Wing Design

Aerospace Design

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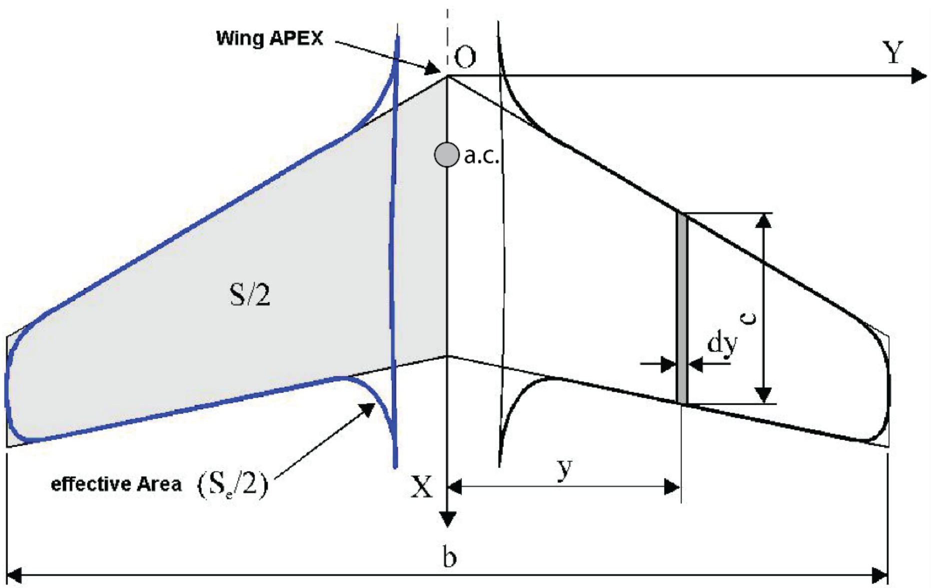
Wing Area and Aerodynamic Center

- ***Wing area*** (or ***planform area***) is the area projected from the top of aircraft
- Usually the wing is assumed to join at centerline
- For simplicity, we can use an equivalent trapezoid to simulate wing
- The aerodynamic center can be thought of the point at which the lift could be said to act if replaced by a point load



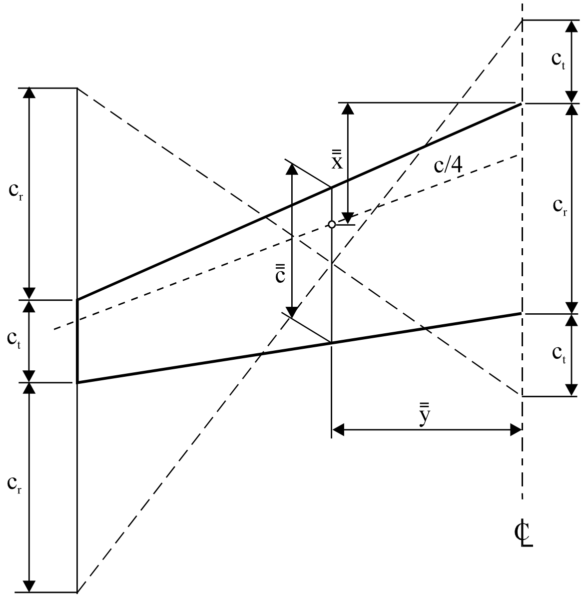
- ***Chord*** is distance from leading edge to trailing edge at some point on the surface
- ***Mean Aerodynamic Chord (MAC)*** is the weighted average of the wing chord based on local pitching moment
- For simplicity, we can use a geometric approximation for the MAC or use a simplified formula

$$\bar{c} = c_r - \frac{2(c_r - c_t)(\frac{c_r}{2} + c_t)}{3(c_r + c_t)}$$

$\bar{c}$, mean aerodynamic chord

c_r, root chord

c_t, tip chord



- ***Wing span*** is the distance from wing tip to wing tip
- ***Wing semi-span*** is the distance from wing tip to wing root
- ***Aspect ratio*** is a measure of how skinny the wing is
- ***Wing taper*** is the ratio of the tip chord to the root chord

$$AR = \frac{b}{\bar{c}} = \frac{b^2}{S}$$

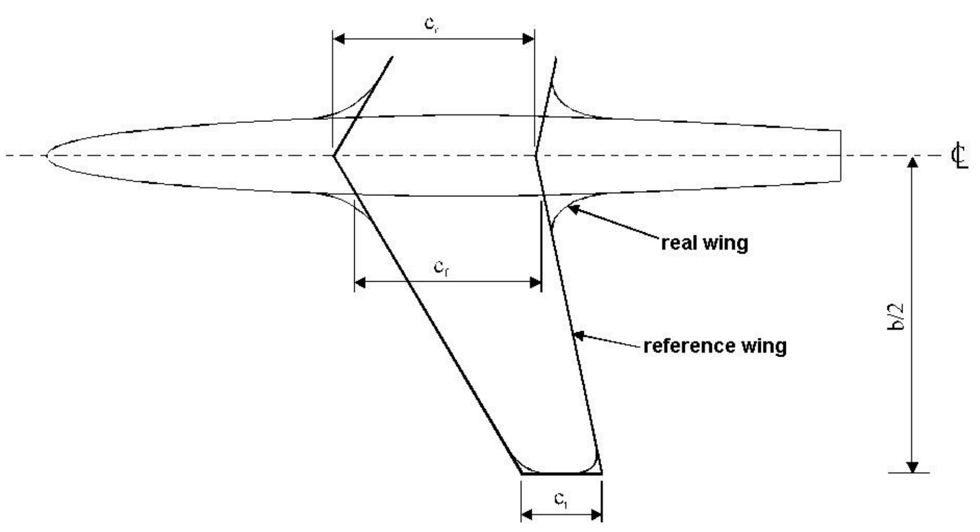
$$\lambda = \frac{c_t}{c_r}$$

b, span

S, planform area

AR, aspect ratio

λ, taper ratio



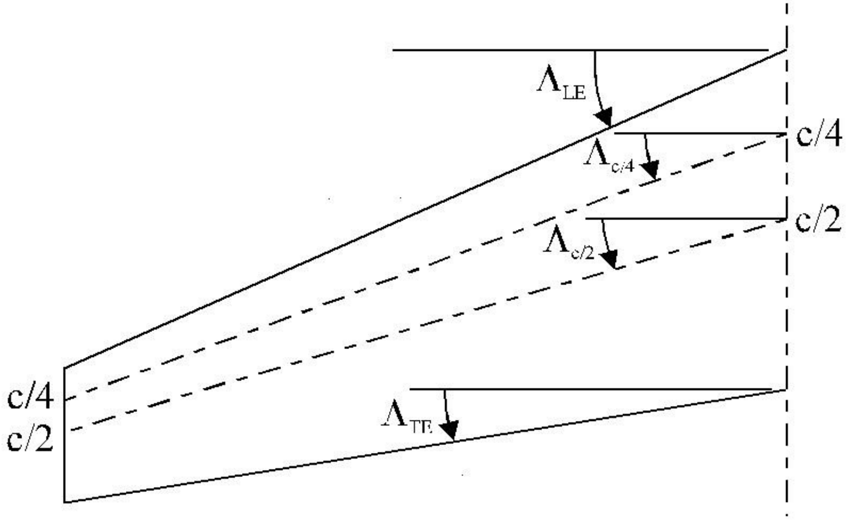
Sweep

- ***Wing sweep*** is the angle that a wing is raked back at
- If the wing is tapered, the sweep will differ from the leading edge to the trailing edge
- Usually we use the sweep of the leading edge or of a line that connects the quarter chord at all stations
- A conversion between the two is

