Aprendizagem 2021/22

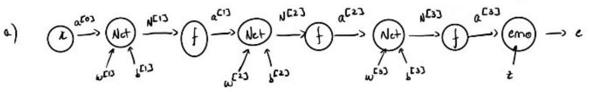
Homework III – Group 17

# I. Pen-and-paper

1)

**a.** 1) $w^{[1]} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$ $\quad w^{[2]} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$ $\quad w^{[3]} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

$b^{[1]} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $\quad b^{[2]} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\quad b^{[3]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

a) 


$x = [1\ 1\ 1\ 1\ 1]^T$

$z = [1\ -1]^T$

$N^{[1]} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$

$a^{[2]} = f(N^{[2]}) = \begin{pmatrix} f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) \end{pmatrix}$

$a^{[1]} = f(N^{[1]}) = \begin{pmatrix} f(6) \\ f(1) \\ f(6) \end{pmatrix}$

$N^{[2]} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} f(6) \\ f(1) \\ f(6) \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2f(6)+f(1)+1 \\ 2f(6)+f(1)+1 \end{pmatrix}$

$N^{[3]} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$a^{[3]} = f(N^{[3]}) = \begin{pmatrix} f(0) \\ f(0) \end{pmatrix}$

$$E(z, a^{[3]}) = \frac{1}{2} \sum_{t=1}^{2} (z_t - a_t^{[3]})^2$$

$$\frac{\partial E}{\partial a_i^{[3]}} = \frac{1}{2} \sum_{t=1}^{2} \frac{\partial}{\partial a_i^{[3]}} (z_t - a_t^{[3]})^2 = \cancel{\tfrac{1}{2}}\,\cancel{2}\,(-1).(z_i - a_i^{[3]}) = a_i^{[3]} - z_i \quad ; \quad \nabla_{a^{[3]}} E = \begin{pmatrix} a_1^{[3]} - z_1 \\ a_2^{[3]} - z_2 \end{pmatrix} = \begin{pmatrix} 0 - 1 \\ 0 + 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$a^{[3]} = \begin{pmatrix} f(0) \\ f(0) \end{pmatrix} ; N^{[3]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$

$$\nabla_{N^{[3]}} a^{[3]} = \begin{pmatrix} \frac{\partial a_1^{[3]}}{\partial N_1^{[3]}} & \frac{\partial a_2^{[3]}}{\partial N_1^{[3]}} \\ \frac{\partial a_1^{[3]}}{\partial N_2^{[3]}} & \frac{\partial a_2^{[3]}}{\partial N_2^{[3]}} \end{pmatrix} = \begin{bmatrix} \frac{\partial f(N_1^{[3]})}{\partial N_1^{[3]}} & \frac{\partial f(N_2^{[3]})}{\partial N_1^{[3]}} \\ \frac{\partial f(N_1^{[3]})}{\partial N_2^{[3]}} & \frac{\partial f(N_2^{[3]})}{\partial N_2^{[3]}} \end{bmatrix} = \begin{bmatrix} f'(N_1^{[3]}) & 0 \\ 0 & f'(N_2^{[3]}) \end{bmatrix} = \begin{bmatrix} \operatorname{sech}^2(0) & 0 \\ 0 & \operatorname{sech}^2(0) \end{bmatrix}$$

$$f'(x) = \left(\frac{e^x - e^{-x}}{e^x + e^{-x}}\right)' = \left(\frac{e^{2x} - 1}{e^{2x} + 1}\right)' = \frac{(2e^{2x})(e^{2x}+1) - (e^{2x}-1)(2e^{2x})}{(e^{2x}+1)^2}, \quad y = e^{2x}$$

$$= \frac{2y(y+1) - (y-1)(2y)}{(y+1)^2}$$

$$= \frac{\cancel{2y^2} + 2y - \cancel{2y^2} + 2y}{(y+1)^2}$$

$$= \frac{4y}{(y+1)^2} = \frac{4e^{2x}}{(e^{2x}+1)^2}$$

$$= \operatorname{sech}^2(x)$$

$$\frac{\partial f(N_i^{[3]})}{\partial N_j^{[3]}} = \begin{cases} f'(N_i^{[3]}), & i = j \\ 0, & i \neq j \end{cases}$$

