebuilt	530
General aviation (single engine)	815
General aviation (twin engine)	1245
Twin turboprop	1910
STOL (short take-off and landing)	3215
Short/medium range jet	4700
Long range	6185
Trainer jet	2395
Fighter jet	3355
Jet transport/bomber/intercept	5745
High altitude	2225

Wing Loading

- Once the weight estimate for the conceptual aircraft is completed for each phase of the flight plan, the next step in the design is the selection of the wing loading
- Wing loading is selected by considering the principle mission objectives of the aircraft
- Wing loading affects stall speed, climb rate, take-off and landing distances and turn performance
- In some cases, the wing loading that optimizes one of these parts has a detrimental effect on another, which requires compromises in wing loading selection
- Alternatives may lead to variable wing geometries such as flap extensions used at take-off and landing

Stall Speed

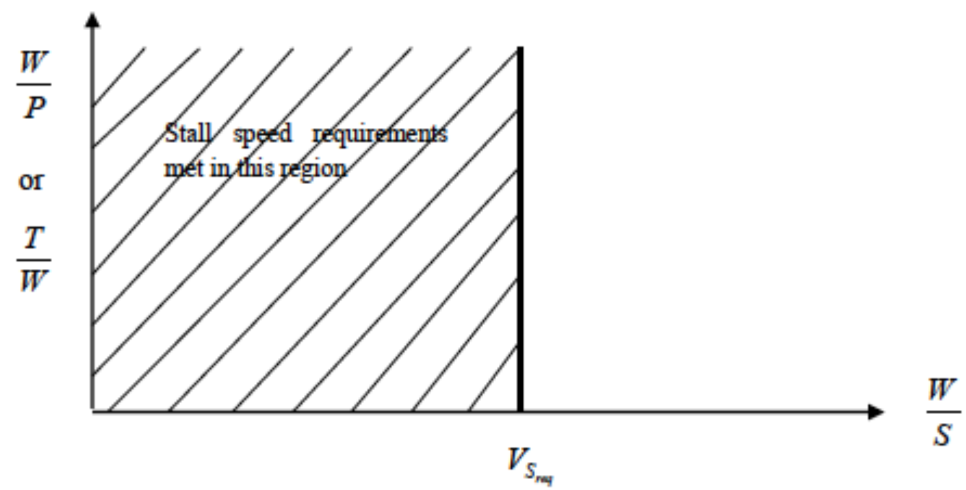
- The stall speed of an aircraft is directly determined by the wing loading and the maximum lift coefficient

$$W = L = q_{\text{stall}} S C_{L\text{max}} = \frac{1}{2}\rho V_{\text{stall}}^2 S C_{L\text{max}}$$

$$\frac{W}{S} = \frac{1}{2}\rho V_{\text{stall}}^2 C_{L\text{max}}$$

Mission Requirement	$\left(C_{L\text{max}}\right)$
Long range	1.6-2.2
Short/medium range	1.6-2.2
STOL (short take-off and landing)	3.0-7.0
Light civil	1.2-1.8
Combat fighter	1.4-2.0

