

# **Learnable Sparsity and Weak Supervision for Data-efficient, Transparent, and Compact Neural Models**

**Gonalo M. Correia**

Jury: Andr  Martins, M rio Figueiredo, Ivan Titov, Wilker Aziz, Isabel Trancoso

# Deep learning successes

# Deep learning successes

- Subset of machine learning that uses **neural networks**

# Deep learning successes

- Subset of machine learning that uses **neural networks**
- Powerful tool for learning representations of any data

# Deep learning successes

- Subset of machine learning that uses **neural networks**
- Powerful tool for learning representations of any data
- Remarkable results

# Deep learning successes

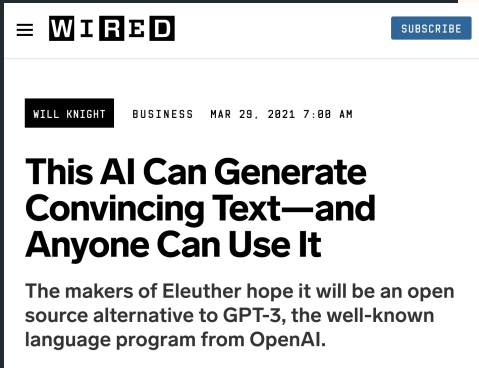
# Deep learning successes

A robot wrote this  
entire article. Are you  
scared yet, human?

*GPT-3*

**The  
Guardian**  
News website of the year

# Deep learning successes



Wired

SUBSCRIBE

WILL KNIGHT BUSINESS MAR 29, 2021 7:00 AM

## This AI Can Generate Convincing Text—and Anyone Can Use It

The makers of Eleuther hope it will be an open source alternative to GPT-3, the well-known language program from OpenAI.



A robot wrote this entire article. Are you scared yet, human?

*GPT-3*

**The Guardian**  
News website of the year



# Deep learning successes



WILL KNIGHT

BUSINESS

MAR 29, 2021 7:00 AM

## This AI Can Generate Convincing Text—And Anyone Can Use It

The makers of Eleuther hope it's a source alternative to GPT-3, the language program from OpenAI

### A robot wrote this entire article. Are you scared yet, human?

GPT-3

### SCIENCE

## Danny's workmate is called GPT-3. You've probably read its work without realising it's an AI

[ABC Science](#) / By technology reporter [James Purtill](#)

Posted Sat 28 May 2022 at 7:30pm

# Deep learning successes

W I R E D

SUBSCRIBE

WILL KNIGHT

BUSINESS MAR 29, 2021 7:00 AM

**This AI Can Generate  
Convincing Text—** SCIENCE

Forbes

INNOVATION

**Are AI Systems About To  
Outperform Humans?**

A robot wrote this  
entire article. Are you  
scared yet, human?

CDT-2

**orkmate is called  
e probably read its  
ut realising it's an AI**

hnology reporter [James Purtill](#)

Posted Sat 28 May 2022 at 7:30pm

# Deep learning successes

## Artificial intelligence beats eight world champions at bridge

**Victory marks milestone for AI as bridge requires more human skills than other strategy games**

INNOVATION

## Are AI Systems About To Outperform Humans?

robot wrote this entire article. Are you scared yet, human?

DT-2

**Arkmate is called**  
**we probably read its**  
**but realising it's an AI**

technology reporter [James Purtil](#)

Posted Sat 28 May 2022 at 7:30pm

## Deep learning successes

Artificial intelligence  
beats eight world  
champions at bridge

Victory marks milestone for AI  
bridge requires more human skill  
than other strategy games

INNOVATION

Are AI Systems About To  
Outperform Humans?

Posted Sa

robot wrote this

AI 'outperforms'  
doctors  
diagnosing breast  
cancer



**Fergus Walsh**

Medical correspondent

@BBCFergusWalsh

# Deep learning limitations and drawbacks

# Deep learning limitations and drawbacks

- Requires a lot of data

# Deep learning limitations and drawbacks

- Requires a lot of data
- Hard to understand and interpret reasons behind decisions

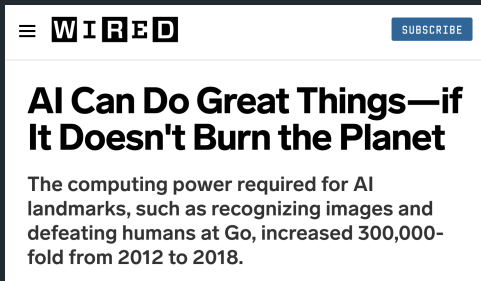
# Deep learning limitations and drawbacks

- Requires a lot of data
- Hard to understand and interpret reasons behind decisions
- Requires a lot of computation



# Deep learning limitations and drawbacks

# Deep learning limitations and drawbacks



# Deep learning limitations and drawbacks

**Forbes**

AI

## Overcoming AI's Transparency Paradox

≡ **WIRED**

SUBSCRIBE

## AI Can Do Great Things—if It Doesn't Burn the Planet

The computing power required for AI landmarks, such as recognizing images and defeating humans at Go, increased 300,000-fold from 2012 to 2018.

# Deep learning limitations and drawbacks

**Forbes**

AI

**Overcoming  
Transparent  
Paradox**

≡ **WIRED**

SUBSCRIBE

**AI Can Do Great Things—if  
It Doesn't Burn the Planet**

The computing power required for AI



**Harvard  
Business  
Review**

**AI Can Outperform  
Doctors. So Why Don't  
Patients Trust It?**

by Chiara Longoni and Carey K. Morewedge

# **Key concepts of this thesis**

**Learnable Sparsity and Weak Supervision  
for Data-efficient, Transparent, and Compact  
Neural Models**

# Key concepts of this thesis

**Learnable Sparsity and Weak Supervision  
for **Data-efficient**, Transparent, and Compact  
Neural Models**

# Key concepts of this thesis

**Learnable Sparsity and Weak Supervision  
for **Data-efficient, Transparent,** and Compact  
Neural Models**

# Key concepts of this thesis

**Learnable Sparsity and Weak Supervision  
for Data-efficient, Transparent, and Compact  
Neural Models**



# Key concepts of this thesis

**Learnable Sparsity** and Weak Supervision  
for **Data-efficient, Transparent, and Compact**  
Neural Models

# Key concepts of this thesis

**Learnable Sparsity** and **Weak Supervision**  
for **Data-efficient, Transparent, and Compact**  
**Neural Models**

# Published work

# Published work

- A Simple and Effective Approach to APE with Transfer Learning
  - weak supervision
  - data-efficiency
  - Poster at ACL 2019

# Published work

- A Simple and Effective Approach to APE with Transfer Learning
  - weak supervision
  - data-efficiency
  - Poster at ACL 2019
- Adaptively Sparse Transformers
  - adaptive sparsity
  - transparency
  - Oral talk at EMNLP 2019

# Published work

- A Simple and Effective Approach to APE with Transfer Learning
  - weak supervision
  - data-efficiency
  - Poster at ACL 2019
- Adaptively Sparse Transformers
  - adaptive sparsity
  - transparency
  - Oral talk at EMNLP 2019
- Efficient Marginalization of Discrete Latent Variables with Sparsity
  - learnable sparsity
  - compactness
  - Spotlight paper at NeurIPS 2020

# Table of Contents

A Simple and Effective Approach to APE with Transfer Learning

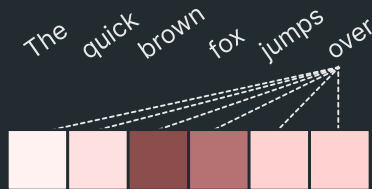
Adaptively Sparse Transformers

Efficient Marg. of Discrete Latent Variables via Sparsity

Future Work and Conclusions

# A bit of context on Transformers

What if... Attention is all you need?

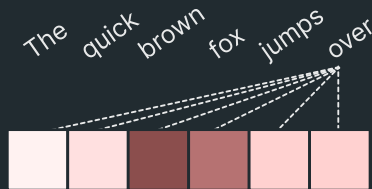




# A bit of context on Transformers

What if... Attention is all you need?

**Key idea:** Let's mainly use attention mechanisms!

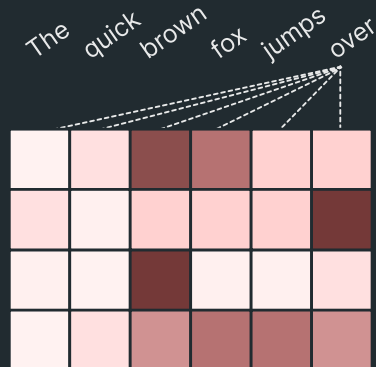


# A bit of context on Transformers

What if... Attention is all you need?

