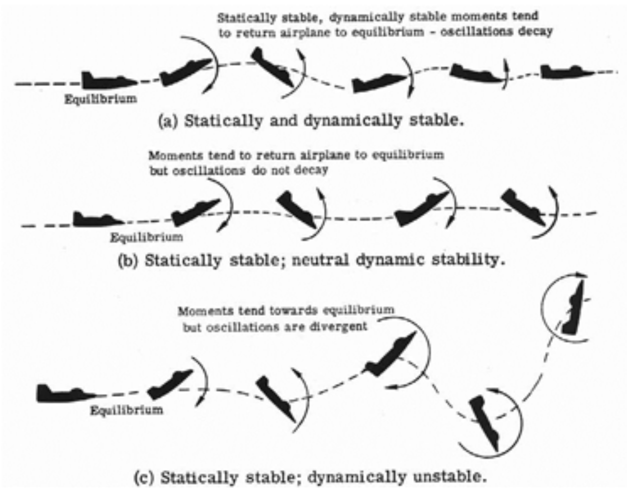
Dynamic Stability

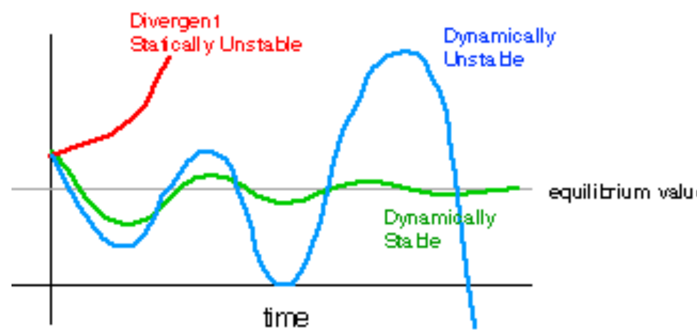
Aerospace Design

TÉCNICO LISBOA

Dynamic Stability

- ***Dynamic stability*** is concerned with the motions of the aircraft, and refers to how the aircraft behaves after it has been disturbed following steady non-oscillating flight
- To be dynamically stable, an aircraft ***must be statically stable***





- Inertia forces must now be considered and included in the equations of motion of the aircraft
- Inertia forces derive from the tendency of mass to resist accelerations
- The mass for rotational accelerations is represented by the ***mass moment of inertia I***
- The mass moment of inertia describes a body's resistance to rotational accelerations

Dynamic Stability

- For aircraft dynamic analysis, the mass moments of inertia about the three principal axes must be determined: I_{xx} about the roll axis, I_{yy} about the pitch axis, and I_{zz} about the yaw axis
- These can be initially determined using historical data based upon the non-dimensional radii of gyration R, aircraft length L and span b

$$I_{xx} = \frac{b^2 W \overline{R}_x^2}{4g}$$

$$I_{yy} = \frac{L^2 W \overline{R}_y^2}{4g}$$

$$I_{zz} = \left(\frac{b+L}{2}\right)^2 \frac{W \overline{R}_z^2}{4g}$$

Table 16.1 Nondimensional radii of gyration[a]

Aircraft class	$\overline{R}_x$	$\overline{R}_y$	$\overline{R}_z$
Single-engine prop	0.25	0.38	0.39
Twin-engine prop	0.34	0.29	0.44
Business jet twin	0.30	0.30	0.43
Twin turboprop transport	0.22	0.34	0.38
Jet transport—Fuselage-mounted engines	0.24	0.36	0.44
—2 wing-mounted engines	0.25	0.38	0.46
—4 wing-mounted engines	0.31	0.33	0.45
Military jet trainer	0.22	0.14	0.25
Jet fighter	0.23	0.38	0.52
Jet heavy bomber	0.34	0.31	0.47
Flying wing (B-49 type)	0.32	0.32	0.51
Flying boat	0.25	0.32	0.41

[a]Typical values see Ref. 11 for examples.

- The equations of motion of a 6 DOF system can be written as

$$m\left(\frac{d}{dt}\mathbf{V} + \Omega\mathbf{V} - \mathbf{S}\mathbf{g}\right) = \mathbf{F}_a$$

$$\mathbf{I}\frac{d}{dt}\omega + \Omega\mathbf{I}\omega = \mathbf{M}_a$$

where F_a and M_a are the aerodynamic forces and moments, respectively

Linearized Equations of Motion

- The previous equations of motion can be expanded and further linearized around a longitudinally trimmed flight condition, resulting in

$$F_x = m(\dot{u} + q\,w_0 + g\,\cos(\theta_0)\,\theta)$$

$$F_y = m(\dot{v} + u_0\,r - p\,w_0 - g\,\cos(\theta_0)\,\phi)$$

$$F_z = m(\dot{w} - u_0\,q + g\,\sin(\theta_0)\,\theta)$$

$$M_x = I_{xx}\,\dot{p} - I_{xz}\,\dot{r}$$

$$M_y = I_{yy}\,\dot{q}$$

$$M_z = I_{xx}\,\dot{r} - I_{xz}\,\dot{p}$$

$$p = \dot{\phi} - \dot{\psi}\,\sin(\theta_0)$$

$$q = \dot{\theta}$$

$$r = \dot{\psi}\,\cos(\theta_0)$$

$u, v, w,$	velocity components
$\phi, \theta, \psi,$	roll, pitch and yaw angles
$p, q, r,$	roll, pitch and yaw rates
$u_0, w_0,$	flight condition velocity
$\theta_0,$	flight condition pitch angle
$I_{xx}, I_{yy}, I_{zz},$	principal moments of inertia
$I_{xz},$	xz product of inertia

- A consequence of the linearization is the uncoupling between longitudinal and lateral motion