$$\tan(\lambda_{LE}) = \tan(\lambda_{c/4}) + \frac{1-\lambda}{AR(1+\lambda)}$$

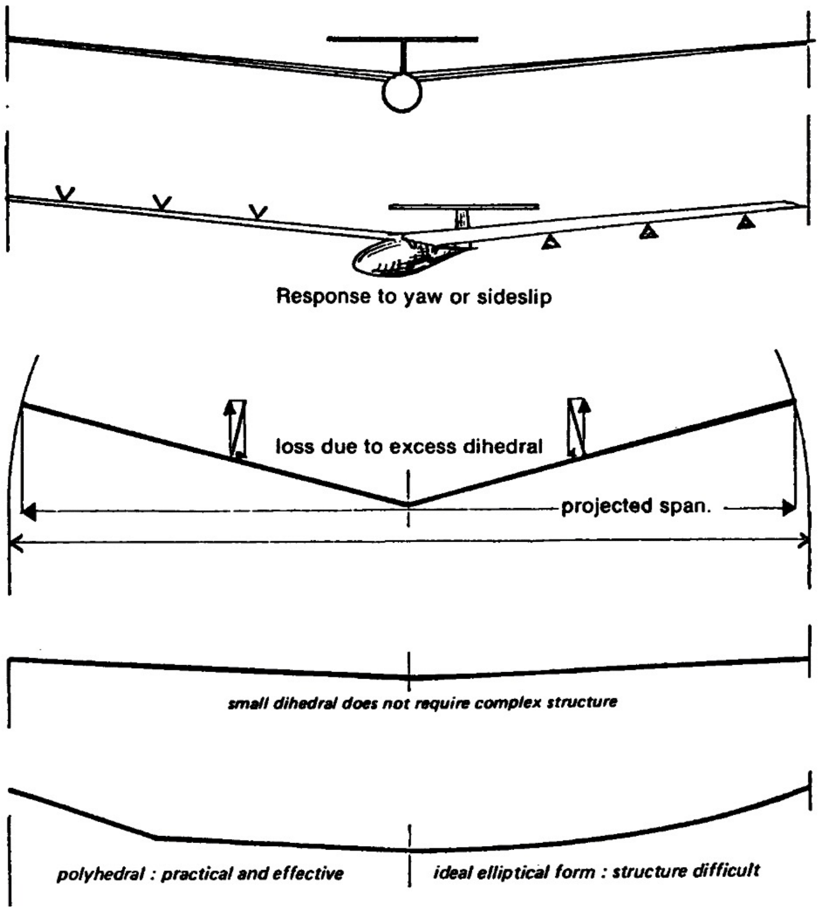
λ_{LE}, leading edge sweep

$\lambda_{c/4}$, quarter chord sweep



Dihedral

- ***Dihedral*** is the angle the wing makes with respect to the horizon when viewed from the front
- Dihedral is used primarily to change the lateral stability of the aircraft
- Large amounts can effect overall lift/drag of wing

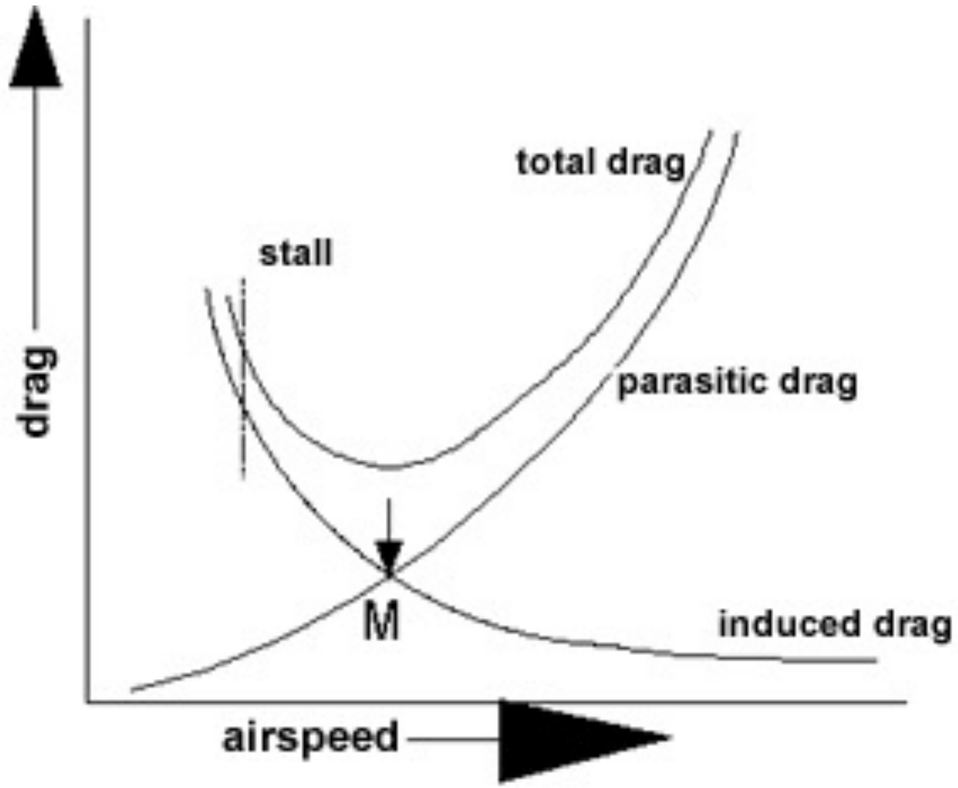


Wing Drag

- Several factors combine to create drag forces on a subsonic wing
    - ***Lift induced drag***
    - ***Form or pressure drag***
    - ***Skin friction or viscous drag***
    - ***Wave Drag***
- Pressure and viscous drag can be minimized through careful choice or airfoils
- Lift induced drag is dependent on your wing geometry
- At high speeds, skin friction and pressure drag are dominant
- At low speeds, it is induced drag that dominates

