VTOL Sizing - MTOW

Aerospace Design

MTOW estimation

The Maximum Take-Off Weight (MTOW) is the summation of the following terms:

$$W_{TO} = W_{Struct} + W_{Subs} + W_{Prop} + W_{Payload} + W_{Energy} \tag{1}$$

where

- **Structural weight** (W_{Struct}) - wings, fuselage, nacelles;
- **Subsystems weight** (W_{Subs}) - flight controls, parachutes, electrical, pneumatic, hydraulics, avionics, cabin furnishing, air-conditioning, and de-icing (if needed);
- **Propulsion** (W_{Prop}) - engine, propeller, electric motor;
- **Payload** ($W_{Payload}$);
- **Energy** (W_{Energy}) - batteries, fuel, fuel cells, hydrogen.

MTOW estimation

Weights are allocated into **two** groups:

- **Linearly scaled with** W_{TO} - W_{Struct}, W_{Prop}, W_{Energy}, W_{Subs};
- **Prescribed/fixed weights** - $W_{Payload}$.

The MTOW equation can be rewritten as:

$$W_{T0} = \frac{W_{Payload}}{1 - (MF_{Struct} + MF_{Subs} + MF_{Prop} + MF_{Energy})}$$

where MF_X represent mass fractions.

The next step is to define the mass fractions in the denominator.

$\mathbf{MF_{Subs}}$ and $\mathbf{MF_{Struct}}$

The empty weight, W_{Empty} is defined as:

$$W_{Empty} = W_{Prop} + W_{Struct} + W_{Subs}$$

Dividing all terms by W_{TO}, and knowing that $W_{Subs} = 0.30 W_{Empty}$ for modern aircraft, and $W_{Struct} = 0.24 W_{TO}$ valid for rotorcraft with a gross take-off weight between 3000 - 100000 lb, the previous equation can be rearranged as:

$$0.70 \frac{W_{Subs}}{W_{TO}} = 0.30 \left(\frac{W_{Prop}}{W_{TO}} + \frac{W_{Struct}}{W_{TO}} \right) \Leftrightarrow$$
$$\Leftrightarrow MF_{Subs} = \frac{3}{7} \left(MF_{Prop} + 0.24 \right)$$

$\mathbf{MF_{Prop}}$

- Although electric motor's efficiency is close to 100%, batteries alone have a limited operation time, and a longer recharching period, with weight and volumetric penalties;