Linearized Equations of Motion

- ***Longitudinal motion***

$$F_x = m(\dot{u} + q\,w_0 + g\,\cos(\theta_0)\,\theta)$$

$$F_z = m(\dot{w} - u_0\,q + g\,\sin(\theta_0)\,\theta)$$

$$M_y = I_{yy}\,\dot{q}$$

$$q = \dot{\theta}$$

- ***Lateral motion***

$$F_y = m(\dot{v} + u_0\,r - p\,w_0 - g\,\cos(\theta_0)\,\phi)$$

$$M_x = I_{xx}\,\dot{p} - I_{xz}\,\dot{r}$$

$$M_z = I_{xx}\,\dot{r} - I_{xz}\,\dot{p}$$

$$p = \dot{\phi} - \dot{\psi}\,\sin(\theta_0)$$

$$r = \dot{\psi}\,\cos(\theta_0)$$

Linearization of Forces

- The longitudinal aerodynamic forces and moments can be linearized and written as a function of the ***state variables***, ***control surface deflections*** and the ***dimensional stability derivatives***

$$\frac{F_x}{m} = X_u u + X_w w + X_{\delta_e}\delta_e + X_{\delta_t}\delta_t$$

$$\frac{F_z}{m} = Z_u u + Z_w w + Z_{\delta_e}\delta_e + Z_{\delta_t}\delta_t$$

$$\frac{M_y}{I_{yy}} = M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta_e}\delta_e + M_{\delta_t}\delta_t$$

- Similarly for the lateral aerodynamic forces and moments

$$\frac{F_y}{m} = Y_v v + Y_p p + Y_r r + Y_{\delta_r}\delta_r$$

$$\frac{M_x}{I_{xx}} = L_v v + L_p p + L_r r + L_{\delta_a}\delta_a + L_{\delta_r}\delta_r$$

$$\frac{M_z}{I_{zz}} = N_v v + N_p p + N_r r + N_{\delta_a}\delta_a + N_{\delta_r}\delta_r$$

Linearization of Forces

- *u*, *v*, *w*, *p*, *q*, *r* are the ***state variables*** and δ_e, δ_t, δ_a, δ_r are the ***control surface deflections***
- The longitudinal and lateral ***dimensional stability derivatives*** are given in the following tables

$X_u = -(C_{D_u} + 2C_{D0})\frac{1}{2\mu}{}^2$	$Z_u = -(C_{L_u} + 2C_{L0})\frac{1}{2\mu}$	$M_u = C_{m_u}\frac{m\bar{c}}{2\mu I_{yy}}$
$X_w = -(C_{D_\alpha} - C_{L0})\frac{1}{2\mu}$	$Z_w = -(C_{L_\alpha} + C_{D0})\frac{1}{2\mu}$	$M_w = C_{m_\alpha}\frac{m\bar{c}}{2\mu I_{yy}}$
$Z_{\dot{w}} = -C_{L_{\dot{\alpha}}}\frac{\bar{c}}{4\mu U_0}$	$Z_q = -C_{L_q}\frac{\bar{c}}{4\mu U_0}$	$M_q = C_{m_q}\frac{m\bar{c}^2}{4\mu I_{yy}}$
$M_{\dot{w}} = C_{m_{\dot{\alpha}}}\frac{m\bar{c}^2}{4\mu I_{yy} U_0}$	$Z_{\delta_E} = -C_{L_{\delta_E}}\frac{U_0}{2\mu}$	$M_{\delta_E} = C_{m_{\delta_E}}\frac{mU_0\bar{c}}{2\mu I_{yy}}$

$Y_v = C_{Y_\beta}\frac{1}{2\mu}$	$L_\beta = C_{l_\beta}\frac{mU_0 b}{2\mu I_{xx}}$	$N_\beta = C_{n_\beta}\frac{mU_0 b}{2\mu I_{zz}}$
$Y_p = C_{Y_p}\frac{b}{4\mu}$	$L_p = C_{l_p}\frac{mb^2}{4\mu I_{xx}}$	$N_p = C_{n_p}\frac{mb^2}{4\mu I_{zz}}$
$Y_r = C_{Y_r}\frac{b}{4\mu}$	$L_r = C_{l_r}\frac{mb^2}{4\mu I_{xx}}$	$N_r = C_{n_r}\frac{mb^2}{4\mu I_{zz}}$
$Y_\delta = C_{Y_\delta}\frac{U_0}{2\mu}$	$L_\delta = C_{l_\delta}\frac{mU_0 b}{2\mu I_{xx}}$	$N_\delta = C_{n_\delta}\frac{mU_0 b}{2\mu I_{zz}}$

$$\mu = \frac{m}{\rho S U_0}$$

Form of Solution of Equations

- The previous equations of motion are in the form

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x} + \mathbf{f}_c$$

- Where **x** is the ***state vector***, **A** is the ***system matrix*** and $\mathbf{f}_c$ is the ***vector of control forces and moments***
- When the control force is zero, the equation to be studied is

$$\dot{\mathbf{x}} = \mathbf{A}\,\mathbf{x}$$

- Solutions of this first-order differential equation are of the form

$$x(t) = a_1\, e^{\lambda_1 t} + a_2\, e^{\lambda_2 t} + \ldots \qquad \lambda = n \pm i\omega$$

- This describes an oscillatory mode, of period *T*, that either grows or decays, depending on the sign of *n*

Form of Solution of Equations

- ***Parameters of interest***

Period, $T = \dfrac{2\pi}{\omega}$

Time to double or half:

$$t_{\text{double}} \text{ or } t_{\text{half}} = \frac{.693}{|n|} = \frac{.693}{|\zeta|\omega_n}$$

Cycles to double or half:

$$N_{\text{double}} \text{ or } N_{\text{half}} = .110\,\frac{\omega}{|n|} = .110\,\frac{\sqrt{1-\zeta^2}}{|\zeta|}$$

