Learnable Sparsity and Weak Supervision for Data-efficient, Transparent, and Compact Neural Models

Gonçalo M. Correia

Jury: André Martins, Mário Figueiredo, Ivan Titov, Wilker Aziz, Isabel Trancoso

Deep learning successes

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- Subset of machine learning that uses **neural networks**

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- Remarkable results

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GPT-3

The Guardian

News website of the year

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- Subs
- Pow
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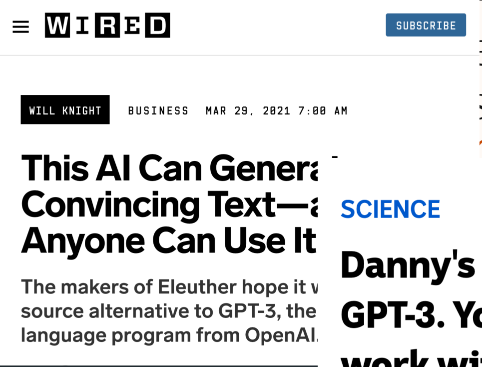
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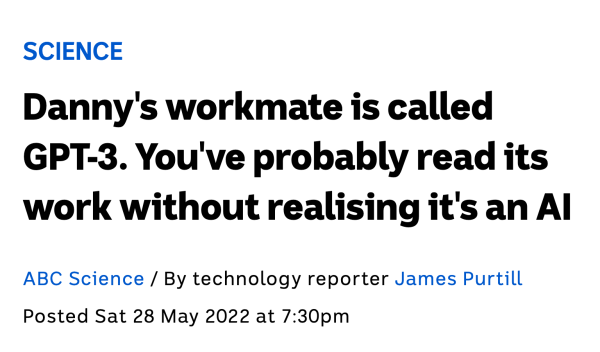
SCIENCE

Danny's workmate is called GPT-3. You've probably read its work without realising it's an AI

ABC Science / By technology reporter James Purtill

Posted Sat 28 May 2022 at 7:30pm

Deep learning successes

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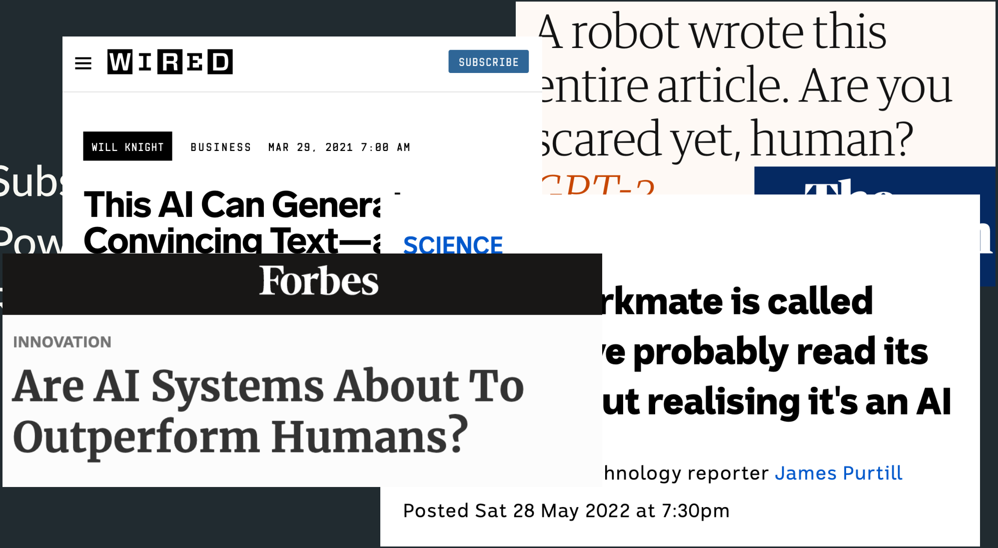
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PT-2

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INNOVATION

Are AI Systems About T… Outperform Humans?

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AI 'outperforms' doctors diagnosing breast cancer

Fergus Walsh
Medical correspondent
@BBCFergusWalsh

Deep learning limitations and drawbacks

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AI

Overcoming AI's Transparency Paradox

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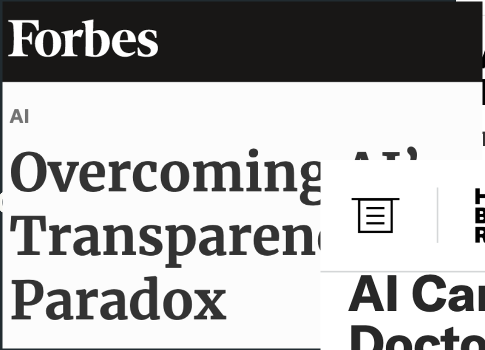
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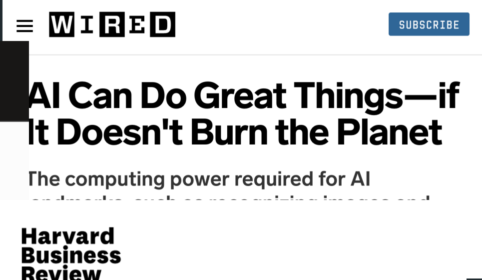
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The computing power required for AI


Harvard Business Review

AI Can Outperform Doctors. So Why Don't Patients Trust It?

by Chiara Longoni and Carey K. Morewedge

Key concepts of this thesis

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Learnable Sparsity and Weak Supervision

for Data-efficient, Transparent, and Compact

Neural Models

Published work of this thesis

(Correia and Martins, 2019)

Published work of this thesis

- Automatic Post-Editing using weak supervision for data-efficiency (ACL)

(Correia, Niculae, and Martins, 2019)

Published work of this thesis

- Automatic Post-Editing using weak supervision for data-efficiency (ACL)
- Letting transformer learn sparsity of its attentions for transparency (EMNLP)

(Correia, Niculae, Aziz, and Martins, 2020)

Published work of this thesis

- Automatic Post-Editing using weak supervision for data-efficiency (ACL)
- Letting transformer learn sparsity of its attentions for transparency (EMNLP)
- General strategy for efficiently training discrete latent variable models, to have compactness (NeurIPS)

Table of Contents

A bit of context on transformers

What if... Attention is all you need?

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Key idea: Let's mainly use attention mechanisms!