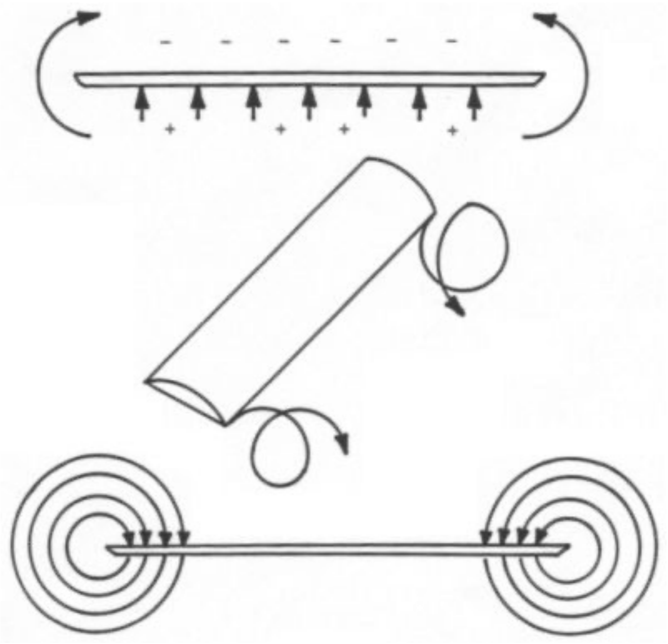
Wing Geometry and Drag

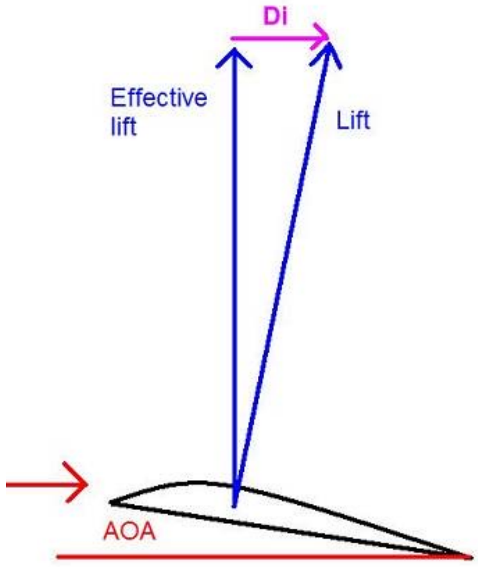
- At high speeds, skin friction and pressure drag are dominant
- At low speeds, it is induced drag that dominates



Finite Wings and Induced Drag

- Recall, at the tips high pressure air from bottom of the wing wants to escape to lower pressure top
- The net effect is a downward component of velocity along the wing





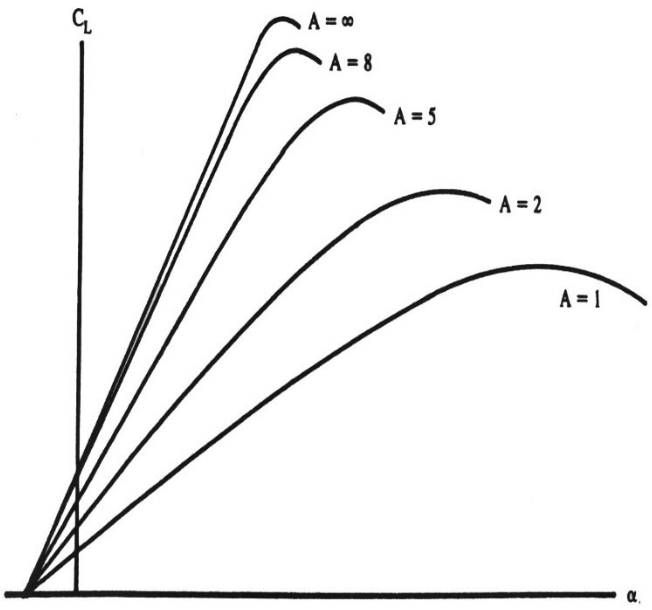
Finite Wings and Induced Drag

- In order to reduce induced drag we must minimize the effect of the downwash
- It can be shown that minimum induced drag occurs when the ***aerodynamic loading across the wing is elliptical***
- The choice of airfoils, the shape of the wing planform, the twist throughout the wing and other geometric factors will all effect the wing loading and therefore the induced drag

Effect of Aspect Ratio

- As the angle of attack (AOA) of a wing increases so does the downwash
- As a result the airfoil of a finite wing sees less local AOA than the aircraft
- therefore the wing's lift slope becomes less steep (C_{L_α} gets smaller)
- However, the effect of aspect ratio on weight is

$$W_{\text{wing}} \propto AR^{0.5}, \text{ for constant } S$$



Effect of Aspect Ratio

- The effect of aspect ratio on the lift slope can be calculated as

$$\frac{dC_L}{d\alpha} = \frac{2\pi AR}{2 + \sqrt{4 + (AR\,\beta)^2\left(1 + \frac{\tan^2(\Lambda)}{\beta^2}\right)}}, \quad \text{where } \beta = \sqrt{1 - M^2_{\text{effective}}}$$

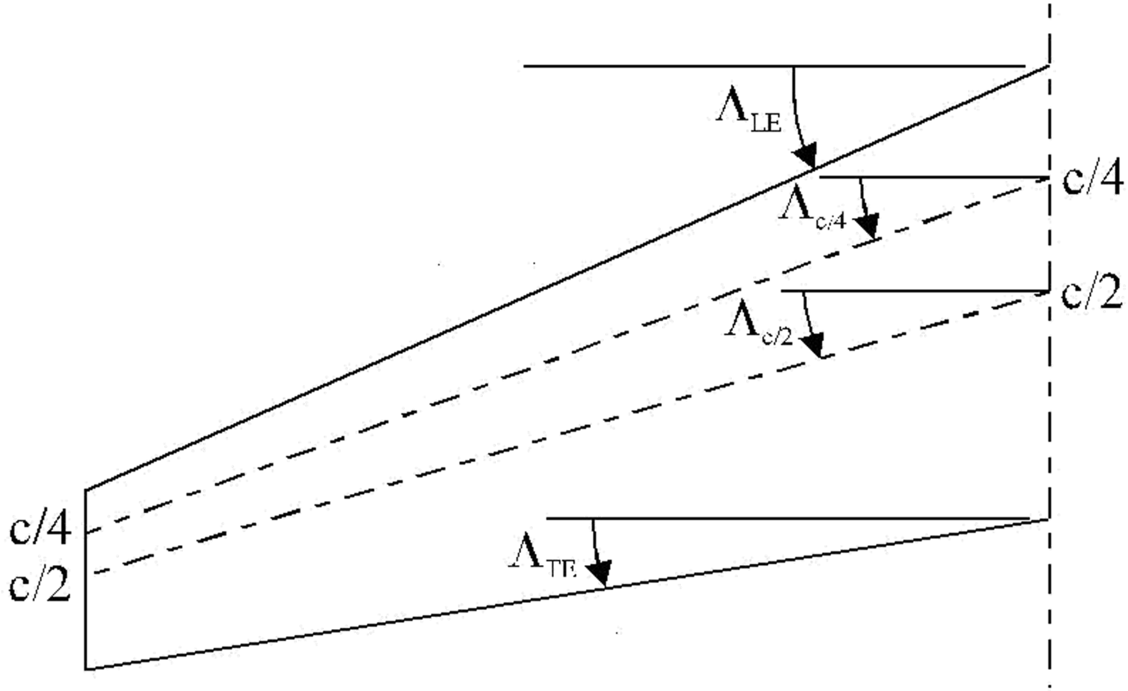
- Based on the drag polar a first order approximation can be used for lift calculations

$$C_L = C_{L_\alpha}\,\alpha + C_{L_{\alpha=0}}$$

Equivalent aspect ratio for Gliders = $4{,}464(L/D)_{max}^{0{,}69}$		
Propeller Aircrafts	Equivalent aspect ratio	
Homebuilt	6,0	
General Aircraft – single engine	7,6	
General Aircraft – twin engine	7,8	
Agriculture	7,5	
Turboprop twin engine	9,2	
Hydroplane	8,0	
	Equivalent aspect ratio = aM_{max}^{C}	
Jets	a	C
Training	4,737	-0,979
Fighter (dog fight)	5,416	-0,622
Fighter (bombers, etc)	4,110	-0,622
Military transportation/bomber	5,570	-1,075
Civil transportation	7,500	0,000

