Static Stability

Aerospace Design

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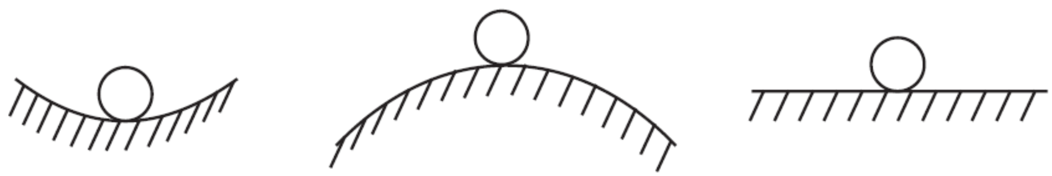
Static Stability

- ***Static stability*** is all about the initial tendency of a body to ***return to its equilibrium state*** after being disturbed
- In the design, it is generally important that the aircraft be statically stable in flight
- However, ***excessive stability*** can have ***adverse effects*** on ***maneuverability*** and ***performance***

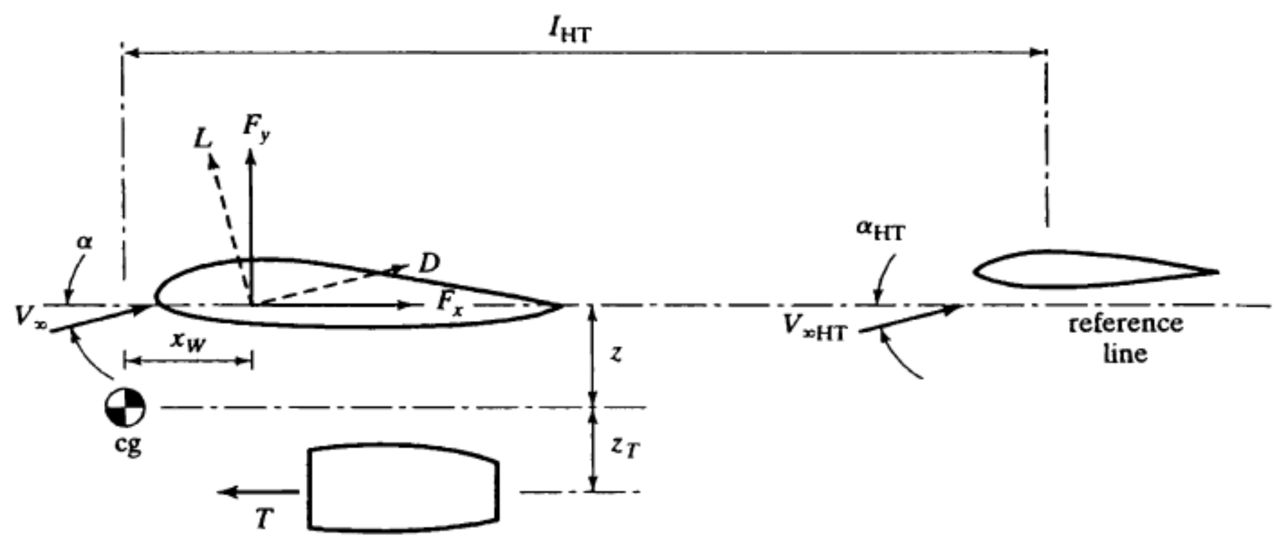


- To have a statically stable equilibrium point, the vehicle must develop a ***restoring force/moment*** to bring it back to the equilibrium condition
- Later on we will also deal with ***dynamic stability***, which is concerned with the ***time history of the motion*** after the disturbance
- A system can be statically stable but not dynamically stable
- To be dynamically stable, a system must be statically stable
- Hence ***static stability is a necessary, but not sufficient, condition for dynamic stability***

Longitudinal Stability

- An aircraft is subjected to some disturbance, say a gust, a cross wind or turbulence
- Will it recover automatically, without pilot's intervention, and resume its original direction of flight? If so, the aircraft is longitudinally stable
- In order to derive the longitudinal stability coefficient, we start by writing an equation that represents the sum of the moments about the aircraft center of gravity, the ***trim equation***

$$C_{M_{cg}} = \left(C_L \frac{x_W}{\bar{c}} + C_D \frac{z}{\bar{c}} + C_{M_{ac}}\right)_W + \frac{T z_T}{q_\infty S_W \bar{c}} - C_{L_{HT}} \frac{l_{HT} S_{HT}}{S_W \bar{c}} \frac{q_{\infty T}}{q_\infty} - \left(C_{M_{cg}}\right)_f$$



