Learnable Sparsity and Weak Supervision for Data-efficient, Transparent, and Compact Neural Models

Gonçalo M. Correia

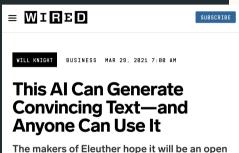
• Subset of machine learning that uses **neural networks**

- Subset of machine learning that uses neural networks
- Powerful tool for learning representations of any data

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- Powerful tool for learning representations of any data
- Remarkable results

A robot wrote this entire article. Are you scared yet, human?

GPT-3



source alternative to GPT-3, the well-known

language program from OpenAI.

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Convincing Text—a SCIENCE **Anvone Can Use It**

The makers of Eleuther hope it v source alternative to GPT-3, the language program from OpenAI

Danny's workmate is called GPT-3. You've probably read its work without realising it's an AI

ABC Science / By technology reporter James Purtill Posted Sat 28 May 2022 at 7:30pm



INNOVATION

Are AI Systems About To Outperform Humans?

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robot wrote this

doctors bridge requires more human sk diagnosing breast cancer



Fergus Walsh Medical correspondent @BBCFergusWalsh

• Requires a lot of data

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- Hard to understand and interpret reasons behind decisions

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- Requires a lot of computation



Forbes

ΔΙ

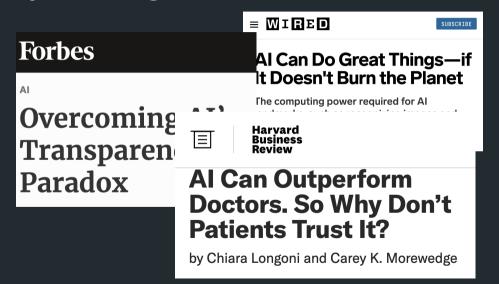
Overcoming AI's Transparency Paradox

■ WIRED

SUBSCRIBE

Al Can Do Great Things—if It Doesn't Burn the Planet

The computing power required for AI andmarks, such as recognizing images and defeating humans at Go, increased 300,000-fold from 2012 to 2018.



- A Simple and Effective Approach to APE with Transfer Learning
 - weak supervision
 - data-efficiency
 - Poster at ACL 2019

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 - learnable sparsity
 - compactness
 - Spotlight paper at NeurIPS 2020

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Adaptively Sparse Transformers

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Future Work and Conclusions

What if... Attention is all you need?



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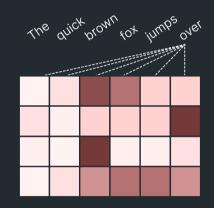
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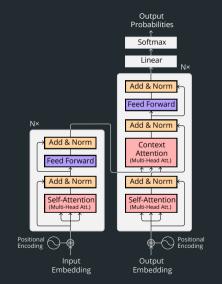
• Do attention with multiple heads (i.e. attention mechanisms in parallel)



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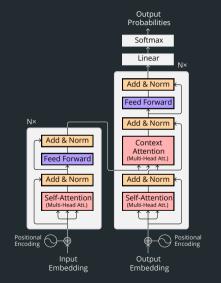
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What if... Attention is all you need?

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- Do attention with multiple heads (i.e. attention mechanisms in parallel)
- ... and do it through several layers
- Inspiration for big general-purpose models like BERT and GPT-3!