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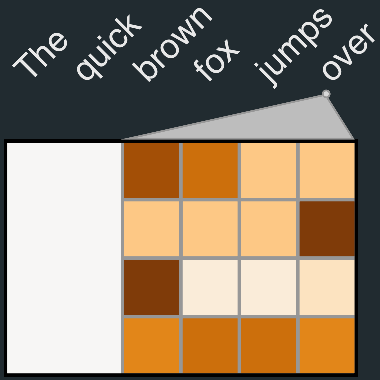

(Vaswani et al., 2017)

A bit of context on transformers

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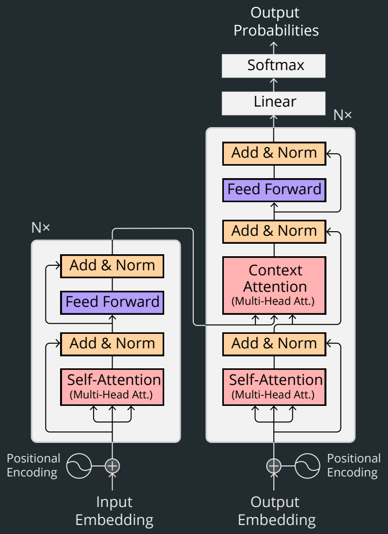
- ... and do it through several layers



(Devlin et al., 2019)

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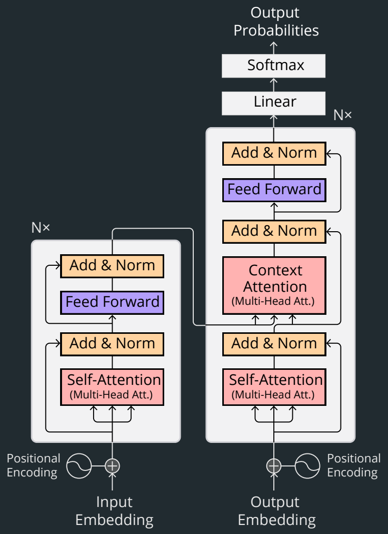

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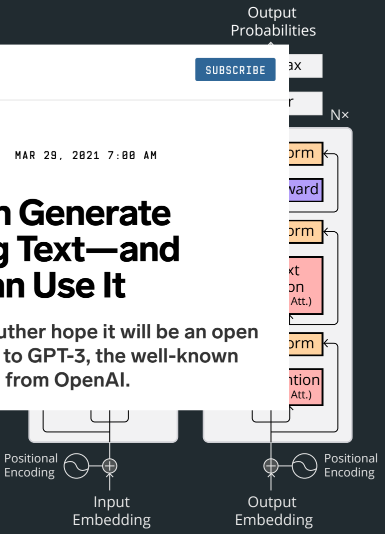

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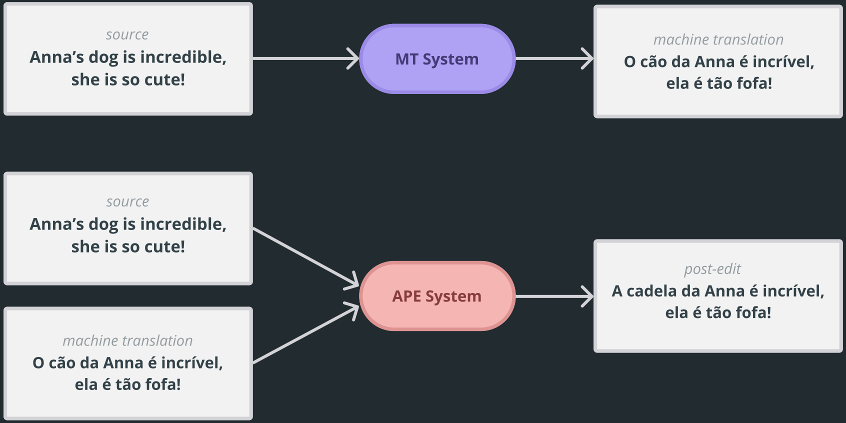
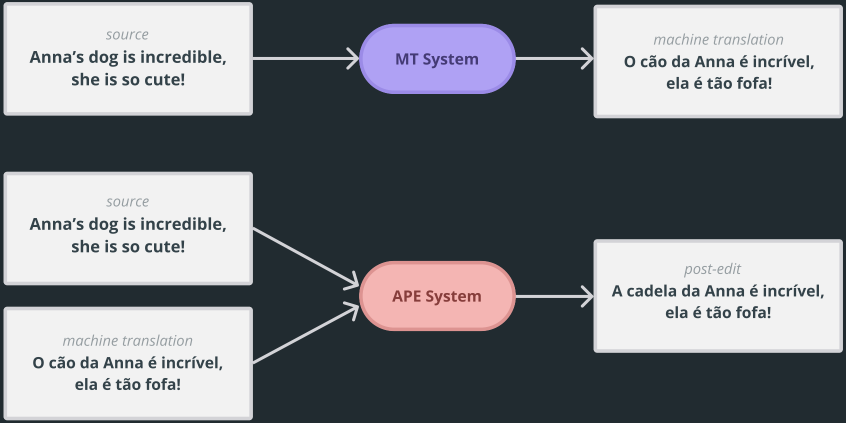
What is APE?

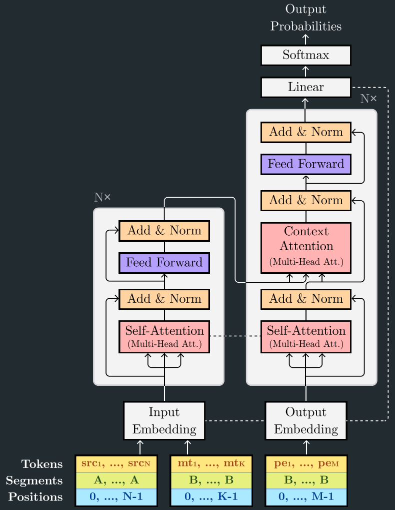

source
Anna's dog is incredible, she is so cute!
MT System
machine translation
O cão da Anna é incrível, ela é tão fofa!
source
Anna's dog is incredible, she is so cute!
machine translation
O cão da Anna é incrível, ela é tão fofa!
APE System
post-edit
A cadela da Anna é incrível, ela é tão fofa!

What is APE?



Challenge: APE data is very scarce! Need to create artificial data.

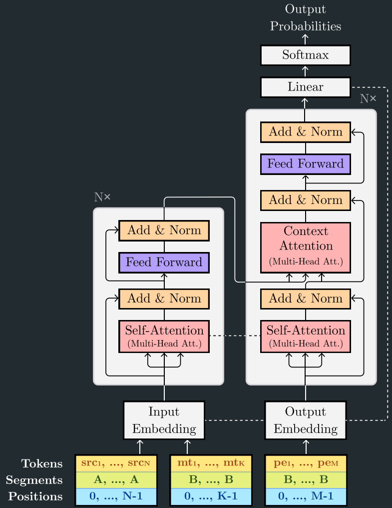
BERT for APE



Output Probabilities
Softmax
Linear
N×
Add & Norm
Feed Forward
Add & Norm
Context Attention (Multi-Head Att.)
Add & Norm
Self-Attention (Multi-Head Att.)
N×
Add & Norm
Feed Forward
Add & Norm
Self-Attention (Multi-Head Att.)
Input Embedding
Output Embedding
Tokens
Segments
Positions
src1, ..., srcN
A, ..., A
0, ..., N-1
mt1, ..., mtK
B, ..., B
0, ..., K-1
pe1, ..., peM
B, ..., B
0, ..., M-1

BERT for APE

Key idea: Use BERT to do APE



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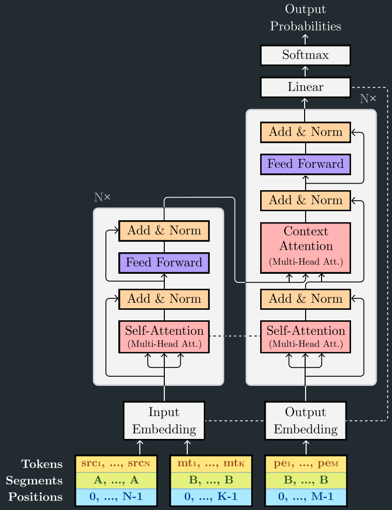

