Aircraft Sizing

Aerospace Design

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The design methods may be divided into different levels:

- The simplest ones use ***historical data***: for instance the initial weight estimation value may be assumed equal to the weight of the aircraft to be replaced by the present design
- The more elaborate ones use all types of ***software*** as well as ***correlations with wind tunnel testing results***, *etc.*
- Between these lower and higher levels there several other methods for the most part of the design process subtasks – as an introduction, an expedite method will be presented to determine the maximum take off weight from an initial sketch.

Take-off Weight Estimation

- The first step in the conceptual design phase of a new aircraft is to ***obtain an estimate of the take-off weight***
- This is a ***crucial estimate***, since it is used throughout the design
- ***Little is known*** about the aircraft at this point so ***historical trends*** of other aircraft must be used
- The take-off weight can be divided into ***crew weight***, ***fuel weight***, ***payload weight*** and ***empty weight***:

$$W_0 = W_{\text{crew}} + W_{\text{payload}} + W_{\text{fuel}} + W_{\text{empty}}$$

- The payload weight includes ***nonexpendable*** and ***expendable*** weights:

$$W_{\text{payload}} = W_{\text{expendable}} + W_{\text{nonexpendable}}$$

Take-off Weight Estimation

- Fuel and Structure weights may be considered fractions of take-off weight:

$$W_0 = W_{crew} + W_{payload} + \left(\frac{W_{fuel}}{W_0}\right) W_0 + \left(\frac{W_{empty}}{W_0}\right) W_0$$

- Solving in order to obtain the take-off weight:

$$W_0 = \frac{W_{crew} + W_{payload}}{1 - \left(\frac{W_{fuel}}{W_0}\right) - \left(\frac{W_{empty}}{W_0}\right)}$$

Empty Weight Estimation

- A statistical estimation is used from historical data, using the tendency values for the empty weight fraction for different aircraft types
- Exponential representation:

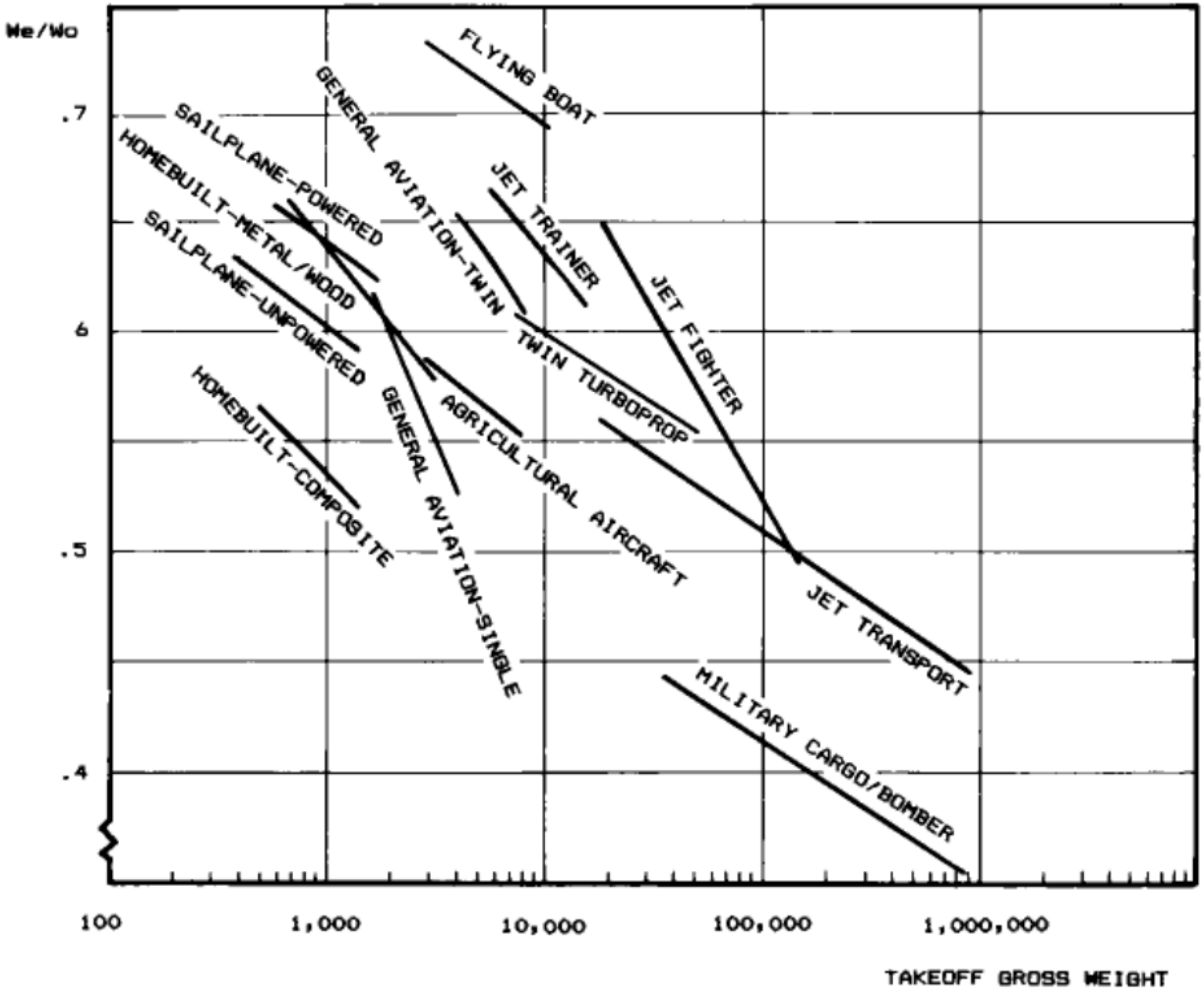
$$\frac{W_{empty}}{W_0} = K\, A\, W_0{}^{C}$$

- Where:

K = 0.95, composite
C = 1.00, others

Aircraft Type	A	C
Sailplane (unpowered)	0.83	-0.05
Sailplane (powered)	0.88	-0.05
Homebuilt (metal/wood)	1.11	-0.09
Homebuilt (composite)	1.07	-0.09
General aviation (single engine)	2.05	-0.18
General aviation (twin engine)	1.40	-0.10
Agricultural aircraft	0.72	-0.03
Twin turboprop	0.92	-0.05
Flying boat	1.05	-0.05
Jet trainer	1.47	-0.10
Jet fighter	2.11	-0.13
Military cargo/bomber	0.88	-0.07
Jet transport	0.97	-0.06