Logarithmic decrement (log of ratio of successive peaks):

$$\delta = \log_e \frac{e^{nt}}{e^{n(t+T)}} = -nT = 2\pi\,\frac{\zeta}{\sqrt{1-\zeta^2}}$$

$$= -.693/N_{\text{double}} \qquad \text{or} \qquad .693/N_{\text{half}}$$

In the preceding equations,

$\omega_n = (\omega^2 + n^2)^{1/2}$, the "undamped" circular frequency

$\zeta = -n/\omega_n$, the damping ratio

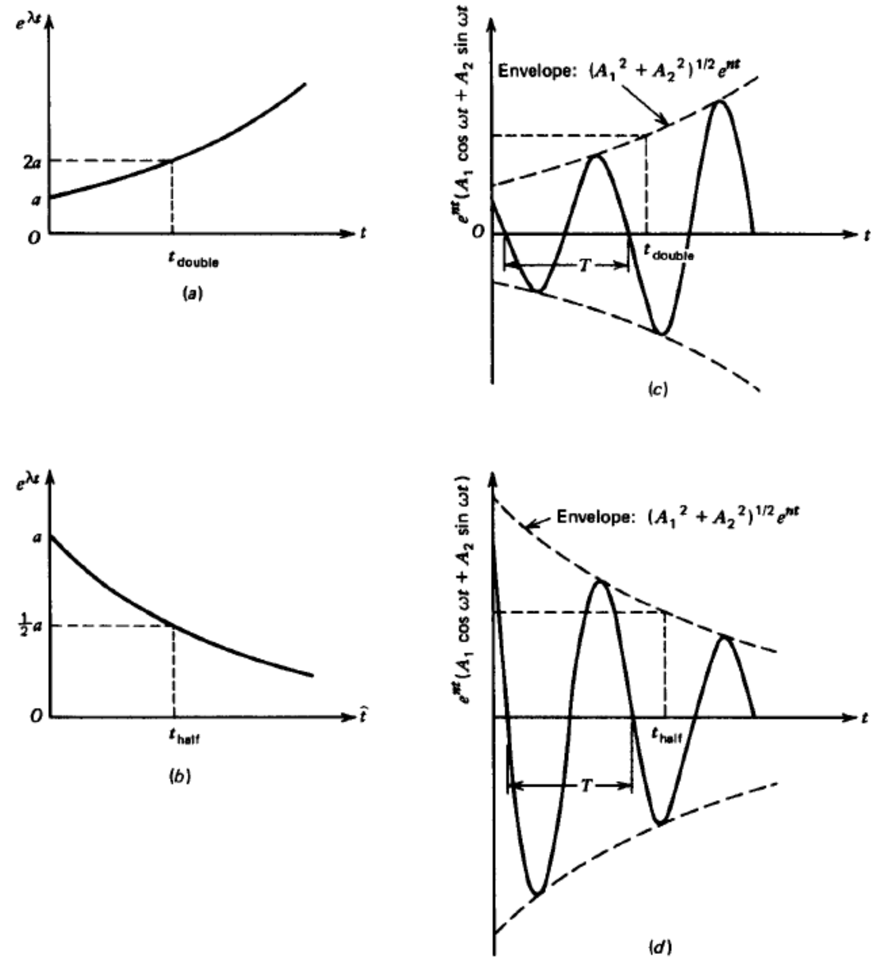


Figure 6.1 Types of solution. (a) λ real, positive. (b) λ real, negative. (c) λ complex, $n > 0$. (d) λ complex, $n < 0$.

Longitudinal Approximations

- After linearization of the aerodynamic forces and moments, the equations of motion can be further simplified to represent the longitudinal and lateral response modes of the aircraft by neglecting some terms in the equations
- ***Short period mode***
  - Usually a heavily damped oscillation with a period of only a few seconds
  - The motion is a rapid pitching of the aircraft about the center of gravity
  - The period is so short that the speed does not have time to change, so the oscillation is essentially an ***angle-of-attack variation***
  - *Approximations*

    $u = 0$, $\theta_0 = 0$, only equations in w and q with input in δ_e
  - *Simplified equations of motion*

$$\dot{w} = Z_w w + u_0 q + Z_{\delta_e}\delta_e \qquad \tilde{M}_w = M_w + M_{\dot{w}} Z_w$$