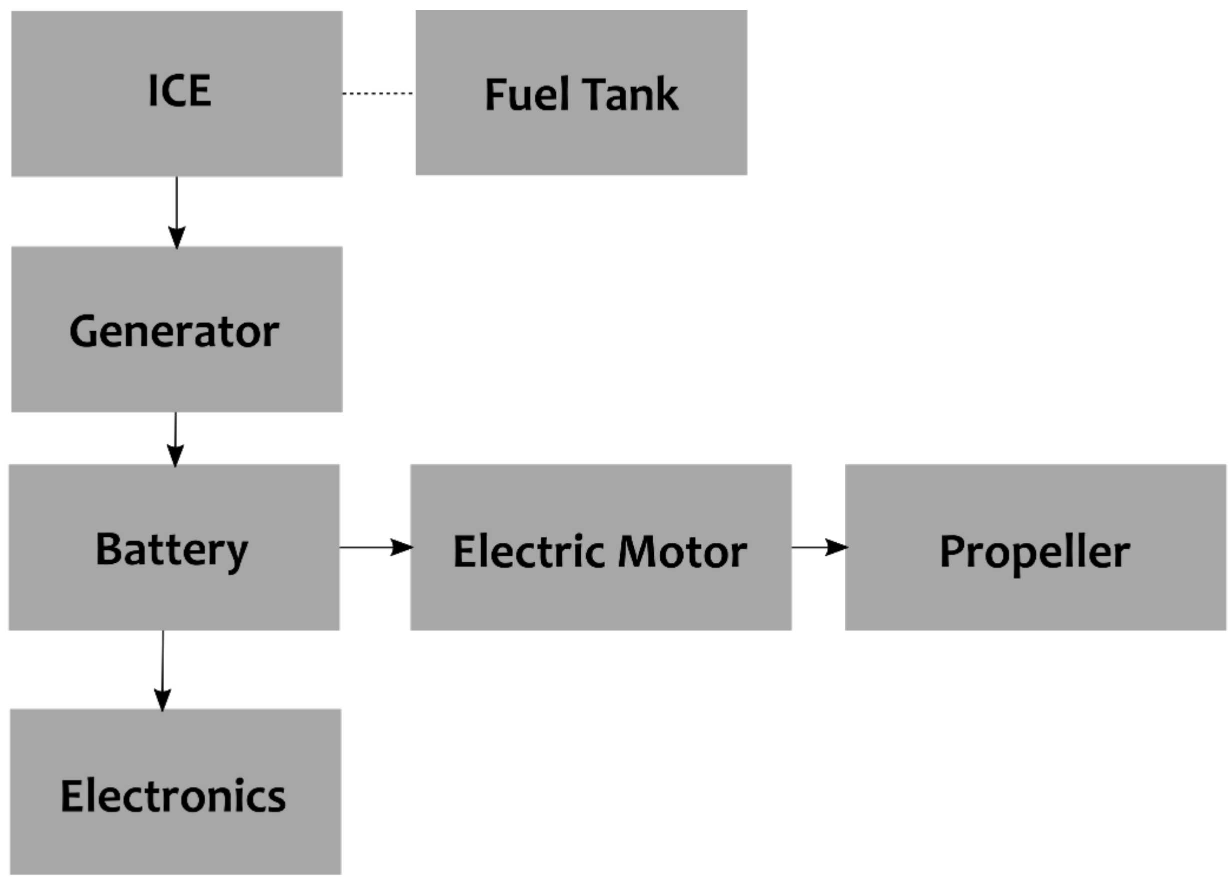
- Hybrid electric propulsion systems decrease the specific fuel consumption, improve the range, and reduce both acoustic and thermal signatures of an aircraft.

$\mathbf{MF_{Prop}}$

- **Series**: the electric motor (EM) is the only means of providing power to the propeller, and the Internal Combustion Engine (ICE) works as an auxiliar power unit.

Pros	Best for low-speed, high-torque applications (urban vehicles) ICE can be sized smaller Electrically and mechanically simpler
Cons	Significant mechanical-electrical conversion losses EM and battery need to be sized larger to accomodate the peak power demands