A bit of context on Transformers

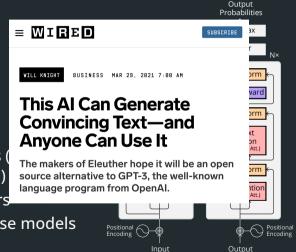
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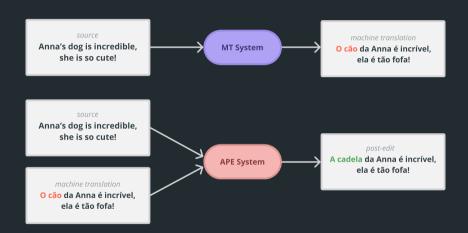
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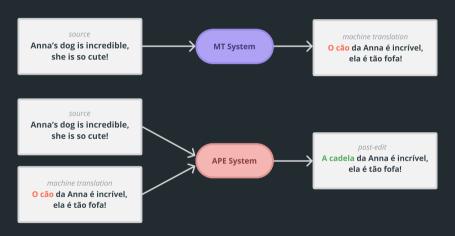


Embedding

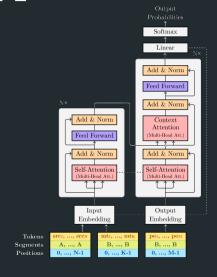
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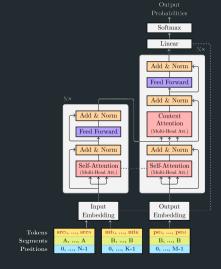




Challenge: APE data is very scarce! Need to create artificial data.



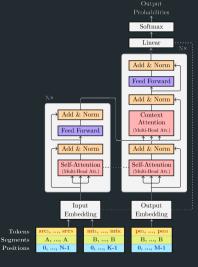
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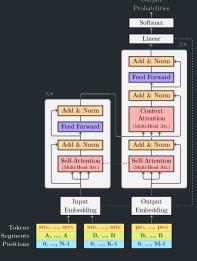
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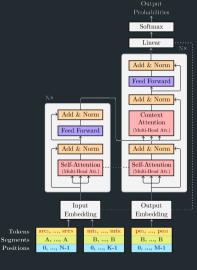
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- We introduced an effective method to use BERT in a generation task (APE)



Key idea: Use BERT to do APE



- Prior to this work, BERT was mainly used for simple classification tasks
- We introduced an effective method to use BERT in a generation task (APE)
- Smart parameter sharing between encoder and decoder



model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49

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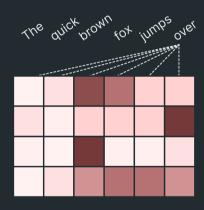
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Getting to know attention heads better

Attention heads may aid visualization but they are completely dense.

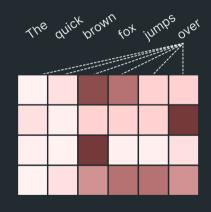


Getting to know attention heads better

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Our solution is to bet on sparsity:

- for interpretability
- for discovering linguistic structure
- for efficiency

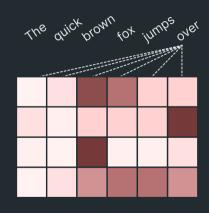


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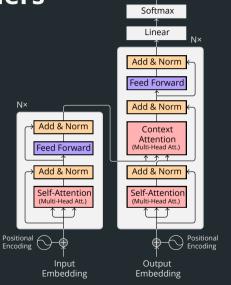
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In each attention head:

$$\bar{\mathbf{V}} = \mathbf{softmax} \left(\frac{\mathbf{Q} \mathbf{K}^{\mathsf{T}}}{\sqrt{d_k}} \right) \mathbf{V}.$$



(Vaswani et al., 2017)

Output

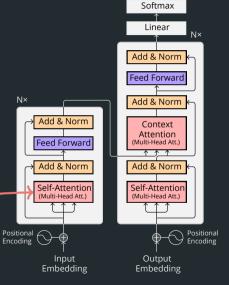
Probabilities

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Attention in three places:

• Self-attention in the encoder



(Vaswani et al., 2017)

Output

Probabilities

N×

Positional

Encoding

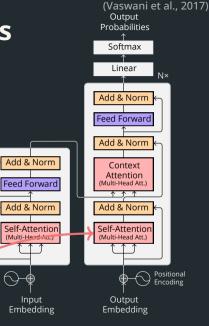
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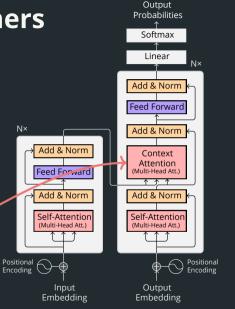


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(Vaswani et al., 2017)

Sparse Transformers

Sparse Transformers

Key idea: replace softmax in attention heads by a sparse normalizing function!