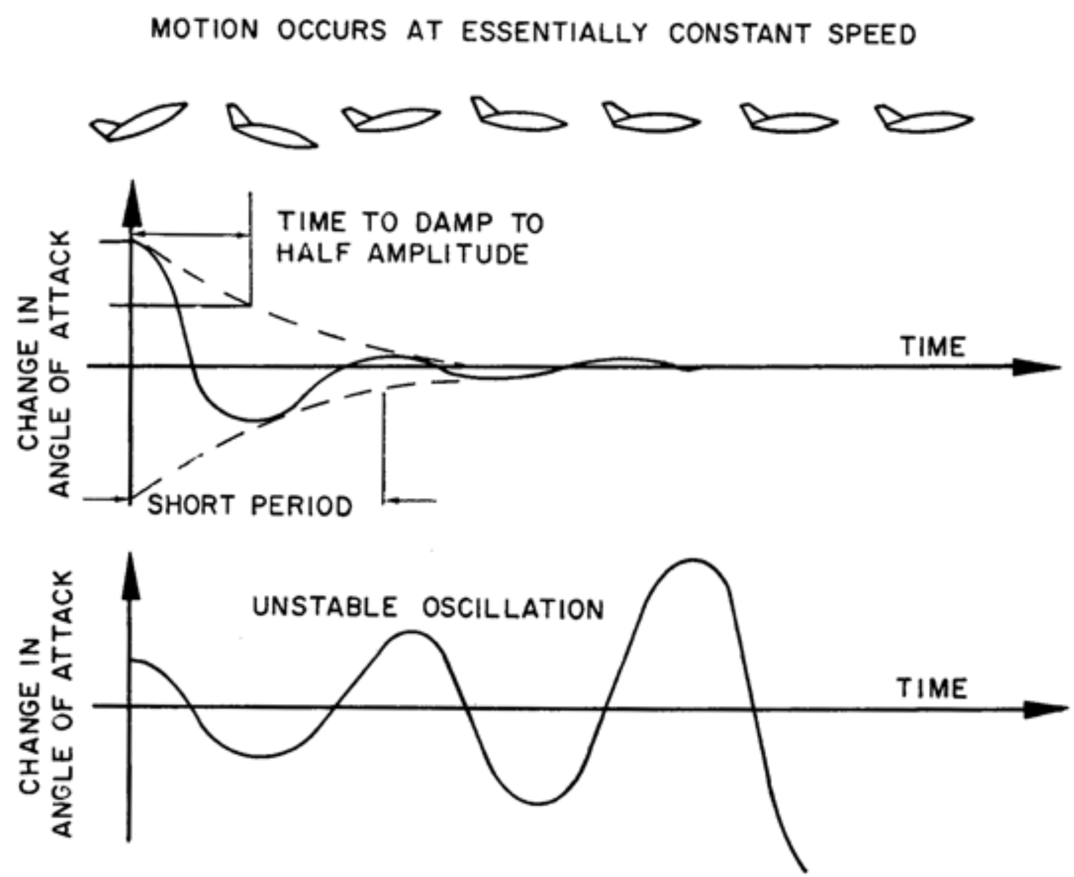
$$\dot{q} = \tilde{M}_w w + \tilde{M}_q q + \tilde{M}_{\delta_e}\delta_e \qquad \tilde{M}_q = M_q + M_{\dot{w}} u_0$$

  - *Natural frequency and damping ratio*

$$\omega_n = \sqrt{-\tilde{M}_w u_0 + \tilde{M}_q Z_w} \qquad \xi = -\frac{Z_w + \tilde{M}_q}{2\omega_n}$$

Longitudinal Approximations

- After linearization of the aerodynamic forces and moments, the equations of motion can be further simplified to represent the longitudinal and lateral response modes of the aircraft by neglecting some terms in the equations
- ***Short period mode***



- ***Phugoid mode***
  - Large-amplitude variation of air-speed, pitch angle, and altitude, but almost no angle-of-attack variation
  - Slow interchange of kinetic energy (velocity) and potential energy (height) about some equilibrium energy level
  - The motion is so slow that the effects of inertia forces and damping forces are very low
  - *Approximations*

    $w = \dot{w} = 0$ (or $\alpha = \dot{\alpha} = 0$), only equations in u and θ with input in δ_t
  - *Simplified equations of motion*

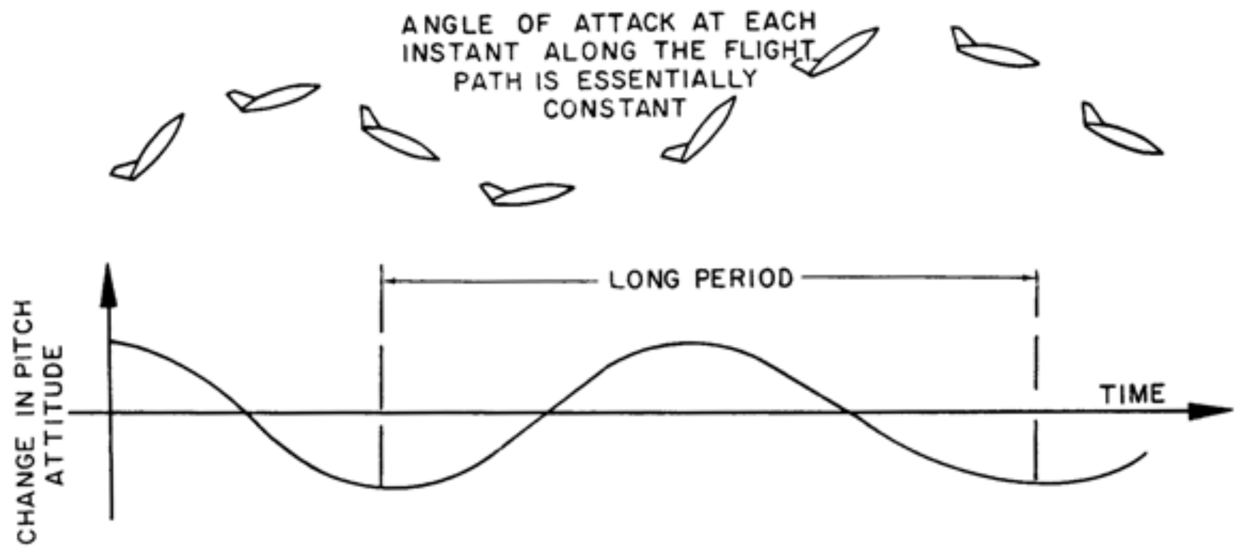
$$\dot{u} = X_u u + g\theta + X_{\delta_t}\delta_t$$

$$\dot{\theta} = -\frac{Z_u}{u_0}u - \frac{Z_{\delta_t}}{u_0}\delta_t$$

  - *Natural frequency and damping ratio*

$$\omega_n = \sqrt{-\frac{Z_u g}{u_0}} \qquad \xi = -\frac{X_u}{2\omega_n}$$

- ***Phugoid mode***



- ***Roll mode***
  - Rotation about longitudinal axis
  - No direct aerodynamic moment created tending to directly restore wings-level
  - Roll mode can be improved by dihedral effects coming from design characteristics, such as high wings, dihedral angles or sweep angles
  - *Approximations*

$$v \text{ (or } \beta) = r = 0, \text{ only equations in } p \text{ with input in } \delta_a$$