**Key idea:** Let's mainly use attention mechanisms!

- Do attention with multiple heads (i.e. attention mechanisms in parallel)

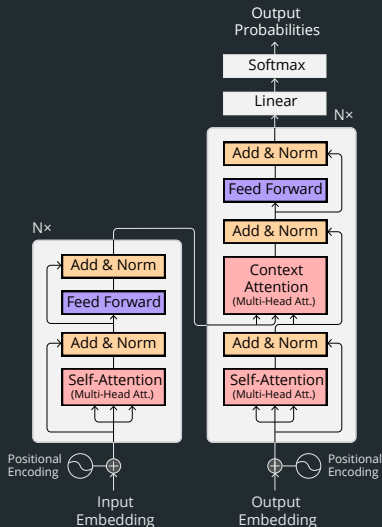


# A bit of context on Transformers

What if... Attention is all you need?

**Key idea:** Let's mainly use attention mechanisms!

- Do attention with multiple heads (i.e. attention mechanisms in parallel)
- ... and do it through several layers

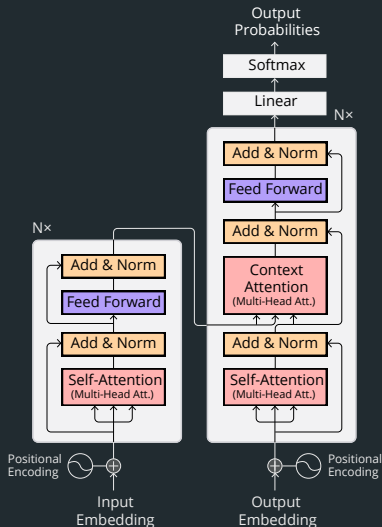


# A bit of context on Transformers

What if... Attention is all you need?

**Key idea:** Let's mainly use attention mechanisms!

- Do attention with multiple heads (i.e. attention mechanisms in parallel)
- ... and do it through several layers
- Inspiration for big general-purpose models like BERT and GPT-3!

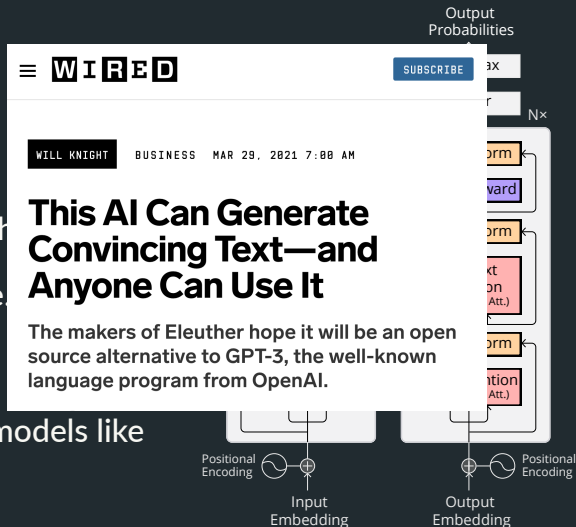


# A bit of context on Transformers

What if... Attention is all you need?

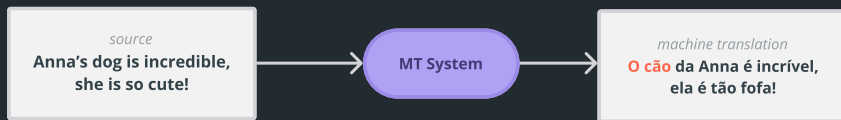
**Key idea:** Let's mainly use attention mechanism

- Do attention with multiple heads (i.e. mechanisms in parallel)
- ... and do it through several layers
- Inspiration for big general-purpose models like BERT and GPT-3!

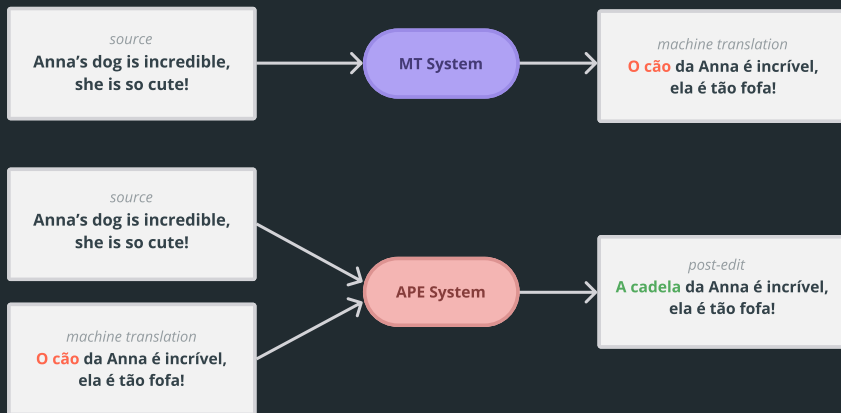


# What is APE?

# What is APE?

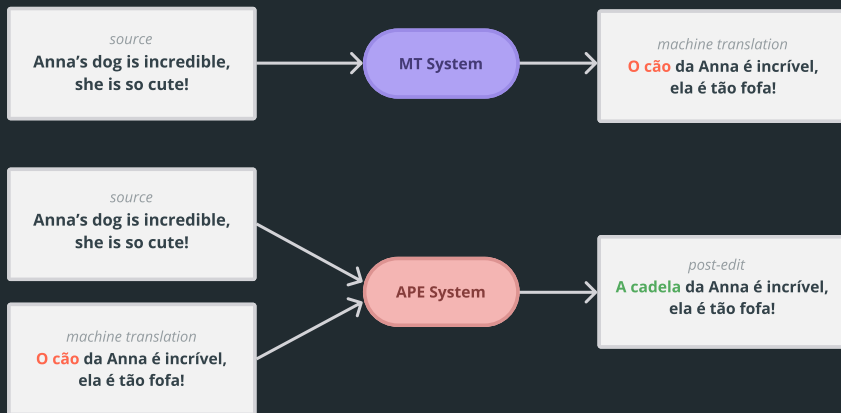


# What is APE?



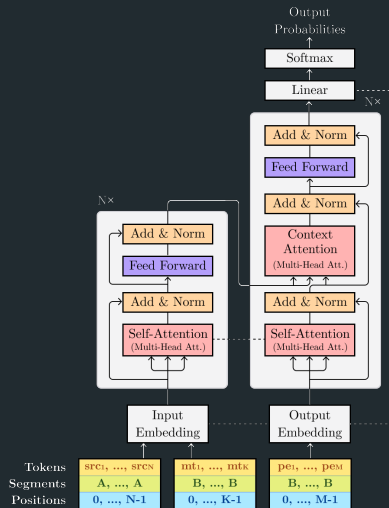


# What is APE?



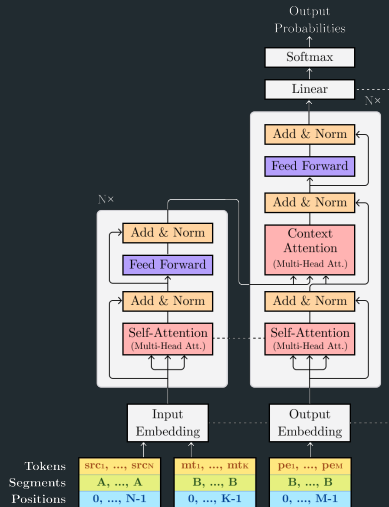
**Challenge:** APE data is very scarce! Need to create artificial data.