Maximum lift coefficient trends

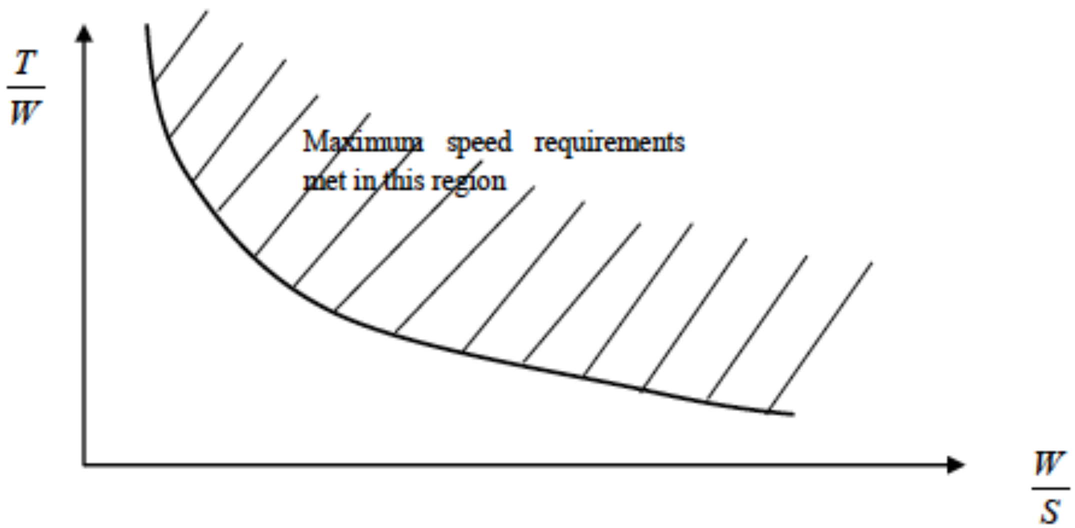


Low stall speed is desirable

Maximum Speed

- Important for fighter jets

$$\left(\frac{T}{W}\right)=\frac{aV_{\max}^{2}}{\left(\frac{W}{S}\right)}+\frac{b}{V_{\max}^{2}}\left(\frac{W}{S}\right)$$



Take-off Distance

- The velocity required for take-off is defined as

$$V_{\text{take-off}} = 1.2V_{\text{stall}} = 1.2\sqrt{\left(\frac{W}{S}\right)_{\text{take-off}} \frac{2}{\rho C_{L_{\max}}}}$$

- The take-off parameter (TOP) correlates the take-off distance for a wide range of aircraft

$$\text{TOP} = \left(\frac{W}{S}\right)_{\text{take-off}} \frac{1}{C_{L_{\text{take-off}}}} \left(\frac{W}{T}\right)_{\text{take-off}} \frac{1}{\sigma}$$

$$\left(\frac{W}{S}\right)_{\text{take-off}} = \text{TOP}\,\sigma\, C_{L_{\text{take-off}}} \left(\frac{T}{W}\right)_{\text{take-off}}$$

- Where

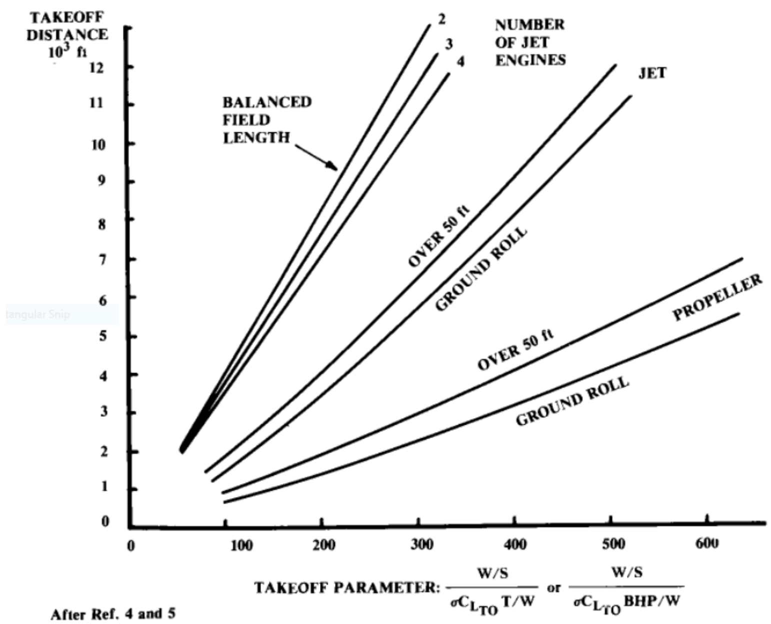
$$\sigma = \frac{\rho_{\text{take-off}}}{\rho_{\text{sea level}}}$$