Longitudinal Stability

- The quantity $C_{L_{HT}}$ refers to the 3D lift coefficient for the horizontal tail
- The effective velocity or angle of attack of the air stream at the location of the horizontal tail can be affected by any upstream components such as the fuselage or the main wing
- A measure of the upstream influence is the tail efficiency

$$\eta_{\mathrm{HT}} = \frac{q_{\infty \mathrm{HT}}}{q_\infty} \leq 1$$

- The only difference in the trim equation for a ***canard configuration*** is that the ***sign on the moment*** produced by the ***horizontal tail*** is ***reversed***
- The effective angle of attack of the horizontal tail is

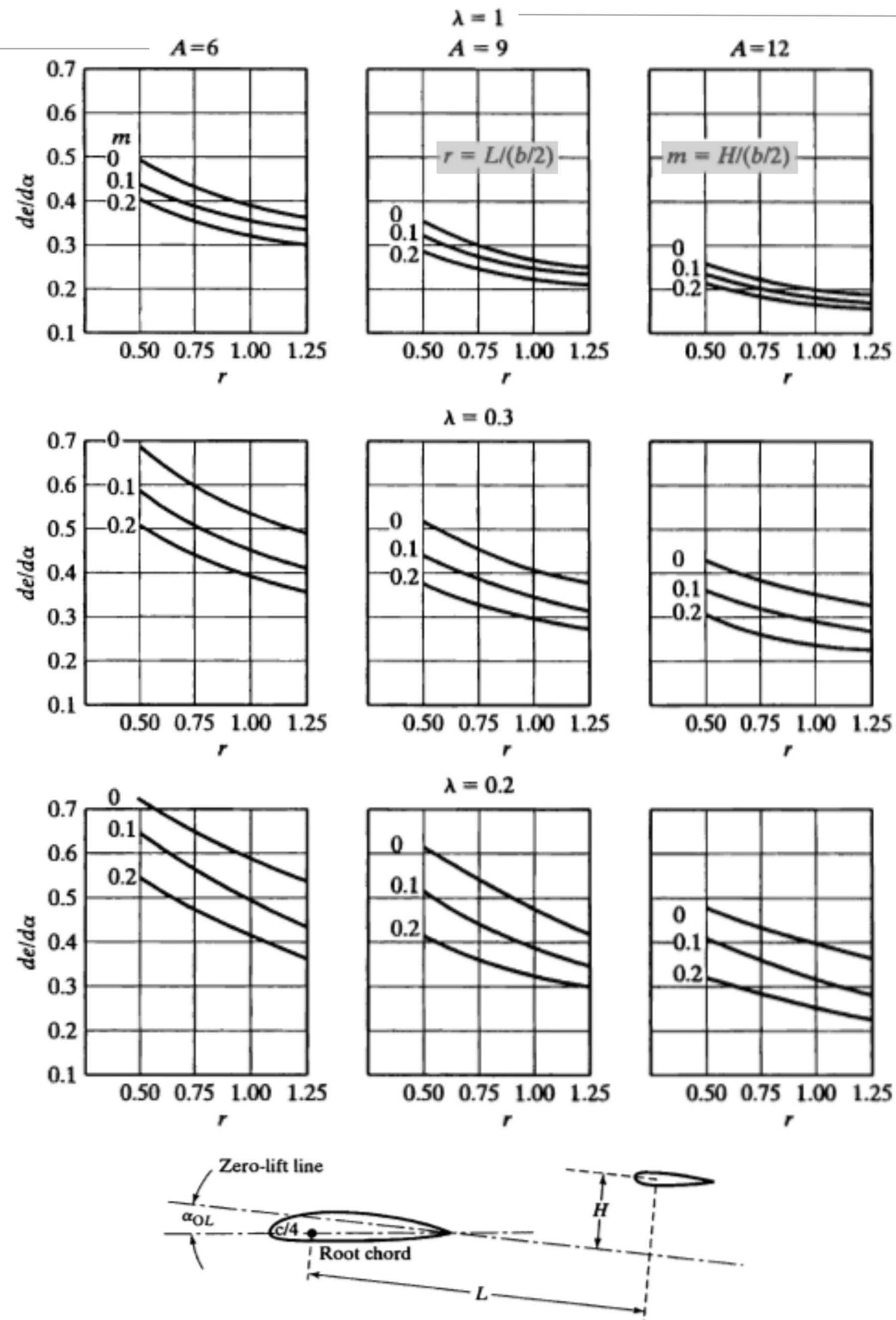
$$\alpha_{\mathrm{HT}} = \left(1 - \frac{de}{d\alpha}\right)\alpha.$$

