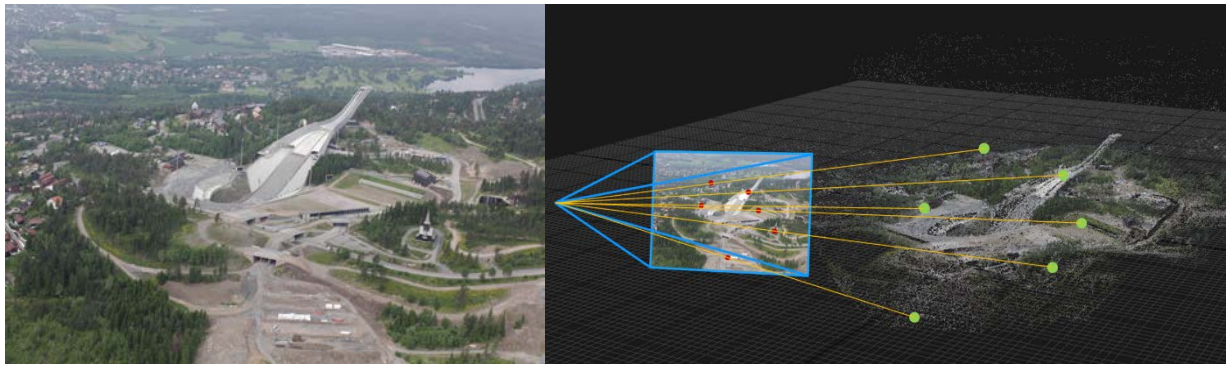
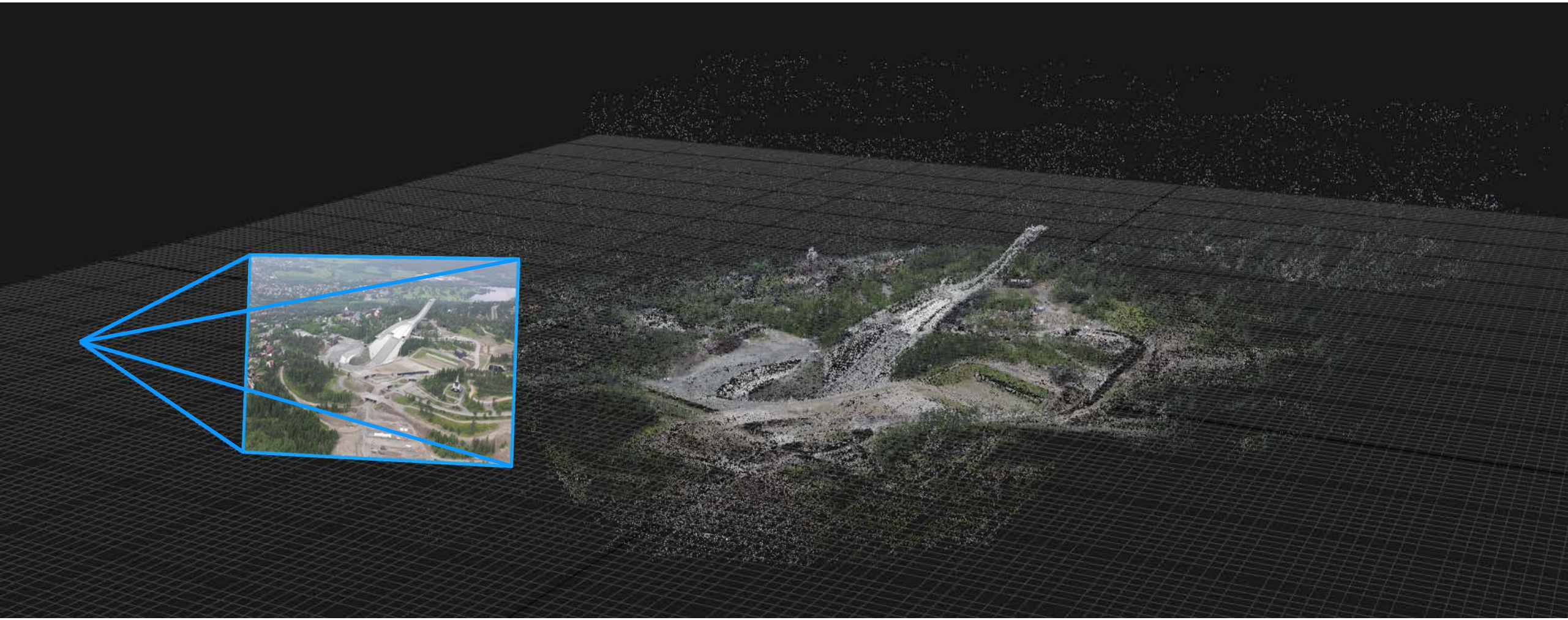