Take-off Distance

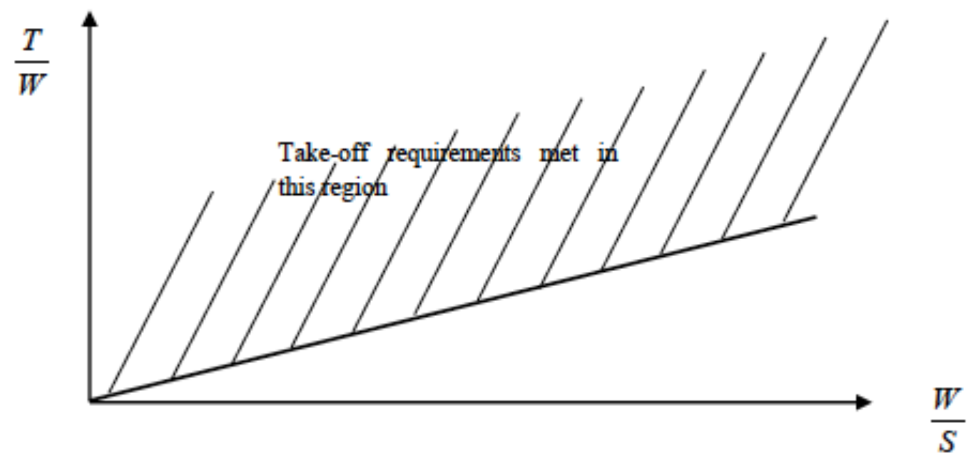
- An empirical estimate of the take-off distance is

$$
s_{\text{take-off}} = \begin{cases} 20.9\,\text{TOP} + 87\sqrt{\text{TOP}\left(\frac{T}{W}\right)}, & [\text{ft}] \\ 1.3\,\text{TOP} + 21.7\sqrt{\text{TOP}\left(\frac{T}{W}\right)}, & [\text{m}] \end{cases}
$$

$$
\left(\frac{T}{W}\right)_{S_{TO}} = \frac{\mu - \left(\mu + \frac{C_{D_G}}{C_{L_R}}\right)\left[\exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)\right]}{1 - \exp\left(0.6\rho g C_{D_G} S_{TO} \frac{1}{W/S}\right)}
$$



Take-off distance and parameter trends



Landing Distance

- The landing parameter relates the wing loading to the landing distance

$$\mathrm{LP} = \left(\frac{W}{S}\right)_{\text{landing}} \frac{1}{\sigma\, C_{L_{\max}}}$$

$$\left(\frac{W}{S}\right)_{\text{landing}} = \mathrm{LP}\, \sigma\, C_{L_{\max}}$$

- An empirical estimate of the take-off distance is

$$s_{\text{landing}} = \begin{cases} 80\,\mathrm{LP} + S_a, & [\mathrm{ft}] \\ 5\,\mathrm{LP} + S_a, & [\mathrm{m}] \end{cases}$$

Aircraft Type	S_a [m]	S_a [m]
Airliner (3 deg glidescope)	305	1000
General aviation power-off approach	183	600
STOL (7 deg glidescope)	137	450

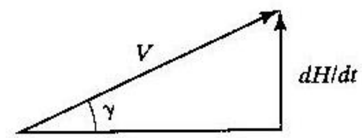
Obstacle clear distance for typical aircraft types

Wing Loading

Rate of Climb

- The rate of climb of an aircraft is the vertical velocity given as

$$\frac{dH}{dt} = V\sin\gamma = \frac{P_{\text{excess}}}{1 + \frac{V}{g}\frac{dV}{dH}} \qquad P_{\text{excess}} = V\frac{(T-D)}{W}$$



- If the aircraft climbs at a constant speed then the climb gradient *G* is given by