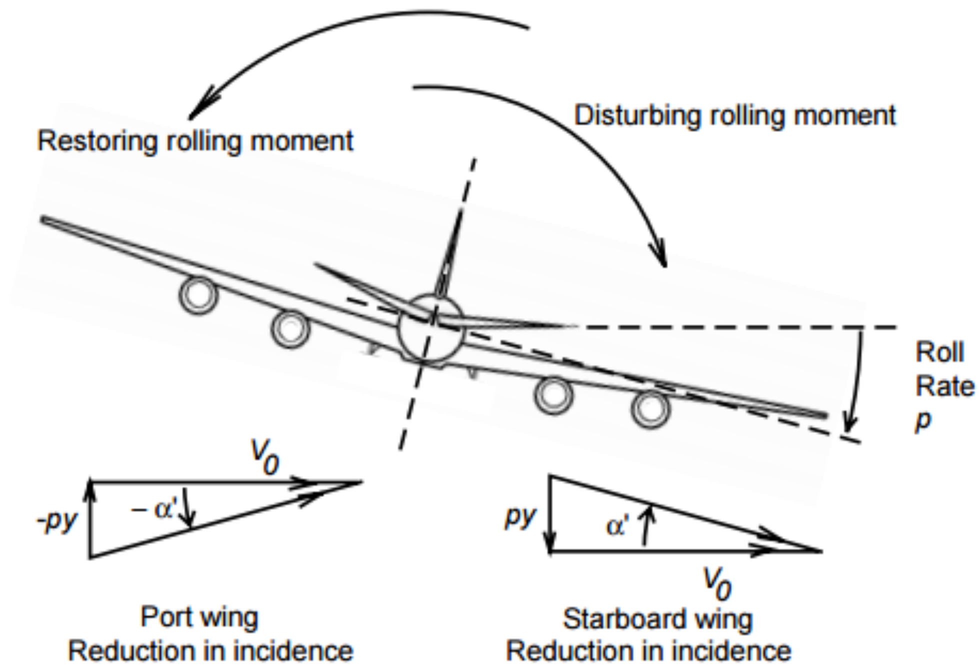
  - *Simplified equations of motion*

$$\dot{p} = L'_p p + L'_{\delta_a} \delta_a$$

$$L'_p = L_p + \frac{I_{xz}}{I_{xx}} N_v$$

$$L'_{\delta_a} = L_{\delta_a} + \frac{I_{xz}}{I_{xx}} N_{\delta_a}$$

- ***Roll mode***


Restoring rolling moment
Disturbing rolling moment
Roll
Rate
p
V_0
-py
− α'
py
α'
V_0
Port wing
Reduction in incidence
Starboard wing
Reduction in incidence

- ***Dutch roll mode***
  - Yaw and roll to the right, followed by a recovery towards the equilibrium condition, then an overshooting of this condition and a yaw and roll to the left, then back past the equilibrium attitude, and so on
  - The period is usually on the order of 3–15 seconds, but it can vary from a few seconds for light aircraft to a minute or more for airliners
  - Damping is increased by large directional stability and small dihedral and decreased by small directional stability and large dihedral.
  - *Approximations*

    $p = 0$, only equations in v (or β) and r with input in δ_r
  - *Simplified equations of motion*

$$\dot{\beta} = Y_\beta \beta + \left(\frac{Y_r}{u_0} - 1\right) r + \frac{Y_{\delta_r}}{u_0} \delta_r$$

$$\dot{r} = N'_\beta \beta + N'_r r + N'_{\delta_r} \delta_r$$

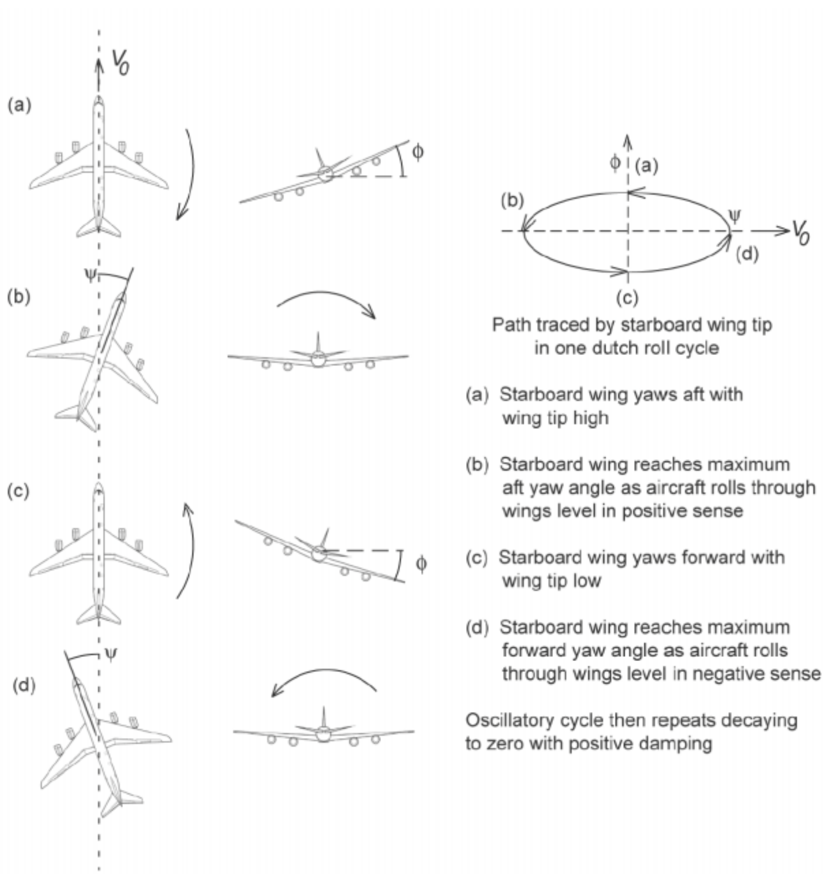
$$N'_\beta = N_\beta + \frac{I_{xz}}{I_{zz}} L_\beta$$