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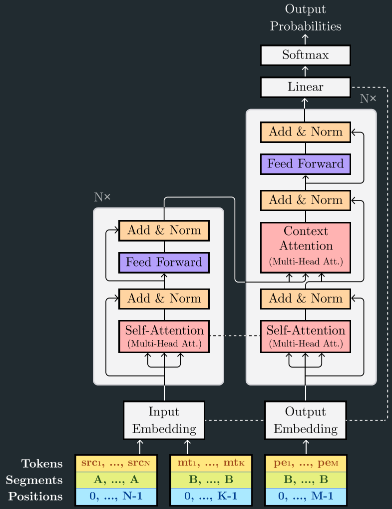
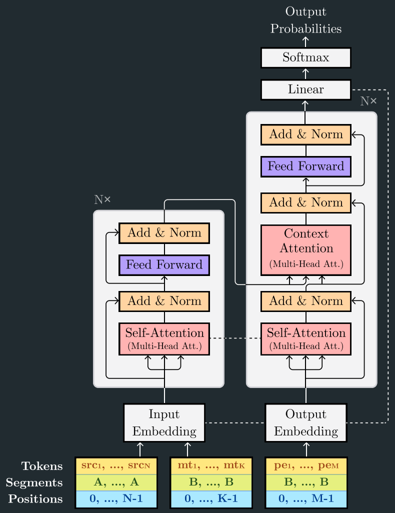
- We introduced an effective method to use BERT in a generation task (APE)



BERT for APE

Key idea: Use BERT to do APE

- Prior to this work, BERT was mainly used for simple classification tasks
- We introduced an effective method to use BERT in a generation task (APE)
- Smart parameter sharing between encoder and decoder



Key results

model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49

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ours (8M)	17.26	73.42

Conclusions and impact

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- One of pioneers in using pre-trained transformer encoders for a generation task

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- Massive improvement in low-resource scenario (data-efficiency)

(Lee et al., 2020)

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(Kodama et al., 2020)

Conclusions and impact

- One of pioneers in using pre-trained transformer encoders for a generation task
- Massive improvement in low-resource scenario (data-efficiency)
- Steered SOTA of APE towards using weak supervision through pre-trained models
- Inspired other works that use scarse data (*e.g.*, dialogue with metadata)

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Getting to know attention heads better

Attention heads may aid visualization but they are completely dense.

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- for interpretability
- for discovering linguistic structure
- for efficiency

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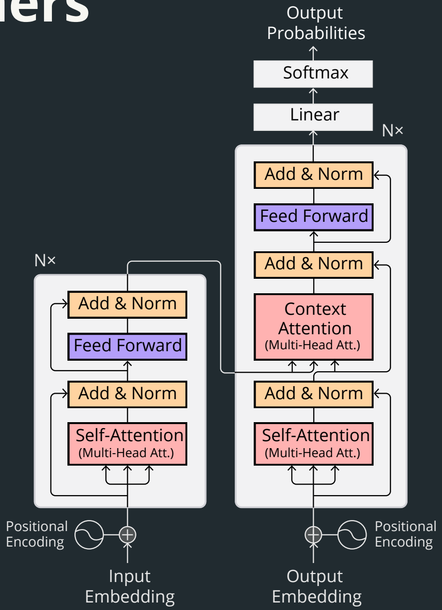
Transformers

In each attention head:

$$\bar{V} = \textbf{softmax}\left(\frac{QK^{\mathsf{T}}}{\sqrt{d_k}}\right)V.$$



Transformers

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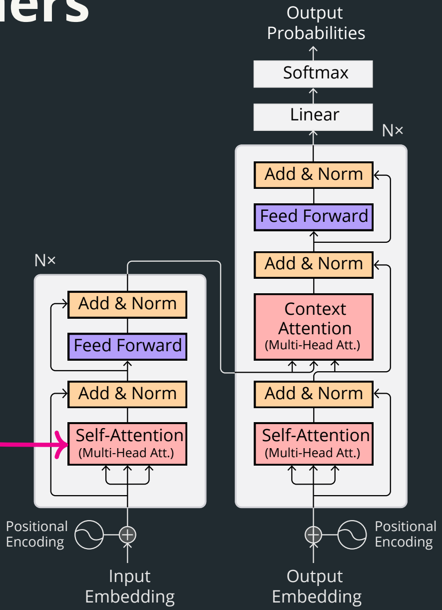
Attention in three places:

- Self-attention in the encoder



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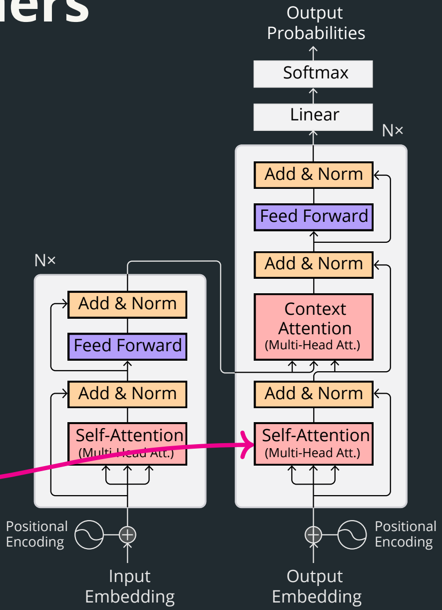

Transformers

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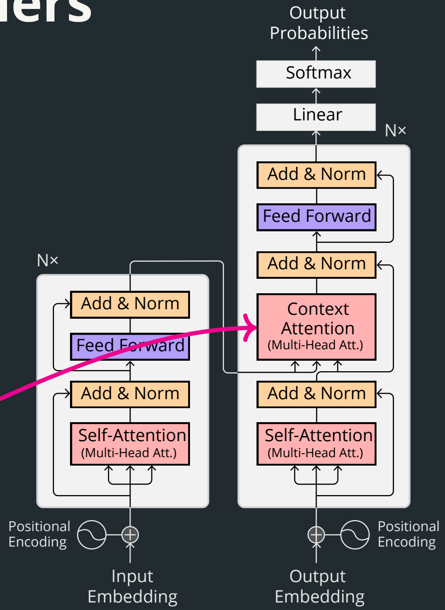
Attention in three places:

- Self-attention in the encoder
- Self-attention in the decoder
- Contextual attention



Sparse Transformers

Sparse Transformers

Key idea: replace softmax in attention heads by a sparse normalizing function! 🙌

Adaptively Sparse Transformers

Key idea: replace softmax in attention heads by a sparse normalizing function! 🙌

Another key idea: use a normalizing function that is adaptively sparse via a learnable α! 🙌 🙌 🙌

What is softmax?

Softmax exponentiates and normalizes:

$$[\mathbf{softmax}(z)]_i := \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

What is softmax?