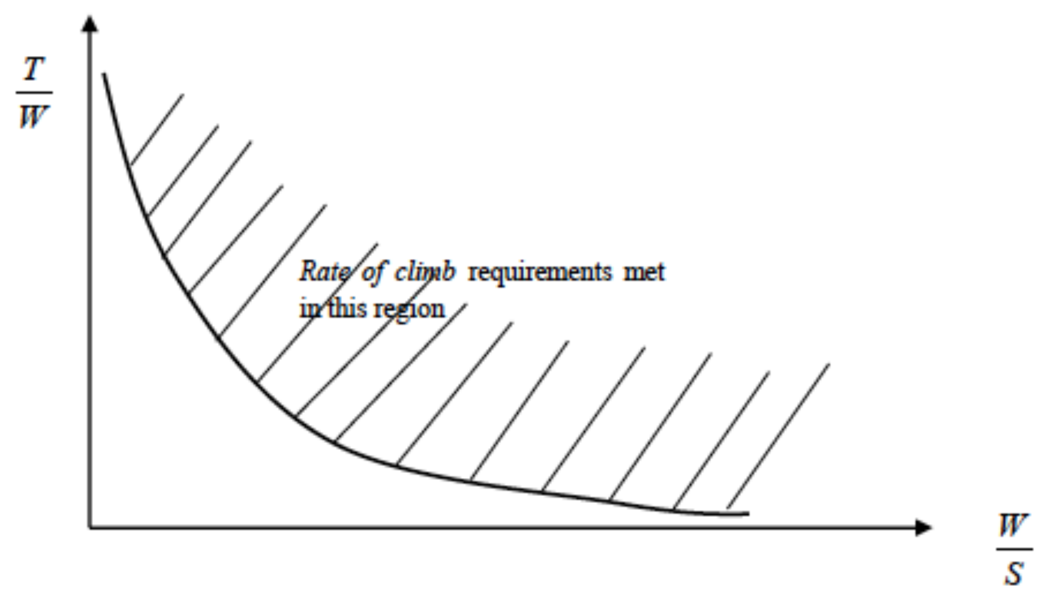
$$\frac{dV}{dH} = 0 \qquad G = \sin\gamma = \frac{(T-D)}{W} \quad \text{or} \quad \frac{D}{W} = \frac{T}{W} - G$$

- *D/W* can also be expressed as

$$\frac{D}{W} = \frac{qSC_{D_0} + qS\left(\frac{C_L^2}{\pi Ae}\right)}{W} = \frac{qC_{D_0}}{\left(\frac{W}{S}\right)} + \left(\frac{W}{S}\right)\frac{1}{q\pi Ae} \quad \text{with} \quad C_L = \frac{W}{qS}$$

Rate of Climb (Jet Aircraft)

$$\left(\frac{T}{W}\right)_{ROC} = \frac{ROC}{\sqrt{\dfrac{2}{\rho\sqrt{\dfrac{C_{D_o}}{K}}}\left(\dfrac{W}{S}\right)}} + \frac{1}{(L/D)_{max}}$$



Rate of Climb

- FAR or military specifications specify the rate of climb for different aircraft types, under different conditions

Aircraft Type	MIL-C5011A Military	FAR Part 23 Civil	FAR Part 25 Commercial
Gear Up, AEO	500 ft/min at sea level	300 ft/min at sea level	-
Gear Up, OEI	100 ft/min at sea level	-	3 % at V_{climb}
Gear Down, OEI	-	-	0.5 % at V_{climb}

AEO - all engines operating

OEI - one engine inoperative

Take-off climb rate specifications

Acceleration

- Acceleration is dependent on excess power

$$P_{\text{excess}} = P_{\text{available}} - P_{\text{required}} = \frac{VT}{W} - \frac{VD}{W}$$

- Usually the available power is limited so, in order to maximize the excess power, drag must be minimized

$$D = qS\left[C_{D_0} + kC_L^2\right]$$

- Introducing the load factor we obtain

$$\frac{D}{W} = q\frac{S}{W}\left[C_{D_0} + k\left(\frac{n}{q}\frac{W}{S}\right)^2\right] \quad \text{with} \quad n = \frac{L}{W} = \frac{C_L qS}{W}$$

Acceleration

- Minimizing *D/W* with respect to *W/S* we obtain

$$\frac{\partial\,(D/W)}{\partial\,(W/S)} = 0 = -\left[C_{D_0} + k\left(\frac{n}{q}\frac{W}{S}\right)^2\right]\left(\frac{W}{S}\right)^{-1} + 2k\left(\frac{W}{S}\right)\left(\frac{n}{q}\right)^2$$

