Exercise 1.

a) X = {A♣, A♦}
A = {peek, guessA♣, guessA♦}
Z = {seeA♣, seeA♦, seeNothing}

b) $P_{peek} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$P_{A♣} = P_{A♦} = \begin{bmatrix} 0.5 & 0.5 \\ 0.5 & 0.5 \end{bmatrix}$

$O_{peek} = \begin{bmatrix} 0.9 & 0.1 & 0 \\ 0.1 & 0.9 & 0 \end{bmatrix}$

$O_{A♣} = O_{A♦} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

$C = \begin{bmatrix} 0.5 & 0 & 1 \\ 0.5 & 1 & 0 \end{bmatrix}$

c) $$b_{t+1} = \frac{b_t P_{peek} diag(O_{peek}, seeA♦)}{\|b_t P_{peek} diag(O_{peek}, seeA♦)\|_1} =$$

$$\frac{\begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 & 0 \\ 0 & 0.9 \end{bmatrix}}{\| \begin{bmatrix} 0.7 & 0.3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0.1 & 0 \\ 0 & 0.9 \end{bmatrix} \|_1} = \frac{\begin{bmatrix} 0.07 & 0.27 \end{bmatrix}}{0.34} = \begin{bmatrix} 0.206 & 0.794 \end{bmatrix}$$