Softmax exponentiates and normalizes:

$$[\mathbf{softmax}(z)]_i := \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

It's fully dense: $\mathbf{softmax}(z) > \mathbf{0}$

(Peters et al., 2019)

α-entmax

Parametrized by $\alpha \geq 0$:

α-entmax

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- **Argmax** corresponds to $\alpha \to \infty$

α-entmax

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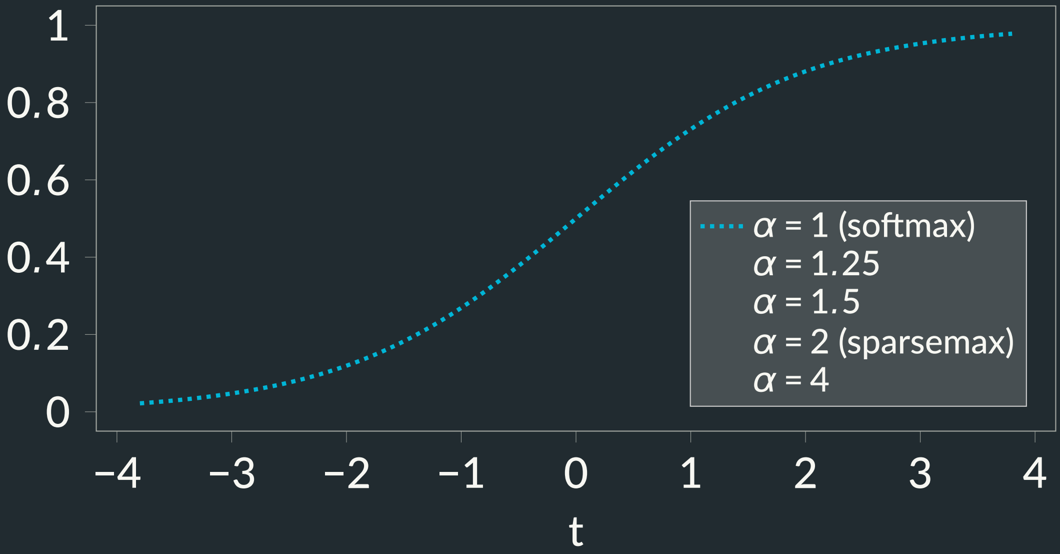
- **Softmax** amounts to $\alpha \to 1$

α-entmax

Parametrized by $\alpha \geq 0$:

- **Argmax** corresponds to $\alpha \to \infty$
- **Softmax** amounts to $\alpha \to 1$

Key result: **can be sparse for** $\alpha > 1$, propensity for sparsity increases with α.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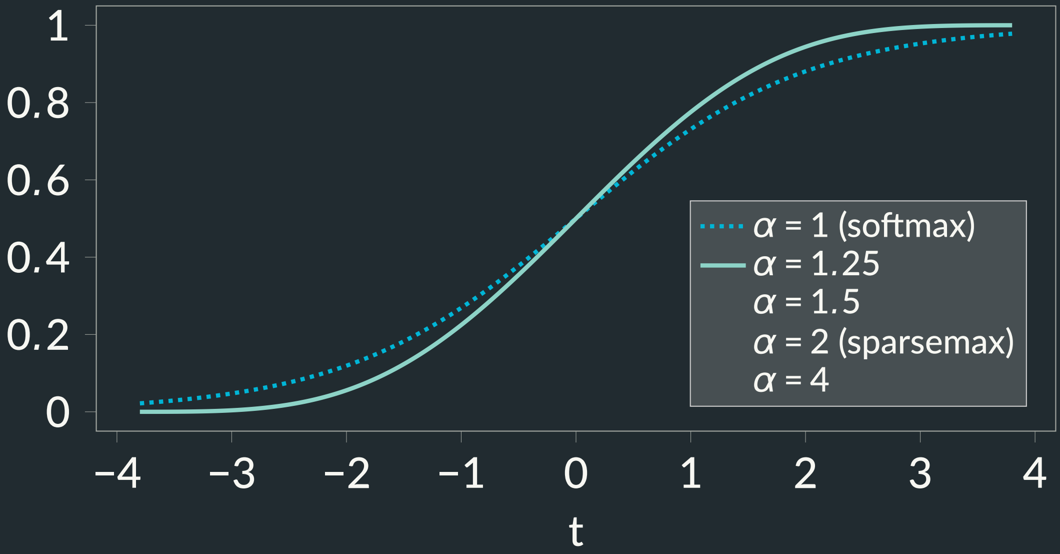
α = 1 (softmax)
α = 1.25
α = 1.5
α = 2 (sparsemax)
α = 4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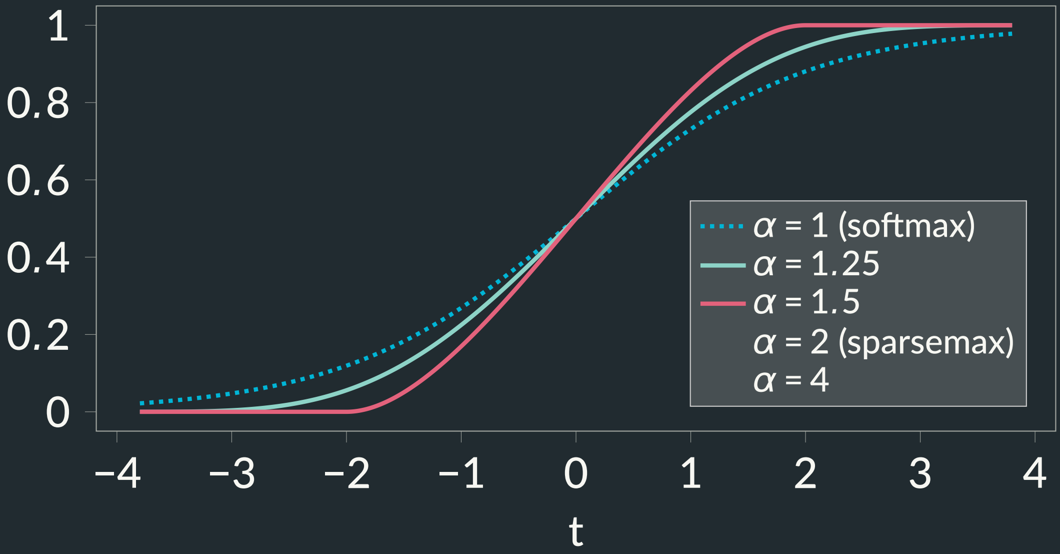