- The condition that satisfies this relation is

$$\frac{W}{S} = \frac{q}{n}\sqrt{\frac{C_{D_0}}{k}}$$

Wing Loading

Range

- To maximize range during cruise, the wing loading should be selected to provide a high *L/D* at the cruise condition
- A ***propeller aircraft***, which loses thrust efficiency as speed increases, gets the maximum range when flying at the ***speed for best L/D***. Therefore, it should fly such that

$$qSC_{D_0} = qS\frac{C_L^2}{\pi Ae} \quad \text{or} \quad \frac{W}{S} = q\sqrt{\frac{C_{D_0}}{k}} = q\sqrt{\pi AeC_{D_0}}$$

- A ***jet aircraft*** maximizes range at a higher speed where ***L/D is slightly reduced***, such that the parasite drag is three times the induced drag. Therefore, it should fly such that

$$\frac{W}{S} = q\sqrt{\frac{C_{D_0}}{3k}} = q\sqrt{\frac{\pi AeC_{D_0}}{3}}$$

Instantaneous turn

- The instantaneous turn rate is the highest turn rate possible, only limited by the maximum lift that the aircraft is able to generate, without maintaining altitude or velocity
- For a level turn, the turn rate is defined as

$$\dot{\psi} = \frac{g\sqrt{n^2 - 1}}{V} \quad \text{where} \quad n = \frac{qC_L}{\left(\frac{W}{S}\right)}$$

- Instantaneous turn rate is limited only by the maximum lift
- The required wing loading is then given by

$$\frac{W}{S} = \frac{qC_{L_{\max}}}{n} \quad \text{where} \quad n = \sqrt{\left(\frac{\dot{\psi}V}{g}\right)^2 + 1}$$

Sustained turn

- The sustained turn rate is the maximum turn rate when thrust and lift are able to compensate drag and weight, so that the flight level and speed are maintained

$$\frac{T}{W} = \frac{D}{W} = q\frac{S}{W}\left[C_{D_0} + \frac{C_L^2}{\pi Ae}\right]$$

- This can be put in terms of the load factor to obtain

$$\frac{T}{W} = \frac{qC_{D_0}}{\left(\frac{W}{S}\right)} + \frac{W}{S}\left(\frac{n^2}{q\pi Ae}\right) \quad \text{where} \quad n = \frac{L}{W} = \frac{C_L qS}{W}$$

Sustained turn

- Solving for the wing loading yields

$$\frac{W}{S} = \frac{\left(\frac{T}{W}\right) \pm \sqrt{\left(\frac{T}{W}\right)^2 - \left(\frac{4n^2 C_{D_0}}{\pi A e}\right)}}{\frac{2n^2}{q\pi A e}}$$

- The term inside the square root cannot go below zero and so

$$\frac{T}{W} \geq 2n\sqrt{\frac{C_{D_0}}{\pi A e}}$$

Wing Loading

Sustained turn

- The maximum sustained load factor is given by

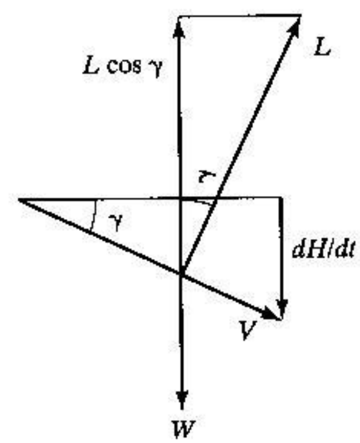
$$n^2_{\max-s} = \frac{q\pi Ae}{\frac{W}{S}}\left[\left(\frac{T}{W}\right)_{\max} - \frac{qC_{D_0}}{\frac{W}{S}}\right]$$

- The maximum sustained turn rate is given by

$$\dot{\psi}_{\max-s} = fracg\sqrt{\frac{q\pi Ae}{\frac{W}{S}}\left[\left(\frac{W}{S}\right)_{\max} - \frac{qC_{D_0}}{\frac{W}{S}}\right] - 1V}$$