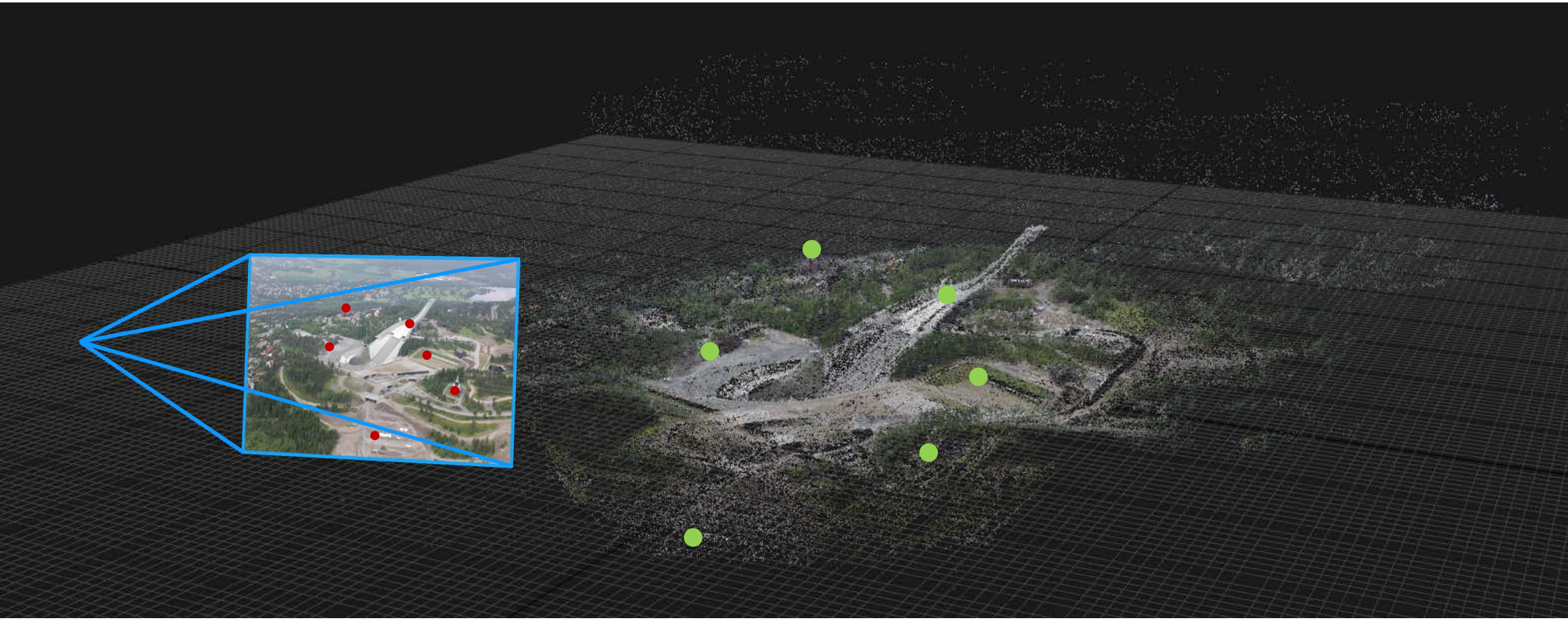
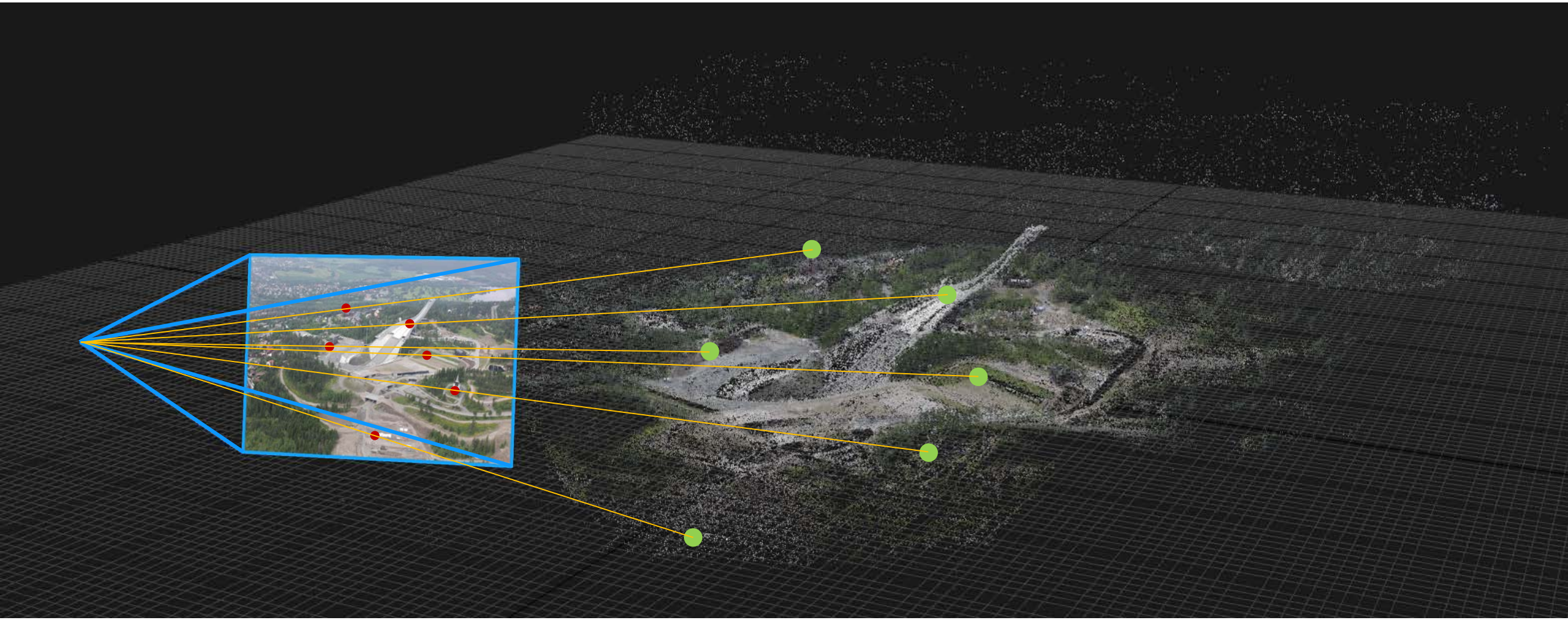