# BERT for APE



# BERT for APE

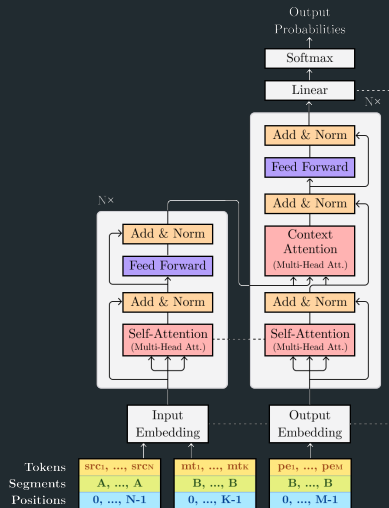
Key idea: Use BERT to do APE 🙌



# BERT for APE

**Key idea:** Use BERT to do APE 🙌

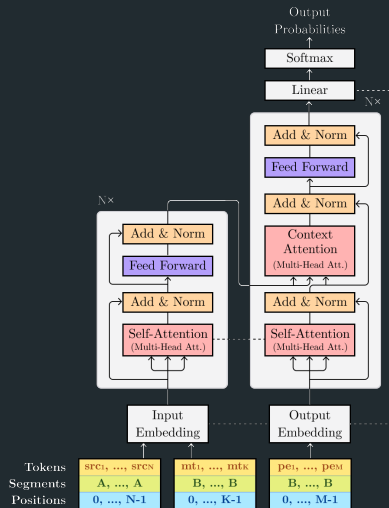
- Prior to this work, BERT was mainly used for simple classification tasks



# BERT for APE

**Key idea:** Use BERT to do APE 🙌

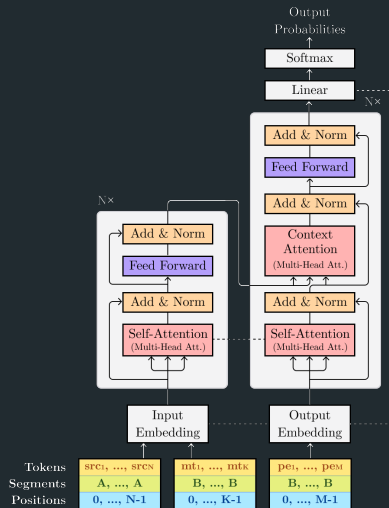
- Prior to this work, BERT was mainly used for simple classification tasks
- We introduced an effective method to use BERT in a generation task (APE)



# BERT for APE

**Key idea:** Use BERT to do APE 🙌

- Prior to this work, BERT was mainly used for simple classification tasks
- We introduced an effective method to use BERT in a generation task (APE)
- Smart parameter sharing between encoder and decoder



# Key results

model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49

# Key results

model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49
dual-source transformer (8M)	18.10	71.72



# Key results

model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49
dual-source transformer (8M)	18.10	71.72
dual-source transformer (23K)	27.73	59.78

# Key results

model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49
dual-source transformer (8M)	18.10	71.72
dual-source transformer (23K)	27.73	59.78
ours (23K)	19.03	70.66

# Key results

model (data size)	TER↓	BLEU↑
mt baseline	24.48	62.49
dual-source transformer (8M)	18.10	71.72
dual-source transformer (23K)	27.73	59.78
ours (23K)	19.03	70.66
ours (8M)	17.26	73.42

# Key takeaways

# Key takeaways

- One of pioneers in using pre-trained Transformer encoders for a generation task

# Key takeaways

- One of pioneers in using pre-trained Transformer encoders for a generation task
- Massive improvement in low-resource scenario (data-efficiency)

# Key takeaways

- One of pioneers in using pre-trained Transformer encoders for a generation task
- Massive improvement in low-resource scenario (data-efficiency)
- Steered SOTA of APE towards using weak supervision through pre-trained models

# Table of Contents

A Simple and Effective Approach to APE with Transfer Learning

Adaptively Sparse Transformers

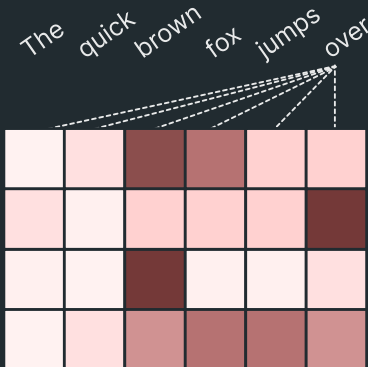
Efficient Marg. of Discrete Latent Variables via Sparsity

Future Work and Conclusions



# Getting to know attention heads better

Attention heads may aid visualization but they are completely **dense**.

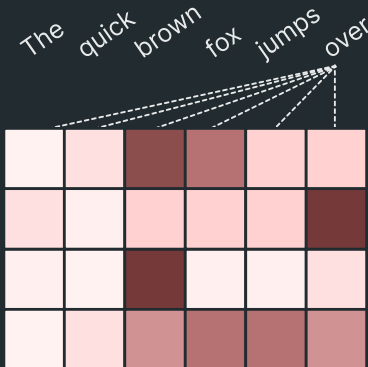


# Getting to know attention heads better

Attention heads may aid visualization but they are completely **dense**.

Our solution is to bet on **sparsity**:

- for interpretability
- for discovering linguistic structure
- for efficiency

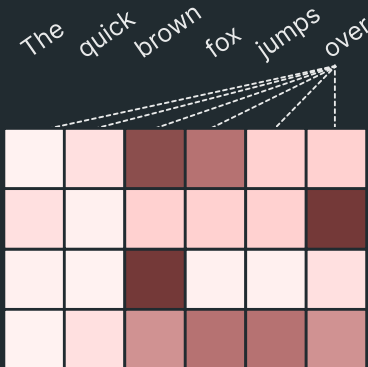


# Getting to know attention heads better

Attention heads may aid visualization but they are completely **dense**.

Our solution is to bet on **sparsity**:

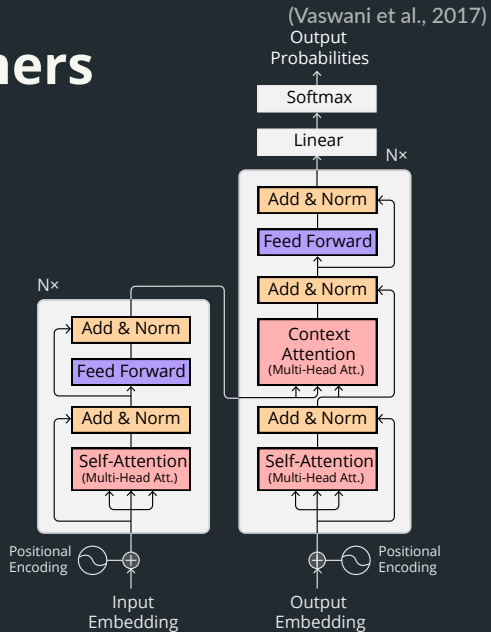
- **for interpretability**
- **for discovering linguistic structure**
- **for efficiency**



# Transformers

In each attention head:

$$\bar{\mathbf{V}} = \text{softmax}\left(\frac{\mathbf{QK}^T}{\sqrt{d_k}}\right)\mathbf{V}.$$



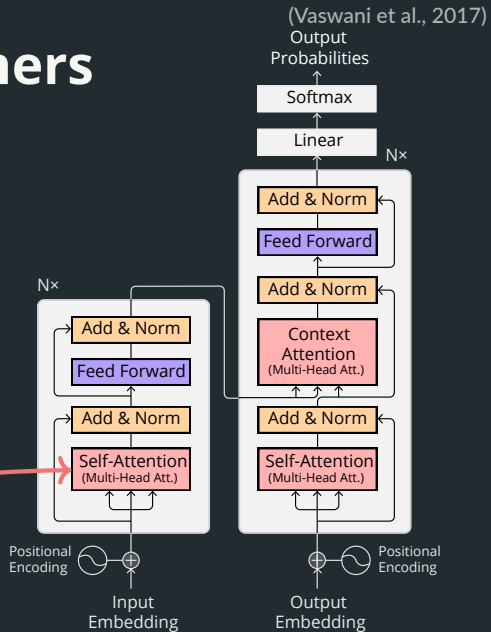
# Transformers

In each attention head:

$$\bar{\mathbf{V}} = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d_k}}\right)\mathbf{V}.$$

Attention in three places:

- Self-attention in the encoder



# Transformers

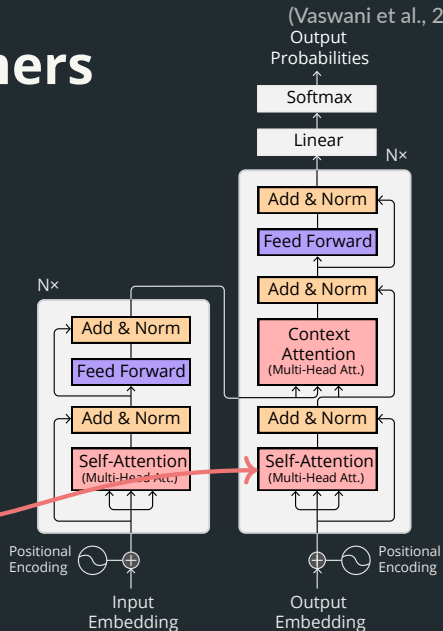
(Vaswani et al., 2017)

In each attention head:

$$\bar{\mathbf{V}} = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d_k}}\right)\mathbf{V}.$$

Attention in three places:

- Self-attention in the encoder
- Self-attention in the decoder



# Transformers

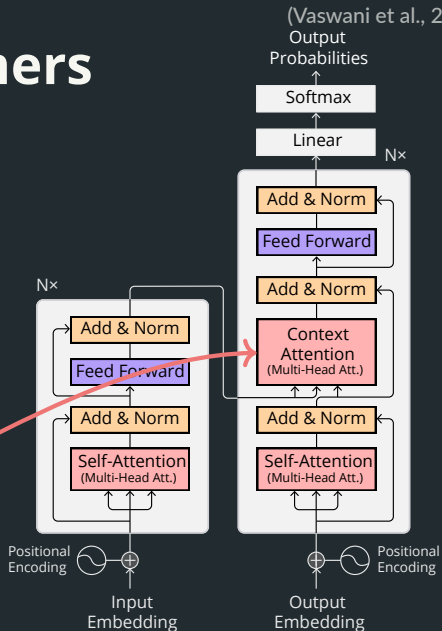
(Vaswani et al., 2017)

In each attention head:

$$\bar{\mathbf{V}} = \text{softmax}\left(\frac{\mathbf{Q}\mathbf{K}^T}{\sqrt{d_k}}\right)\mathbf{V}.$$

Attention in three places:

- Self-attention in the encoder
- Self-attention in the decoder
- Contextual attention



# Sparse Transformers



# Sparse Transformers

**Key idea:** replace softmax in attention heads by a sparse normalizing function! 🙌

# Adaptively Sparse Transformers

**Key idea:** replace softmax in attention heads by a sparse normalizing function! 🙌

**Another key idea:** use a normalizing function that is adaptively sparse via a learnable  $\alpha$ ! 🙌 🙌 🙌

# What is softmax?

Softmax exponentiates and normalizes:

$$[\mathbf{softmax}(z)]_i := \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

# What is softmax?

Softmax exponentiates and normalizes:

$$[\mathbf{softmax}(z)]_i := \frac{\exp(z_i)}{\sum_j \exp(z_j)}$$

It's fully dense:  **$\mathbf{softmax}(z) > 0$**

# $\alpha$ -entmax

Parametrized by  $\alpha \geq 0$ :

# $\alpha$ -entmax

Parametrized by  $\alpha \geq 0$ :

- **Argmax** corresponds to  $\alpha \rightarrow \infty$

# $\alpha$ -entmax

Parametrized by  $\alpha \geq 0$ :

- **Argmax** corresponds to  $\alpha \rightarrow \infty$
- **Softmax** amounts to  $\alpha \rightarrow 1$

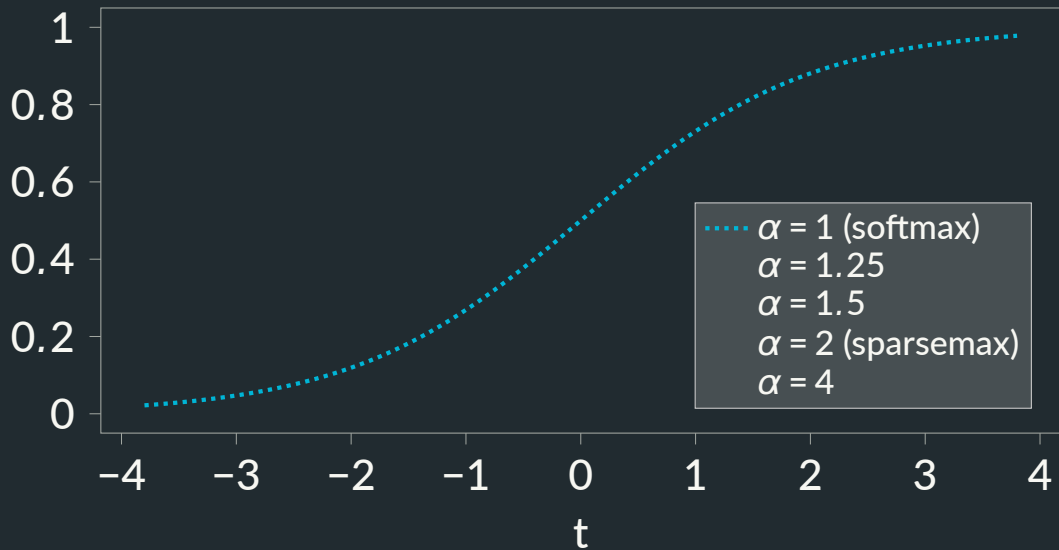
# $\alpha$ -entmax

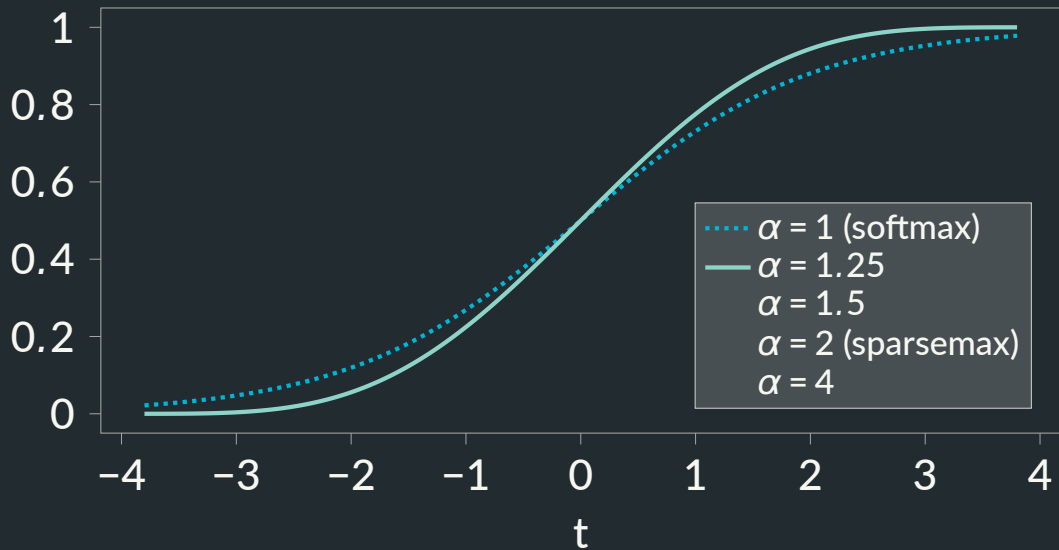
Parametrized by  $\alpha \geq 0$ :

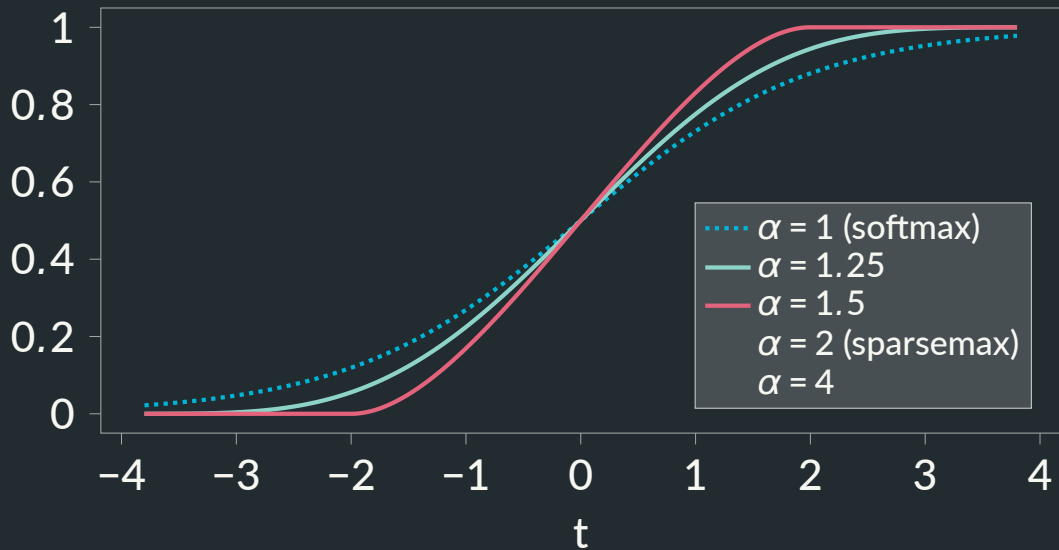
- **Argmax** corresponds to  $\alpha \rightarrow \infty$
- **Softmax** amounts to  $\alpha \rightarrow 1$