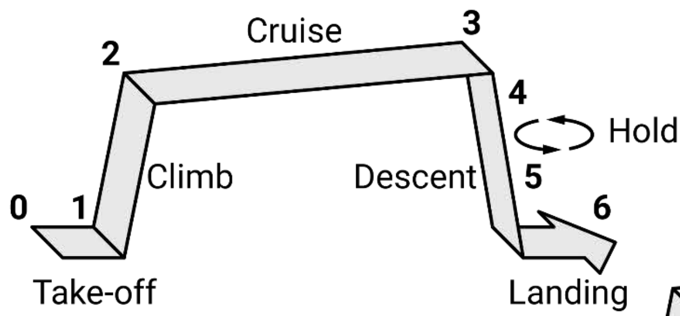
Empty Weight Estimation



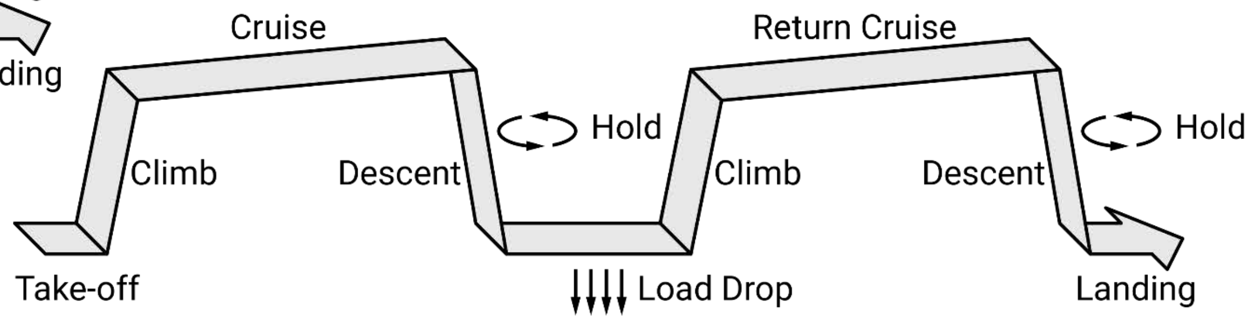
Empty weight fraction trends

Mission Segments

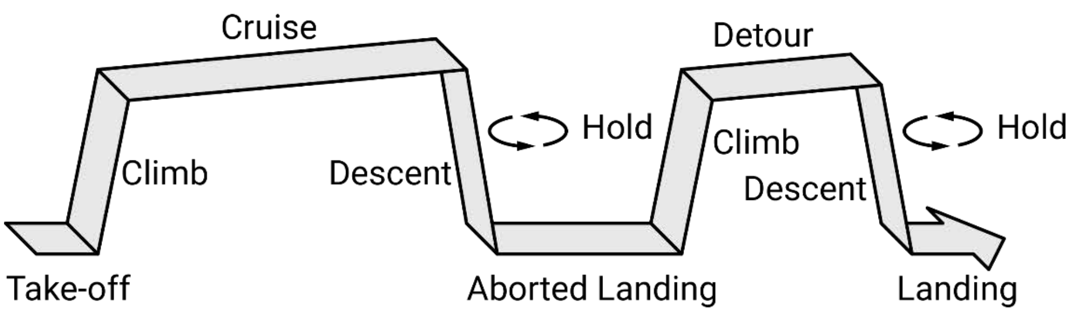
Simple Cruise



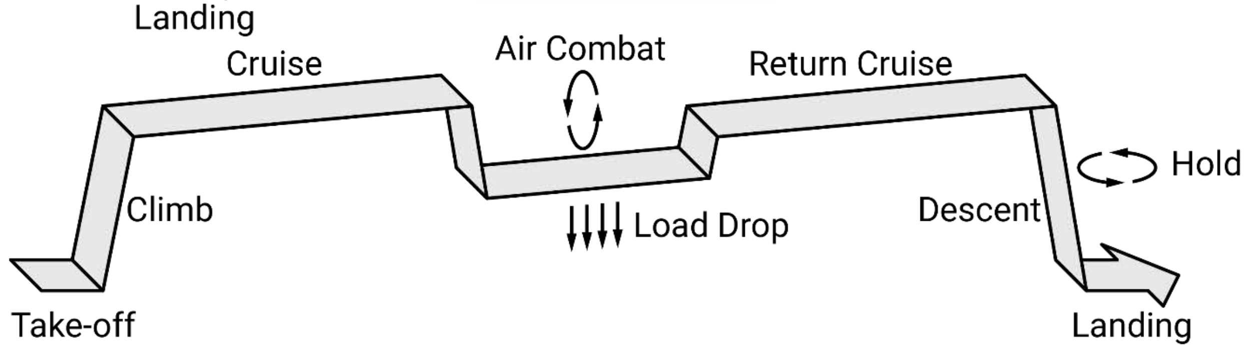
Ground Attack



Transport Aircraft



Air Combat and Ground Attack



Fuel Fraction Estimation

- Fuel fractions correspondent to the different mission segments
- For any mission segment *i*, the mission segment weight fraction can be expressed as:

$$\text{Mission Segment } i \text{ Weight Fraction} = \left(\frac{W_i}{W_{i-1}}\right)$$

Segment	Fuel fraction
1. Engine start-up (taxi) and take-off	0.970
2. Climb and acceleration to cruise	0.985
3. Cruise (range)	Breguet Range Equation
4. Hold or loiter (endurance)	Breguet Endurance Equation
5. Descent (may be included in 3 or 4)	1.000
6. Landing	0.995

Example of segments and fuel fractions of a mission profile

$$\frac{W_7}{W_0} = \left(\frac{W_1}{W_0}\right)\left(\frac{W_2}{W_1}\right)\left(\frac{W_3}{W_2}\right)\left(\frac{W_4}{W_3}\right)\left(\frac{W_5}{W_4}\right)\left(\frac{W_6}{W_5}\right)\left(\frac{W_7}{W_6}\right)$$

Engine start-up (taxi) and take-off

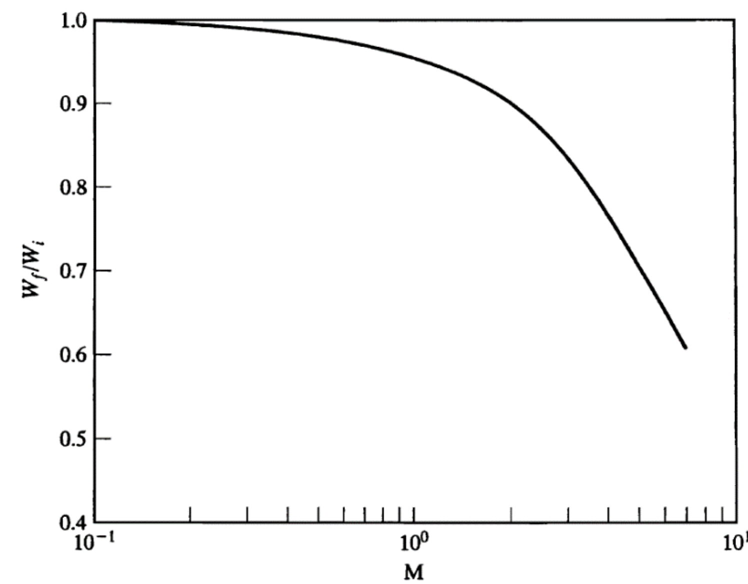
$$0.97 \leq \frac{W_i}{W_{i-1}} \leq 0.975$$

Climb and acceleration to cruise

$$\frac{W_i}{W_{i-1}} = \begin{cases} 1 - 0.04M, & M < 1 \\ 0.96 - 0.03\,(M - 1)\,, & M \geq 1 \end{cases}$$