Lecture 5.2
Pose from known 3D points

Trym Vegard Haavardsholm







World geometry from correspondences

	Structure (scene geometry)	Motion (camera geometry)	Measurements
Pose estimation	**Known**	**Estimate**	3D to 2D correspondences
Triangulation, Stereo	**Estimate**	**Known**	2D to 2D correspondences
Reconstruction, Structure from Motion	**Estimate**	**Estimate**	2D to 2D correspondences

Pose estimation from known 3D points

Given at least 6 3D to 2D correspondences

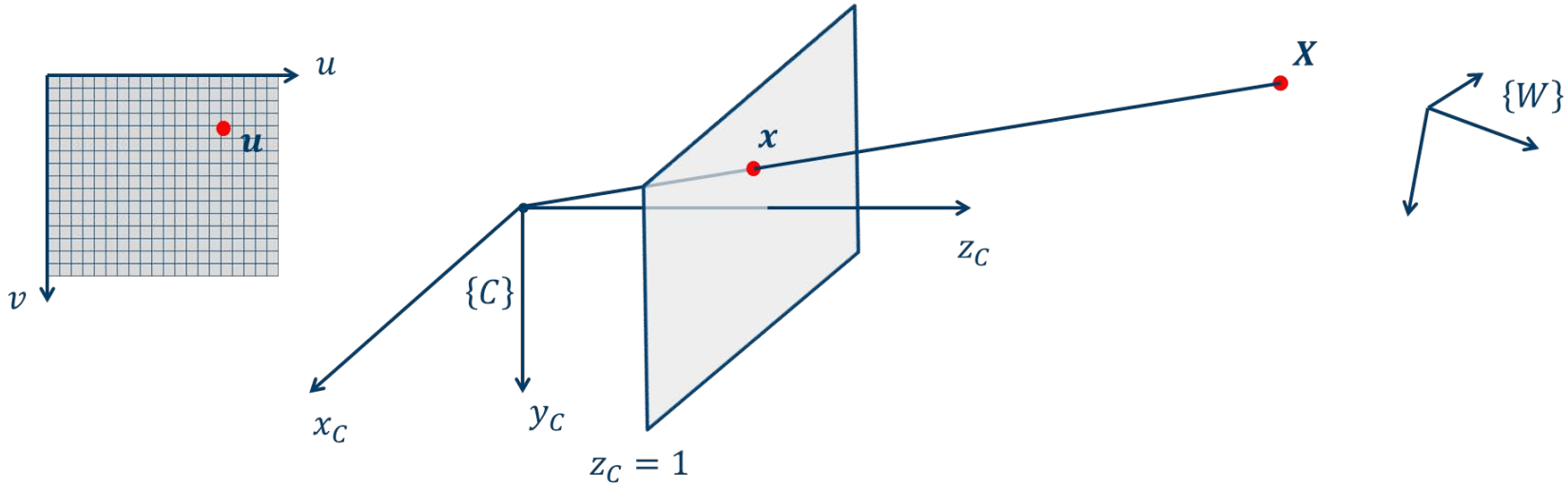
$$\left\{ \tilde{\boldsymbol{X}}_i, \tilde{\boldsymbol{u}}_i \right\}$$