Glide

- For a maximum range during a gliding descent, a minimum glide angle is required
- To find the condition that maximizes *L/D*, we find the C_L that minimizes drag



$$D = q\,S\,C_{D_0} + q\,S\left(\frac{C_L^2}{\pi A e}\right) = W\frac{C_{D_0}}{C_L} + W\,k\,C_L \quad \text{with} \quad q = \frac{1}{C_L}\frac{W}{S}$$

- We seek the condition where

$$\frac{\partial D}{\partial C_L} = 0 = \frac{-C_{D_0}}{C_L^2} + k$$

Wing Loading

Glide

- This gives the condition for C_L that minimizes the drag

$$C_{L_{\text{min drag}}} = \sqrt{\frac{C_{D_0}}{k}}$$

- The velocity for minimum drag is given by

$$V_{\text{min drag}} = \sqrt{\frac{2}{\rho C_{L_{\text{min drag}}}} \frac{W}{S}} = \sqrt{\frac{2}{\rho} \frac{W}{S}} \left[\frac{k}{C_{D_0}}\right]^{0.25}$$

- The rate of descent of the aircraft is given by

$$\frac{dH}{dt} = V \sin\gamma = \sin\gamma \sqrt{\frac{W}{S} \frac{2}{\rho \frac{C_L}{\cos\gamma}}} = \sqrt{\frac{W}{S} \frac{2\cos^3\gamma}{\rho \frac{C_L^3}{C_D^2}}} \approx \sqrt{\frac{W}{S} \frac{2}{\rho \frac{C_L^3}{C_D^2}}}$$

Glide

- Substituting the total drag into the previous expression

$$\frac{dH}{dt} = \sqrt{\frac{W}{S}\frac{2}{\rho}\frac{\left(C_{D_0} + kC_L^2\right)^2}{C_L^3}}$$

- The condition for the minimum descent velocity is found by taking the partial derivative with respect to C_L

$$C_{L_{\text{min descent}}} = \sqrt{\frac{3C_{D_0}}{k}}$$

- The minimum descent velocity is given by

$$\left(\frac{dH}{dt}\right)_{\text{min}} = 4\left(\frac{2}{\rho}\frac{W}{S}\right)^{\frac{1}{2}}\left(\frac{k}{3}\right)^{\frac{3}{4}}\left(C_{D_0}\right)^{\frac{1}{4}}$$

Wing Loading

Requirement	Thrust Performance	Observations
Stall speed	$\frac{W}{S} \leq \frac{1}{2}\rho V_{\text{stall}}^2 C_{L\text{max}}$	
Take-off	$\frac{T}{W} \geq \frac{A_1^2}{2C_{L\text{max}}}\left[\frac{2}{\rho g s_{\text{take-off}}}\left(\frac{W}{S}\right) + \left(C_{D_0} - \mu C_L + kC_L^2\right)\right] + \mu$	$A_1 = 1.2$ $0.04 \leq \mu \leq 0.08$
Climb gradient	$\frac{W}{S} \leq \left\{\frac{dH}{dt} - \left[\left(\frac{T}{W}\right)k_2 - \frac{\rho C_{D_0}}{2}k_2^3 - \frac{2k}{\rho k_2}\right]\right\}^2$	$k_2 = \sqrt{\frac{1}{3\rho C_{D_0}}\left[\frac{T}{W} + \sqrt{\left(\frac{T}{W}\right)^2 - 12C_{D_0}k}\right]}$
Climb angle	$\frac{T}{W} \geq \sin\gamma + 2\sqrt{C_{D_0}k}$	
Cruise speed	$\frac{T}{W} \geq \frac{\rho V^2 C_{D_0}}{2(\frac{W}{S})} + \frac{2k}{\rho V^2}\left(\frac{W}{S}\right)$	
Range	$\frac{W}{S} = \frac{1}{2}\rho V^2\sqrt{\frac{C_{D_0}}{3k}}$	
Endurance	$\frac{W}{S} = \frac{1}{2}\rho V^2\sqrt{\frac{C_{D_0}}{k}}$	
Maximum ceiling	$\frac{W}{S} \leq \frac{1}{2}\rho V^2\sqrt{\frac{C_{D_0}}{k}}$ and $\frac{W}{S} = \frac{1}{2}\rho V^2 C_L$	
Instantaneous turn rate	$\frac{W}{S} \leq \frac{1}{2}\rho C_{L\text{max}}\left(\frac{g}{\dot{\psi}}\right)^2\frac{n^2-1}{n}$	
Sustained turn rate	$\frac{W}{S} \leq \frac{(\frac{T}{W}) \pm \sqrt{(\frac{T}{W})^2 - 4n^2 C_{D_0}k}}{\frac{4kn^2}{\rho V^2}}$ and $\frac{T}{W} \geq 2n\sqrt{C_{D_0}k}$	
Landing	$\frac{W}{S} \leq \frac{\rho g}{2}\left(s_{\text{landing}} - \frac{h}{\tan\gamma}\right)\left[\frac{2\mu C_{L\text{max}}}{A_2^2} + \left(C_{D_0} - \mu C_L + kC_L^2\right)\right]$	$1.15 \leq A_2 \leq 1.30$ $0.08 \leq \mu \leq 0.40$ $3\,\text{deg} \leq \gamma \leq 5\,\text{deg}$