- For a canard, $\eta_{HT} = 1$ and $\alpha_{HT} = \alpha$, namely, the main wing has no effect
- $(C_{M_{cg}})_I$ represents the force in the *y* direction, which is produced by the turning of the air at the inlet of an engine

$$(C_M)_I = \frac{(F_y)_I l_I}{q S_W \bar{c}} \simeq \frac{2\dot{m} l_I \beta}{\rho V_\infty S_W \bar{c}}$$

Aspect Ratio of Main Wing

Taper Ratio of Main Wing



Longitudinal Stability

- The coefficient of longitudinal stability, C_{M_α}, is found by differentiating the terms in the trim equation with respect to angle of attack, $C_{M_\alpha} = dC_{M_{cg}}/d\alpha$

$$C_{M_\alpha} = \frac{dC_L}{d\alpha}\frac{x_W}{\bar{c}} + \frac{dC_D}{d\alpha}\frac{z}{\bar{c}} - \frac{dC_{L_{HT}}}{d\alpha}\left(1 - \frac{de}{d\alpha}\right)\eta_T \bar{V}_{HT} - \left(\frac{dC_{M_{cg}}}{d\alpha}\right)_I$$

- Where V_{HT} is called the horizontal tail volume coefficient

$$\bar{V}_{HT} = \frac{l_{HT} S_{HT}}{S_W \bar{c}}$$

- Within the drag bucket, $dC_D/d\alpha = 0$ and so

$$C_{M_\alpha} = (C_{L_\alpha})_W \frac{x_W}{\bar{c}} - (C_{L_\alpha})_{HT}\left(1 - \frac{de}{d\alpha}\right)\eta_T \bar{V}_{HT} - \left(\frac{dC_{M_{cg}}}{d\alpha}\right)_I$$

- The effect of the engine inlet on the longitudinal stability coefficient is

$$\left(\frac{dC_{M_{cg}}}{d\alpha}\right)_I = \frac{2\dot{m} l_I}{\rho V_\infty S_W \bar{c}}\frac{d\beta}{d\alpha}$$

- The value of $d\beta/d\alpha$ depends on the Mach number and the location of the engine inlet
- ***Supersonic flow***

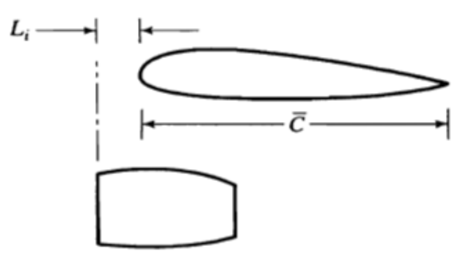
$$\frac{d\beta}{d\alpha} = 1$$

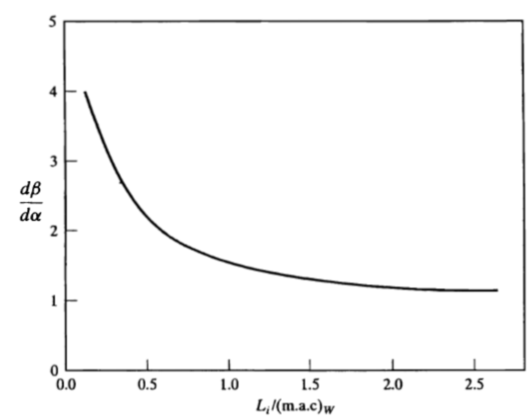
- ***Subsonic flow***
    - For engine inlets located downstream of the trailing edge of the main wing

$$\frac{d\beta}{d\alpha} \simeq \left(1 - \frac{de}{d\alpha}\right) \frac{x_I}{l_{HT}}$$

where x_I is the distance from the wing trailing edge to the engine inlet and l_{HT} is the distance between the wing trailing edge to the horizontal tail