Effect of Sweep

- Sweep can be given at several points on wing
- Many of the following equations require sweep at thickest part of the airfoil
- For our purposes assume that this occurs at 25% chord



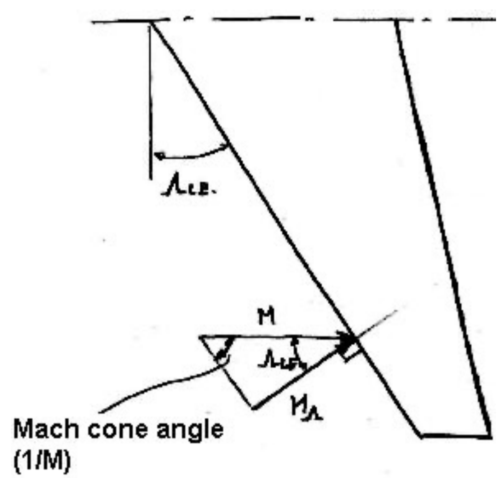
Effect of Sweep

- Sweep is primarily used to reduce the effects of wave drag
- As Mach number increases some parts of aircraft may experience local flows approaching $M = 1$
- The speed at which this occurs is M_{critical}. At this point the drag increases sharply!
- Effective mach number (or the M that the wing sees) is the is reduced with increasing sweep

$$M_{\text{effective}} = M_{\infty} \cos(\Lambda_{\text{LE}})$$

- This means that the aircraft can fly faster before reaching M_{critical}

$$M_{\text{critical}} = f\left(\frac{1}{\cos^{m}(\Lambda)}\right)$$



Effect of Sweep

- The following formula will ensure your wing is inside of your Mach cone

$$\Lambda = \frac{\pi}{2} - \arcsin\left(\frac{1}{M_\infty}\right)$$

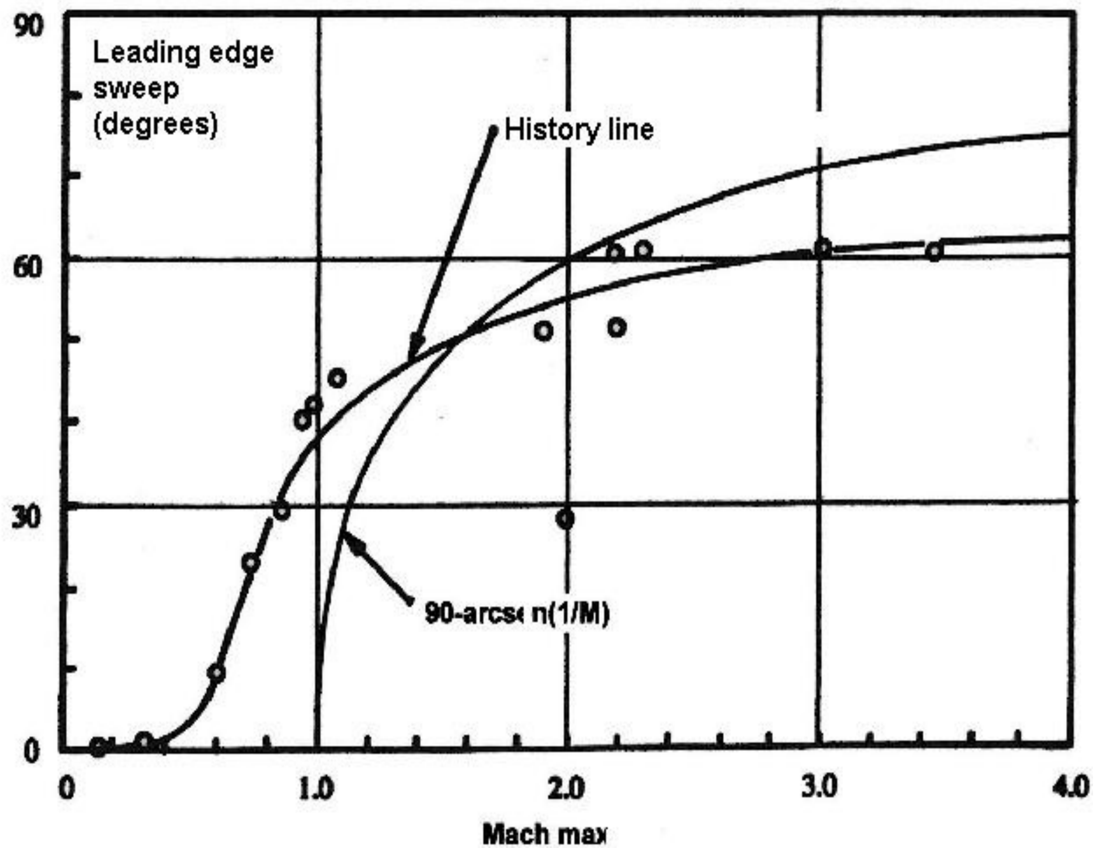
- However, lift will also be reduced due to lower effective dynamic pressure

$$q_{\text{effective}} = q_\infty \cos^2(\Lambda)$$

- And wing weight will increase with the sweep

$$W_{\text{wing}} = f\left(\tan^2(\Lambda)\right)$$

- The following is a historical trend of sweep angles chosen for different design M_{max}



Effect of Sweep

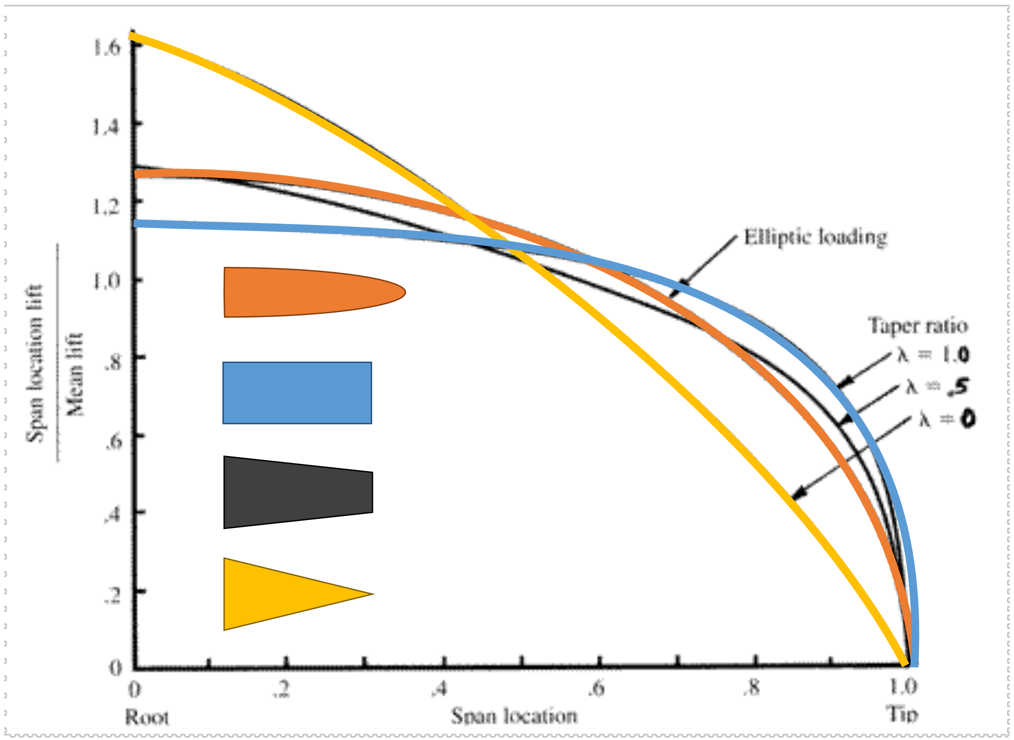
- Some other aspects to consider when choosing the sweep