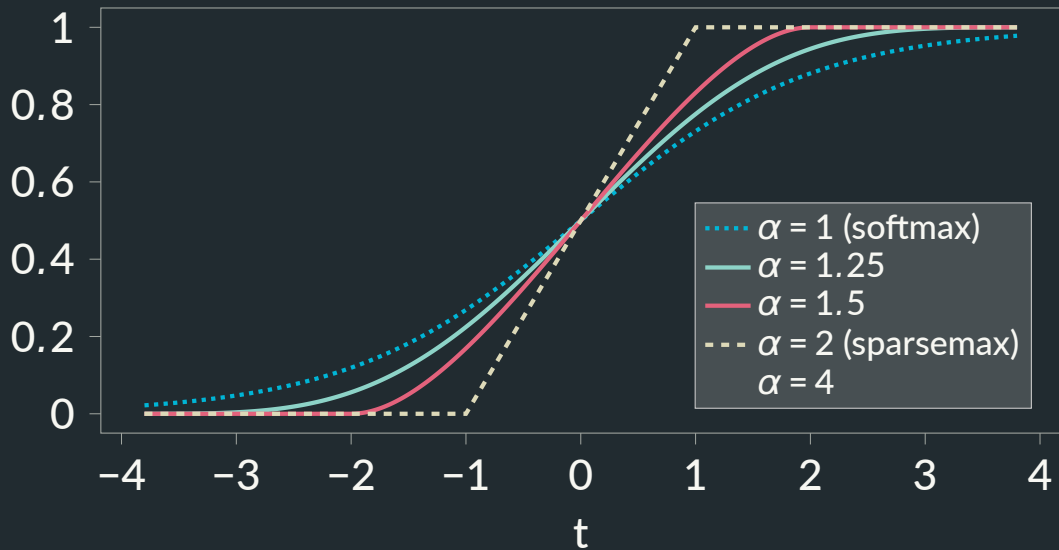
**Key result:** can be sparse for  $\alpha > 1$ , propensity for sparsity increases with  $\alpha$ .

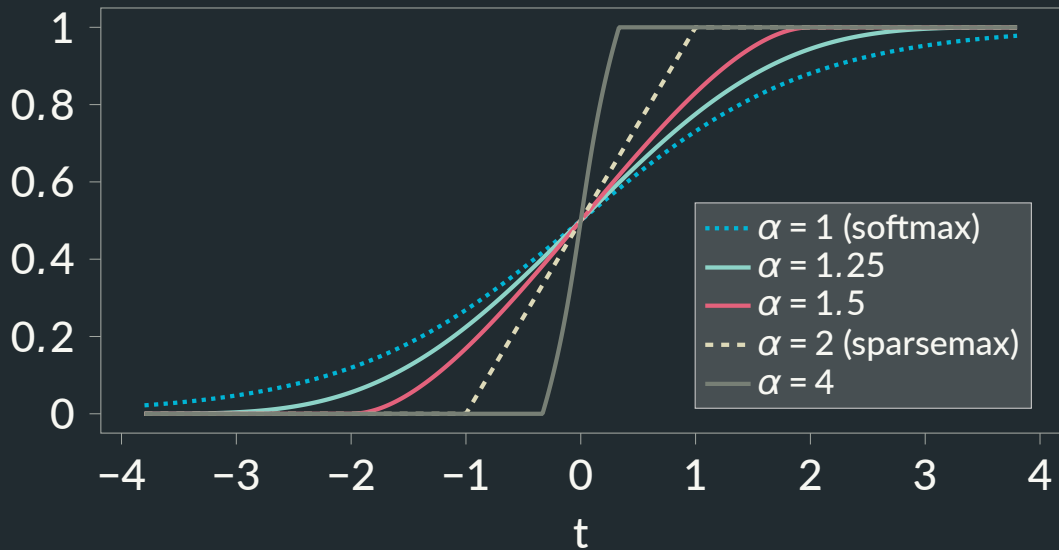




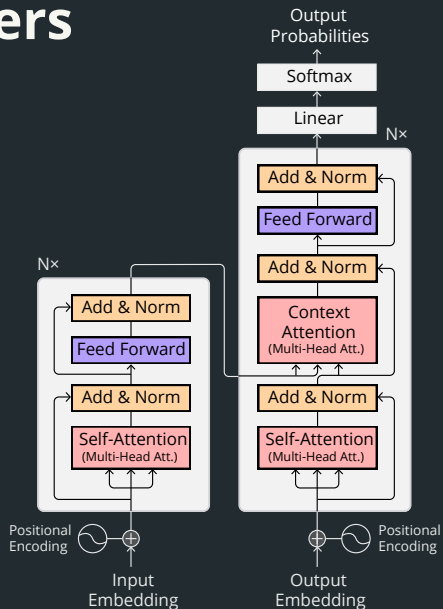








# Transformers

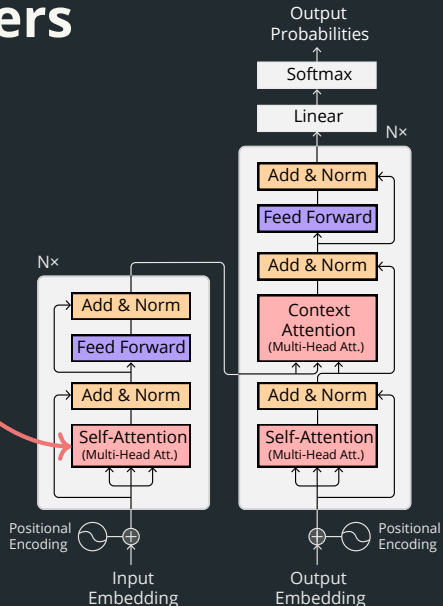


# Transformers

Attention in three places:

- Self-attention in the encoder

6 layers  $\times$  8 attention heads = 48

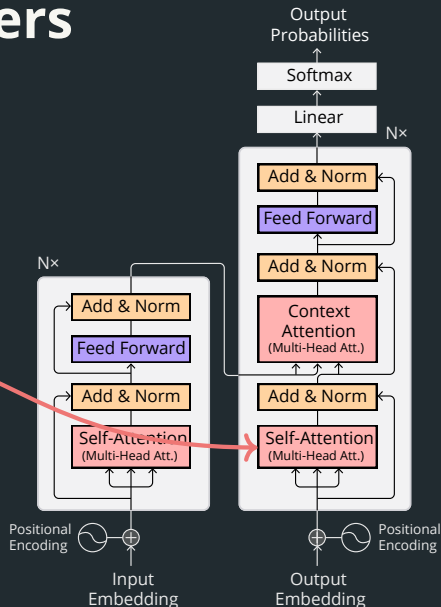


# Transformers

Attention in three places:

- Self-attention in the encoder
- Self-attention in the decoder

$$6 \text{ layers} \times 8 \text{ attention heads} = 48 \\ + 48$$



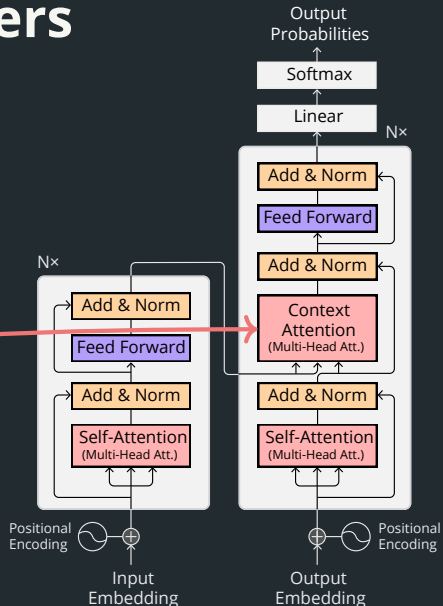


# Transformers

Attention in three places:

- Self-attention in the encoder
- Self-attention in the decoder
- Contextual attention

$$\begin{aligned}
 6 \text{ layers} \times 8 \text{ attention heads} &= 48 \\
 &+ 48 \\
 &+ 48 = 144
 \end{aligned}$$



# Learning $\alpha$

# Learning $\alpha$

Key contribution:

a closed-form expression for  $\frac{\partial \alpha\text{-entmax}(\mathbf{z})}{\partial \alpha}$  🤯

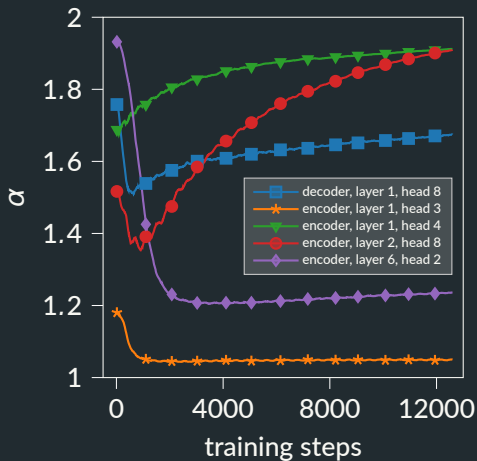
# Learning $\alpha$

Key contribution:

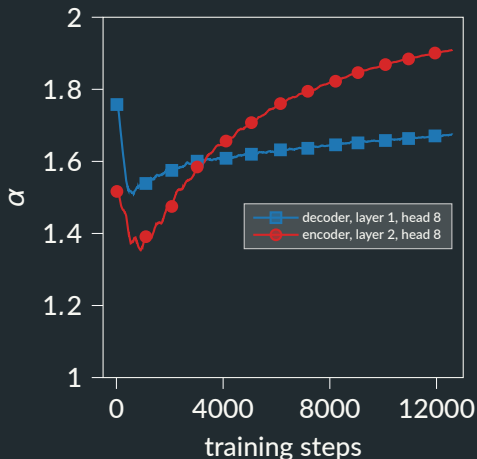
a closed-form expression for  $\frac{\partial \alpha\text{-entmax}(\mathbf{z})}{\partial \alpha}$  🤖

Not trivial! Requires implicit differentiation

# Trajectories of $\alpha$ during training

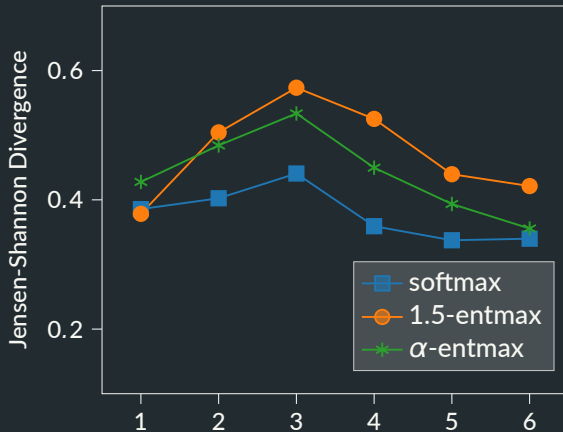


# Trajectories of $\alpha$ during training

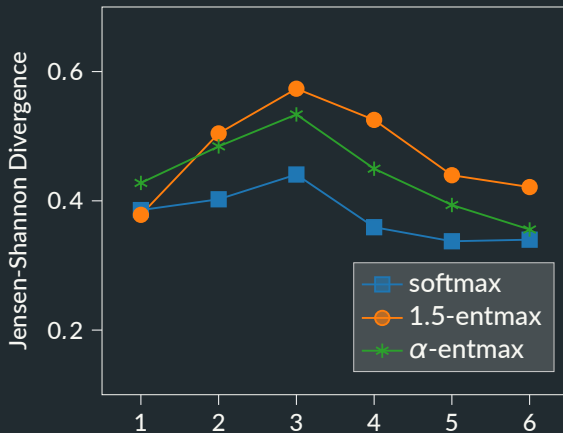


Some heads choose to start dense before becoming sparse.

# Head Diversity per Layer



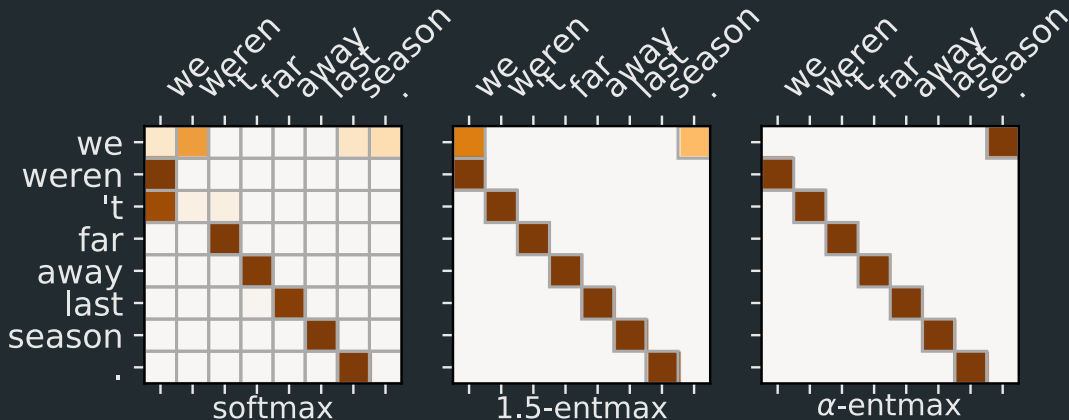
# Head Diversity per Layer



Specialized heads are important as seen in Voita et al. (2019)!

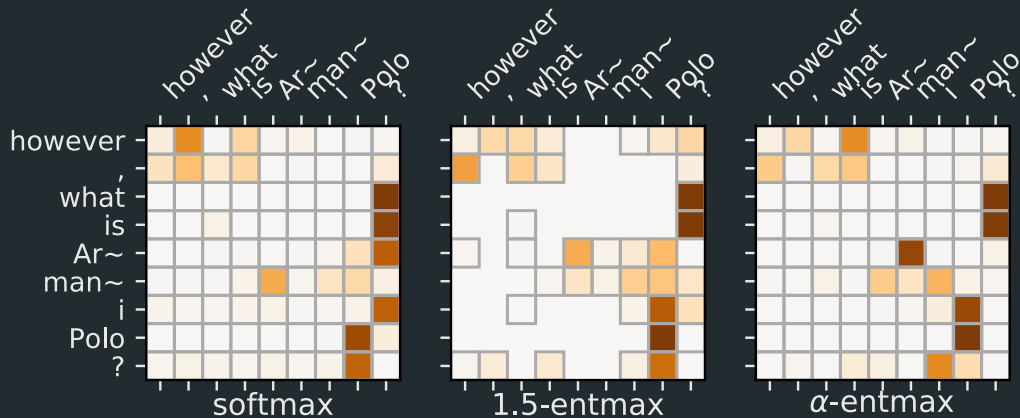


# Previous position head



This head role was also found in Voita et al. (2019)! Learned  $\alpha = 1.91$ .

# Interrogation-detecting head



Learned  $\alpha = 1.05$ .

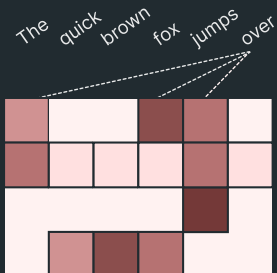
# Key takeaways

Introduce **adaptive sparsity**  
for Transformers via  $\alpha$ -entmax with a gradient learnable  $\alpha$ ,  
improving **transparency**.

# Key takeaways

Introduce **adaptive sparsity**  
for Transformers via  $\alpha$ -entmax with a gradient learnable  $\alpha$ ,  
improving **transparency**.

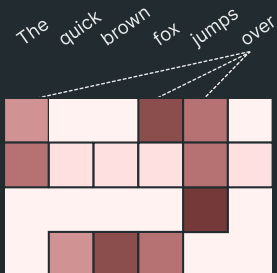
## *adaptive sparsity*



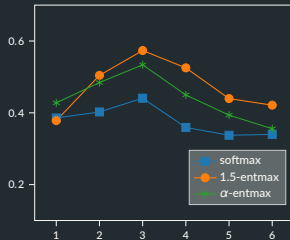
# Key takeaways

Introduce **adaptive sparsity**  
for Transformers via  $\alpha$ -entmax with a gradient learnable  $\alpha$ ,  
improving **transparency**.

*adaptive sparsity*



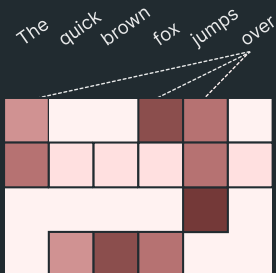
*reduced head redundancy*



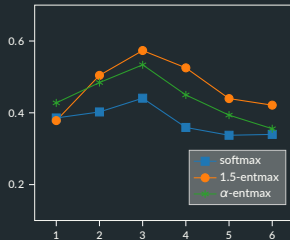
# Key takeaways

Introduce **adaptive sparsity**  
for Transformers via  $\alpha$ -entmax with a gradient learnable  $\alpha$ ,  
improving **transparency**.

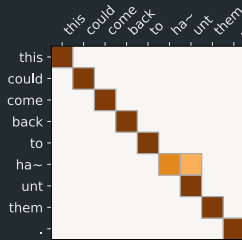
*adaptive sparsity*



*reduced head redundancy*



*clearer head roles*



# Table of Contents

A Simple and Effective Approach to APE with Transfer Learning

Adaptively Sparse Transformers

Efficient Marg. of Discrete Latent Variables via Sparsity

Future Work and Conclusions

# Latent variable models

We focus on latent variables  $z$  that are



# Latent variable models

We focus on latent variables  $z$  that are **discrete**



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$

$\ell(x, z; \theta)$ : downstream loss: ELBO, Log-Likelihood, (...)