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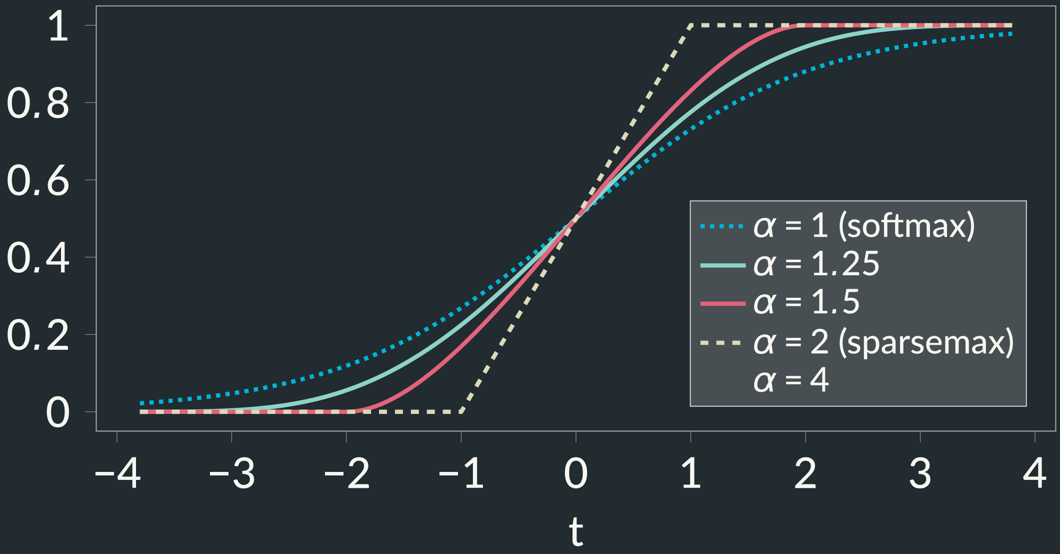
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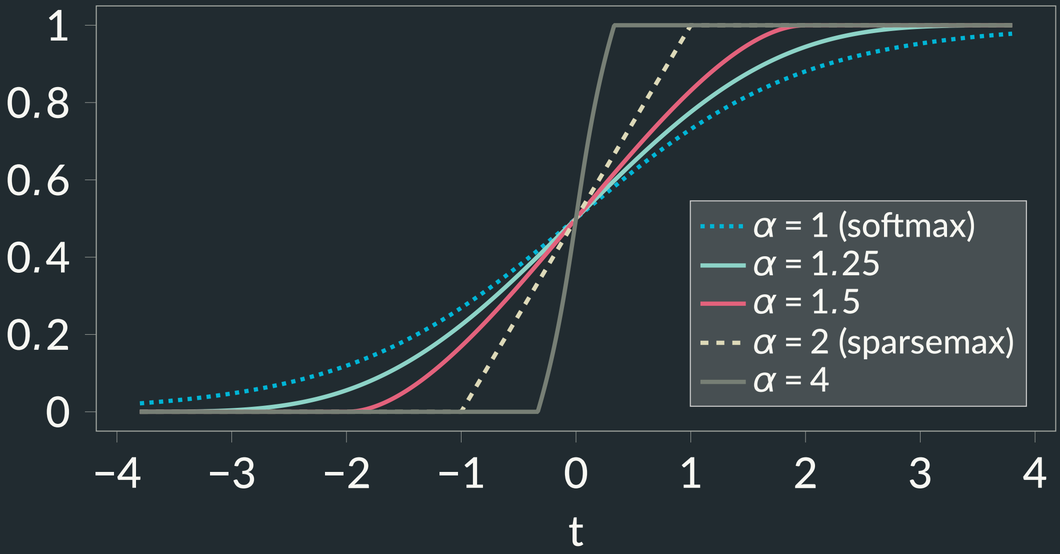
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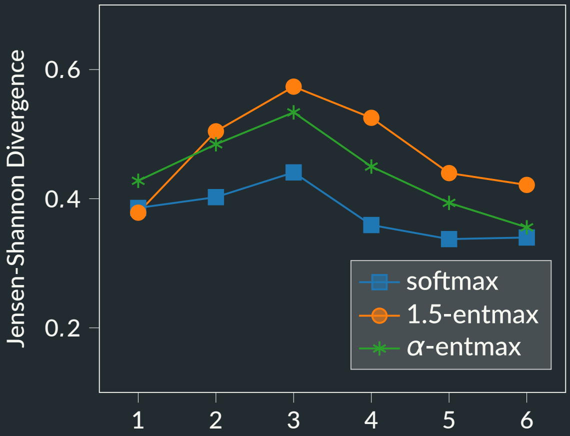
Learning α

Learning α

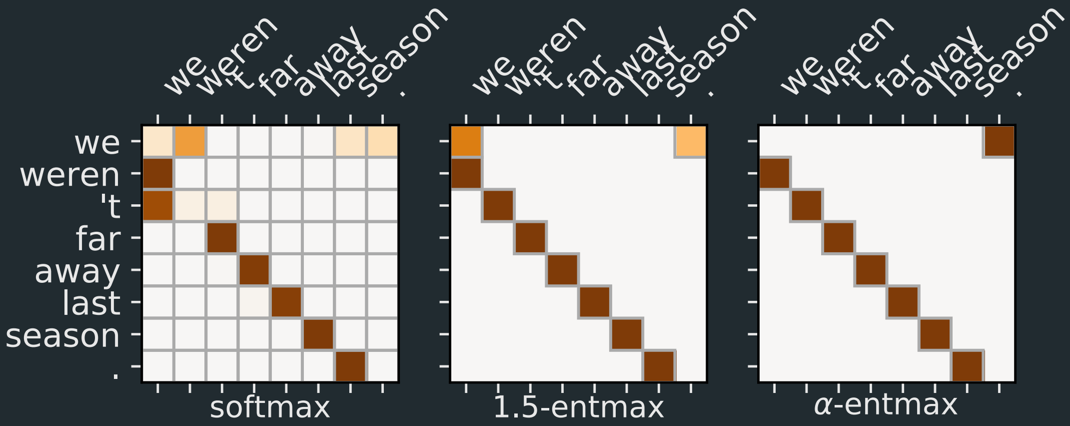
Key contribution:

a closed-form expression for $\dfrac{\partial\, \alpha\text{-entmax}(\mathbf{z})}{\partial \alpha}$ 😱

Head diversity per layer

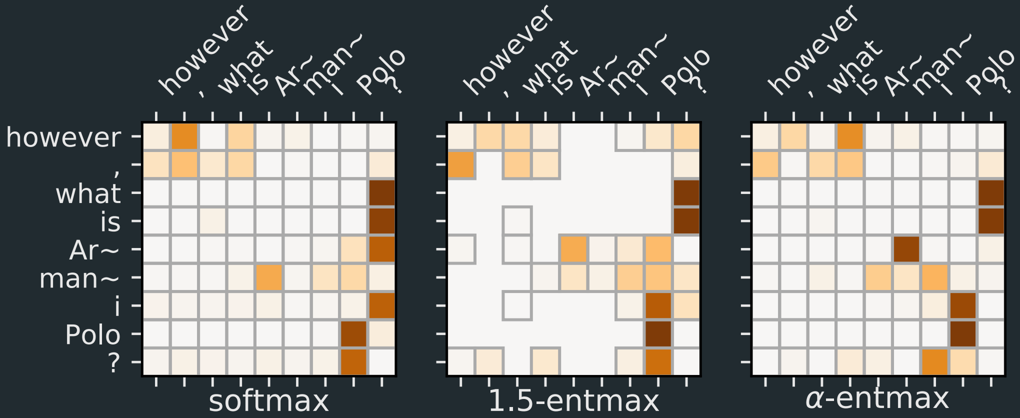


Previous position head



This head role was also found in Voita et al. (2019)! Learned $\alpha = 1.91$.

Interrogation-detecting head



Learned $\alpha = 1.05$.