    - For engine inlets located upstream of the wing leading edge





Criterion for Longitudinal Stability

- Longitudinal (or pitch) stability requires

$$C_{M_\alpha} < 0.$$

- An appropriate range of values for C_{M_α} is likely to be

$$-1.5 \le C_{M_\alpha} \le -0.16.$$

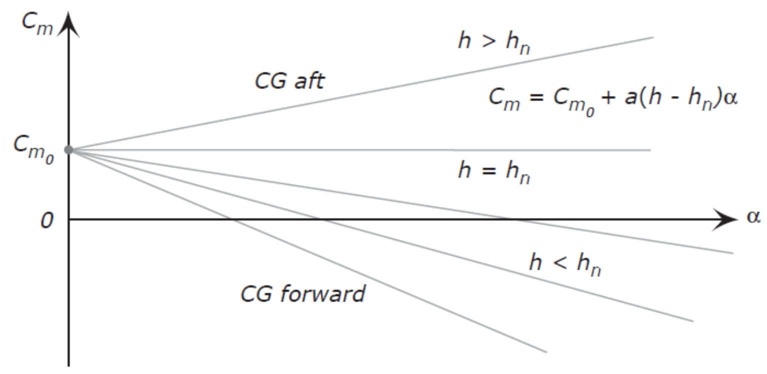
- Too large of a negative coefficient will result in an excessive amount of trim drag in aft tail designs
- The ***neutral point*** is the point where the ***moment coefficient*** about the center of gravity ***does not vary*** with the ***angle of attack*** and can be found by

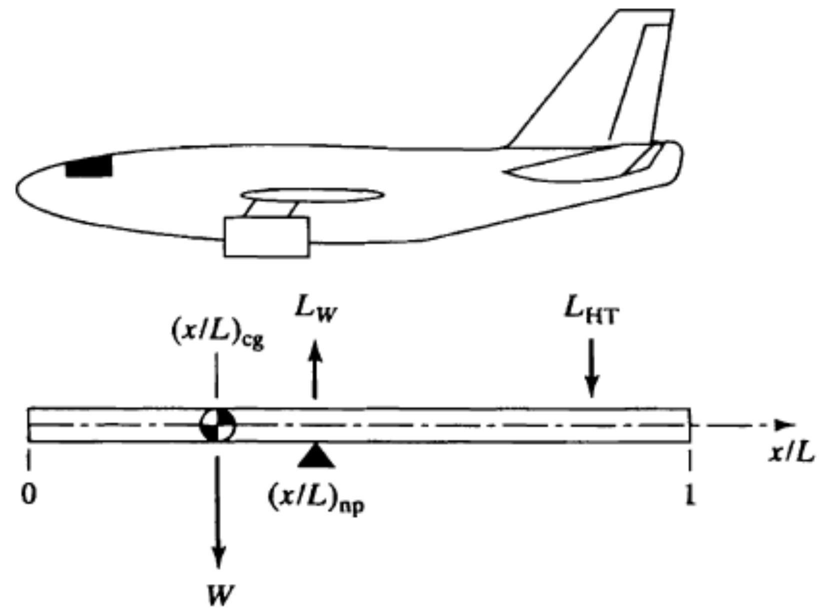
$$C_{M_\alpha} = 0.$$

- The ***static margin*** is the normalized ***difference*** between the location of the ***neutral point*** x_{np} and the ***center of gravity*** x_{cg}

$$\mathrm{SM} = \frac{x_{\mathrm{np}} - x_{\mathrm{cg}}}{\bar{c}} > 0 \ \text{for stability}$$

$$\frac{x_{\mathrm{np}}}{\bar{c}} = h_n \qquad \frac{x_{\mathrm{cg}}}{\bar{c}} = h$$

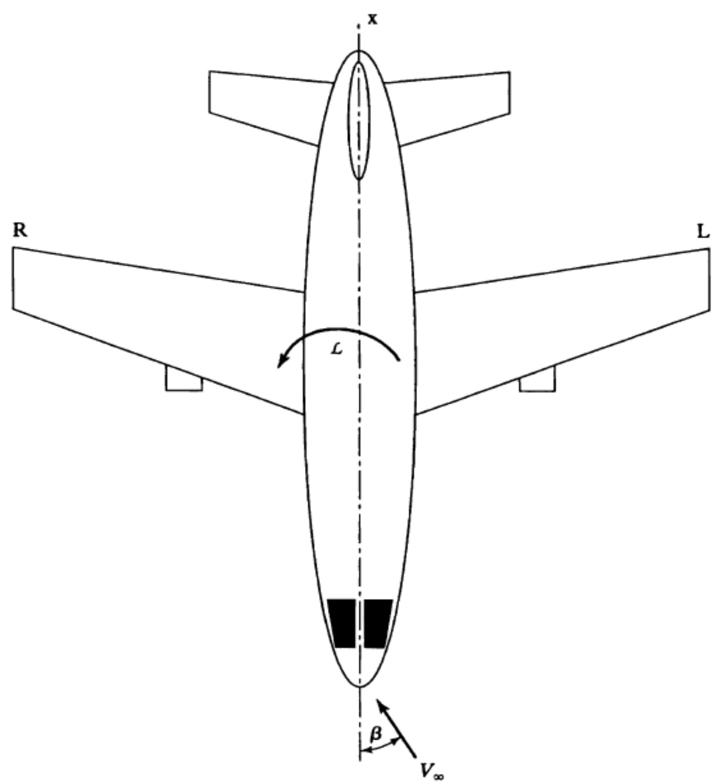




Lateral Stability

- The rolling moment is defined as *L*, with a positive value corresponding to the right wing tip moving down.
- The dimensionless rolling moment coefficient is defined as