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$

$\ell(x, z; \theta)$ : downstream loss: ELBO, Log-Likelihood, (...)

To train, we need to compute the following expectation:

0.2 0.6 0.1



$\begin{bmatrix} \text{purple} & \text{green} & \text{orange} \end{bmatrix}$  0.4  
 $\begin{bmatrix} \text{orange} & \text{purple} & \text{green} \end{bmatrix}$  0.05

...

$\begin{bmatrix} \text{green} & \text{purple} & \text{orange} \end{bmatrix}$  0.3

# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$

$\ell(x, z; \theta)$ : downstream loss: ELBO, Log-Likelihood, (...)

To train, we need to compute the following expectation:

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x, \theta) \ell(x, z; \theta)$$

0.2 0.6 0.1



$\begin{bmatrix} \text{purple} & \text{green} & \text{orange} \end{bmatrix}$  0.4  
 $\begin{bmatrix} \text{orange} & \text{purple} & \text{green} \end{bmatrix}$  0.05

...

$\begin{bmatrix} \text{green} & \text{purple} & \text{orange} \end{bmatrix}$  0.3



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

$\pi(z|x, \theta)$ : distribution over possible  $z$

$\ell(x, z; \theta)$ : downstream loss: ELBO, Log-Likelihood, (...)

To train, we need to compute the following expectation:

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x, \theta) \ell(x, z; \theta)$$

If  $\mathcal{Z}$  is **large**, this sum can get very expensive due to  $\ell(x, z; \theta)$ !



# Latent variable models

We focus on latent variables  $z$  that are **discrete** or **structured**

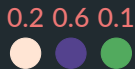
$\pi(z|x, \theta)$ : distribution over possible  $z$

$\ell(x, z; \theta)$ : downstream loss: ELBO, Log-Likelihood, (...)

To train, we need to compute the following expectation:

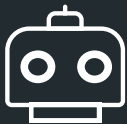
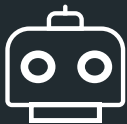
$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x, \theta) \ell(x, z; \theta)$$

If  $\mathcal{Z}$  is **combinatorial**, this can be intractable to compute!



# Current solutions

Using emergent communication as example



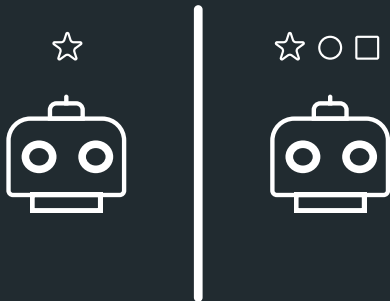
Method	success (%)	# messages
--------	-------------	------------

<i>Monte Carlo</i>		
--------------------	--	--

<i>Marginalization</i>		
------------------------	--	--

# Current solutions

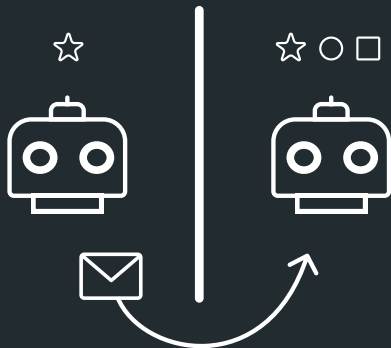
Using emergent communication as example



Method	success (%)	# messages
Monte Carlo		
Marginalization		

# Current solutions

Using emergent communication as example



Method	success (%)	# messages
Monte Carlo		
Marginalization		

# Current solutions

Using emergent communication as example



---

Method	success (%)	# messages
--------	-------------	------------

---

<i>Monte Carlo</i>		
--------------------	--	--

---

<i>Marginalization</i>		
------------------------	--	--

---

# Current solutions

Using emergent communication as example



Method	success (%)	# messages
--------	-------------	------------

<i>Monte Carlo</i>		
--------------------	--	--

<i>Marginalization</i>		
------------------------	--	--

Dense	93.37 $\pm 0.42$	256
-------	------------------	-----

# Current solutions

Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256



# Current solutions

Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256

# Current solutions

Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
Gumbel	23.51 $\pm 16.19$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256

# Our solution

Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
Gumbel	23.51 $\pm 16.19$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256
Sparse		

# Our solution

Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
Gumbel	23.51 $\pm 16.19$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256
Sparse		

# Our solution

Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
Gumbel	23.51 $\pm 16.19$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256
<b>Sparse</b>	93.35 $\pm 0.50$	

# Our solution

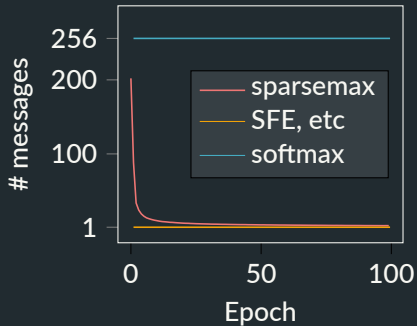
Using emergent communication as example



Method	success (%)	# messages
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
Gumbel	23.51 $\pm 16.19$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256
<b>Sparse</b>	93.35 $\pm 0.50$	3.13 $\pm 0.48$

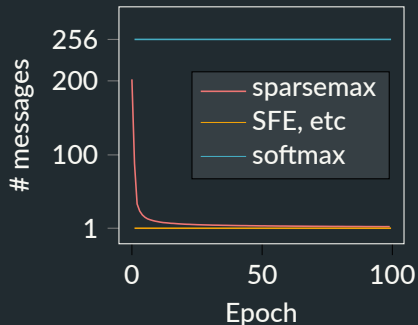
# Our solution

Using emergent communication as example



# Our solution

Using emergent communication as example



We use **sparsemax**, **top-k sparsemax** and **SparseMAP** to allow efficient marginalization



# Results

We test our methods for models with discrete latent variables,

# Results

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE

# Results

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE
- Emergent communication

# Results

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE
- Emergent communication

but also in models with an exponentially large set of  $\mathcal{Z}$ ,

# Results

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE
- Emergent communication

but also in models with an exponentially large set of  $\mathcal{Z}$ ,

- Bit-vector VAE

# Results

We test our methods for models with discrete latent variables,

- Semi-Supervised VAE
- Emergent communication

but also in models with an exponentially large set of  $\mathcal{Z}$ ,

- Bit-vector VAE

Our methods are top-performers and efficient!

# Key takeaways

We introduce a new method  
to train **compact** latent variable models,  
using **sparsity**.

# Key takeaways

We introduce a new method  
to train **compact** latent variable models,  
using **sparsity**.

*discrete and structured*

0.2 0.6 0.1  


[    ] 0.4  
[    ] 0.05

...

[    ] 0.3



# Key takeaways

We introduce a new method  
to train **compact** latent variable models,  
using **sparsity**.

*discrete and structured*



*deterministic, yet efficient*



# Key takeaways

We introduce a new method to train **compact** latent variable models, using **sparsity**.

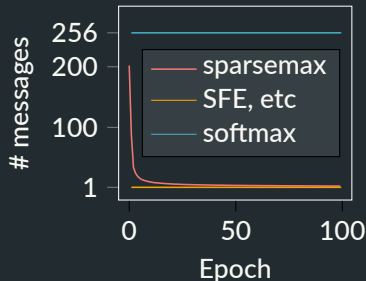
*discrete and structured*



*deterministic, yet efficient*



*sparse, as needed*



# Table of Contents

A Simple and Effective Approach to APE with Transfer Learning

Adaptively Sparse Transformers

Efficient Marg. of Discrete Latent Variables via Sparsity

Future Work and Conclusions

# Future directions

# Future directions

- Semi-supervised learning: data-efficiency and compactness

# Future directions

- Semi-supervised learning: **data-efficiency** and **compactness**
- Learning  $\pi(z|x)$  without learning  $\ell(x, z)$ : **compactness**

# Future directions

- Semi-supervised learning: **data-efficiency** and **compactness**
- Learning  $\pi(z|x)$  without learning  $\ell(x, z)$ : **compactness**
- Latent draft translations: **transparency** and **compactness**

# Conclusions

Using **learned sparsity** and **weak supervision**  
we took steps to take neural models closer to version 2.0



# Conclusions

Using **learned sparsity** and **weak supervision**  
we took steps to take neural models closer to version 2.0

*data-efficiency*

model (data size)	BLEU↑
dual-source transformer (8M)	71.72
dual-source transformer (23K)	59.78
ours (23K)	70.66

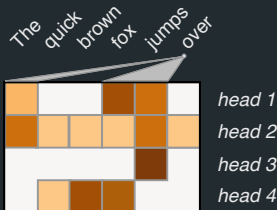
# Conclusions

Using **learned sparsity** and **weak supervision**  
we took steps to take neural models closer to version 2.0

*data-efficiency*

model (data size)	BLEU↑
dual-source transformer (8M)	71.72
dual-source transformer (23K)	59.78
ours (23K)	70.66

*transparency*



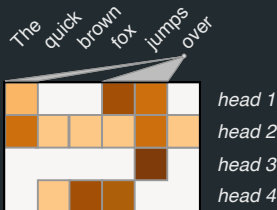
# Conclusions

Using **learned sparsity** and **weak supervision**  
we took steps to take neural models closer to version 2.0

## *data-efficiency*

model (data size)	BLEU↑
dual-source transformer (8M)	71.72
dual-source transformer (23K)	59.78
ours (23K)	70.66

## *transparency*



## *better & efficient compactness*



# References I



Child, Rewon, Scott Gray, Alec Radford, and Ilya Sutskever (2019). "Generating long sequences with sparse transformers". In: *arXiv preprint arXiv:1904.10509*.