Conclusions and impact

Introduce adaptive sparsity
for Transformers via α-entmax with a gradient learnable α.

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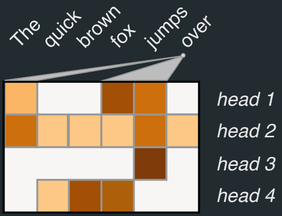
adaptive sparsity



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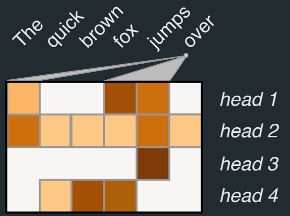
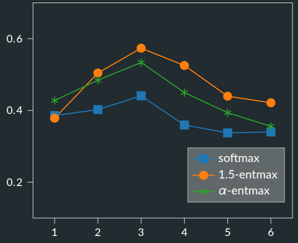


reduced head redundancy



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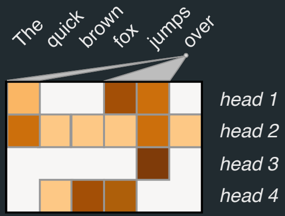
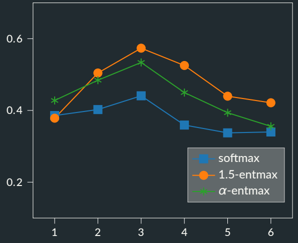
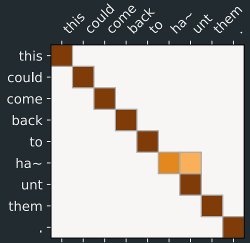


reduced head redundancy



clearer head roles



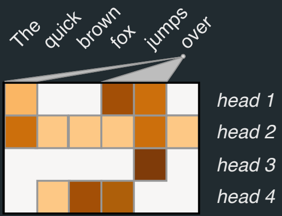
(Treviso et al., 2022; Zhang et al., 2021; Raganato et al., 2020)

Conclusions and impact

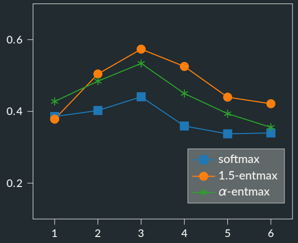
Subsequent work has:

- focused on taking computational advantage of sparsity
- proposed other sparse activations (*e.g.*, ReLU)
- incorporated fixed attention patterns that we found.

adaptive sparsity



reduced head redundancy



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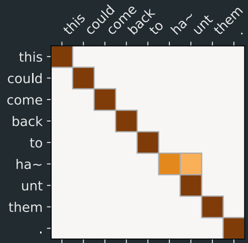


Table of Contents

Latent variable models

Latent variable z can be

Latent variable models

Latent variable z can be continuous

Source: Bouges et al., 2013

Latent variable models

Latent variable z can be continuous, discrete

Source: valleyeyecareaz.com

Latent variable models

Latent variable z can be continuous, discrete, or structured

Source: Liu et al., 2015

Training discrete or structured latent variable models

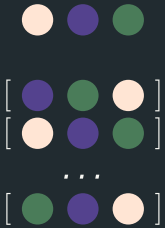
Latent variable z can be

Training discrete or structured latent variable models

Latent variable z can be discrete

Training discrete or structured latent variable models

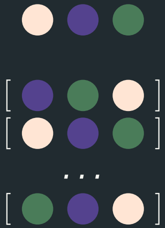
Latent variable z can be discrete or structured



Training discrete or structured latent variable models

Latent variable z can be discrete or structured

$\pi(z|x, \theta)$: distribution over possible z



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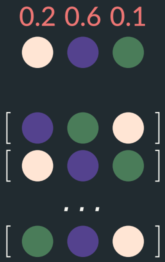

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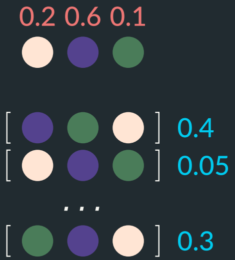

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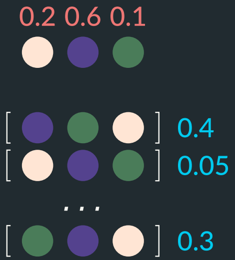
Training discrete or structured latent variable models



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If $\mathcal{Z}$ is large, this sum can get very expensive due to $\ell(x, z; \theta)$!

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If $\mathcal{Z}$ is combinatorial, this can be intractable to compute! 😱 😱 😱

Current solutions

Using emergent communication as example

Method	success (%)	Dec. calls
Monte Carlo		
Marginalization		
Dense	93.37 ±0.42	256

Current solutions

Using emergent communication as example

- SFE → unbiased but high variance

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Sparse	93.35 ±0.50	3.13±0.48

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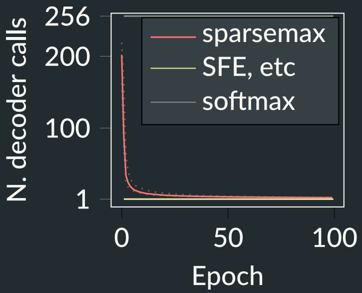

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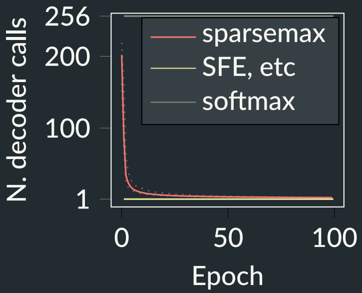

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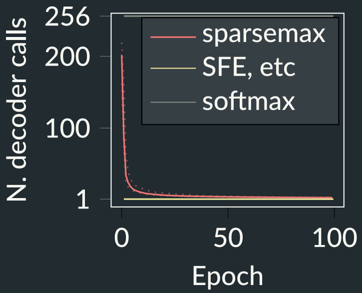
no continuous relaxation



Taking a step back...

Does the expectation over possible z need to be expensive?

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&= \pi(z_1|x, \theta)\, \ell(x, z_1; \theta) + \pi(z_2|x, \theta)\, \ell(x, z_2; \theta) + \ldots \\
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Usually we normalize π with softmax $\propto \exp(s) \Rightarrow \pi(z_i|x,\theta) > 0$

(Martins and Astudillo, 2016; Niculae, Martins, et al., 2018)

Sparse normalizers

We use sparsemax, top-k sparsemax and SparseMAP to allow efficient marginalization

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No need for computing $\ell(x,z;\theta)$ for all $z\in\mathcal{Z}$!

Results

We test our methods for models with discrete latent variables,

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Our methods are top-performers and efficient!