$$C_{\mathcal{L}} = \frac{\mathcal{L}}{qS_wb}$$



Lateral Stability

- The process of rotating the right wing tip down will result in a sideslip motion of the aircraft
- The sideslip angle is denoted as β
- The rolling motion of the aircraft in response to a gust at an angle β with respect to the flight direction is a measure of the lateral stability.
- Therefore, the lateral stability coefficient is defined as

$$C_{\mathcal{L}_\beta} = \frac{dC_{\mathcal{L}}}{d\beta}$$

- For lateral stability, the right wing tip should rotate up to counter a positive sideslip angle. Therefore

$$C_{\mathcal{L}_\beta} < 0$$

Lateral Stability

- The total lateral stability is primarily considered to be the sum of the contributions of the ***main wing***, ***vertical stabilizer***, and the ***wing-fuselage***

$$C_{\mathcal{L}_\beta} = (C_{\mathcal{L}_\beta})_{\mathbf{W}} + (C_{\mathcal{L}_\beta})_{\mathbf{VS}} + (C_{\mathcal{L}_\beta})_{\mathbf{W-F}}$$

- The contribution of the main wing is further subdivided into three elements consisting of the ***basic wing***, ***sweep angle***, and ***dihedral angle***

$$(C_{\mathcal{L}_\beta})_{\mathbf{W}} = (C_{\mathcal{L}_\beta})_{\mathbf{BW}} + (C_{\mathcal{L}_\beta})_{\Lambda} + (C_{\mathcal{L}_\beta})_{\Gamma}$$

- ***Basic wing***
  - Generally, is stabilizing
  - Greater stability for smaller aspect ratios

$$-0.9 \leq (C_{\mathcal{L}_\beta})_{\mathrm{BW}}/C_L \leq -0.05.$$

- ***Taper ratio***
  - Small effect on lateral stability
- ***Sweep angle***
  - If large aspect ratios are used for the main wing, the lowered lateral stability can be increased by applying a positive sweep angle

Lateral Stability

- ***Dihedral angle***
  - Adding a positive (upward) dihedral angle greatly improves the lateral stability
  - This is one of the most commonly used approaches for improving lateral stability
- ***Vertical stabilizer***
  - By definition has a stabilizing effect to lateral motions
  - The stabilizing effect increase with the increase in $(C_{L_\alpha})_{\mathrm{VT}}$, S_{VT} and z_{VT}
- ***Wing-fuselage placement***
  - A ***high wing*** configuration mounts the wing above the horizontal centerline of the fuselage. This arrangement is stabilizing to lateral motions
  - A ***mid wing*** locates the wing on the fuselage centerline and has a neutral effect on the lateral stability
  - A ***low wing*** places the wing below the fuselage centerline. This is destabilizing to lateral motions
- It is difficult to accurately estimate the lateral stability coefficient for a conceptual aircraft. As a first approximation, the approach is to take