$N^{[3]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad a^{[2]} = \begin{pmatrix} f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) \end{pmatrix}$

$$\frac{\partial N_j^{[3]}}{\partial a_i^{[2]}} = \frac{\partial}{\partial a_i^{[2]}} \sum_{t=1}^{2} w_{jt}^{[3]} . a_t^{[2]} + b_j^{[3]} = w_{ji}^{[3]}$$

$$\nabla_{a^{[2]}} N^{[3]} = \begin{bmatrix} \frac{\partial N_1^{[3]}}{\partial a_1^{[2]}} & \frac{\partial N_2^{[3]}}{\partial a_1^{[2]}} \\ \frac{\partial N_1^{[3]}}{\partial a_2^{[2]}} & \frac{\partial N_2^{[3]}}{\partial a_2^{[2]}} \end{bmatrix} = \begin{bmatrix} w_{11}^{[3]} & w_{21}^{[3]} \\ w_{12}^{[3]} & w_{22}^{[3]} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial N_j^{[3]}}{\partial a_i^{[2]}} = \frac{\partial}{\partial a_i^{[2]}} \sum_{t=1}^{2} w_{jt}^{[3]} . a_t^{[2]} + b_j^{[3]} = w_{ji}^{[3]}$$

$$\nabla_{N^{[2]}} a^{[2]} = \begin{bmatrix} \frac{\partial a_1^{[2]}}{\partial N_1^{[2]}} & \frac{\partial a_2^{[2]}}{\partial N_1^{[2]}} \\ \frac{\partial a_1^{[2]}}{\partial N_2^{[2]}} & \frac{\partial a_2^{[2]}}{\partial N_2^{[2]}} \end{bmatrix} = \begin{bmatrix} f'(N_1^{[2]}) & 0 \\ 0 & f'(N_2^{[2]}) \end{bmatrix} = \begin{bmatrix} \operatorname{sech}^2(2\tanh(6) + \tanh(1)+1) & 0 \\ 0 & \operatorname{sech}^2(2\tanh(6) + \tanh(1)+1) \end{bmatrix}$$

$$\nabla_{a^{[1]}} N^{[2]} = \begin{bmatrix} w_{11}^{[2]} & w_{21}^{[2]} \\ w_{12}^{[2]} & w_{22}^{[2]} \\ w_{13}^{[2]} & w_{23}^{[2]} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\nabla_{N^{[1]}} a^{[1]} = \begin{bmatrix} \frac{\partial a_1^{[1]}}{\partial N_1^{[1]}} & \frac{\partial a_2^{[1]}}{\partial N_1^{[1]}} & \frac{\partial a_3^{[1]}}{\partial N_1^{[1]}} \\ \frac{\partial a_1^{[1]}}{\partial N_2^{[1]}} & \frac{\partial a_2^{[1]}}{\partial N_2^{[1]}} & \frac{\partial a_3^{[1]}}{\partial N_2^{[1]}} \\ \frac{\partial a_1^{[1]}}{\partial N_3^{[1]}} & \frac{\partial a_2^{[1]}}{\partial N_3^{[1]}} & \frac{\partial a_3^{[1]}}{\partial N_3^{[1]}} \end{bmatrix} = \begin{bmatrix} \operatorname{sech}^2(\tanh(6)) & 0 & 0 \\ 0 & \operatorname{sech}^2(\tanh(1)) & 0 \\ 0 & 0 & \operatorname{sech}^4(\tanh(6)) \end{bmatrix}$$

$$\delta^{[3]} = \nabla^{e}_{N^{[3]}} = \nabla^{a^{[3]}}_{N^{[3]}} \cdot \nabla^{e}_{a^{[3]}} = \underset{2\times 2}{\begin{pmatrix} \operatorname{sech}^2(0) & 0 \\ 0 & \operatorname{sech}^2(0) \end{pmatrix}} \underset{2\times 1}{\begin{pmatrix} -1 \\ 1 \end{pmatrix}} = \begin{pmatrix} -\operatorname{sech}^2(0) \\ \operatorname{sech}^2(0) \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$\nabla_{w^{[3]}} e = \delta^{[3]} \cdot a^{[2]^T} = \begin{pmatrix} -1 \\ 1 \end{pmatrix} \begin{pmatrix} f(2f(6)+f(1)+1) & f(2f(6)+f(1)+1) \end{pmatrix} = \begin{bmatrix} -f(2f(6)+f(1)+1) & -f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) & f(2f(6)+f(1)+1) \end{bmatrix}$$