Acceleration to high speed (intercept)

$$\frac{W_i}{W_{i-1}} = \begin{cases} 1, & M_{i-1} = M_i \\ 1 - 0.04M_i, & M_i < 1 \\ 0.96 - 0.03\,(M_i - 1)\,, & M_i \geq 1 \end{cases}$$



Fuel weight fractions for different Mach numbers during a climb and acceleration

Fuel Fraction Estimation

Cruise (range)

$$\frac{W_i}{W_{i-1}} = \exp \frac{-R\,C}{V\,(L/D)}$$

Hold or loiter (endurance)

$$\frac{W_i}{W_{i-1}} = \exp \frac{-E\,C}{(L/D)}$$

R, range [ft]
E, endurance [s]
C, specific fuel consumption [$\mathrm{lb_{fuel}/lb_{thrust}\ s}$]
V, velocity [ft/s]
L/D, gliding ratio
η_p, propeller efficiency
P, shaft (brake) horsepower [ft lb/s]

For a propeller engine, the equivalent thrust SFC is:

$$C = C_{\text{bhp}} \frac{V}{\eta_p} \qquad \eta_p = \frac{T\,V}{P}$$

C_{bhp} [$\mathrm{lb_{fuel}/bhp\ hr}$] and η_p	Cruise	Loiter
Piston-prop (fixed pitch)	0.4/0.8	0.5/0.7
Piston-prop (variable pitch)	0.4/0.8	0.5/0.8
Turboprop	0.5/0.8	0.6/0.8

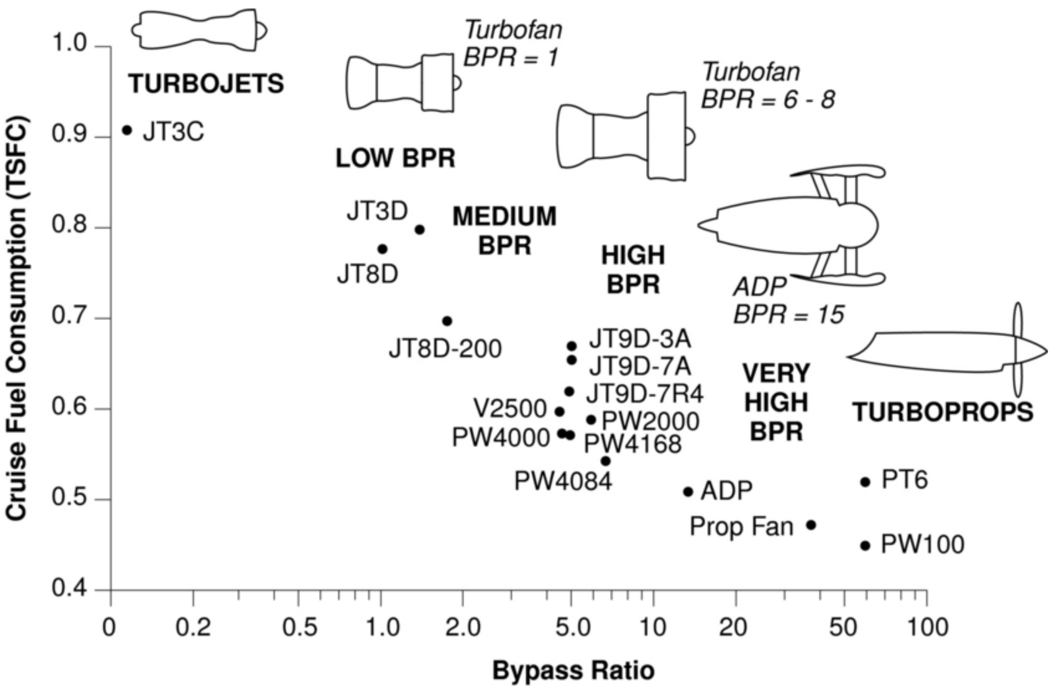
Typical SFC and efficiency of propeller engines

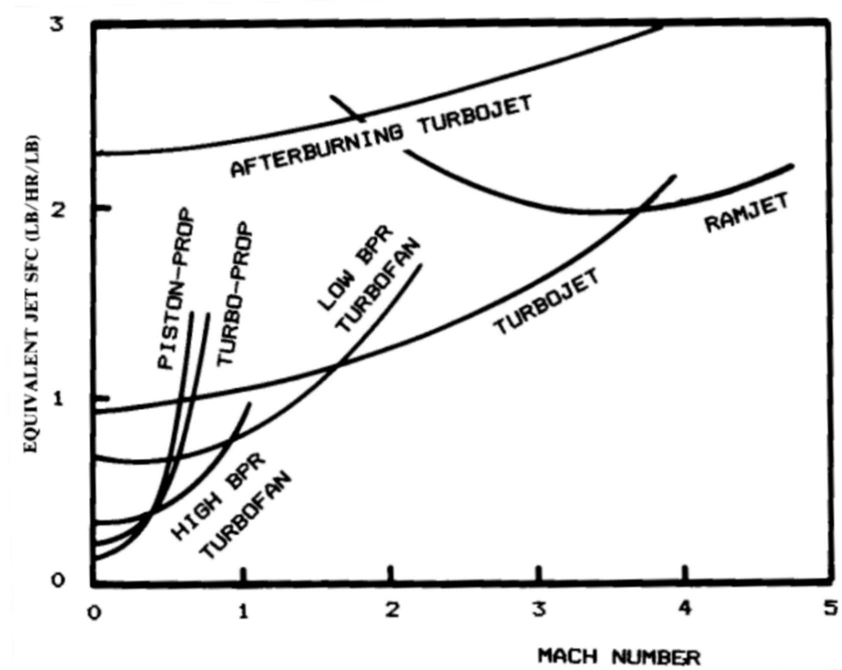
C [$\mathrm{lb_{fuel}/lb_{thrust}\ hr}$]	Cruise	Loiter
Turbojet	0.9	0.8
Low-bypass turbofan	0.8	0.7
High-bypass turbofan	0.5	0.4

Typical SFC of jet engines

Fuel Fraction Estimation

- The specific fuel consumption can be estimated by looking at historical trends, for cruise speed at typical altitudes and for each engine type