Adaptively Sparse Transformers

Key idea: replace softmax in attention heads by a sparse normalizing function!

Another key idea: use a normalizing function that is adaptively sparse via a learnable α !

What is softmax?

Softmax exponentiates and normalizes:

$$[\mathbf{softmax}(z)]_i := \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

What is softmax?

Softmax exponentiates and normalizes:

$$[\mathbf{softmax}(z)]_i := \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

It's fully dense: softmax(z) > 0

Parametrized by $\alpha \geq 0$:

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• **Argmax** corresponds to $\alpha \rightarrow \infty$

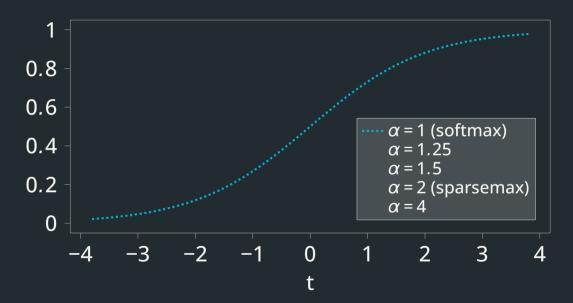
Parametrized by $\alpha \geq 0$:

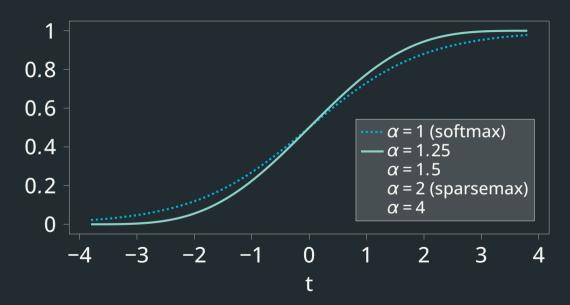
- **Argmax** corresponds to $\alpha \rightarrow \infty$
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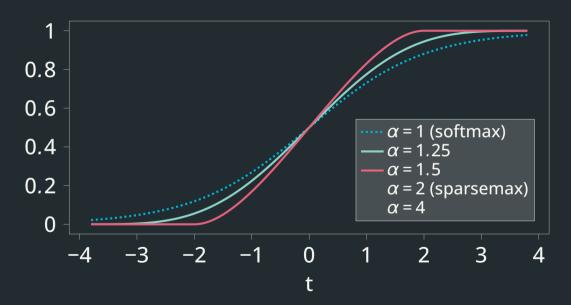
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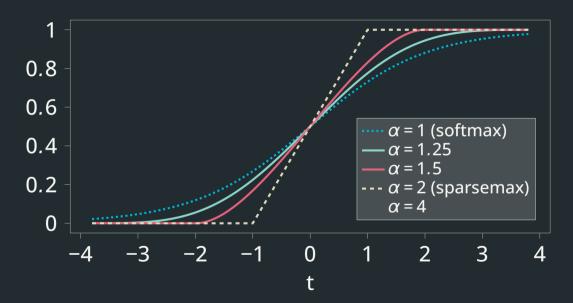
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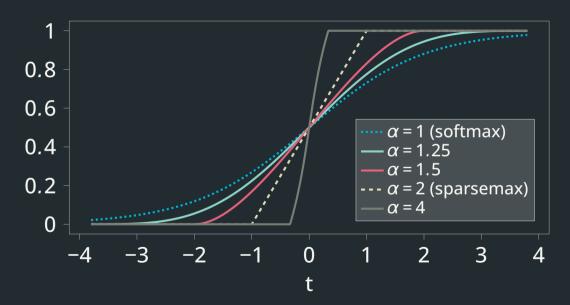
Key result: can be sparse for $\alpha > 1$, propensity for sparsity increases with α .