$$w^{[3]} = w^{[3]} - \eta \cdot \nabla^{e}_{w^{[3]}} = w^{[3]} - 0.1 \times \begin{bmatrix} -f(2f(6)+f(1)+1) & -f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) & f(2f(6)+f(1)+1) \end{bmatrix} \simeq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} -0{,}09989 & -0{,}09989 \\ 0{,}09989 & 0{,}09989 \end{pmatrix} = \begin{pmatrix} 0{,}09989 & 0{,}09989 \\ -0{,}09989 & -0{,}09989 \end{pmatrix}$$

$$\delta^{[2]} = \nabla^{e}_{N^{[2]}} = \underset{2\times 2}{\nabla^{a^{[2]}}_{N^{[2]}}} \cdot \underset{2\times 2}{\nabla^{N^{[3]}}_{a^{[2]}}} \cdot \underset{2\times 1}{\nabla^{e}_{N^{[3]}}} = \begin{bmatrix} \operatorname{sech}^2(2\tanh(6)+\tanh(1)+1) & 0 \\ 0 & \operatorname{sech}^2(2\tanh(6)+\tanh(1)+1) \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\nabla^{e}_{w^{[2]}} = \delta^{[2]} \cdot a^{[1]^T} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} f(6) & f(1) & f(6) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$w^{[2]} = w^{[2]} - \eta \cdot \nabla^{e}_{w^{[2]}} = w^{[2]} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\delta^{[1]} = \nabla^{e}_{N^{[1]}} = \nabla^{a^{[1]}}_{N^{[1]}} \cdot \nabla^{N^{[2]}}_{a^{[1]}} \cdot \nabla^{e}_{N^{[2]}} = \begin{bmatrix} \operatorname{sech}^2(\tanh(6)) & 0 & 0 \\ 0 & \operatorname{sech}^2(\tanh(1)) & 0 \\ 0 & 0 & \operatorname{sech}^2(\tanh(6)) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \cdot \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\nabla^{e}_{w^{[1]}} = \delta^{[1]} a^{[0]^T} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$w^{[1]} = w^{[1]} - \eta \cdot \nabla^{e}_{w^{[1]}} = w^{[1]} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$b^{[3]} = b^{[3]} - \eta \cdot \delta^{[3]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 0{,}1 \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0{,}1 \\ -0{,}1 \end{pmatrix}$$

$$b^{[2]} = b^{[2]} - \eta \cdot \delta^{[2]} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 0{,}1 \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$b^{[1]} = b^{[1]} - \eta \cdot \delta^{[1]} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - 0{,}1 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

b.

**b)**

$$F : \mathbb{R}^n \to \mathbb{R}^n$$

$$F(X) = \begin{pmatrix} f(x_1) \\ \vdots \\ f(x_n) \end{pmatrix}$$

$$f(x_k) = \frac{e^{x_k}}{\sum_{i=1}^{n} e^{x_i}}$$

$$L(z, s) = -\sum_{i=1}^{c} z_i \log(s_i)$$

- Forward

$x = [1\ 1\ 1\ 1\ 1]^T$

$z = [1\ \ 0]^T$

$$N^{[1]} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 6 \end{pmatrix}$$

$$a^{[1]} = F(N^{[1]}) = \begin{pmatrix} f(6) \\ f(1) \\ f(6) \end{pmatrix} \simeq \begin{pmatrix} 0{,}4983 \\ 0{,}0036 \\ 0{,}4983 \end{pmatrix}$$

$$N^{[2]} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} f(6) \\ f(1) \\ f(6) \end{pmatrix} + \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2f(6)+f(1)+1 \\ 2f(6)+f(1)+1 \end{pmatrix}$$

$$a^{[2]} = F(N^{[2]}) = \begin{pmatrix} f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) \end{pmatrix} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

$$N^{[3]} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} f(2f(6)+f(1)+1) \\ f(2f(6)+f(1)+1) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$a^{[3]} = F(N^{[3]}) = \begin{pmatrix} f(0) \\ f(0) \end{pmatrix}$$

$$\nabla_{a^{[3]}} e = \begin{bmatrix} -z_1 \cdot \frac{1}{a_1^{[3]}} \\ -z_2 \cdot \frac{1}{a_2^{[3]}} \end{bmatrix} = \begin{bmatrix} -\frac{1}{f(0)} \\ 0 \end{bmatrix}$$