Specific fuel consumption trends

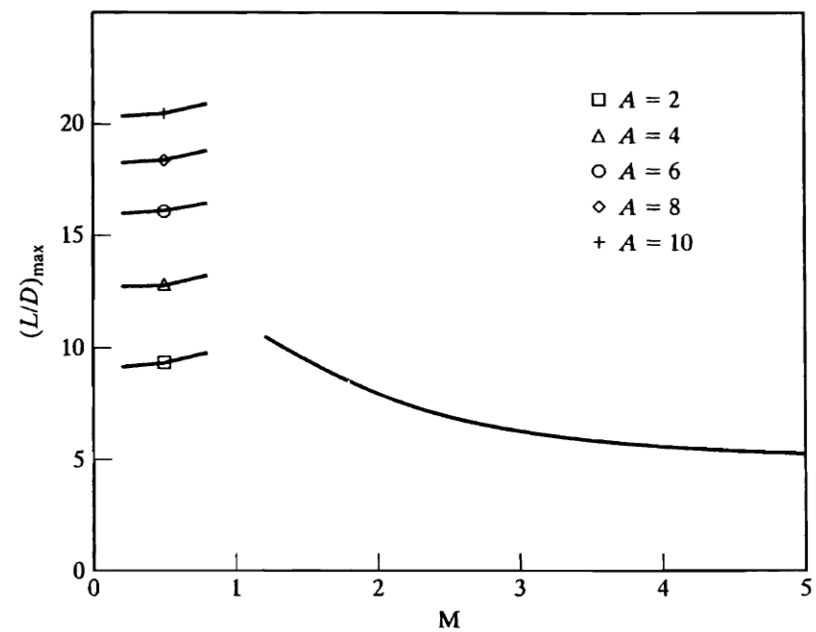
Fuel Fraction Estimation

- Gliding ratio depends on:
  - Induced drag (depends on span and aspect ratio)
  - Parasite drag (depends on wetted area)
  - Wetted aspect ratio
- Thus, gliding ratio strongly depends on the ***aircraft configuration***
- Gliding ratio also depends on the mission phase

(L/D)	**Cruise**	**Loiter**
Jet	$0.866\ (L/D)_{max}$	$(L/D)_{max}$
Propeller	$(L/D)_{max}$	$0.866\ (L/D)_{max}$

Most efficient gliding ratios for different engine types and mission phases

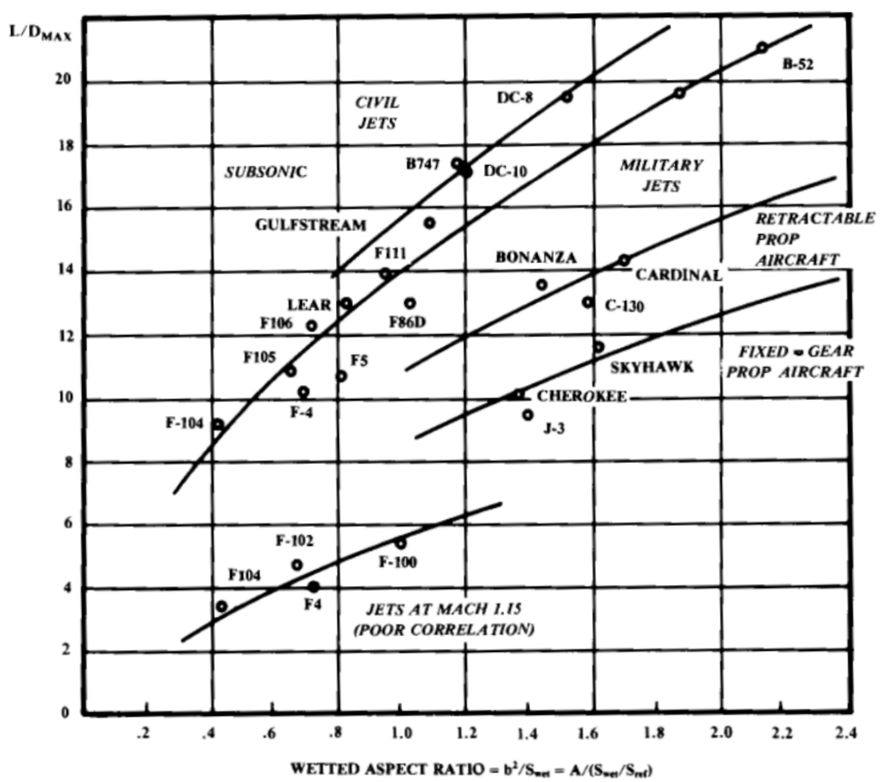
$$(L/D)_{max} = \begin{cases} AR + 10, & M < 1 \\ 11M^{-1/2}, & M \geq 1 \end{cases}$$



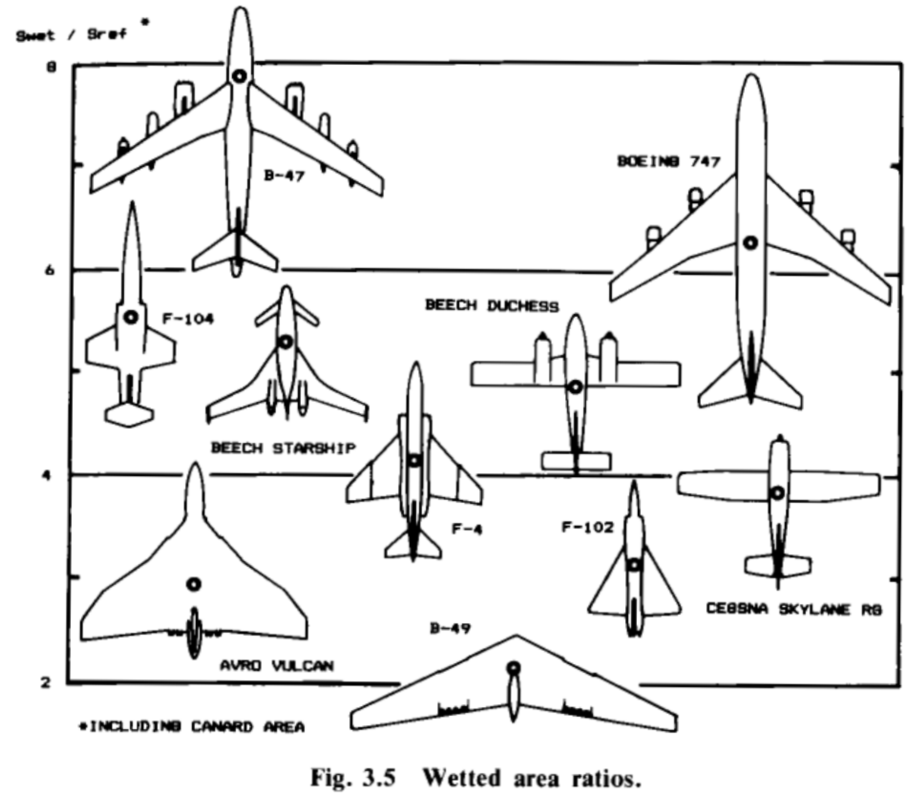
Variation of gliding ratio with Mach number and aspect ratio

Fuel Fraction Estimation

- The maximum gliding ratio can also be initially estimated by looking at historical trends



Maximum gliding ratio trends



Wetted are ratio trends

$$AR = \frac{b^2}{S} \qquad AR_{wet} = \frac{b^2}{S_{wet}} = \frac{A}{S_{wet}/S}$$

Combat

$$W_{i-1} - W_i = C_{\max} T_{\max} t_{\text{combat}}$$

$$\frac{W_i}{W_{i-1}} = 1 - \frac{C_{\max} T_{\max} t_{\text{combat}}}{W_{i-1}}$$