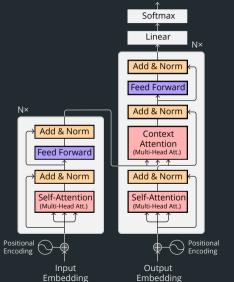






Output Probabilities

Transformers



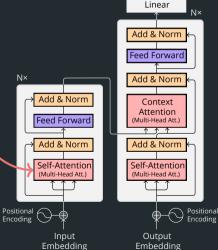
Output Probabilities ↑
Softmax

Transformers

Attention in three places:

• Self-attention in the encoder

6 layers \times 8 attention heads = 48



Output Probabilities

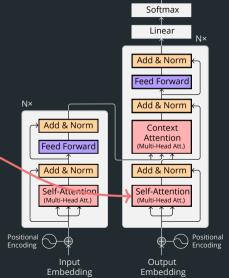
Transformers

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+ 48



Output Probabilities

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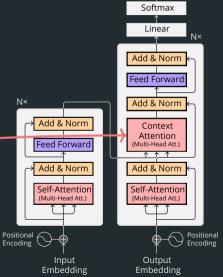
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+ 48

+48 = 144



Learning α

Learning α

Key contribution:

a closed-form expression for $\frac{\partial \alpha - \text{entmax}(z)}{\partial \alpha}$



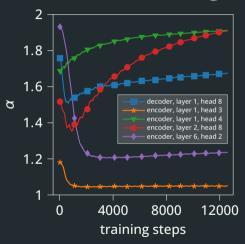
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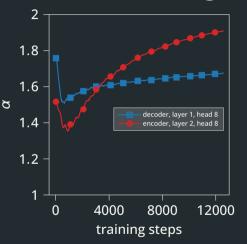
a closed-form expression for $\frac{\partial \alpha - \text{entmax}(z)}{\partial \alpha}$

Not trivial! Requires implicit differentiation

Trajectories of α during training

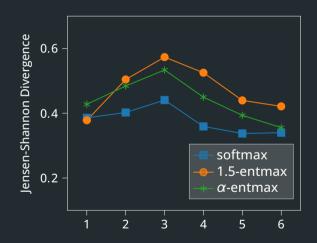


Trajectories of α during training

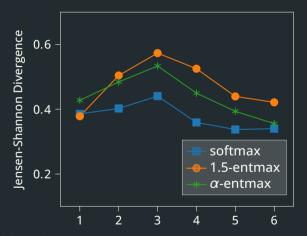


Some heads choose to start dense before becoming sparse.

Head Diversity per Layer

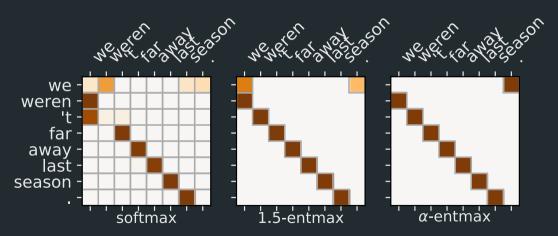


Head Diversity per Layer



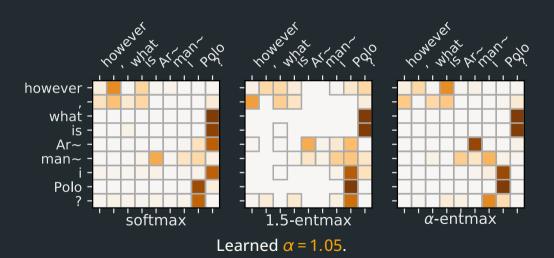
Specialized heads are important as seen in Voita et al. (2019)!