$$\frac{\partial e}{\partial a_i^{[3]}} = \frac{\partial}{\partial a_i^{[3]}} \left( -\sum_{l=1}^{2} z_l \cdot \log(a_l^{[3]}) \right)$$

$$= -z_i \cdot \frac{1}{a_i^{[3]}}$$

$$\nabla_{N^{[3]}} a^{[3]} = \begin{bmatrix} \frac{\partial}{\partial N_1^{[3]}} f(N_1^{[3]}) & \frac{\partial}{\partial N_1^{[3]}} f(N_2^{[3]}) \\ \frac{\partial}{\partial N_2^{[3]}} f(N_1^{[3]}) & \frac{\partial}{\partial N_2^{[3]}} f(N_2^{[3]}) \end{bmatrix} = \begin{bmatrix} f(N_1^{[3]})(1-f(N_1^{[3]})) & -f(N_2^{[3]})f(N_1^{[3]}) \\ -f(N_1^{[3]})f(N_2^{[3]}) & f(N_2^{[3]})(1-f(N_2^{[3]})) \end{bmatrix}$$

$$a^{[3]} = F(N^{[3]})$$

$$a^{[3]} = \begin{pmatrix} f(N_1^{[3]}) \\ f(N_2^{[3]}) \end{pmatrix}$$

$$\frac{\partial}{\partial x_i} f(x_j) = \frac{\partial}{\partial x_i} \left( \frac{e^{x_j}}{\sum_{k=1}^{n} e^{x_k}} \right) = \frac{\left(\frac{\partial x_j}{\partial x_i} \cdot e^{x_j}\right)\left(\sum_{k=1}^{n} e^{x_k}\right) - \left(e^{x_j}\right)\left(\sum_{k=1}^{n} \frac{\partial x_k}{\partial x_i} \cdot e^{x_i}\right)}{\left(\sum_{k=1}^{n} e^{x_k}\right)^2}, \quad \frac{\partial x_k}{\partial x_i} = \begin{cases} 0, & i \neq k \\ 1, & i = k \end{cases}$$

$$\frac{\partial}{\partial x_i} f(x_j) = \begin{cases} \dfrac{e^{x_j} \cdot \sum_{k=1}^{n}(e^{x_k}) - e^{2x_j}}{\left(\sum_{k=1}^{n} e^{x_k}\right)^2}, & i = j \rightarrow f(x_j)(1-f(x_j)) \\ \dfrac{-e^{x_j}\left(e^{x_i}\right)}{\left(\sum_{k=1}^{n} e^{x_k}\right)^2}, & i \neq j \\ \quad \hookrightarrow -f(x_j) \cdot f(x_i) & \end{cases}$$

$$\frac{\partial}{\partial x_i} f(x_j) = \begin{cases} f(x_j)(1-f(x_j)), & i = j \\ -f(x_j) \cdot f(x_i), & i \neq j \end{cases}$$

$$\nabla_{a^{[2]}} N^{[3]} = \begin{bmatrix} \frac{\partial N_1^{[3]}}{\partial a_1^{[2]}} & \frac{\partial N_2^{[3]}}{\partial a_1^{[2]}} \\ \frac{\partial N_1^{[3]}}{\partial a_2^{[2]}} & \frac{\partial N_2^{[3]}}{\partial a_2^{[2]}} \end{bmatrix} = \begin{bmatrix} w_{11}^{[3]} & w_{21}^{[3]} \\ w_{12}^{[3]} & w_{22}^{[3]} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\frac{\partial N_j^{[3]}}{\partial a_i^{[2]}} = w_{ji}^{[3]}$$

$$\nabla_{N^{[2]}} a^{[2]} = \begin{bmatrix} \frac{\partial a_1^{[2]}}{\partial N_1^{[2]}} & \frac{\partial a_2^{[2]}}{\partial N_1^{[2]}} \\ \frac{\partial a_1^{[2]}}{\partial N_2^{[2]}} & \frac{\partial a_2^{[2]}}{\partial N_2^{[2]}} \end{bmatrix} = \begin{bmatrix} f(N_1^{[2]})(1-f(N_1^{[2]})) & -f(N_2^{[2]})f(N_1^{[2]}) \\ -f(N_1^{[2]})f(N_2^{[2]}) & f(N_2^{[2]})(1-f(N_2^{[2]})) \end{bmatrix}$$