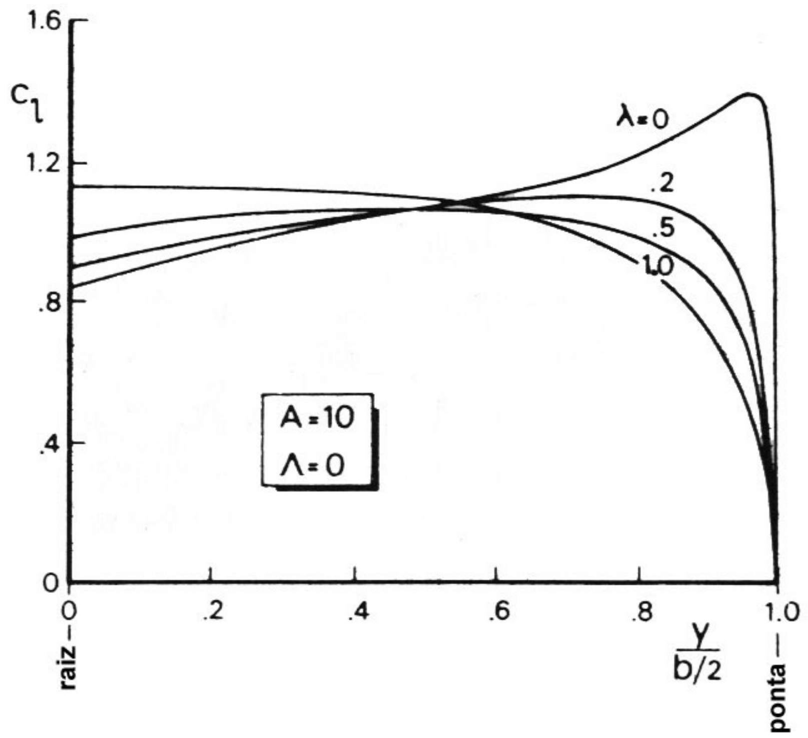
$$C_{L_{\max}} = C_{L_{\max_{\Lambda=0}}} \cos(\Lambda)$$

- Effect on the structure and attachment of the wing to the fuselage
- Lateral stability: 10 deg of sweep correspond to 1 deg of dihedral
- Stability at high angles of attack decreases
- ac and cg locations are shifted back
- High speeds – high sweep
- Slow cruise, take off and landing – low sweep

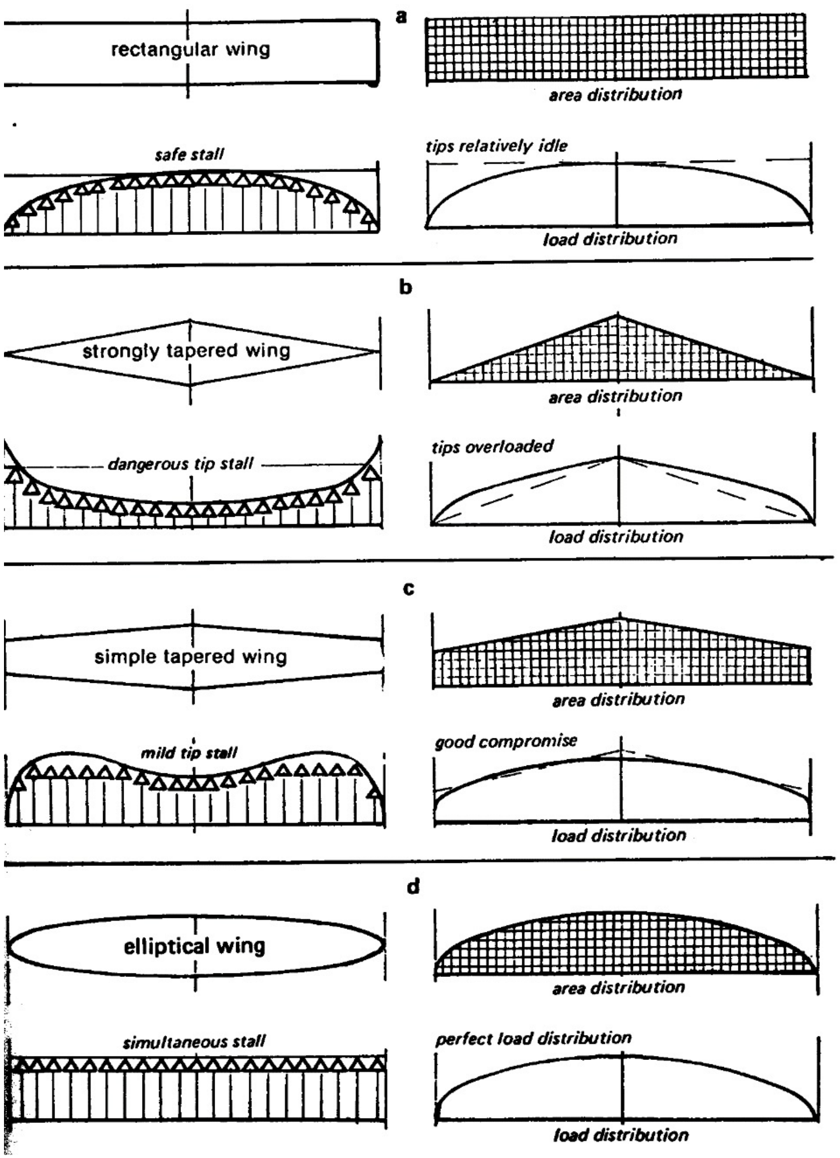
Effect of Taper

- Wing taper directly affect the wing loading distribution
- This can effect the efficiency (recall that elliptical wing loading is theoretical maximum)
- Also has strong effect on stall characteristics