Key takeaways

We introduce a new method
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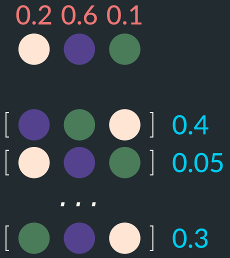
discrete and structured



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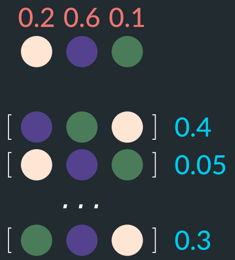

deterministic, yet efficient

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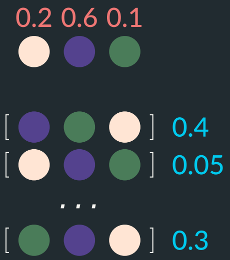

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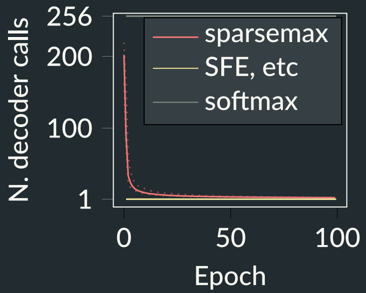
sparse, as needed



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Conclusions

Using learned sparsity and weak supervision
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data-efficiency

model (data size)	BLEU↑
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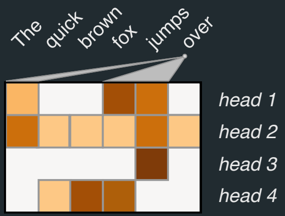

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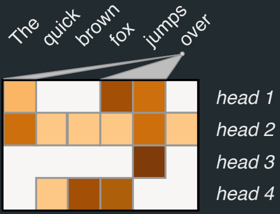
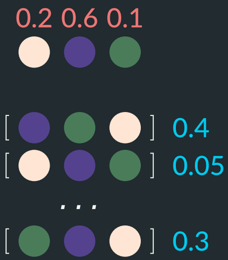


better compactness



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Parameter sharing analysis

	TER↓	BLEU↑
MT Baseline	24.76	62.11
Transformer	27.80	60.76
Transformer decoder	20.33	69.31
Pre-trained BERT	20.83	69.11
with CA ← SA	18.91	71.81
and SA ⟷ Encoder SA	**18.44**	**72.25**
and CA ⟷ SA	18.75	71.83
and FF ⟷ Encoder FF	19.04	71.53

(Niculae and Blondel, 2017)

Ω-Regularized Argmax

For convex Ω, define the **Ω-regularized argmax transformation:**

$$\mathbf{argmax}_{\Omega}(\boldsymbol{z}) := \arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{z}^{\top}\boldsymbol{p} - \Omega(\boldsymbol{p})$$

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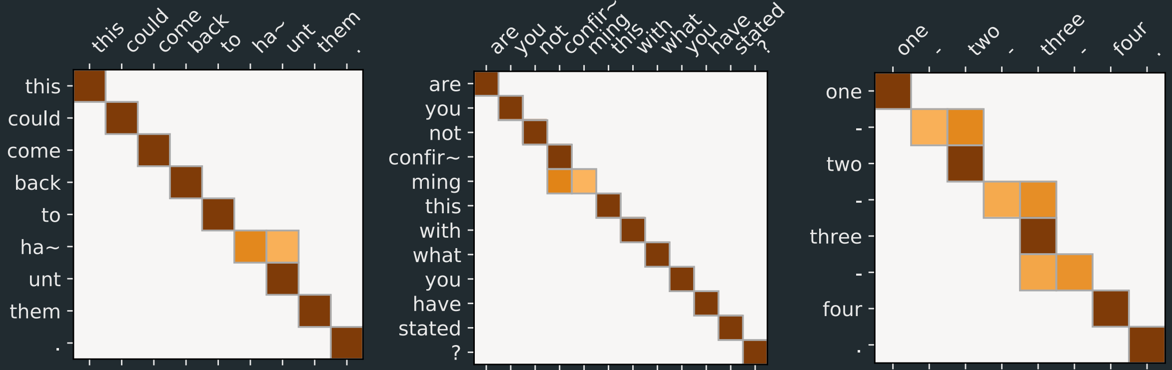
Is there something in-between?

Subword-Merging Head



Learned $\alpha = 1.91$.

Semi-Supervised VAE

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$
$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

- Semi-Supervised VAE on MNIST: z is one of 10 categories

Semi-Supervised VAE

Gaussian VAE

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classification network

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Semi-Supervised VAE

sum over the 10 digits

Gaussian VAE

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Gaussian VAE

sum over
the 10 digits

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$

classification network

$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

- Semi-Supervised VAE on MNIST: z is one of 10 categories
- Train this with 10% labeled data

Semi-Supervised VAE

Method	Accuracy (%)	Dec. calls
Monte Carlo		
SFE	$94.75_{\pm .002}$	1
SFE+	$96.53_{\pm .001}$	2
NVIL	$96.01_{\pm .002}$	1
Gumbel	$95.46_{\pm .001}$	1
Marginalization		
Dense	$96.93_{\pm .001}$	10

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Sparse	96.87 $\pm$.001	1.01 $\pm$0.01

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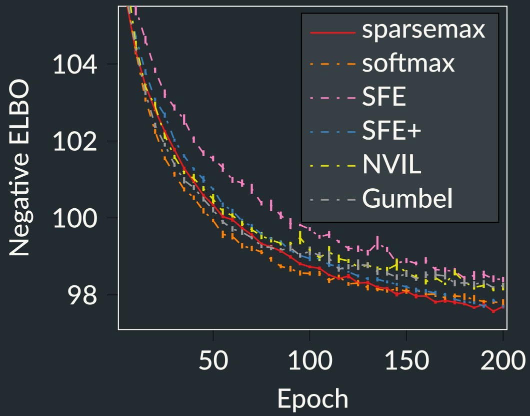

Emergent communication

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$
$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

Emergent communication

sender

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$

receiver

$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

- receiver picks image from a set $\mathcal{V}$ based on message

Emergent communication

sum over
all possible messages
in the vocabulary

sender

receiver

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$

$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

- receiver picks image from a set $\mathcal{V}$ based on message
- images come from ImageNet

(Lazaridou et al., 2017; Correia, Niculae, Aziz, et al., 2020)

Emergent Communication

... but make it harder: $|\mathcal{Z}| = 256$, $|\mathcal{V}| = 16$

Method	success (%)	Dec. calls
Monte Carlo		
SFE	33.05 ±2.84	1
SFE+	44.32 ±2.72	2
NVIL	37.04 ±1.61	1
Gumbel	23.51 ±16.19	1
ST Gumbel	27.42 ±13.36	1
Marginalization		

(Lazaridou et al., 2017; Correia, Niculae, Aziz, et al., 2020)

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Marginalization		
Dense	93.37 ±0.42	256

(Lazaridou et al., 2017; Correia, Niculae, Aziz, et al., 2020)

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Marginalization		
Dense	93.37 ± 0.42	256
Sparse	93.35 ± 0.50	3.13 ± 0.48

(Lazaridou et al., 2017; Correia, Niculae, Aziz, et al., 2020)

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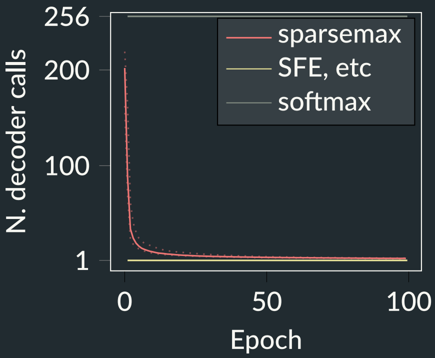
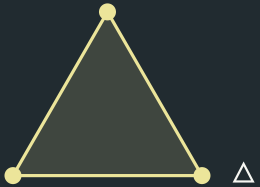
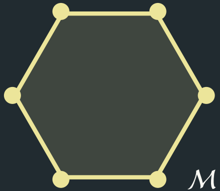
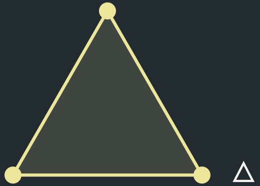
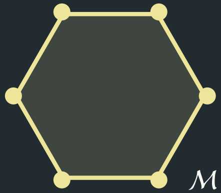