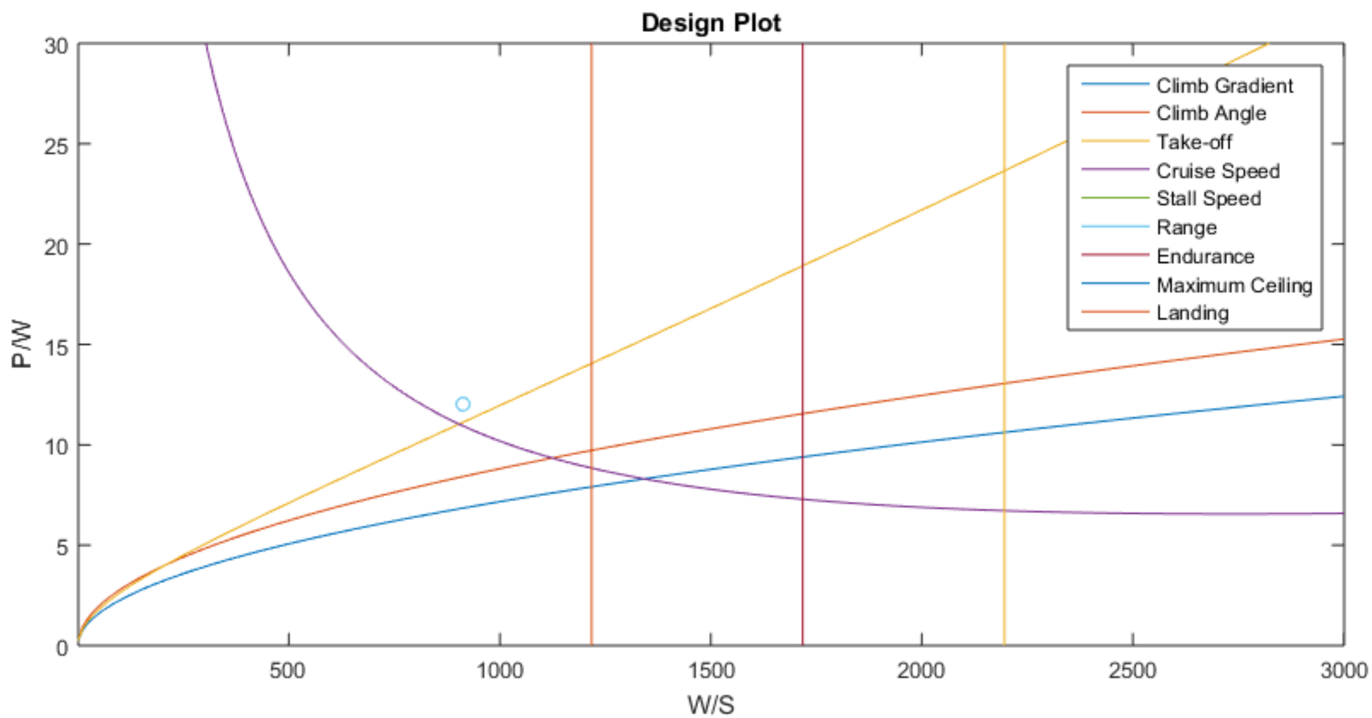
Wing loading requirements for aircraft with jet engine