we can estimate a camera matrix $\hat{P}$ that fits with the camera projection model

$$\tilde{\boldsymbol{u}}_i = P\tilde{\boldsymbol{X}}_i$$

where P is given by

$$P = K[R \mid \boldsymbol{t}]$$



Pose from *P*

- Decomposition of the Camera Matrix

$$
\begin{aligned}
P &= K[R \mid \boldsymbol{t}] \\
&= K[R \mid -R\boldsymbol{C}] \\
&= [M \mid -M\boldsymbol{C}]
\end{aligned}
$$

Pose from *P*

- Decomposition of the Camera Matrix

$$P = K[R \mid \boldsymbol{t}]$$
$$= K[R \mid -R\boldsymbol{C}]$$
$$= [M \mid -M\boldsymbol{C}]$$

Find the camera center ***C***

Find intrinsic *K* and rotation *R*

Pose from *P*

- Decomposition of the Camera Matrix

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Find the camera center ***C***

$$P\boldsymbol{C} = 0$$

SVD of *P*!
C is the right singular vector
corresponding to the smallest singular value

Find intrinsic *K* and rotation *R*

Pose from *P*

- Decomposition of the Camera Matrix

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Find intrinsic *K* and rotation *R*

$$M = KR$$

RQ decomposition

RQ decomposition

- RQ-factorization is a decomposition of a matrix M into the product $M = RQ$ where R is an upper triangular matrix and Q is an orthogonal matrix

- The RQ in ‘’RQ-decomposition’’ corresponds to KR in the expression $M = KR$,
  Upper triangular matrix R ←→ Camera calibration matrix K
  Orthogonal matrix Q ←→ Rotation matrix R

- In addition we know:
  - *K* has positive diagonal: $(KD)(D^{-1}R)$ where
  - det(R) = 1: Enforce det(M) > 0

RQ decomposition

- When only QR decomposition is available:

- In matlab:

```
function [R,Q] = rq(M)
[Q,R] = qr(rot90(M,3));
R = rot90(R,2)';
Q = rot90(Q);
```

Pose estimation from known 3D points

1. Extract local features
2. Establish 3D to 2D correspondences $\{\widetilde{\boldsymbol{X}}_i, \widetilde{\boldsymbol{u}}_i\}$
3. Estimate P from $\{\widetilde{\boldsymbol{X}}_i, \widetilde{\boldsymbol{u}}_i\}$
4. Let M be the first 3×3 matrix of P
5. Normalize P by multiplying all elements with sign(det(M))
6. Compute the camera center $\boldsymbol{C}$ by SVD
7. QR-decomposition of M gives us $M = \hat{K}\hat{R}$
   where $\hat{K}$ is an upper triangular like K and $\hat{R}$ is an orthogonal matrix like R
8. Define $D = diag(sign(K_{11}), sign(K_{22}), sign(K_{33}))$
9. Then $K = \hat{K}D$ and $R = D\hat{R}$ and $\boldsymbol{t} = -R\boldsymbol{C}$