$$\nabla_{a^{[1]}} N^{[2]} = \begin{bmatrix} w_{11}^{[2]} & w_{21}^{[2]} \\ w_{12}^{[2]} & w_{22}^{[2]} \\ w_{13}^{[2]} & w_{23}^{[2]} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix}$$

$$\nabla_{N^{[1]}} a^{[1]} = \begin{bmatrix} \frac{\partial a_1^{[1]}}{\partial N_1^{[1]}} & \frac{\partial a_2^{[1]}}{\partial N_1^{[1]}} & \frac{\partial a_3^{[1]}}{\partial N_1^{[1]}} \\ \frac{\partial a_1^{[1]}}{\partial N_2^{[1]}} & \frac{\partial a_2^{[1]}}{\partial N_2^{[1]}} & \frac{\partial a_3^{[1]}}{\partial N_2^{[1]}} \\ \frac{\partial a_1^{[1]}}{\partial N_3^{[1]}} & \frac{\partial a_2^{[1]}}{\partial N_3^{[1]}} & \frac{\partial a_3^{[1]}}{\partial N_3^{[1]}} \end{bmatrix} = \begin{bmatrix} f(N_1^{[1]})(1-f(N_1^{[1]})) & -f(N_2^{[1]})f(N_1^{[1]}) & -f(N_3^{[1]})f(N_1^{[1]}) \\ -f(N_1^{[1]})f(N_2^{[1]}) & f(N_2^{[1]})(1-f(N_2^{[1]})) & -f(N_3^{[1]})f(N_2^{[1]}) \\ -f(N_1^{[1]})f(N_3^{[1]}) & -f(N_2^{[1]})f(N_3^{[1]}) & f(N_3^{[1]})(1-f(N_3^{[1]})) \end{bmatrix}$$

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$$\delta^{[3]} = \nabla_{N^{[3]}} e = \nabla_{N^{[3]}} a^{[3]} \cdot \nabla_{a^{[3]}} e = \begin{bmatrix} f(N_1^{[3]})(1-f(N_1^{[3]})) & -f(N_2^{[3]})f(N_1^{[3]}) \\ -f(N_1^{[3]})f(N_2^{[3]}) & f(N_2^{[3]})(1-f(N_2^{[3]})) \end{bmatrix} \begin{bmatrix} -\frac{1}{f(0)} \\ 0 \end{bmatrix}$$

$$= f(0)(1-f(0)) \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} -\frac{1}{f(0)} \\ 0 \end{bmatrix} = \cancel{f(0)}(1-f(0))(-1) \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} f(0)-1 \\ 1-f(0) \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$\nabla_{w^{[3]}} e = \delta^{[3]} \cdot a^{[2]T} = \begin{bmatrix} -\frac{1}{2} \\ 1/2 \end{bmatrix} \begin{bmatrix} f(2f(6)+f(1)+1) & f(2f(6)+f(1)+1) \end{bmatrix} = \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \begin{pmatrix} \frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{bmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} \end{bmatrix}$$

$$w^{[3]} = w^{[3]} - \eta \cdot \nabla_{w^{[3]}}^{e} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} - \begin{pmatrix} -0{,}025 & -0{,}025 \\ 0{,}025 & 0{,}025 \end{pmatrix} = \begin{pmatrix} 0{,}025 & 0{,}025 \\ -0{,}025 & -0{,}025 \end{pmatrix}$$

$$\delta^{[2]} = \nabla_{N^{[2]}}^{e} = \nabla_{N^{[2]}} a^{[2]} \cdot \nabla_{a^{[2]}} N^{[3]} \cdot \nabla_{N^{[3]}}^{e} = \begin{bmatrix} f(N_1^{[2]})(1-f(N_1^{[2]})) & -f(N_2^{[2]})f(N_1^{[2]}) \\ -f(N_1^{[2]})f(N_2^{[2]}) & f(N_2^{[2]})(1-f(N_2^{[2]})) \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1/2\,(1-1/2) & (-\frac{1}{2})\,1/2 \\ (-1/2)\,\frac{1}{2} & 1/2\,(1-1/2) \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} -\frac{1}{2} \\ 1/2 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\nabla_{w^{[2]}}^{e} = \delta^{[2]} \cdot a^{[1]T} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} f(6) & f(1) & f(6) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$w^{[2]} = w^{[2]} - \eta \cdot \nabla_{w^{[2]}}^{e} = w^{[2]} = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