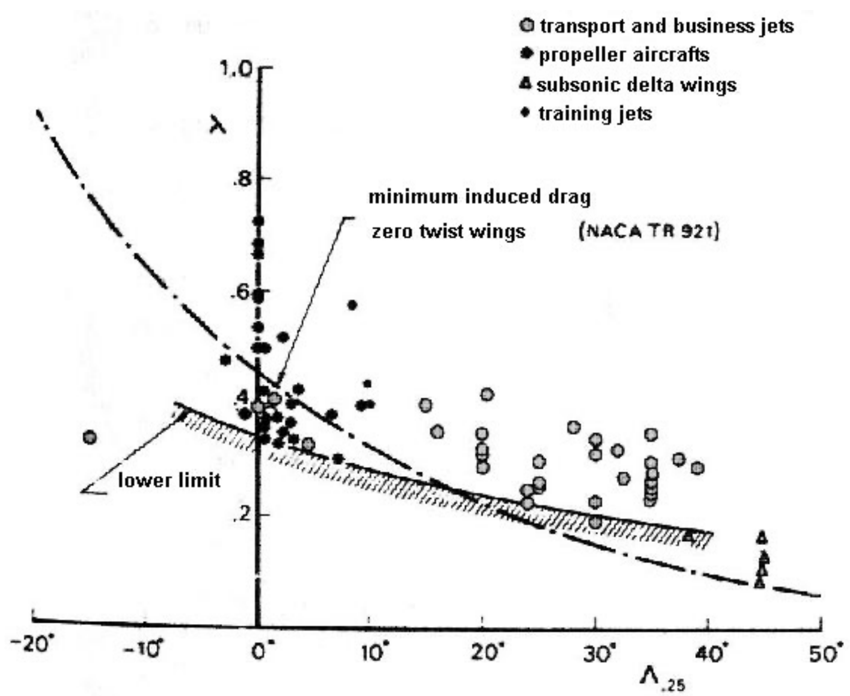


Effect of Taper

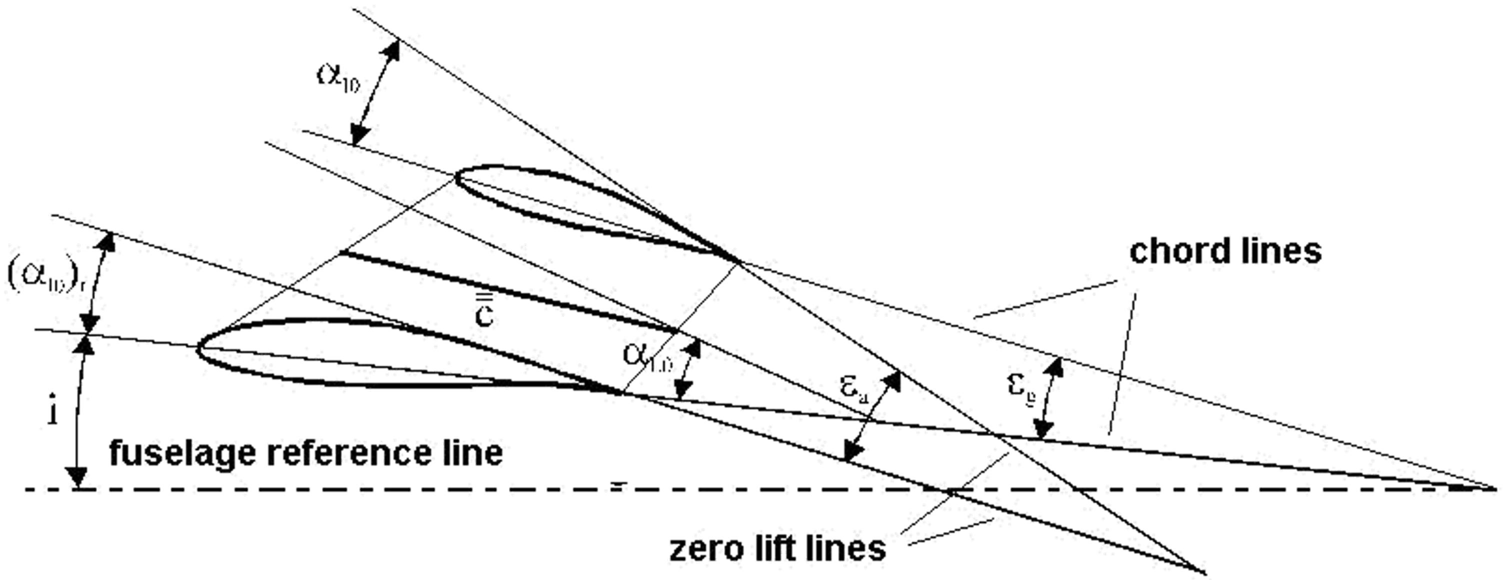


Effect of Taper

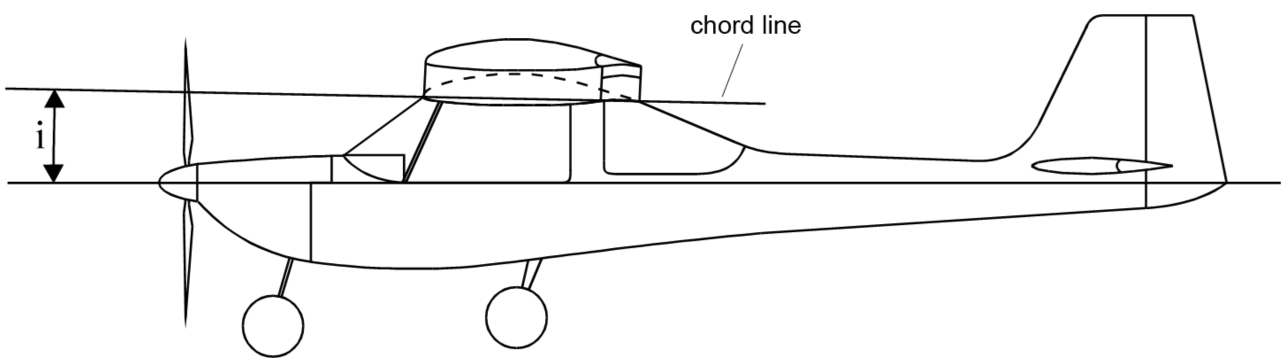
- For a trapezoidal wing with a constant chord and twist, the ideal taper ratio is around 0.4
- Wing sweep affects the choice of taper



- Wing twist along span can be used to achieve elliptical wing loading at one operating point
- Twist can also be used to prevent tip stalling
- Typically around -3 deg for tapered wings
- ***Note:*** We don't focus on twist in this class



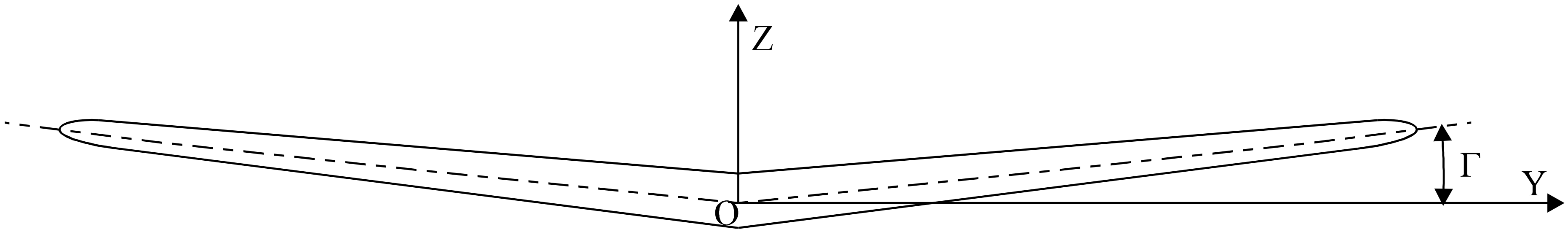
- The global twist on a wing with respect to the fuselage is called ***incidence***
- The incidence on the wing root is chosen in order to optimize *L/D* on a certain flight phase, accounting for the fuselage line to be nearly horizontal;
- For a zero twist wing the typical values for (root) incidence are:
  - General aircraft: 2 deg
  - Transport aircraft: 1 deg
  - Military aircraft: 0 deg