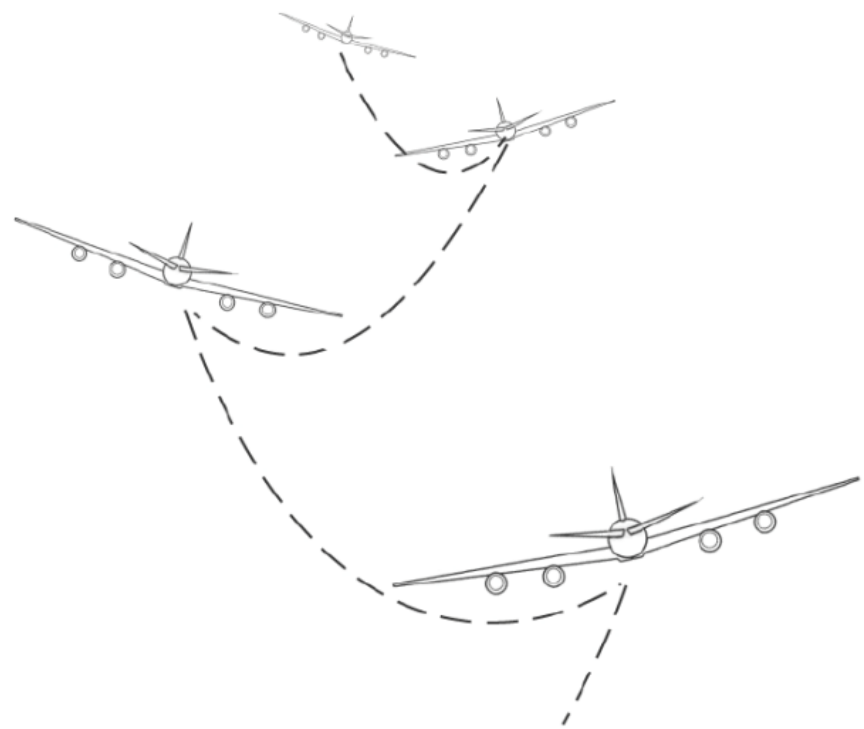
$$N'_r = N_r + \frac{I_{xz}}{I_{zz}} L_r$$

$$N'_{\delta_r} = N_{\delta_r} + \frac{I_{xz}}{I_{zz}} L_{\delta_r}$$

Lateral Approximations

- ***Dutch roll mode***





- ***Spiral mode***
  - All aircraft trimmed for straight-and-level flight, if flown stick-fixed, will spiral. Some types will spiral-dive
  - This is more likely with some configurations, such as low wing. With other configurations, such as high wing and relatively low power, the spiral does not become a dive
  - *Approximations*

$$v \text{ (or } \beta) = r = 0, \text{ only equations in } p \text{ with input in } \delta_a$$

  - *Simplified equations of motion*

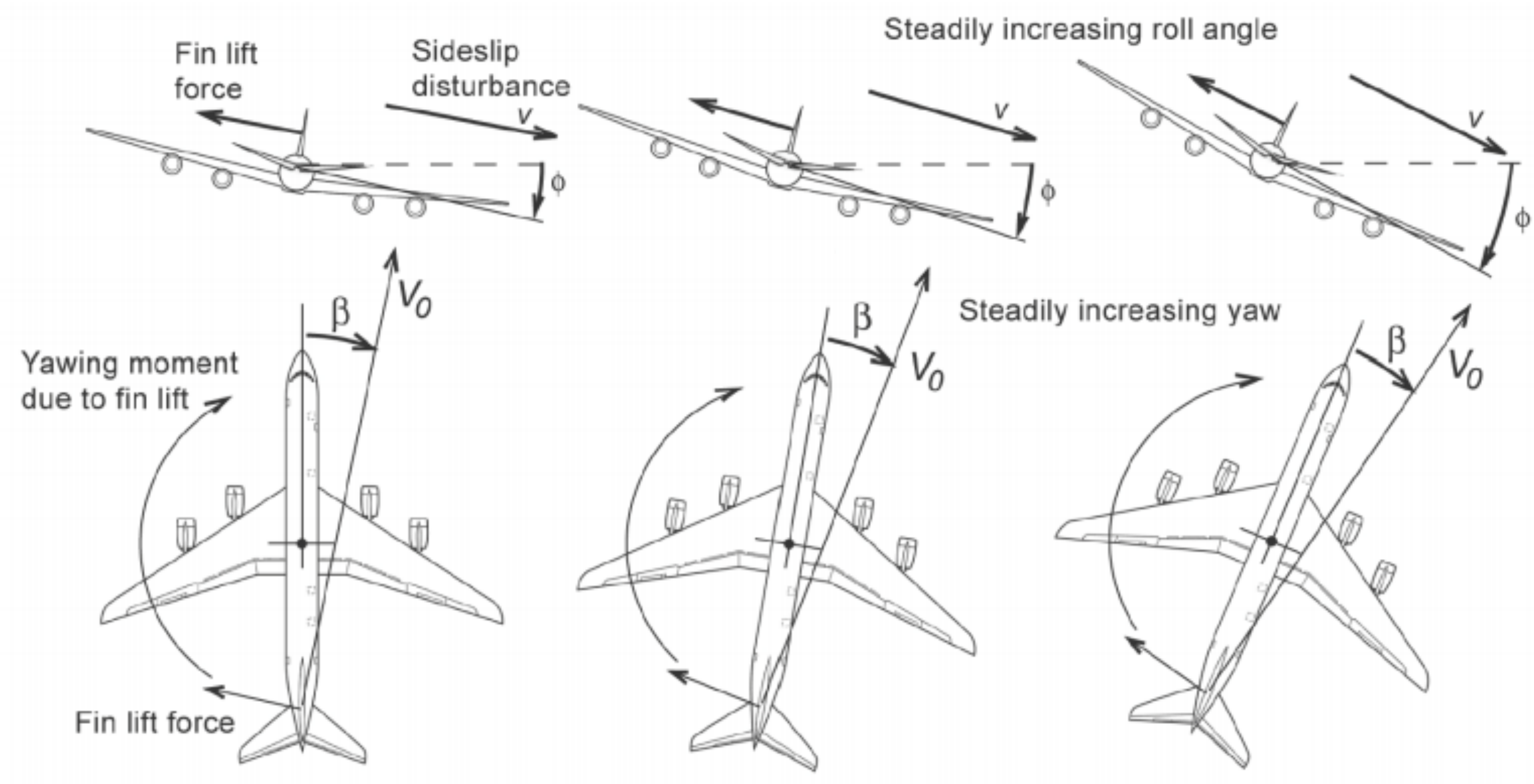
$$\dot{p} = L'_p p + L'_{\delta_a} \delta_a$$