$\mathbf{MF_{Prop}}$



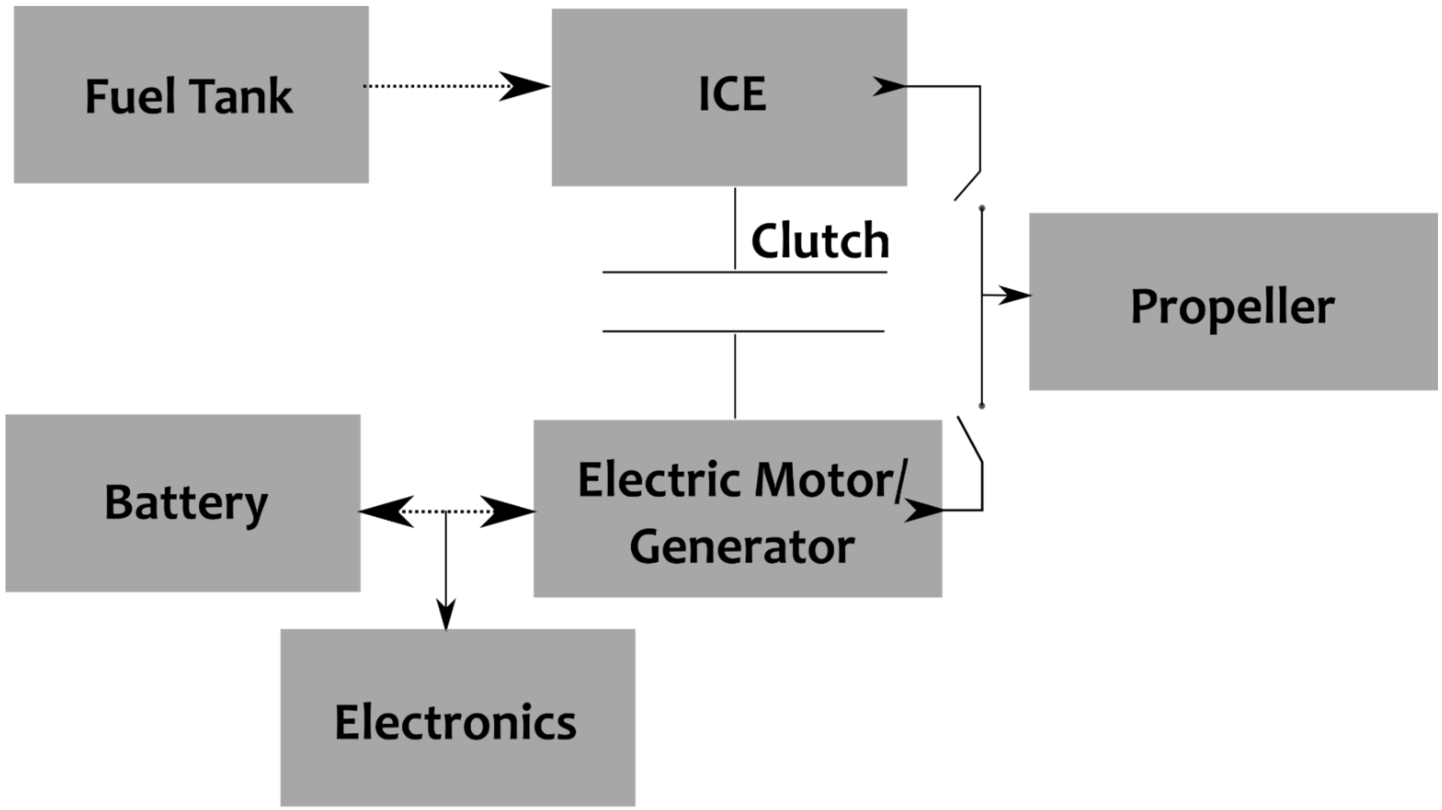
a) Series configuration

$\mathbf{MF_{Prop}}$

- **Parallel**: can work on **three** different modes: ***ICE only*** (provides extra energy to recharge the batteries while driving the propeller), ***EM only*** or ***ICE coupled with EM*** (ICE is provided with assisted torque by the electric motor(s)).

Pros	Can work in separate modes More lightweight, more compact design (e.g. EM also functions as a generator) Less energy losses
Cons	More complicated clutch and control system

$\mathbf{MF_{Prop}}$



b) Parallel configuration

$\mathbf{MF_{Prop}}$

For the **series** configuration, MF_{Prop} is given by:

$$MF_{Prop} = \frac{W_{ICE} + W_{Generator} + W_{EM} + W_{Propeller}}{W_{TO}}$$

For the **parallel** configuration:

$$MF_{Prop} = \frac{W_{ICE} + W_{EM} + W_{Propeller}}{W_{TO}}$$

However, a ***rubber engine sizing*** is generally employed, where the **power-to-weight ratio** is applied as a parameter for series and parallel configurations, respectively:

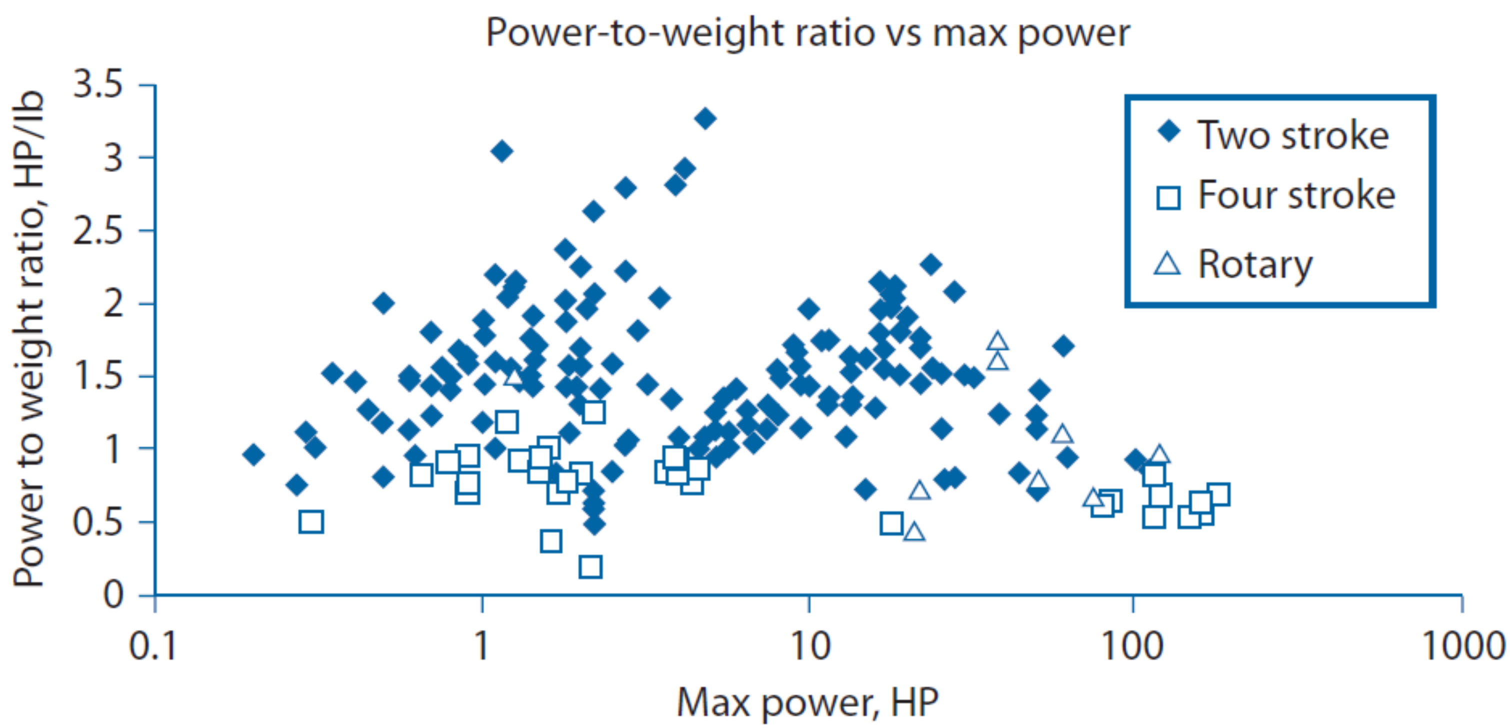
$$MF_{Prop} = f_{install} \cdot \frac{P/W_{TO}}{P/W_{ICE} + P/W_{Generator} + P/W_{EM} + P/W_{Propeller}}$$

$$MF_{Prop} = f_{install} \cdot \frac{P/W_{TO}}{P/W_{ICE} + P/W_{EM} + P/W_{Propeller}}$$

where P is the maximum shaft power, and $f_{install}$ is a factor bigger than 1 that accounts for all auxiliary systems (e.g. digital engine controls) if their weights are not exactly known.