$$\delta^{[1]} = \nabla_{N^{[1]}}^{e} = \nabla_{N^{[1]}} a^{[1]} \cdot \nabla_{a^{[1]}} N^{[2]} \cdot \nabla_{N^{[2]}}^{e} = \begin{bmatrix} f(6)(1-f(6)) & -f(1).\,f(6) & -f(6)^2 \\ -f(6)\,f(1) & f(1)(1-f(1)) & -f(6)\,f(1) \\ -f(6)^2 & -f(1)\,f(6) & f(6)(1-f(6)) \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\nabla_{w^{[1]}}^{e} = \delta^{[1]} \cdot a^{[0]T} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \end{bmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$w^{[1]} = w^{[1]} - \eta \cdot \nabla_{w^{[1]}}^{e} = w^{[1]} = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix}$$

$$b^{[3]} = b^{[3]} - \eta \cdot \delta^{[3]} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} - 0{,}1 \begin{pmatrix} -\frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0{,}05 \\ -0{,}05 \end{pmatrix}$$

$$b^{[2]} = b^{[2]} - \eta \cdot \delta^{[2]} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} - 0{,}1 \begin{pmatrix} 0 \\ 0 \end{pmatrix} = b^{[2]} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$b^{[1]} = b^{[1]} - \eta \cdot \delta^{[1]} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} - 0{,}1 \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

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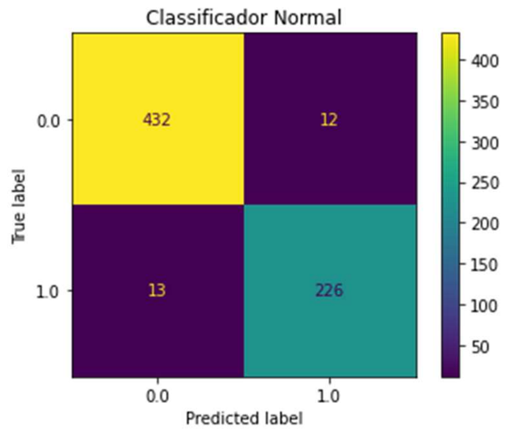
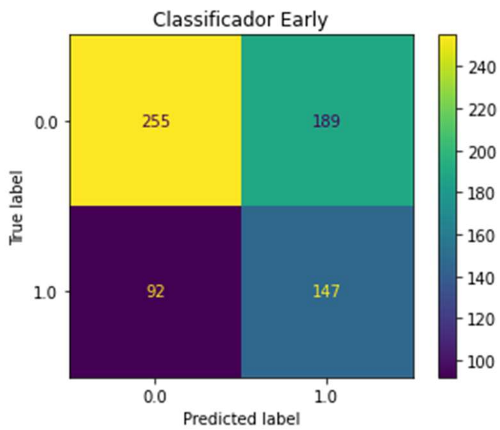
## II. Programming and critical analysis

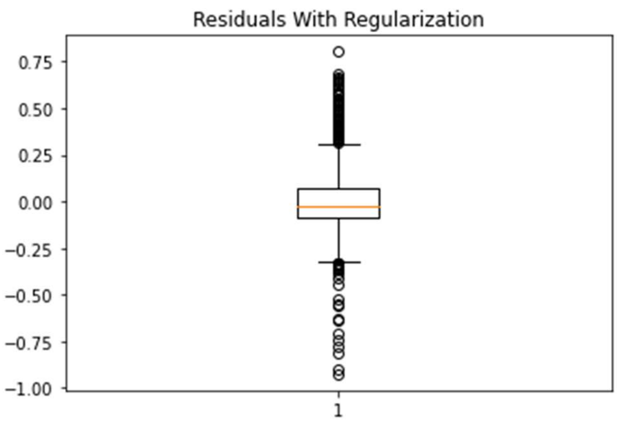
2)