$$\dot{\phi} = p$$

$$L'_p = L_p + \frac{I_{xz}}{I_{xx}} N_v$$

$$L'_{\delta_a} = L_{\delta_a} + \frac{I_{xz}}{I_{xx}} N_{\delta_a}$$

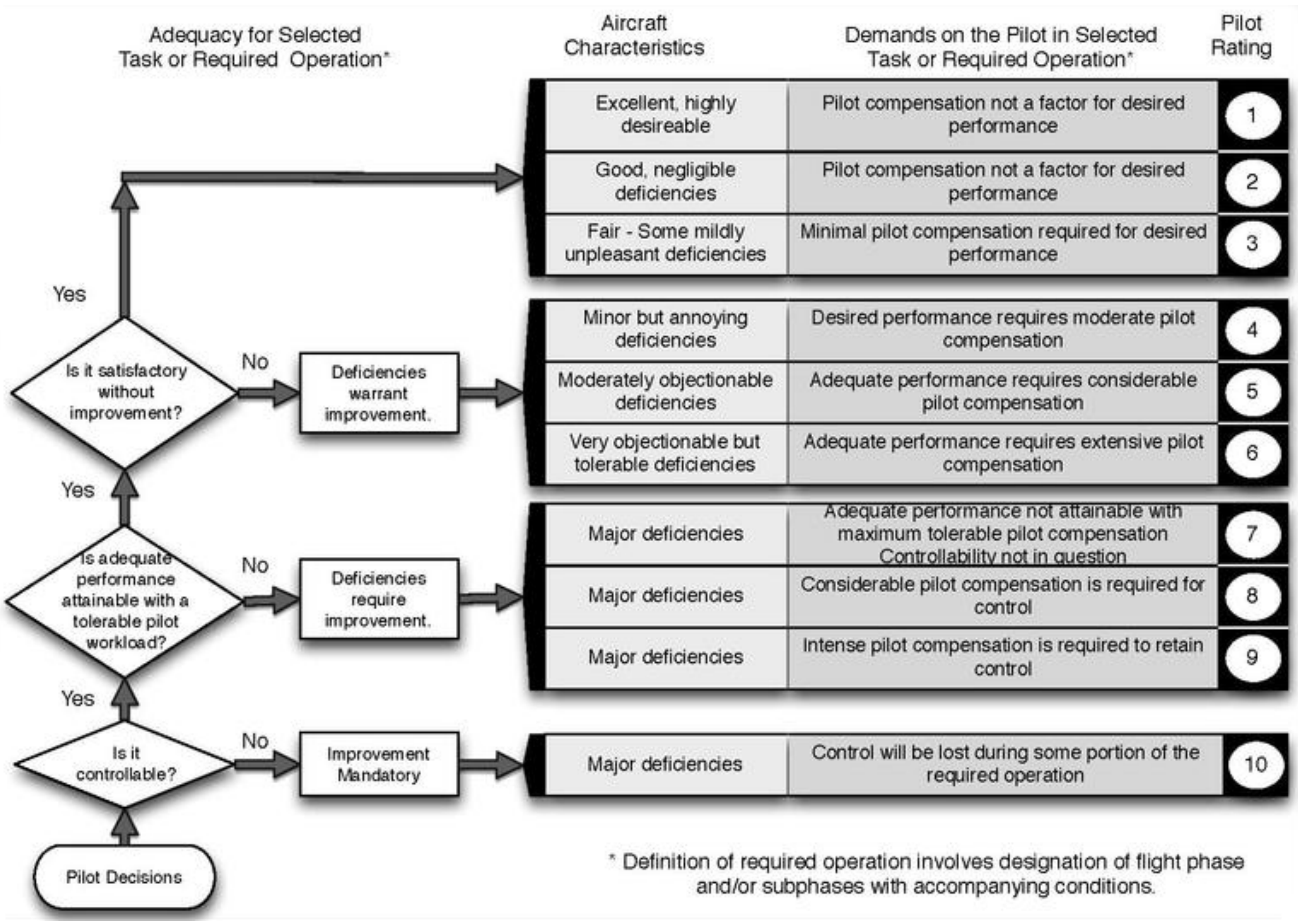
- ***Spiral mode***


Fin lift
force
Sideslip
disturbance
v
ϕ
v
ϕ
Steadily increasing roll angle
v
ϕ
β
V_0
Yawing moment
due to fin lift
Fin lift force
β
V_0
Steadily increasing yaw
β
V_0

Flying/Handling Qualities

- The primary reason for conducting flying quality investigations is to determine if the pilot-aircraft combination can safely and precisely perform the various tasks and maneuvers required by the total aircraft mission
- In order to exhibit satisfactory flying qualities, an aircraft must be both stable and controllable
- A certain stability is necessary if the aircraft is to be easily controlled by a human pilot
- However, too much stability can severely degrade the pilot’s ability to perform maneuvering tasks