Effect of Dihedral

- Dihedral contributes for lateral stability
- Dihedral may also be achieved through the configuration and geometry of the aircraft
- Excessive dihedral could cause Dutch-Roll

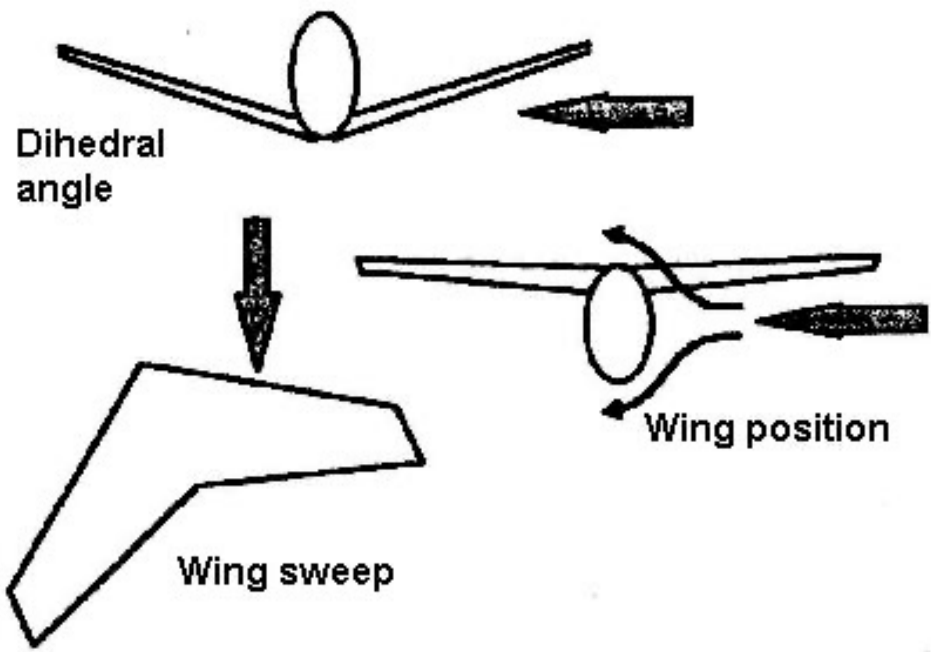
	Wing position		
	Low	Medium	High
No sweep (civilian)	5 a 7	2 a 4	0 a 2
Subsonic with sweep	3 a 7	-2 a 2	-5 a -2
Supersonic with sweep	0 a 5	-5 a 0	-5 a 0



Effect of Dihedral

- Contributions for dihedral effect

Contribution	Equivalent dihedral angle [degrees]
High wing	2,0
Low wing	-3,0
10 º of sweep	1,0
Vert Stab over fuselage	+
Vert Stab under fuselage	-



Effect of Vertical Position

- The choice of the wing’s vertical position must account for
  - Cabin space
  - Landing gear location and volume
  - Landing gear height
  - Structure
  - Engine position
  - Landing clearance
  - Flotation ability in case of emergency landing on water
  - Crashworthiness
  - Visibility in turns
  - *etc.*

Effect of Wing Drag

- Recall that drag can be broken down into individual components

$$C_D = \underbrace{C_{D_0}}_{\text{base}} + \underbrace{kC_L^2}_{\text{lift induced}} + \underbrace{k'\left(C_L - C_{L_{\min_D}}\right)}_{\text{losses}}$$

$$k = \frac{1}{\pi A\!\!R e} \qquad \underset{\substack{\text{no sweep,} \\ \text{large LE}}}{0.02} \leq k' \leq \underset{\substack{\text{sharp LE,} \\ \text{delta wings}}}{0.16}$$

- Previously we assumed *e* to be approximately 0.8
- A better approximation is based on the fuselage diameter, *d*, and span, *b*

$$e = e'\left[1 - \left(\frac{d}{b}\right)^2\right], \quad e' \approx 0.98$$

Base Drag

- $C_{D_0} > C_{d_0}$ due to non elliptical loading, interference, etc.
- C_{D_0} composed by three factors
  - C_f – Coefficient of friction (like a flat plate)
  - F – Form factor to account for flow separation
  - Q – Interference factor to account for interference between wing/fuselage, *etc.*
- Combining we get an estimate of C_{D_0}

$$C_{D_0} = C_f \mathcal{F} \mathcal{Q} \frac{S_{\text{wet}}}{S}$$

- The wetted area can be measured from a CAD model or approximated using a formula such as this one

$$S_{\text{wet}} \approx \begin{cases} S\left[1.977 + 0.52\,(t/c)_{\max}\right], & (t/c)_{\max} > 0.05 \\ 2.003S, & (t/c)_{\max} \leq 0.05 \end{cases}$$

Base Drag

- ***Friction coefficient*** is different depending on weather the flow is laminar (Re < 5 x 10^5) or turbulent (Re > 5 x 10^5)

$$C_f = \begin{cases} \dfrac{1.328}{\sqrt{\mathrm{Re}_x}}, & \text{laminar} \\ \dfrac{0.455}{(\log_{10} \mathrm{Re}_x)^{2.58}(1+0.144M^2)^{0.65}}, & \text{turbulent} \end{cases}$$

- Where the Reynolds is an averaged value based on the mean aerodynamic chord and the flow normal to the leading edge

$$V_{0_{\text{normal}}} = V_0 \cos(\Lambda_{\text{LE}})$$

$$\overline{\mathrm{Re}_x} = \frac{V_{0_{\text{normal}}} \bar{c}}{\nu}$$

- The ***form drag*** can be calculated based on the *t/c* the Mach number, the location of the thickest point on the airfoil and sweep

$$\mathcal{F} = \left[1 + \frac{0.6}{(x/c)_m}\left(\frac{t}{c}\right) + 100\left(\frac{t}{c}\right)^4\right]\left[1.34M^{0.18}\left(\cos(\Lambda_{t/c_{max}})^{0.28}\right)\right]$$

- ***Interference drag*** can be estimated empirically using tables such as the one here

TABLE 4.2: Values of Interference Factor, Q, for different arrangements.