MTOW estimation

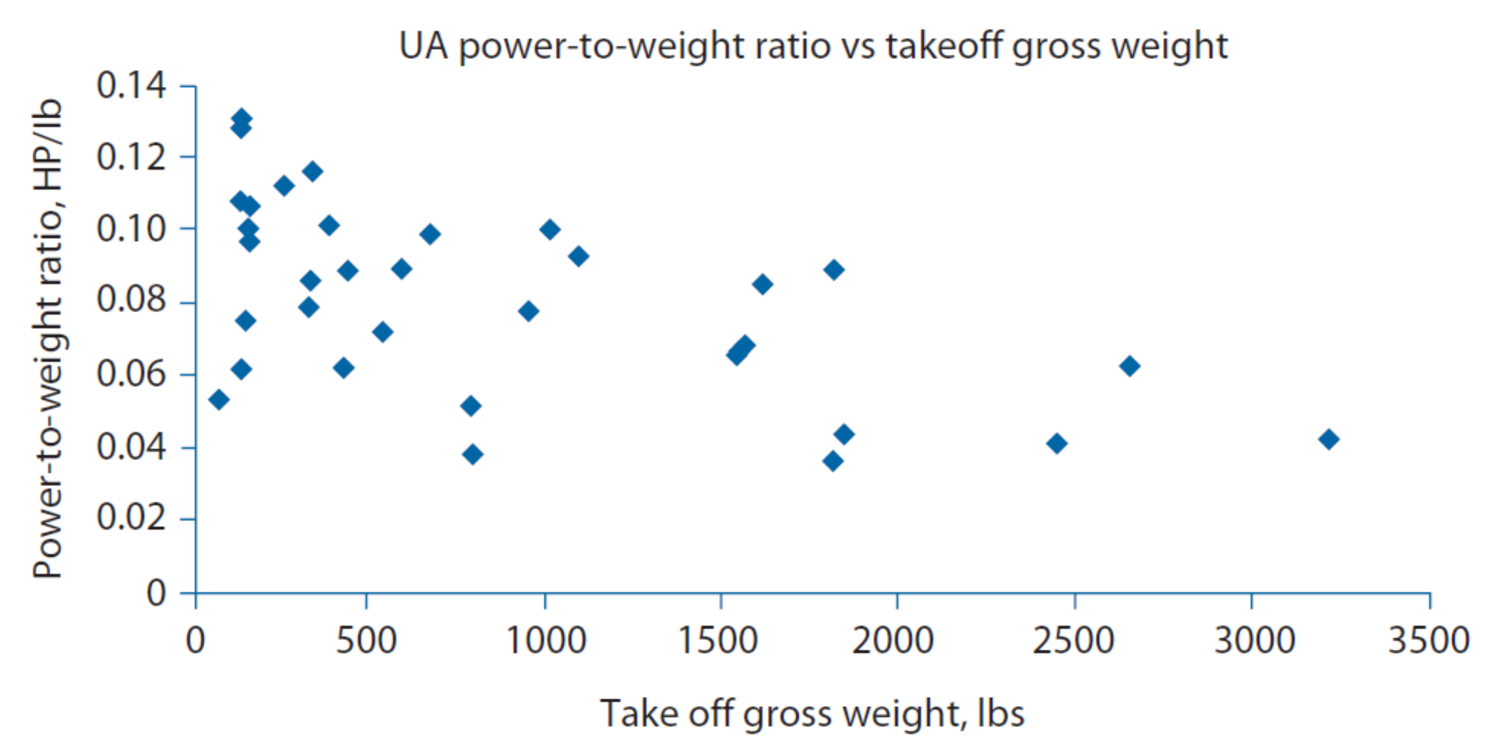
$$\mathbf{MF_{Prop}}$$



$$\mathbf{P/W_{ICE} = f(P)}$$

MTOW estimation

$$\mathbf{MF_{Prop}}$$



$$\mathbf{P/W_{TO} = f(W_{TO})}$$

$\mathbf{MF_{Energy}}$

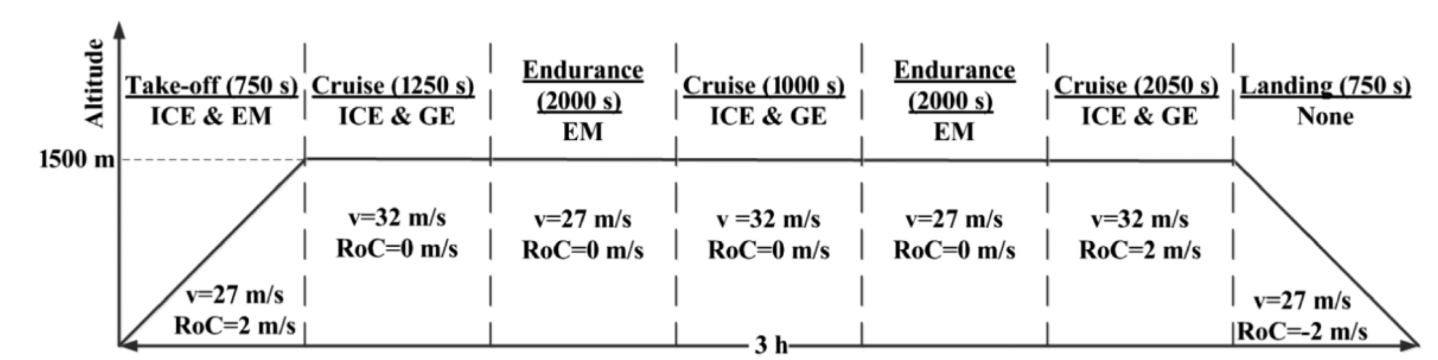
For the case of hybrid-electric propulsion systems, two forms of energy must be considered in the total mass fraction:

$$MF_{Energy} = MF_{Batt} + MF_{Fuel}$$

Tipically, these two forms of energy are used in the following flight stages:

- **Climb and Descent**: ICE and EM (parallel configuration) or EM only;
- **Hover**: EM only;
- **Cruise**: ICE only (parallel configuration).

An example of a mission profile for a fixed-wing aircraft is shown below.



$\mathbf{MF_{Batt}}$

The general equation for the battery's mass fraction considering a specific mission segment, i, is given by:

$$MF_{Batt,i} = \frac{tP}{M.E_{spec}\left(\prod \eta\right)\eta_{batt} f_{usable}}$$

where

- t - Mission segment's time duration;
- P - Power required to perform the segment;
- M - Aircraft's mass;
- E_{spec} - Battery's specific energy;
- η_{batt} - Battery's efficiency;
- f_{usable} - Usable capacity.
- $\prod \eta$ - includes propeller efficiency (η_p), motor gearbox efficiency (η_{gear}), electric motor efficiency (η_{motor}), electronic speed control efficiency (η_{ESC}), and efficiency of the power distribution system (η_{Dist}).

$\mathbf{MF_{Batt}}$

The previous equation can be rearranged in order to be function of the **power loading** $(PL = W/P)$:

$$MF_{Batt,i} = \frac{t.g}{E_{spec}\left(\prod\eta\right)\eta_{batt} f_{usable}} \frac{1}{PL}$$

The power loading term can be replaced by the expressions to be presented in the following slides regarding **hover, climb/descent** and **cruise** operations.