Return cruise (range)

$$\frac{W_i}{W_{i-1}}_{\text{return cruise}} = \frac{W_i}{W_{i-1}}_{\text{cruise}}$$

Descent

$$\frac{W_i}{W_{i-1}} = 1.000$$

Landing

$$0.97 \leq \frac{W_i}{W_{i-1}} \leq 0.975$$

Fuel Fraction Estimation

- Reserve and trapped fuel corrections

$$\begin{aligned} 5\,\%, &\quad \text{reserve fuel} \\ 1\,\%, &\quad \text{trapped fuel} \end{aligned}$$

- Total fuel fraction

$$\frac{W_{\text{fuel}}}{W_0} = 1.06\left(1 - \frac{W_N}{W_0}\right)$$

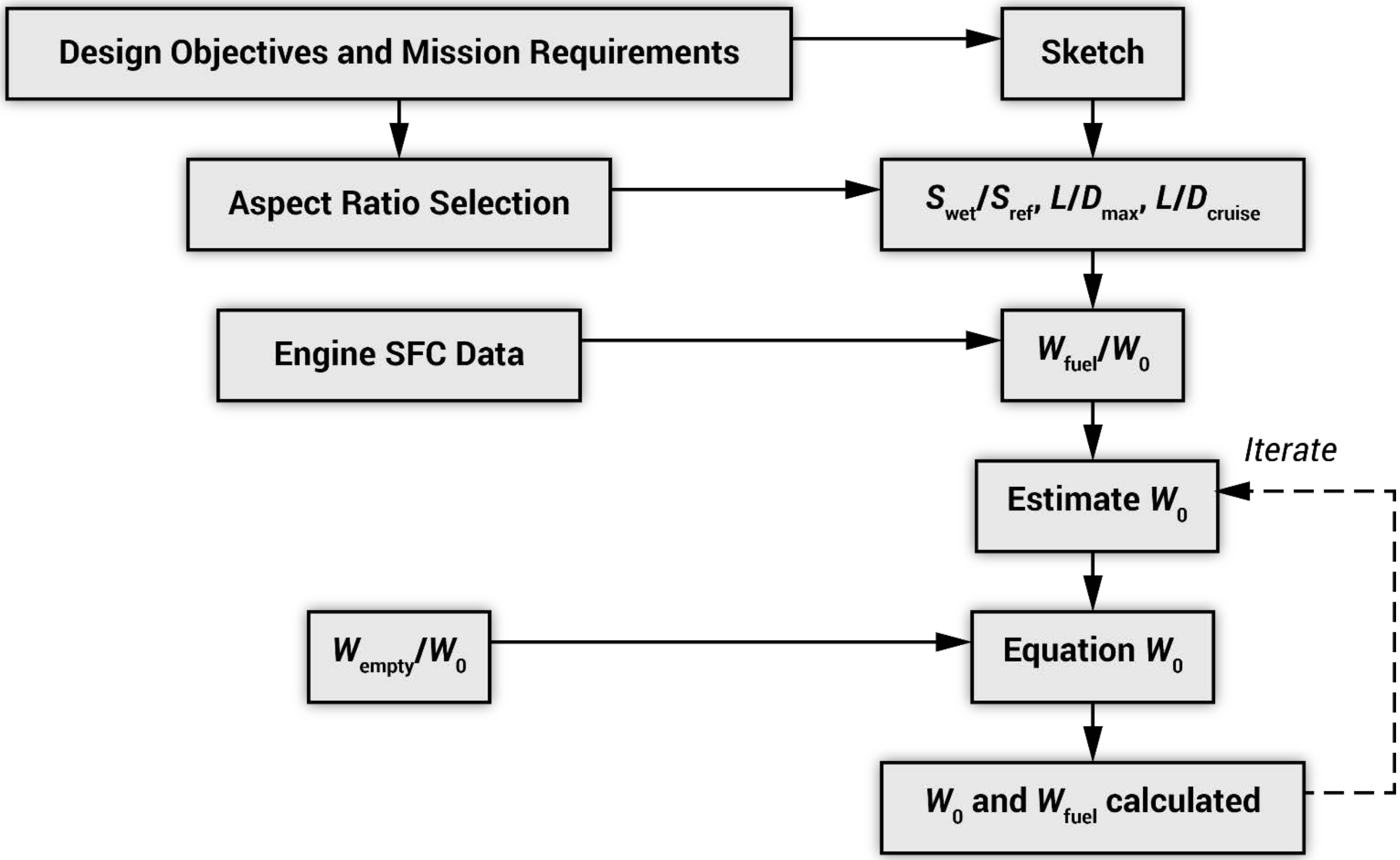
Where N is the number of mission phases and

$$\frac{W_N}{W_0} = \left(\frac{W_n}{W_{n-1}}\right)\cdots\left(\frac{W_i}{W_{i-1}}\right)\cdots\left(\frac{W_1}{W_0}\right)$$

Take-off Weight Estimation

- The take-off weight may be obtained by solving iteratively

$$W_0 = \frac{W_{crew} + W_{payload}}{1 - \left(\frac{W_{fuel}}{W_0}\right) - \left(\frac{W_{empty}}{W_0}\right)} = \frac{W_{crew} + W_{payload}}{1 - \left(\frac{W_{fuel}}{W_0}\right) - K\,A\,W_0^{\,C}}$$