△

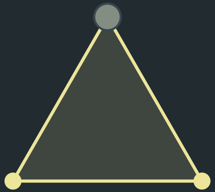

$\mathcal{M}$

$$\begin{aligned}\mathcal{M} &:= \operatorname{conv}\left\{\boldsymbol{a}_z : z \in \mathcal{Z}\right\} \\ &= \left\{\boldsymbol{A}\boldsymbol{p} : \boldsymbol{p} \in \triangle\right\} \\ &= \left\{\mathbb{E}_{Z\sim\boldsymbol{p}}\, \boldsymbol{a}_Z : \boldsymbol{p} \in \triangle\right\}\end{aligned}$$

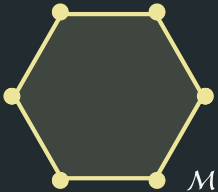




- **argmax** $\displaystyle \arg\max_{\boldsymbol{p} \in \triangle} \boldsymbol{p}^\top \boldsymbol{s}$

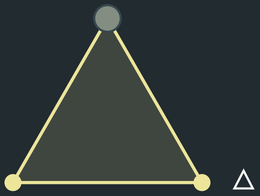


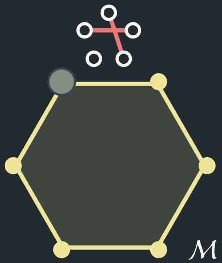
$\triangle$



$\mathcal{M}$

- **argmax** $\underset{\boldsymbol{p}\in\triangle}{\arg\max}\, \boldsymbol{p}^\top \boldsymbol{s}$
- **MAP** $\underset{\boldsymbol{\mu}\in\mathcal{M}}{\arg\max}\, \boldsymbol{\mu}^\top \boldsymbol{t}$

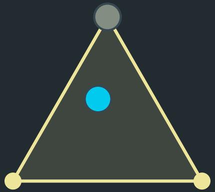


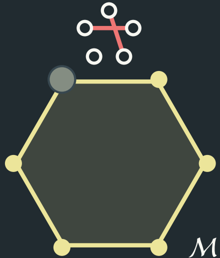


argmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$

softmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$

MAP $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t}$





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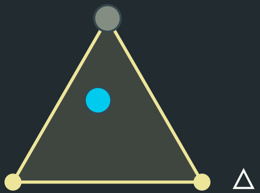
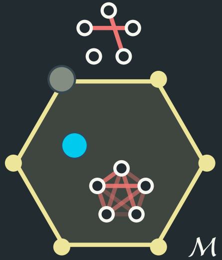
- **softmax** $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$

- **MAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t}$
- **marginals** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$





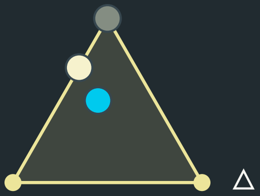
argmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$

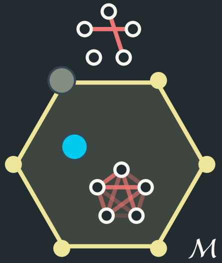
softmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$

sparsemax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - 1/2\|\boldsymbol{p}\|^2$

MAP $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t}$

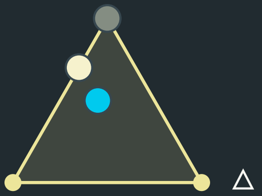
marginals $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$

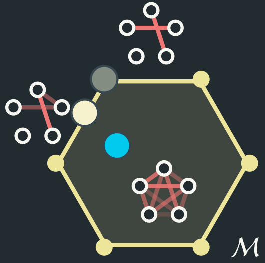




(Niculae, Martins, Blondel, and Cardie, 2018)

argmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s}$	**MAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t}$
softmax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$	**marginals** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t} + \widetilde{\mathrm{H}}(\boldsymbol{\mu})$
sparsemax $\arg\max_{\boldsymbol{p}\in\triangle} \boldsymbol{p}^\top \boldsymbol{s} - 1/2\lVert\boldsymbol{p}\rVert^2$	**SparseMAP** $\arg\max_{\boldsymbol{\mu}\in\mathcal{M}} \boldsymbol{\mu}^\top \boldsymbol{t} - 1/2\lVert\boldsymbol{\mu}\rVert^2$





Bit-vector VAE

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$
$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

Bit-vector VAE

generative network

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x)\, \ell(x, z)$$

inference network

$$= \mathbb{E}_{z \sim \pi(z|x)}\, \ell(x, z)$$

- VAE where z is a collection of D bits

Bit-vector VAE

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- VAE where z is a collection of D bits
- Minimize the negative ELBO

Bit-vector VAE

sum over
an exponetially large
set of structures

generative network

inference network

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Bit-vector VAE

Method	$D = 32$	$D = 128$
Monte Carlo		
SFE	3.74	3.77
SFE+	3.61	3.59
NVIL	3.65	3.60
Gumbel	3.57	3.49
Marginalization		
Top-*k* sparsemax	3.62	3.61
SparseMAP	3.72	3.67
SparseMAP (w/ budget)	3.64	3.66

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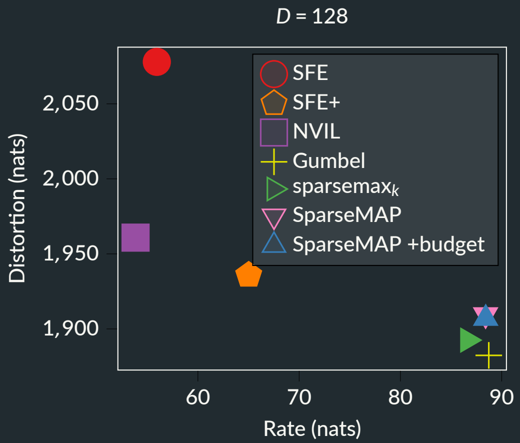

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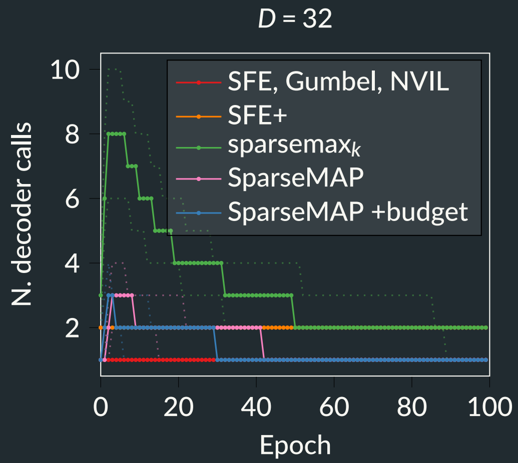

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