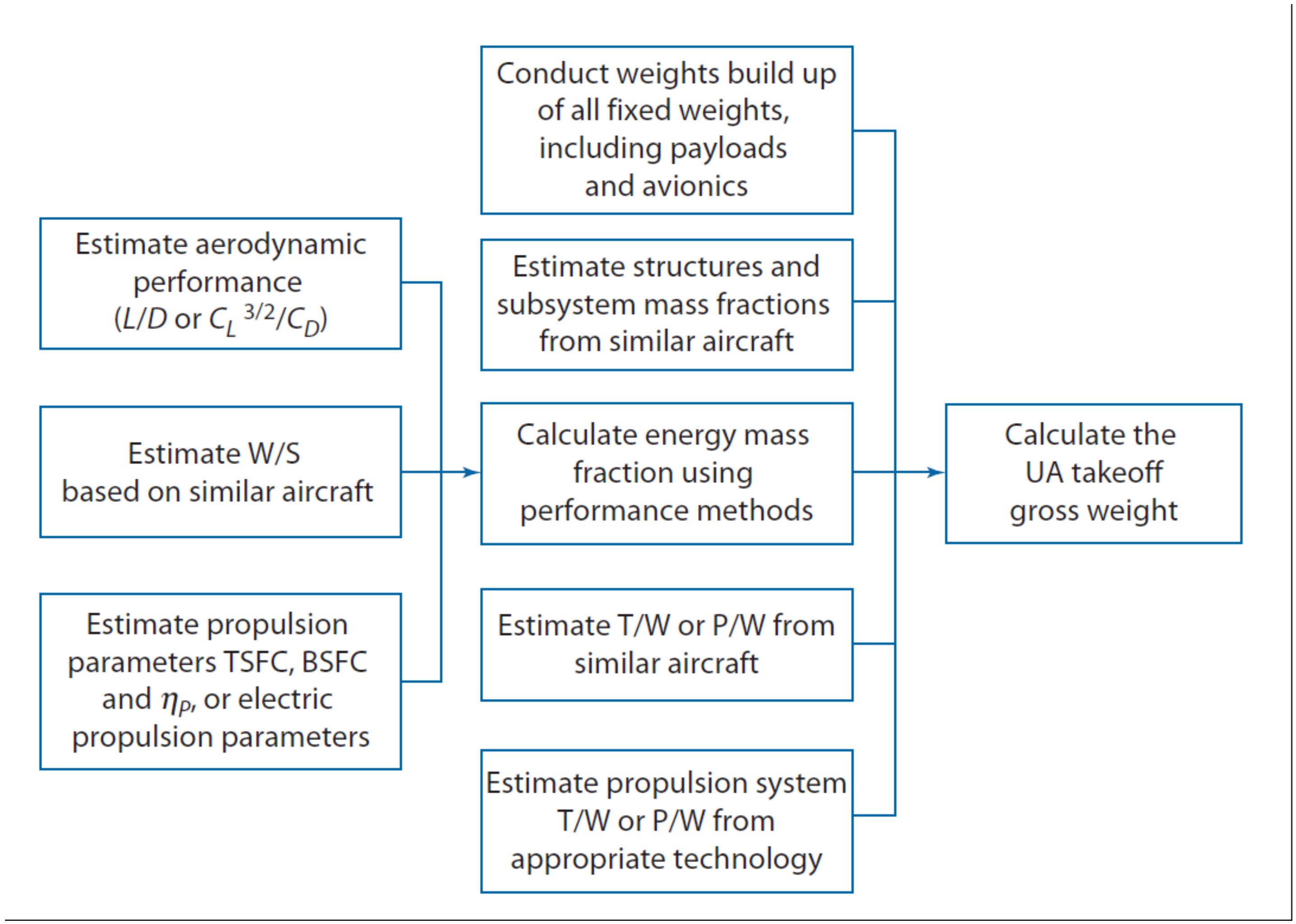
Examples:

- **Maximum range:** $MF_{Batt,i} = \frac{Rg}{E_{spec}(\prod\eta)\eta_{batt} f_{usable}(L/D)_{max}}$
- **Maximum endurance flight:** $MF_{Batt,i} = \frac{t_{loiter}g}{E_{spec}(\prod\eta)\eta_{batt} f_{usable}(C_l^{3/2}/C_D)}\sqrt{\frac{2W/S}{\rho}}$

The battery's mass doesn't change during the flight mission. The battery's total mass fraction is the summation of the individual segment's mass fractions:

$$MF_{Batt} = \sum_{i=1}^{n_{seg}} MF_{Batt,i}$$

In summary:


Conduct weights build up of all fixed weights, including payloads and avionics
Estimate aerodynamic performance (L/D or $C_L^{3/2}/C_D$)
Estimate structures and subsystem mass fractions from similar aircraft
Estimate W/S based on similar aircraft
Calculate energy mass fraction using performance methods
Calculate the UA takeoff gross weight
Estimate propulsion parameters TSFC, BSFC and η_P, or electric propulsion parameters
Estimate T/W or P/W from similar aircraft
Estimate propulsion system T/W or P/W from appropriate technology