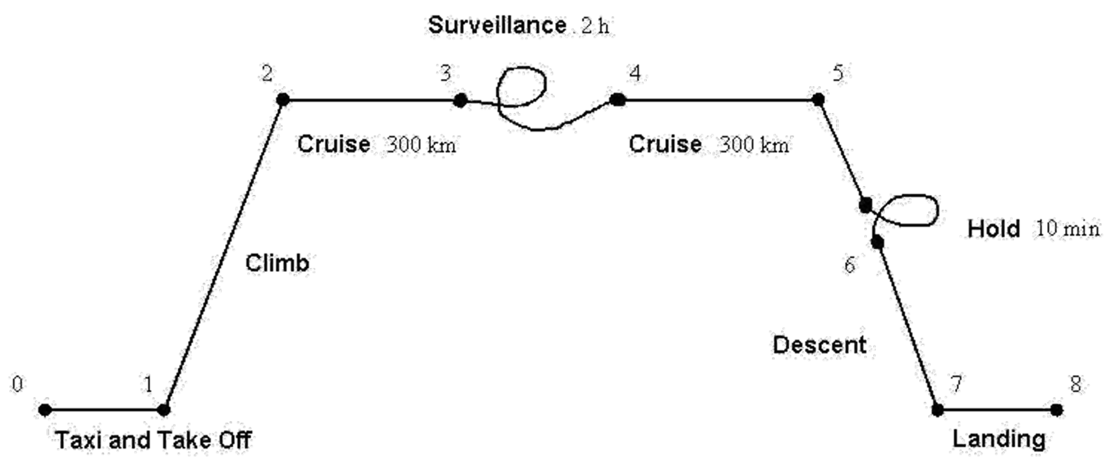


Example: Surveillance Aircraft

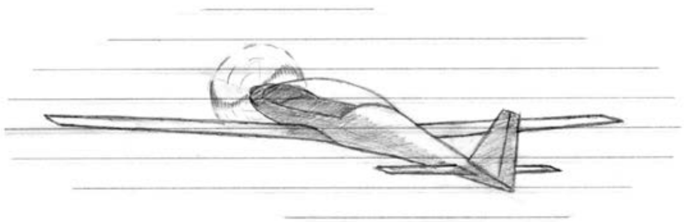
- Mission requirements

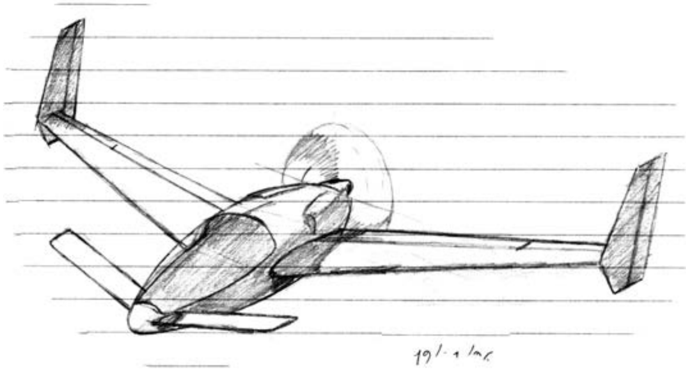
Requirements	
Crew	2 (86 kg each)
Payload	50 kg (camera equipment and parachute)
Cruise speed	180 km/h
Structure	Composite

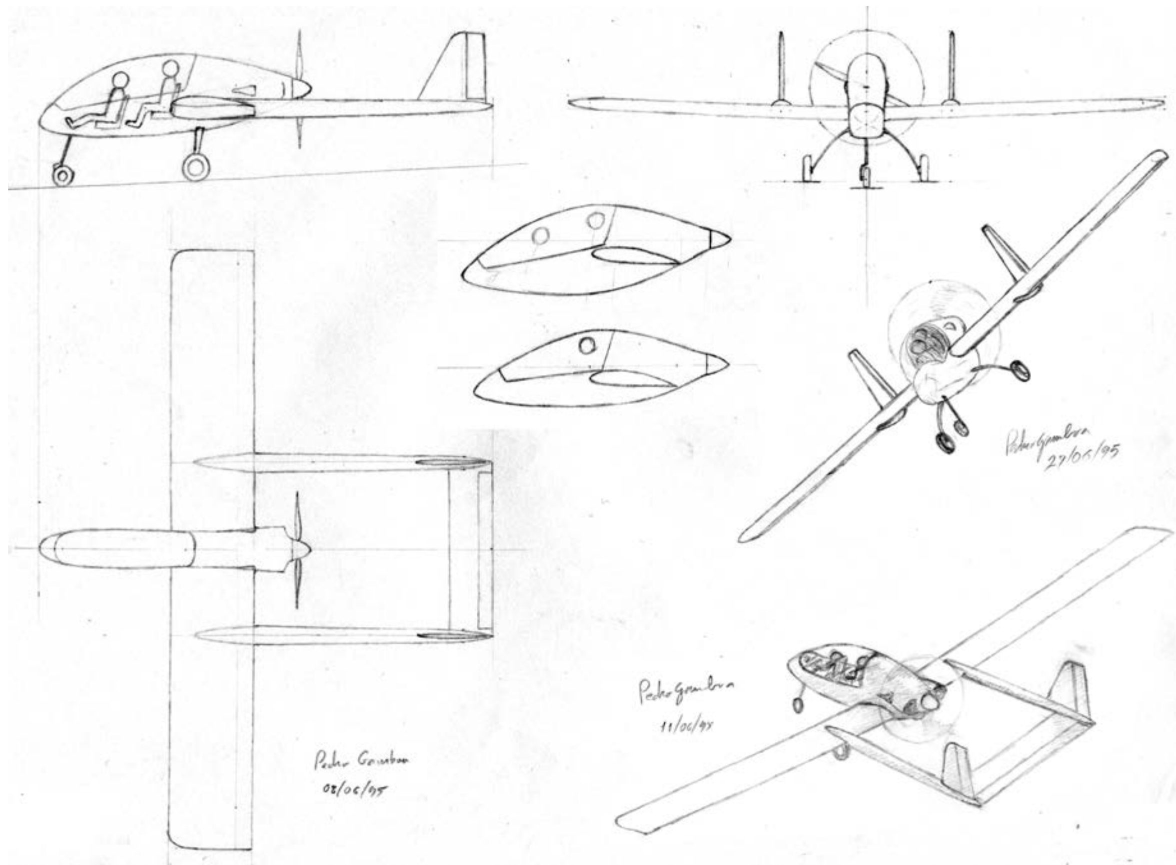
- Mission profile



- Sketches







Example: Surveillance Aircraft

- Glide ratio estimation

$$AR = 8 \qquad S_{wet}/S = 4 \qquad AR_{wet} = \frac{8}{4} = 2 \qquad (L/D)_{max} = 12.5$$

- Mission segment weight fractions
  - Taxi and take-off

$$\frac{W_1}{W_0} = 0.970$$

  - Climb

$$\frac{W_2}{W_1} = 0.985$$

Example: Surveillance Aircraft

- Cruise

$$R = 300\,\text{km} = 0.3 \times 10^6\,\text{m}$$

$$C_{\text{bhp}} = 0.068\,{}^{\text{mg}}\!/_{\text{W s}} = 68.0 \times 10^{-9}\,{}^{\text{kg}}\!/_{\text{W s}}$$

$$V = 180\,{}^{\text{km}}\!/_{\text{h}} = 50\,{}^{\text{m}}\!/_{\text{s}}$$

$$\eta_p = 0.8$$

$$C = C_{\text{bhp}} \frac{V}{\eta_p} = 68.0 \times 10^{-9} \frac{50}{0.8} = 4.25 \times 10^{-6}\,{}^{\text{kg}}\!/_{\text{N s}}$$

$$({}^{L}\!/_{D}) = ({}^{L}\!/_{D})_{\text{max}} = 12.5$$

$$\frac{W_3}{W_2} = \exp \frac{-R\,C\,g}{V\,({}^{L}\!/_{D})} = \exp \frac{(-0.3 \times 10^6)\,(4.25 \times 10^{-6})\,(9.81)}{(50)\,(12.5)} = 0.980$$

Example: Surveillance Aircraft

- Surveillance

$$E = 2\,\text{h} = 7200\,\text{s}$$

$$C_{\text{bhp}} = 0.085\,\text{mg/W s} = 85.0 \times 10^{-9}\,\text{kg/W s}$$

$$V = 130\,\text{km/h} = 36\,\text{m/s}$$

$$\eta_p = 0.7$$

$$C = C_{\text{bhp}} \frac{V}{\eta_p} = 85.0 \times 10^{-9} \frac{36}{0.7} = 4.37 \times 10^{-6}\,\text{kg/N s}$$

$$(L/D) = 0.866\,(L/D)_{\max} = (0.866)\,(12.5) = 10.825$$

$$\frac{W_4}{W_3} = \exp \frac{-E\,C\,g}{(L/D)} = \exp \frac{(-7200)\,(4.37 \times 10^{-6})\,(9.81)}{10.825} = 0.972$$