Matlab example

```
% Estimate P by basic DLT.
est.P = estimateCameraMatrix_DLT(u,X);
est.P = est.P * sign(det(est.P(1:3, 1:3)));

% Estimate C.
[U,S,V] = svd(est.P);
est.C = V(1:3,end) / V(end, end);

% Estimating K and R by RQ decomposition.
[est.K,est.R] = rq(est.P(1:3,1:3));

% Enforce positive diagonal of K by changing signs in both est.K and est.R.
D = diag(sign(diag(est.K)));
est.K = est.K*D;
est.R = D*est.R;
est.K = est.K/est.K(end,end);

%Determine t from the estimated C.
est.t = -est.R*est.C;
```

Real world example

- 6 correspondences from map

Real world example

- 6 correspondences from map
- Position error: ~100m
- Estimated focal length: 52 mm
- True focal length: 50 mm

What if we know parts of *P*?

- Iterative methods
  - Initialize with P from DLT, clamp known parameters
  - Minimize geometric error

$$\sum_i d(\boldsymbol{u}_i, P\boldsymbol{X}_i)^2$$

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  - Better: Soft constraints, for example:

$$\sum_i d(\boldsymbol{u}_i, P\boldsymbol{X}_i)^2 + ws^2 + w(f_x - f_y)^2$$

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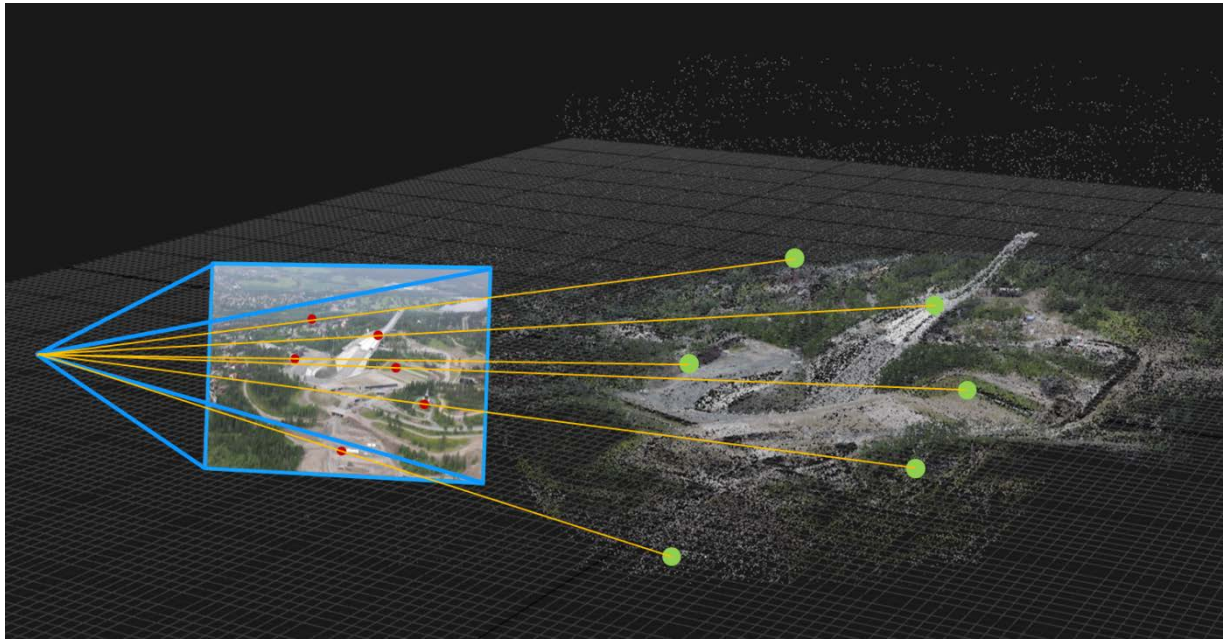
- Use Perspective-*n*-Point (P*n*P)

n-Point Pose Problem (P*n*P)

- Several different methods available
  - Typically fast non-iterative methods
  - Minimal in number of points
  - Accuracy comparable to iterative methods

- Examples
  - P3P
    - Estimate pose, known *K*
  - P4Pf
    - Estimate pose and focal length
  - P6P
    - DLT!
  - R6P
    - Estimate pose with rolling shutter



Summary

- Pose from known 3D points
  - Decomposition of *P* matrix
  - P*n*P

- Further reading:
  - Torstein Sattler,
    CVPR 2015 Tutorial on Large-Scale Visual Place Recognition and Image-Based Localization