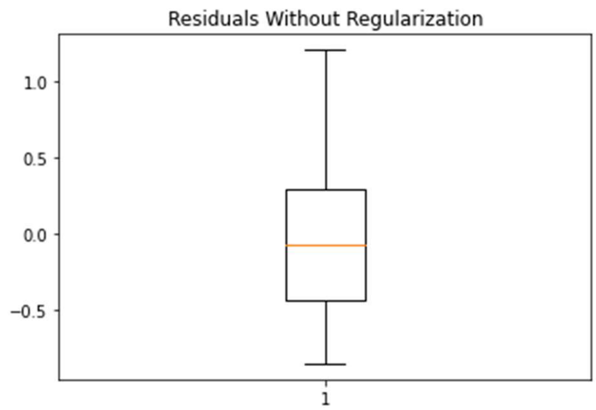



A estratégia de early stopping é geralmente utilizada para evitar overfitting, no entanto, devido à reduzida dimensão da amostra, neste caso prejudica a performance do classificador causando underfitting; para além disso, o early stopping pode levar à paragem num mínimo local, podendo levar assim a classificações erradas.

3)




Utilizar regularização para evitar overfit aos dados de treino; diminuir a learning rate para fazer com que o regressor converja mais lentamente; utilizar gradient descent em vez de stochastic gradient descent, o que permite uma melhor aproximação ao mínimo da função de erro; reduzir o número de features, o que ajuda a diminuir o ruído e evitar overfitting.

Aprendizagem 2021/22
Homework III – Group 17

# III. APPENDIX

```
import numpy as np
import scipy.io.arff
from sklearn.neural_network import MLPClassifier, MLPRegressor
from sklearn.model_selection import KFold
from sklearn.metrics import confusion_matrix, ConfusionMatrixDisplay #cross entropy
import matplotlib.pyplot as plt

arff = scipy.io.arff

breast_data, breast_meta = arff.loadarff("breast.w.new.arff")
kin_data, kin_meta = arff.loadarff("kin8nm.arff")

breast_targets = np.array(list(el["Class"] for el in breast_data))
breast_X = np.array(np.array(list(list(el[i] for i in range(9)) for el in breast_data)))

kin_targets = np.array(list(el["y"] for el in kin_data))
kin_X = np.array(np.array(list(list(el[i] for i in range(8)) for el in kin_data)))

#2
kfold = KFold(n_splits=5, random_state=0, shuffle=True)

classifier = MLPClassifier(hidden_layer_sizes=(3,2), activation="relu", early_stopping=False,
max_iter=2000)
classifier_early = MLPClassifier(hidden_layer_sizes=(3,2), activation="relu", early_stopping=True,
max_iter=2000)
predictions = []
predictions_early = []
truth = []

for train_i, test_i in kfold.split(breast_X):
    train_data, train_target = breast_X[train_i], breast_targets[train_i]
    test_data, test_target = breast_X[test_i], breast_targets[test_i]
    classifier.fit(train_data, train_target)
    classifier_early.fit(train_data, train_target)
    predictions += list(classifier.predict(test_data))
    predictions_early += list(classifier_early.predict(test_data))
    truth += list(test_target)

conf_m = confusion_matrix(np.array(truth), np.array(predictions), labels=classifier.classes_)
conf_m_early = confusion_matrix(np.array(truth), np.array(predictions_early),
labels=classifier_early.classes_)

disp = ConfusionMatrixDisplay(conf_m, display_labels=classifier.classes_)

disp_e = ConfusionMatrixDisplay(conf_m_early, display_labels=classifier_early.classes_)
```

```
disp.plot()
plt.title("Classificador Normal")
plt.show()

disp_e.plot()
plt.title("Classificador Early")
plt.show()

#3
kfold = KFold(n_splits=5, random_state=0, shuffle=True)

regressor = MLPRegressor(hidden_layer_sizes=(3,2), activation="relu", alpha=10, max_iter=2000)
regressor_no_reg = MLPRegressor(hidden_layer_sizes=(3,2), activation="relu", alpha=0, max_iter=2000)
residuals = []
residuals_no_reg = []

for train_i, test_i in kfold.split(breast_X):
    train_data, train_target = breast_X[train_i], breast_targets[train_i]
    test_data, test_target = breast_X[test_i], breast_targets[test_i]
    regressor.fit(train_data, train_target)
    regressor_no_reg.fit(train_data, train_target)
    residuals += list(test_target - regressor.predict(test_data))
    residuals_no_reg += list(test_target - regressor_no_reg.predict(test_data))

plt.boxplot(np.array(residuals))
plt.title("Residuals With Regularization")
plt.show()

plt.boxplot(np.array(residuals_no_reg))
plt.title("Residuals Without Regularization")
plt.show()
```

END