Wing Loading

Requirement	Power Performance	Observations
Stall speed	$\frac{W}{S} \leq \frac{1}{2}\rho V_{\text{stall}}^2 C_{L\text{max}}$	
Take-off	$\frac{P}{W} \geq \frac{A_1}{\eta_p \sqrt{\rho C_{L\text{max}}}} \sqrt{\frac{W}{S}} \left\{ \frac{A_1^2}{2C_{L\text{max}}} \left[\frac{2}{\rho g s_{\text{take-off}}} \left(\frac{W}{S}\right) + \left(C_{D_0} - \mu C_L + kC_L^2\right) \right] + \mu \right\}$	$A_1 = 1.2$ $0.04 \leq \mu \leq 0.08$ $0.3 \leq \eta_p \leq 0.7$
Climb gradient	$\frac{P}{W} \geq \frac{1}{\eta_p} \left[\frac{dH}{dt} + \frac{\rho V^3 C_{D_0}}{2(\frac{W}{S})} + \frac{2k}{\rho V}\left(\frac{W}{S}\right) \right]$	$\eta_p \approx 0.7$ $V = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{k}{3C_{D_0}}}}$
Climb angle	$\frac{P}{W} \geq \frac{V}{\eta_p} \left[\sin\gamma + \frac{\rho v^2 C_{D_0}}{2(\frac{W}{S})} + \frac{2k}{\rho V^2}\left(\frac{W}{S}\right) \right]$	$\eta_p \approx 0.8$ $V = \sqrt{\frac{2}{\rho}\left(\frac{W}{S}\right)\sqrt{\frac{k}{3C_{D_0}}}}$
Cruise speed	$\frac{P}{W} \geq \frac{1}{\eta_p} \left[\frac{\rho V^3 C_{D_0}}{2(\frac{W}{S})} + \frac{2k}{\rho V}\left(\frac{W}{S}\right) \right]$	$\eta_p \approx 0.8$
Range	$\frac{W}{S} = \frac{1}{2}\rho V^2 \sqrt{\frac{C_{D_0}}{k}}$	
Endurance	$\frac{W}{S} = \frac{1}{2}\rho V^2 \sqrt{\frac{3C_{D_0}}{k}}$	
Maximum ceiling	$\frac{W}{S} \leq \frac{1}{2}\rho V^2 \sqrt{\frac{C_{D_0}}{k}}$ and $\frac{W}{S} = \frac{1}{2}\rho V^2 C_L$	
Instantaneous turn rate	$\frac{W}{S} \leq \frac{1}{2}\rho C_{L\text{max}} \left(\frac{g}{\dot\psi}\right)^2 \frac{n^2-1}{n}$	
Sustained turn rate	$\frac{P}{W} \geq \frac{1}{\eta_p} \left[\frac{\rho V^3 C_{D_0}}{2(\frac{W}{S})} + \frac{2kn^2}{\rho V}\left(\frac{W}{S}\right) \right]$ and $\frac{P}{W} \geq \frac{2nV}{\eta_p}\sqrt{C_{D_0} k}$	$0.7 \leq \eta_p \leq 0.8$
Landing	$\frac{W}{S} \leq \frac{\rho g}{2}\left(s_{\text{landing}} - \frac{h}{\tan\gamma}\right) \left[\frac{2\mu C_{L\text{max}}}{A_2^2} + \left(C_{D_0} - \mu C_L + kC_L^2\right) \right]$	$1.15 \leq A_2 \leq 1.30$ $0.08 \leq \mu \leq 0.40$ $3\,\text{deg} \leq \gamma \leq 5\,\text{deg}$

Wing loading requirements for aircraft with propeller engine

Example



Example of a design space plot and design point selection