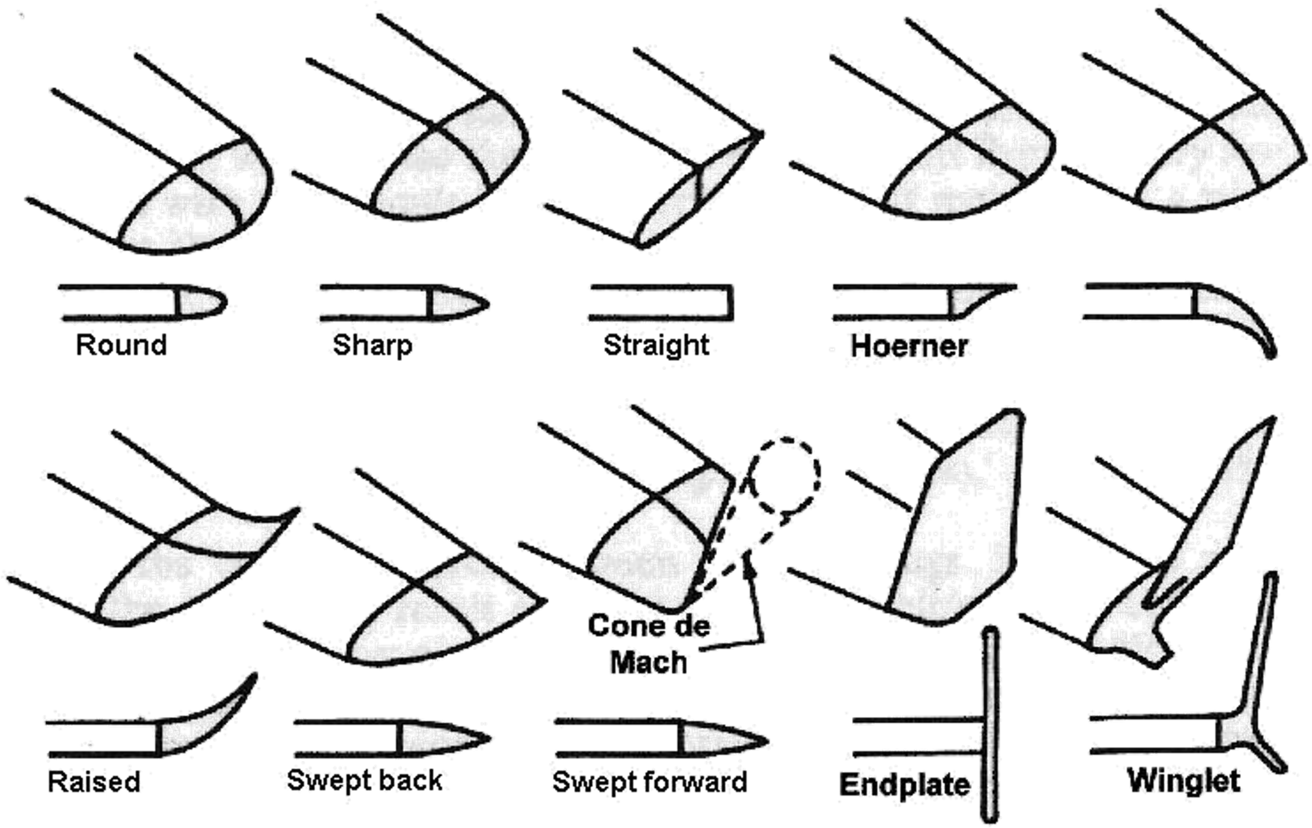
	Q
Wing-Mounted Nacelle or Store	1.5
Wing-Tip Missile	1.25
High-Wing, Mid-Wing	1
Well Filleted Low-Wing	1
Unfilletted Low-Wing	1.25

- Combining all the components

$$C_{D_0} = C_{D_{0_{wing}}} + C_{D_{0_{engine}}} + C_{D_{0_{missile}}} + \ldots$$

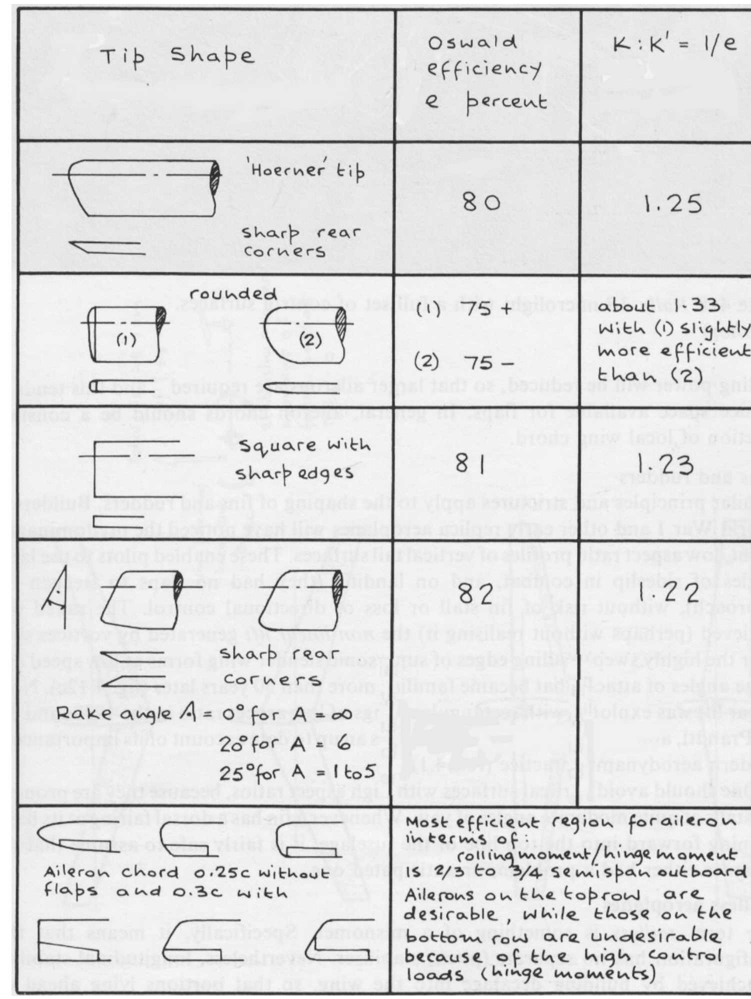
Reducing Drag

- Wing tip geometry can be designed to reduce lift induced drag
- Careful as sometimes this will add to the parasitic and interference drag!



- The wing tip geometry contributes for the aerodynamic efficiency
  - Increase the wetted area (-)
  - Decrease the tip vortex intensity (+)

- Effect on induced drag is summarized in the table



Tip Shape	Oswald efficiency e percent	K:K' = 1/e
'Hoerner' tip sharp rear corners	80	1.25
rounded (1) (2)	(1) 75 + (2) 75 −	about 1.33 with (1) slightly more efficient than (2)
Square with sharp edges	81	1.23
sharp rear corners Rake angle $\Lambda = 0°$ for $A = \infty$ $20°$ for $A = 6$ $25°$ for $A = 1$ to 5	82	1.22
Aileron chord 0.25c without flaps and 0.3c with	Most efficient region for aileron in terms of rolling moment/hinge moment is 2/3 to 3/4 semispan outboard Ailerons on the top row are desirable, while those on the bottom row are undesirable because of the high control loads (hinge moments)	

- For certain C_L, and positions, the winglet creates a forward pointing force (attention to the local tip relative airflow (induced) direction