Previous position head



This head role was also found in Voita et al. (2019)! Learned $\alpha = 1.91$.

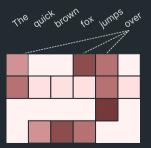
Interrogation-detecting head



Introduce adaptive sparsity for Transformers via α -entmax with a gradient learnable α , improving transparency.

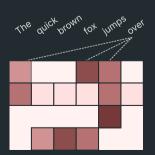
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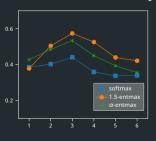


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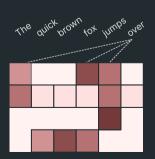


reduced head redundancy

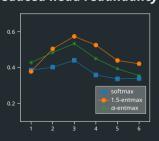


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adaptive sparsity



reduced head redundancy



clearer head roles

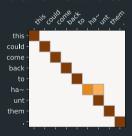


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We focus on latent variables *z* that are discrete or structured



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 $\pi(z|x,\theta)$: distribution over possible z



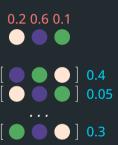
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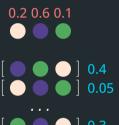
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 $\ell(x, z; \theta)$: downstream loss: ELBO, Log-Likelihood, (...)

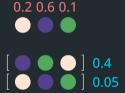


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ood, (...) [• • •] 0.4

0.2 0.6 0.1

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$$\mathcal{L}_{x}(\boldsymbol{\theta}) = \sum_{z \in \mathcal{T}} \pi(z|x, \boldsymbol{\theta}) \; \ell(x, z; \boldsymbol{\theta})$$

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If Z is large, this sum can get very expensive due to $\ell(x, z; \theta)$!



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$$\pi(z|x, \theta)$$
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If Z is combinatorial, this can be intractable to compute!







Current solutions

Using emergent communication as example

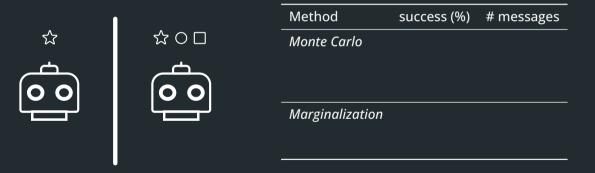


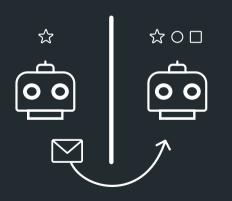


Method	success (%)	# messages
Monte Carlo		
 Marginalization		

Current solutions

Using emergent communication as example





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	\subseteq
\square	

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\square	\bigvee	

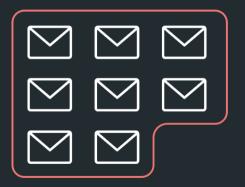
Method	success (%)	# messages
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<i>Marginalization</i> Dense	93.37 ±0.42	256



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<i>Marginalization</i> Dense	93.37 ±0.42	256



Method	success (%)	# messages
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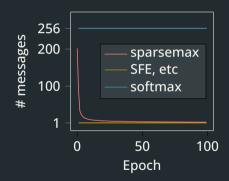


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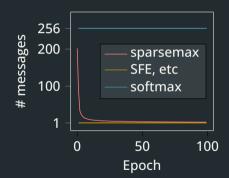
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Using emergent communication as example





We use sparsemax, top-k sparsemax and SparseMAP to allow efficient marginalization

We test our methods for models with discrete latent variables,

We test our methods for models with discrete latent variables,

Semi-Supervised VAE

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE
- Emergent communication

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- Emergent communication

but also in models with an exponentially large set of Z,

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE
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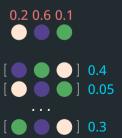
Bit-vector VAE

Our methods are top-performers and efficient!