- ***Cooper-Harper handling qualities rating scale***



- ***Aircraft classes and flight phases***

Table 7.4 MIL-F-8785C aircraft classes

Class	General aircraft types	Specific examples
Class I small, light airplanes	Light utility	T-41
	Primary trainer	T-6
	Light observation	O-1, O-2
Class II medium weight; low-to-medium maneuverability airplanes	Heavy utility/search and rescue	C-21
	Light or medium transport/cargo/tanker	C-130
	Early warning/ECM/Command & control	E-2
	Anti-submarine	S-3A
	Assault transport	C-130
	Reconnaissance	U-2
	Tactical bomber	B-66
	Heavy attack	A-6
	Trainer for Class II	T-1A
Class III large, heavy, low-to-medium maneuverability airplanes	Heavy transport/cargo/tanker	KC-10, C-17
	Heavy bomber	B-52, B-1, B-2
	Patrol/Early warning/ECM/Command & control	P-3, SR-71
	Trainer for Class III	TC-135
Class IV high-maneuverability airplanes	Figher/Intecepter	F-22, F-15, F-16
	Attack	F-15E, A-10
	Tactical reconnaissance	RF-4
	Observation	OV-10
	Trainer for Class IV	T-38

Table 7.5 MIL-F-8785C flight phase categories

Category A — Those nonterminal flight phases that require rapid maneuvering, precision tracking, or precise flight-path control. Included in this category are:
- (a) Air-to-air combat (CO)
- (b) Ground attack (GA)
- (c) Weapon delivery/launch (WD)
- (d) Aerial recovery (AR)
- (e) Reconnaissance (RC)
- (f) In-flight refueling (receiver) (RR)
- (g) Terrain following (TF)
- (h) Antisubmarine search (AS)
- (i) Close formation flying (FF)

Category B — Those nonterminal flight phases that are normally accomplished using gradual maneuvers and without precision tracking, although accurate flight-path control may be required. Included in this category are:
- (a) Climb (CL)
- (b) Cruise (CR)
- (c) Loiter (LO)
- (d) In-flight refueling (tanker) (RT)
- (e) Descent (D)
- (f) Emergency descent (ED)
- (g) Emergency deceleration (DE)
- (h) Aerial delivery (AD)

Category C — Those terminal flight phases that are normally accomplished using gradual maneuvers and which usually require accurate flight-path control. Included in this category are:
- (a) Takeoff (TO)
- (b) Catapult takeoff (CT)
- (c) Approach (PA)
- (d) Wave-off/go-around (WO)
- (e) Landing (L)

- ***Short period dynamic requirements***

Table 7.7 Short period damping ratio (ζ_{SP}) limits

	Category A and C flight phases		Category B flight phases	
	Minimum	Maximum	Minimum	Maximum
Level 1	0.35	1.30	0.30	2.00
Level 2	0.25	2.00	0.20	2.00
Level 3	0.15*	no maximum	0.15*	no maximum

* May be reduced at altitudes above 20,000 ft if approved by the procuring activity.

Table 7.8 Additional short-period dynamic requirements

Flight phase category	Aircraft class	Level 1	Level 2	Level 3
A	All	$\omega_{sp} \geq 1$ rad/s	$\omega_{sp} \geq 0.6$ rad/s	$\zeta_{sp} \geq 0.15$ may be relaxed above 20,000 ft
B	All			T_2, the time to double amplitude based on the unstable root, should be no less than 6 s. In the presence of any other Level 3 flying qualities, ζ_{sp} should be at least 0.05.
C	I, II- C, IV	$\omega_{sp} \geq 0.87$ rad/s $n/\alpha \geq 2.7$ g/rad	$\omega_{sp} \geq 0.6$ rad/s $n/\alpha \geq 1.8$ g/rad	
	II-L, III	$\omega_{sp} \geq 0.7$ rad/s $n/\alpha \geq 2.0$ g/rad	$\omega_{sp} \geq 0.4$ rad/s $n/\alpha \geq 1.0$ rad	

- ***Phugoid dynamic requirements***

Table 7.9 Phugoid damping requirements

Level 1	$\zeta > 0.04$
Level 2	$\zeta > 0$
Level 3	$T_2 \geq 55$ s

- ***Roll dynamic requirements***

Table 7.10 Maximum roll mode time constant (seconds)

Flight phase category	Class	Level 1	Level 2	Level 3
A	I and IV	1.0	1.4	10
	II and III	1.4	3.0	10
B	All	1.4	3.0	10
C	I, II-C, and IV	1.0	1.4	10
	II-L and III	1.4	3.0	10

- ***Spiral dynamic requirements***

Table 7.11 MIL-F-8785C spiral mode minimum time to double amplitude

Flight phase category	Level 1	Level 2	Level 3
A and C	12 s	8 s	4 s
B	20 s	8 s	4 s

- ***Dutch roll dynamic requirements***

Table 7.12 Minimum dutch roll frequency and damping

Level	Flight phase category	Class	Minimum ζ*	Minimum $\zeta\omega_N$,* rad/s	Minimum ω_N, rad/s
	A (CO, GA, RR, TF, RC, FF, and AS)	I, II, III, and IV	0.4	0.4	1.0
	A	I and IV	0.19	0.35	1.0
1		II and III	0.19	0.35	0.4
	B	All	0.08	0.15	0.4
	C	I, II-C, and IV	0.08	0.15	1.0
		II-L and III	0.08	0.10	0.4
2	All	All	0.02	0.05	0.4
3	All	All	0	—	0.4

*The governing damping requirement is that yielding the larger value of ζ_d, except that a ζ_d of 0.7 is the maximum required for Class III aircraft.