- Return cruise (equal to previous cruise phase)

$$\frac{W_5}{W_4} = 0.980$$

Example: Surveillance Aircraft

- Hold

$$E = 10\,\text{min} = 600\,\text{s}$$

$$C = 4.37 \times 10^{-6}\ \text{kg/N s}$$

$$(L/D) = 10.825$$

$$\frac{W_6}{W_5} = \exp\frac{-E\,C\,g}{(L/D)} = \exp\frac{(-600)\,(4.37\times 10^{-6})\,(9.81)}{10.825} = 0.998$$

- Descent

$$\frac{W_7}{W_6} = 1.000$$

- Descent

$$\frac{W_7}{W_6} = 0.995$$

- Total

$$\frac{W_8}{W_0} = \left(\frac{W_1}{W_0}\right)\left(\frac{W_2}{W_1}\right)\left(\frac{W_3}{W_2}\right)\left(\frac{W_4}{W_3}\right)\left(\frac{W_5}{W_4}\right)\left(\frac{W_6}{W_5}\right)\left(\frac{W_7}{W_6}\right)\left(\frac{W_8}{W_7}\right) =$$

$$= (0.970)\,(0.985)\,(0.980)\,(0.972)\,(0.980)\,(0.998)\,(1.000)\,(0.995) =$$

$$= 0.886$$

Example: Surveillance Aircraft

- Fuel weight fraction

$$\frac{W_{\text{fuel}}}{W_0} = 1.06\left(1 - \frac{W_8}{W_0}\right) = 1.06\,(1 - 0.886) = 0.121$$

- Empty weight fraction

$$\frac{W_{\text{empty}}}{W_0} = (0.95)\,(2.05)\,W_0^{-0.18} = 1.95\,W_0^{-0.18}$$

- Take-off weight

$$W_0 = \frac{W_{\text{crew}} + W_{\text{payload}}}{1 - \left(\frac{W_{\text{fuel}}}{W_0}\right) - \left(\frac{W_{\text{empty}}}{W_0}\right)} = \frac{(2)\,(86) + 50}{1 - 0.121 - 1.95\,W_0^{-0.18}}$$

$$= \frac{222}{0.879 - 1.95\,W_0^{-0.18}}$$

Example: Surveillance Aircraft

- The take-off weight may be obtained by solving the previous equation iteratively

W_0 **initial** [N]	$\frac{W_{\text{empty}}}{W_0}$	W_0 **final** [N]
600	0.617	847
800	0.585	755
780	0.588	763
765	0.590	768
768	0.590	**768**

$$\begin{aligned} W_{\text{crew}} &= 172\,\text{N} \\ W_{\text{payload}} &= 50\,\text{N} \\ W_{\text{fuel}} &= 93\,\text{N} \\ W_{\text{empty}} &= 453\,\text{N} \\ W_0 &= 768\,\text{N} \end{aligned}$$

Example: Surveillance Aircraft

- Parametric studies
  - To better understand the effect of chosen parameters on the take-off weight, parametric studies may be developed

  - Surveillance time influence

W_0 **initial** [N]	$\frac{W_{empty}}{W_0}$	W_0 **final** [N]
742	0.593	**742**

$$E = 1\,\text{h} = 3600\,\text{s}$$

$$\frac{W_4}{W_3} = \exp\frac{(-3600)\,(4.37\times10^{-6})\,(9.81)}{10.825} = 0.986$$

$$\frac{W_8}{W_0} = \frac{(0.886)\,(0.986)}{0.972} = 0.899$$

$$\frac{W_{fuel}}{W_0} = 1.06\,(1-0.899) = 0.107$$

$$W_0 = \frac{222}{1-0.107-1.95\,W_0{}^{-0.18}}$$

$$\begin{aligned} W_{crew} &= 172\,\text{N} \\ W_{payload} &= 50\,\text{N} \\ W_{fuel} &= 80\,\text{N} \\ W_{empty} &= 440\,\text{N} \\ W_0 &= 742\,\text{N} \end{aligned}$$

Example: Surveillance Aircraft

- Surveillance time influence

W_0 **initial** [N]	$\frac{W_{\text{empty}}}{W_0}$	W_0 **final** [N]
794	0.586	**794**

$$W_{\text{crew}} = 172\,\text{N}$$
$$W_{\text{payload}} = 50\,\text{N}$$
$$W_{\text{fuel}} = 107\,\text{N}$$
$$W_{\text{empty}} = 405\,\text{N}$$
$$W_0 = 794\,\text{N}$$

$$E = 3\,\text{h} = 10800\,\text{s}$$

$$\frac{W_4}{W_3} = \exp\frac{(-10800)\,(4.37\times10^{-6})\,(9.81)}{10.825} = 0.958$$

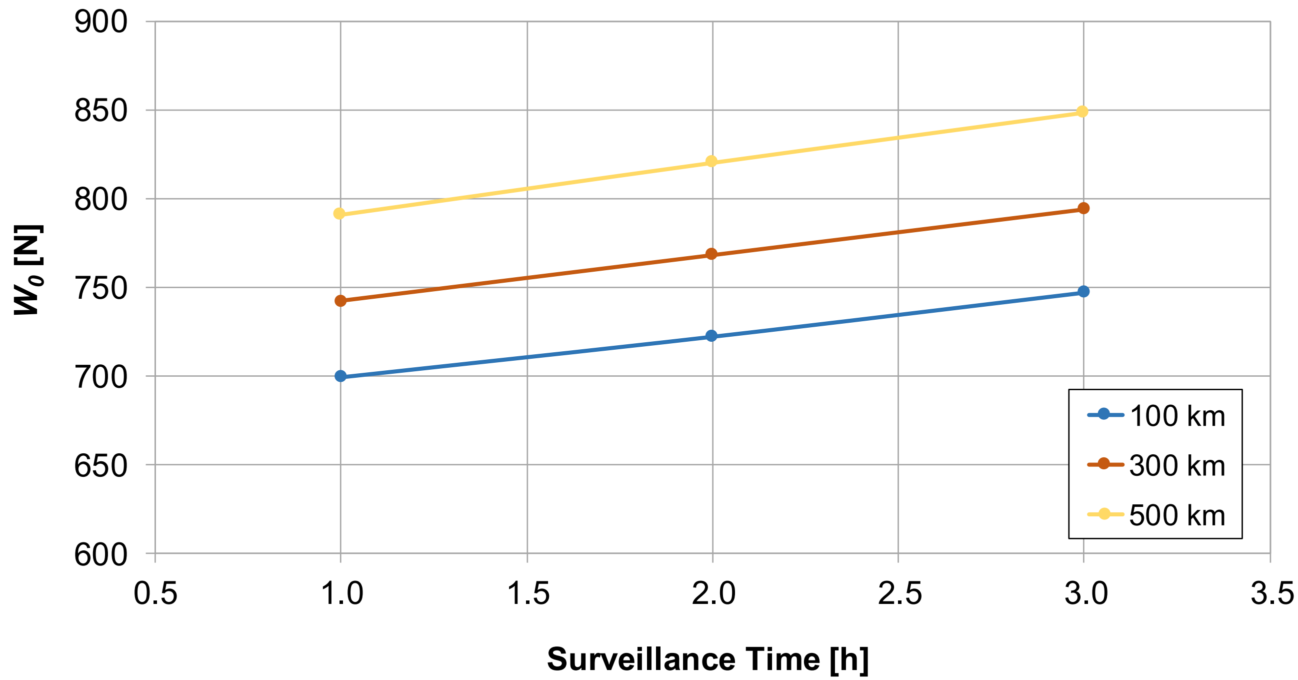
$$\frac{W_8}{W_0} = \frac{(0.886)\,(0.958)}{0.972} = 0.873$$