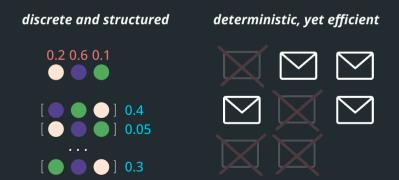
We introduce a new method to train compact latent variable models, using sparsity.

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Adaptively Sparse Transformers

Efficient Marg. of Discrete Latent Variables via Sparsity

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- Semi-supervised learning: data-efficiency and compactness
- Learning $\pi(z|x)$ without learning $\ell(x,z)$: compactness

- Semi-supervised learning: data-efficiency and compactness
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- Latent draft translations: transparency and compactness

Using learned sparsity and weak supervision we took steps to take neural models closer to version 2.0

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data-efficiency

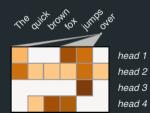
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better & efficient compactness



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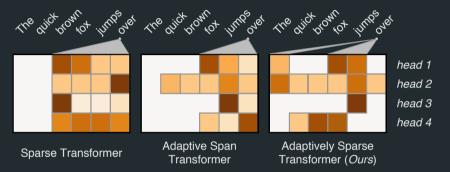
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Parameter sharing analysis

	TER↓	BLEU↑
MT Baseline	24.76	62.11
Transformer	27.80	60.76
Transformer decoder	20.33	69.31
Pre-trained BERT	20.83	69.11
with CA ← SA	18.91	71.81
and SA \longleftrightarrow Encoder SA	18.44	72.25
andCA ↔ SA	18.75	71.83
and FF ↔ Encoder FF	19.04	71.53

Related Work: Other Sparse Transformers



Our model allows non-contiguous attention for each head.

Ω-Regularized Argmax

For convex Ω , define the Ω -regularized argmax transformation:

$$\operatorname{argmax}_{\Omega}(\mathbf{z}) \coloneqq \arg \max_{\mathbf{p} \in \Delta} \mathbf{z}^{\mathsf{T}} \mathbf{p} - \Omega(\mathbf{p})$$

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Is there something in-between?

BLEU Scores

activation	de→en	ja→en	ro→en	en→de
softmax	29.79	21.57	32.70	26.02
1.5-entmax	29.83	22.13	33.10	25.89
α-entmax	29.90	21.74	32.89	26.93

BLEU Scores

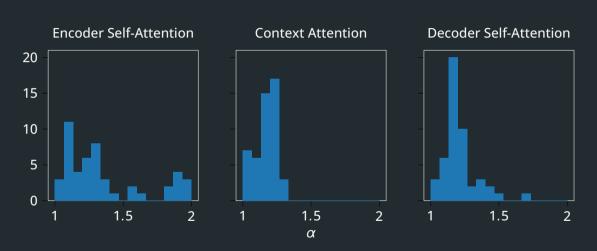
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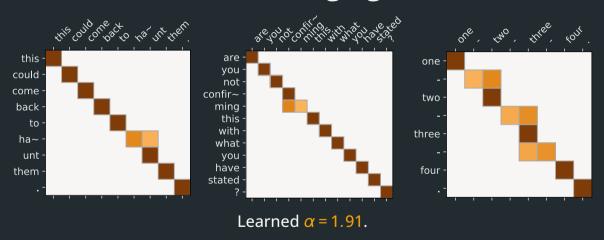
For analysis for other language pairs, see Appendix A.

Learned α



Bimodal for the encoder, mostly unimodal for the decoder.