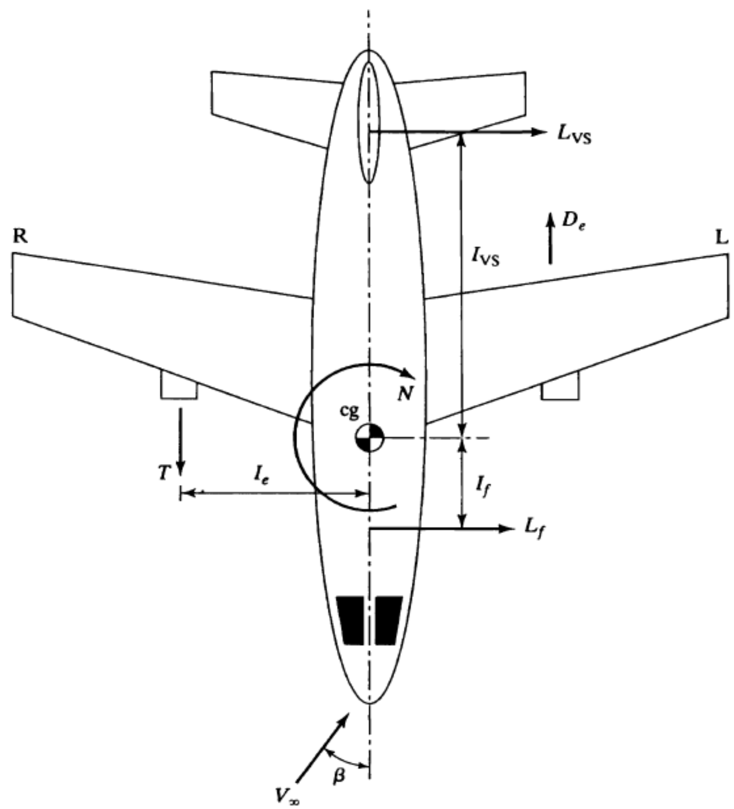
$$C_{\mathcal{L}_\beta} = -C_{n_\beta}$$

where C_{n_β} is the coefficient of directional stability

Directional Stability

- The directional motion of an aircraft is a rotation about its vertical axis
- Defined positive in the clockwise direction (right wing back)
- The directional moment coefficient is defined as

$$C_n = \frac{N}{q S_W b}$$



Directional Stability

- The fuselage, wing and vertical stabilizer contribute to C_n as follows

$$C_n = -\frac{l_F L_F}{qS_W b} + \frac{N_W}{qS_W b} + \frac{l_{VS} L_{VS}}{bS_W}$$

where q_{VS} and α_{VS} account for any upstream influence of the fuselage on the flow approaching the vertical stabilizer

- The lateral force produced by the vertical stabilizer is given by

$$L_{VS} = (C_{L_\alpha})_{VS} \alpha_{VS} \frac{q_{VS}}{q} S_{VS}$$

- The effective angle of attack of the vertical stabilizer is given as

$$\alpha_{VS} = \left(1 + \frac{d\sigma}{d\beta}\right)\beta$$

and an estimate can be obtained from the following empirical relation

$$\left(1 + \frac{d\sigma}{d\beta}\right)\frac{q_{VS}}{q} = 0.724 + \frac{3.06 S_{VS}/S_W}{1 + \cos\Lambda_{VS}} + 0.4\frac{z_W}{h} + 0.009 A_W$$

Directional Stability

- The directional stability coefficient reflects the change in the directional moment coefficient C_n with respect to the sideslip angle β

$$C_{n_\beta} = \frac{dC_n}{d\beta} = (C_{n_\beta})_F + (C_{n_\beta})_W + (C_{n_\beta})_{\text{VS}}$$

- This again emphasizes the separate contributions of the ***fuselage***, ***wing***, and ***vertical stabilizer***
- ***Vertical stabilizer***

$$(C_{n_\beta})_{\text{VS}} = \bar{V}_{\text{VS}} (C_{L_\alpha})_{\text{VS}} \left(1 + \frac{d\sigma}{d\beta}\right) \frac{q_{\text{VS}}}{q}$$

where V_{VS} is the vertical tail volume coefficient, which is similar to the horizontal tail volume coefficient V_{HT} used in the pitch stability analysis

$$\bar{V}_{\text{VS}} = \frac{l_{\text{VS}} S_{\text{VS}}}{b S_W}$$

- ***Main wing***

$$(C_{n_\beta})_W = C_L^2 \left[\frac{1}{4\pi A} - \frac{\tan\Lambda}{\pi A(A + 4\cos\Lambda)}\left(\cos\Lambda - \frac{A}{2} - \frac{A^2}{8\cos\Lambda} + 6\frac{x}{\bar{c}}\frac{\sin\Lambda}{A}\right)\right]$$

Directional Stability

- ***Fuselage***

$$\left(C_{n_\beta}\right)_F = -1.3\frac{(\mathrm{VOL})_F}{S_W b}\frac{h}{w}$$

where $(\mathrm{VOL})_F$ is the total volume of the fuselage and h and w are the average height and width of the fuselage, respectively

- For directional stability, the aircraft should rotate such that the sideslip angle is reduced. In terms of the directional coefficient

$$C_{n_\beta} > 0$$

- A reasonable range for the directional coefficient is

$$0.08 \le C_{n_\beta} \le 0.28$$

- A suggested trend for the directional coefficient as a function of Mach number is on the right figure