$$\frac{W_{\text{fuel}}}{W_0} = 1.06\,(1-0.873) = 0.135$$

$$W_0 = \frac{222}{1-0.135-1.95\,W_0{}^{-0.18}}$$

Example: Surveillance Aircraft

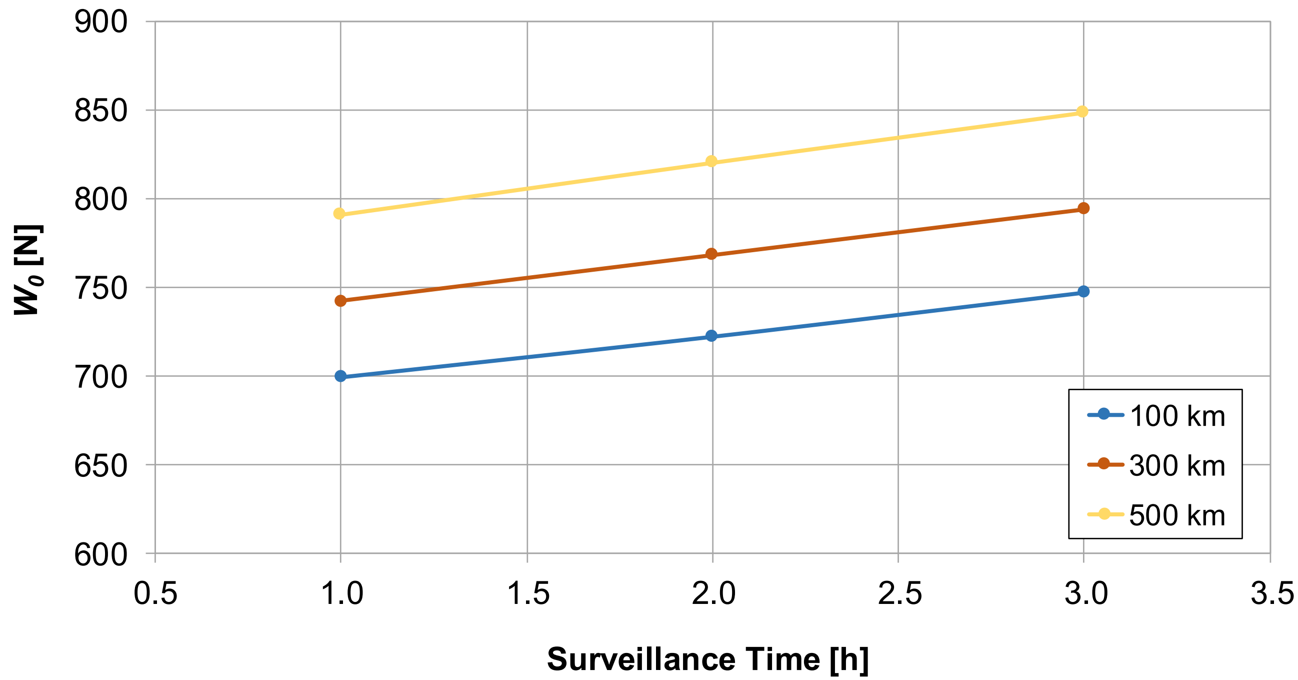
- Cruise distance influence



Influence of range on take-off weight

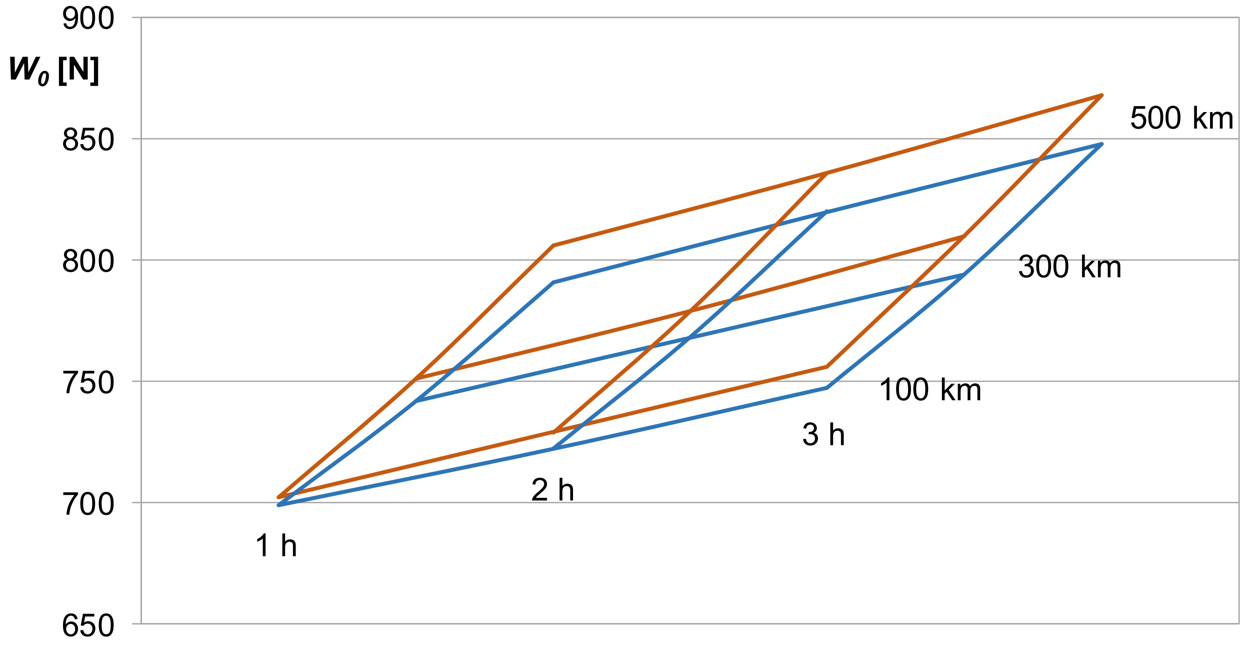
Example: Surveillance Aircraft

- Cruise distance influence



Influence of range on take-off weight

Example: Surveillance Aircraft



Influence of range and endurance on take-off weight

- The influence of other parameters may also be analysed:
  - Payload
  - Aspect Ratio
  - Configuration
  - *etc.*