Subword-Merging Head

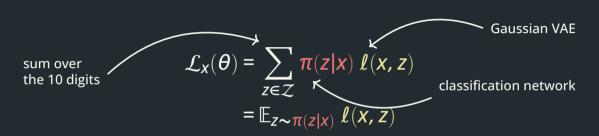


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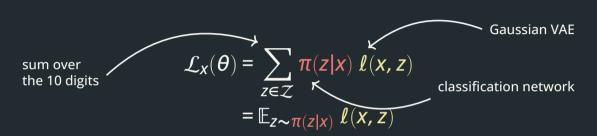
Semi-Supervised VAE on MNIST: z is one of 10 categories

$$\mathcal{L}_{X}(\boldsymbol{\theta}) = \sum_{\boldsymbol{z} \in \mathcal{Z}} \pi(\boldsymbol{z}|\boldsymbol{x}) \; \boldsymbol{\ell}(\boldsymbol{x},\boldsymbol{z})$$
 classification network
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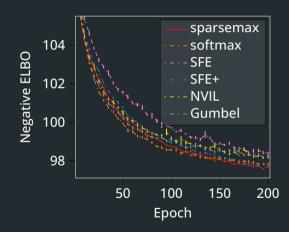


- Semi-Supervised VAE on MNIST: z is one of 10 categories
- Train this with 10% labeled data

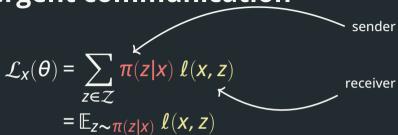
Method	Accuracy (%)	Dec. calls
Monte Ca	rlo	
SFE	$94.75 \pm .002$	1
SFE+	96.53±.001	2
NVIL	$96.01 \pm .002$	1
Gumbel	95.46±.001	1
Marginali	zation	
Dense	96.93±.001	10

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Sparse	$96.87 \scriptstyle{\pm .001}$	$1.01 \scriptstyle{\pm 0.01}$

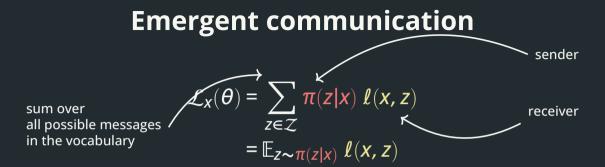
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Sparse	$96.87 \scriptstyle{\pm .001}$	1.01 ±0.01



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ullet receiver picks image from a set ${\mathcal V}$ based on message



- ullet receiver picks image from a set ${\mathcal V}$ based on message
- images come from ImageNet

... but make it harder: |Z| = 256, |V| = 16

Method	success (%)	Dec. calls
Monte Carlo		
SFE	33.05 ±2.84	1
SFE+	44.32 ±2.72	2
NVIL	37.04 ±1.61	1
Gumbel	23.51 ±16.19	1
ST Gumbel	27.42 ±13.36	1

Marginalization

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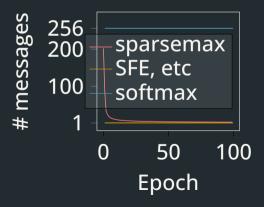
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<i>Marginalization</i> Dense	93.37 ±0.42	256

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- Mostly (and eventually) very sparse. But fully dense worst case.
- For the same reason, sparsemax cannot handle structured z.

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One solution: top-k sparsemax

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$$k$$
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- Non-convex but easy: sparsemax over the k highest scores (Kyrillidis et al., 2013).
- Top-k oracle available for some structured problems.
- Certificate: if at least one of the top-k z gets p(z) = 0, k-sparsemax = sparsemax! thus, biased early on, but it goes away.





$$\mathcal{M} := \operatorname{conv} \left\{ \boldsymbol{a}_{Z} : Z \in \mathcal{Z} \right\}$$
$$= \left\{ \boldsymbol{A} \boldsymbol{p} : \boldsymbol{p} \in \Delta \right\}$$
$$= \left\{ \mathbb{E}_{Z \sim \boldsymbol{p}} \; \boldsymbol{a}_{Z} : \boldsymbol{p} \in \Delta \right\}$$





• $\operatorname{\mathsf{argmax}} \operatorname{\mathsf{argmax}} \operatorname{\mathsf{p}}^\mathsf{T} s$





