Efficient Marginalization of Discrete and Structured Latent Variables via Sparsity

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Latent Variable Models

Latent variable z can be

Latent Variable Models

Latent variable z can be continuous

Source: Bouges et al., 2013

Latent Variable Models

Latent variable z can be continuous, discrete

Source: valleyeyecareaz.com

Latent Variable Models

Latent variable z can be continuous, discrete, or structured

Source: Liu et al., 2015

Training Discrete or Structured Latent Variable Models

Latent variable z can be

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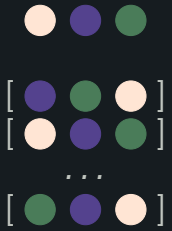
Training Discrete or Structured Latent Variable Models

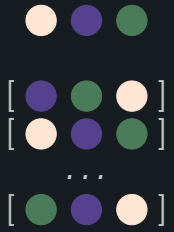
Latent variable z can be discrete or structured



Training Discrete or Structured Latent Variable Models

Latent variable z can be discrete or structured

$\pi(z|x, \theta)$: distribution over possible z



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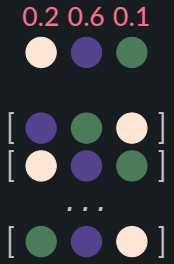

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If $\mathcal{Z}$ is large, this sum can get very expensive due to $\ell(x, z; \theta)$! 😱

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If $\mathcal{Z}$ is combinatorial, this can be intractable to compute! 😱 😱 😱

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no need for sampling –> no variance

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New option: use sparsity! 🙌

no need for sampling –> no variance
no relaxation into the continuous space

Taking a step back...

Does the expectation over possible z need to be expensive?

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$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x, \theta)\, \ell(x, z; \theta) \\ &= \pi(z_1|x, \theta)\, \ell(x, z_1; \theta) + \pi(z_2|x, \theta)\, \ell(x, z_2; \theta) + \ldots \\ &+ \pi(z_i|x, \theta)\, \ell(x, z_i; \theta) + \ldots + \pi(z_N|x, \theta)\, \ell(x, z_N; \theta)\end{aligned}$$

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Usually we normalize π with softmax$\propto \exp(\pi) \Rightarrow \pi(z_i|x, \theta) > 0$

(Martins and Astudillo, 2016; Niculae et al., 2018)

Sparse normalizers

We use sparsemax, top-k sparsemax and SparseMAP to allow efficient marginalization

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No need for computing $\ell(x, z; \theta)$ for all $z \in \mathcal{Z}$!

Results

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Our methods are top-performers and efficient!

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