 $\operatorname{\mathsf{argmax}}_{oldsymbol{p} \in \Delta} \overline{oldsymbol{p}}^\mathsf{T} oldsymbol{s}$

 $\mathsf{MAP} \underset{\boldsymbol{\mu} \in \mathcal{M}}{\operatorname{arg \, max}} \boldsymbol{\mu}^{\mathsf{T}} \boldsymbol{t}$





• argmax $\arg \max_{p \in \Delta} p^{\top} s$

$$\mathsf{MAP}\, \operatorname*{arg\, max} \boldsymbol{\mu}^\mathsf{T} \boldsymbol{t} \\ \boldsymbol{\mu} \boldsymbol{\in} \mathcal{M}$$

 $\iota^{ op} t$

softmax $\underset{\boldsymbol{p} \in \Delta}{\operatorname{arg max}} \boldsymbol{p}^{\mathsf{T}} \boldsymbol{s} + \mathrm{H}(\boldsymbol{p})$





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marginals $\underset{\boldsymbol{\mu} \in \mathcal{M}}{\operatorname{max}} \boldsymbol{\mu}^{\mathsf{T}} \boldsymbol{t} + \widetilde{\operatorname{H}}(\boldsymbol{\mu})$





- argmax $\arg \max p^T s$ $p \in \Delta$
- softmax $\underset{\boldsymbol{p} \in \Delta}{\operatorname{arg max}} \boldsymbol{p}^{\mathsf{T}} \mathbf{s} + \mathrm{H}(\boldsymbol{p})$
- sparsemax $\underset{\boldsymbol{p} \in \Delta}{\operatorname{arg max}} \boldsymbol{p}^{\mathsf{T}} \boldsymbol{s} 1/2 ||\boldsymbol{p}||^2$

MAP
$$\underset{\boldsymbol{\mu} \in \mathcal{M}}{\operatorname{arg max}} \boldsymbol{\mu}^{\mathsf{T}} \boldsymbol{t}$$

marginals $\underset{\boldsymbol{\mu} \in \mathcal{M}}{\operatorname{marginals}} \mathbf{T} + \widetilde{\mathbf{H}}(\boldsymbol{\mu})$





- argmax $arg max p^T s$ $b \in \Delta$
- softmax $\arg \max \boldsymbol{p}^{\mathsf{T}} \mathbf{s} + \mathbf{H}(\boldsymbol{p})$ $p \in \Delta$

• sparsemax $\arg \max p^{\top} s - 1/2 ||p||^2$ $p \in \Delta$

MAP
$$\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^{\mathsf{T}} \boldsymbol{t}$$

$$\mu \in \mathcal{M}$$
marginals $\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^{\mathsf{T}} \boldsymbol{t} + \widetilde{\mathbf{H}}(\boldsymbol{\mu})$

SparseMAP $\arg \max \mu^{\mathsf{T}} t - 1/2 \|\mu\|^2 \bullet$ $\mu \in \mathcal{M}$





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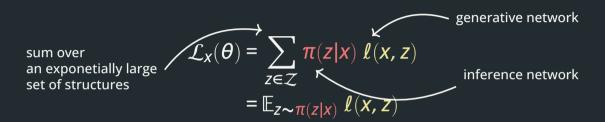
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 inference network

• VAE where *z* is a collection of *D* bits

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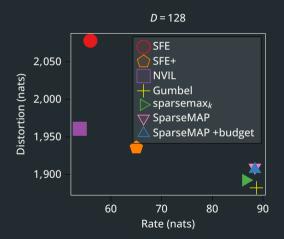
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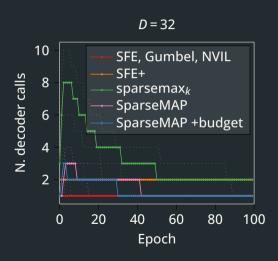
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Method	D = 32	<i>D</i> = 128
Monte Carlo		
SFE	3.74	3.77
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