Correia, Gonalo M., Vlad Niculae, Wilker Aziz, and Andr  F. T. Martins (2020). "Efficient Marginalization of Discrete and Structured Latent Variables via Sparsity". In: *Proc. NeurIPS*. URL: <https://arxiv.org/abs/2007.01919>.



Devlin, Jacob, Ming-Wei Chang, Kenton Lee, and Kristina Toutanova (2019). "BERT: Pre-training of deep bidirectional transformers for language understanding". In: *Proc. NAACL-HLT*.



Junczys-Dowmunt, Marcin and Roman Grundkiewicz (2018). "MS-UEdin Submission to the WMT2018 APE Shared Task: Dual-Source Transformer for Automatic Post-Editing". In: *Proceedings of WMT18*.



Kyrillidis, Anastasios, Stephen Becker, Volkan Cevher, and Christoph Koch (2013). "Sparse projections onto the simplex". In: *Proc. ICML*.



Lazaridou, Angeliki, Alexander Peysakhovich, and Marco Baroni (2017). "Multi-agent cooperation and the emergence of (natural) language". In: *Proc. ICLR*.



Lee, Jihyung, WonKee Lee, Jaehun Shin, Baikjin Jung, Young-Kil Kim, and Jong-Hyeok Lee (2020). "POSTECH-ETRI's Submission to the WMT2020 APE Shared Task: Automatic Post-Editing with Cross-lingual Language Model". In: *Proceedings of WMT*.



Martins, Andr  FT and Ram n Fernandez Astudillo (2016). "From softmax to sparsemax: A sparse model of attention and multi-label classification". In: *Proc. of ICML*.

# References II



Niculae, Vlad and Mathieu Blondel (2017). "A Regularized Framework for Sparse and Structured Neural Attention". In: *arXiv preprint arXiv:1705.07704*.



Niculae, Vlad, André FT Martins, Mathieu Blondel, and Claire Cardie (2018). "SparseMAP: Differentiable sparse structured inference". In: *Proc. of ICML*.



Peters, Ben, Vlad Niculae, and André F. T. Martins (2019). "Sparse Sequence-to-Sequence Models". In: *Proceedings of the Annual Meeting of the Association for Computational Linguistics*.



Sukhbaatar, Sainbayar, Edouard Grave, Piotr Bojanowski, and Armand Joulin (2019). "Adaptive Attention Span in Transformers". In: *arXiv preprint arXiv:1905.07799*.



Vaswani, Ashish, Noam Shazeer, Niki Parmar, Jakob Uszkoreit, Llion Jones, Aidan N. Gomez, Lukasz Kaiser, and Illia Polosukhin (2017). "Attention Is All You Need". In: *Proc. of NeurIPS*.

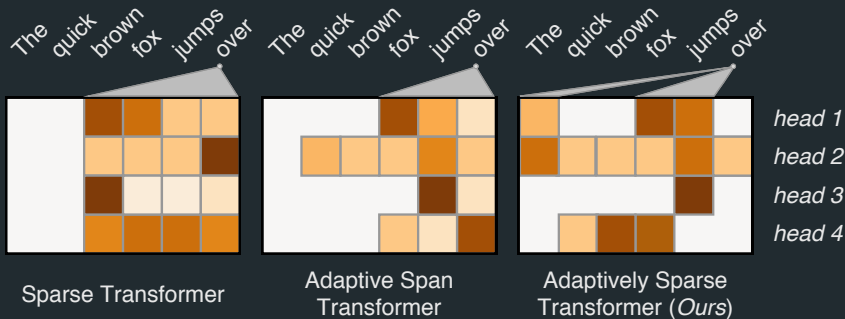


Voita, Elena, David Talbot, Fedor Moiseev, Rico Sennrich, and Ivan Titov (2019). "Analyzing multi-head self-attention: Specialized heads do the heavy lifting, the rest can be pruned". In: *Proc. ACL*.

# Parameter sharing analysis

	TER↓	BLEU↑
MT Baseline	24.76	62.11
Transformer	27.80	60.76
Transformer decoder	20.33	69.31
Pre-trained BERT	20.83	69.11
<i>with</i> CA ← SA	18.91	71.81
<i>and</i> SA ↔ Encoder SA	<b>18.44</b>	<b>72.25</b>
<i>and</i> CA ↔ SA	18.75	71.83
<i>and</i> FF ↔ Encoder FF	19.04	71.53

# Related Work: Other Sparse Transformers



Our model allows **non-contiguous** attention for each head.

# $\Omega$ -Regularized Argmax

For convex  $\Omega$ , define the  $\Omega$ -regularized argmax transformation:

$$\mathbf{argmax}_{\Omega}(\mathbf{z}) := \arg \max_{\mathbf{p} \in \Delta} \mathbf{z}^T \mathbf{p} - \Omega(\mathbf{p})$$



# $\Omega$ -Regularized Argmax

For convex  $\Omega$ , define the  $\Omega$ -regularized argmax transformation:

$$\text{argmax}_{\Omega}(\mathbf{z}) := \arg \max_{\mathbf{p} \in \Delta} \mathbf{z}^T \mathbf{p} - \Omega(\mathbf{p})$$

- **Argmax** corresponds to no regularization,  $\Omega \equiv 0$

# $\Omega$ -Regularized Argmax

For convex  $\Omega$ , define the  $\Omega$ -regularized argmax transformation:

$$\text{argmax}_{\Omega}(\mathbf{z}) := \arg \max_{\mathbf{p} \in \Delta} \mathbf{z}^T \mathbf{p} - \Omega(\mathbf{p})$$

- **Argmax** corresponds to no regularization,  $\Omega \equiv 0$
- **Softmax** amounts to entropic regularization,  $\Omega(\mathbf{p}) = \sum_{i=1}^K p_i \log p_i$

# $\Omega$ -Regularized Argmax

For convex  $\Omega$ , define the  $\Omega$ -regularized argmax transformation:

$$\text{argmax}_{\Omega}(\mathbf{z}) := \arg \max_{\mathbf{p} \in \Delta} \mathbf{z}^T \mathbf{p} - \Omega(\mathbf{p})$$

- **Argmax** corresponds to no regularization,  $\Omega \equiv 0$
- **Softmax** amounts to entropic regularization,  $\Omega(\mathbf{p}) = \sum_{i=1}^K p_i \log p_i$
- **Sparsemax** amounts to  $\ell_2$ -regularization,  $\Omega(\mathbf{p}) = \frac{1}{2} \|\mathbf{p}\|^2$ .

# $\Omega$ -Regularized Argmax

For convex  $\Omega$ , define the  $\Omega$ -regularized argmax transformation:

$$\text{argmax}_{\Omega}(\mathbf{z}) := \arg \max_{\mathbf{p} \in \Delta} \mathbf{z}^T \mathbf{p} - \Omega(\mathbf{p})$$

- **Argmax** corresponds to **no regularization**,  $\Omega \equiv 0$
- **Softmax** amounts to **entropic regularization**,  $\Omega(\mathbf{p}) = \sum_{i=1}^K p_i \log p_i$
- **Sparsemax** amounts to  $\ell_2$ -regularization,  $\Omega(\mathbf{p}) = \frac{1}{2} \|\mathbf{p}\|^2$ .

Is there something in-between?

# BLEU Scores

activation	de→en	ja→en	ro→en	en→de
softmax	29.79	21.57	32.70	26.02
1.5-entmax	29.83	22.13	33.10	25.89
$\alpha$ -entmax	29.90	21.74	32.89	26.93

# BLEU Scores

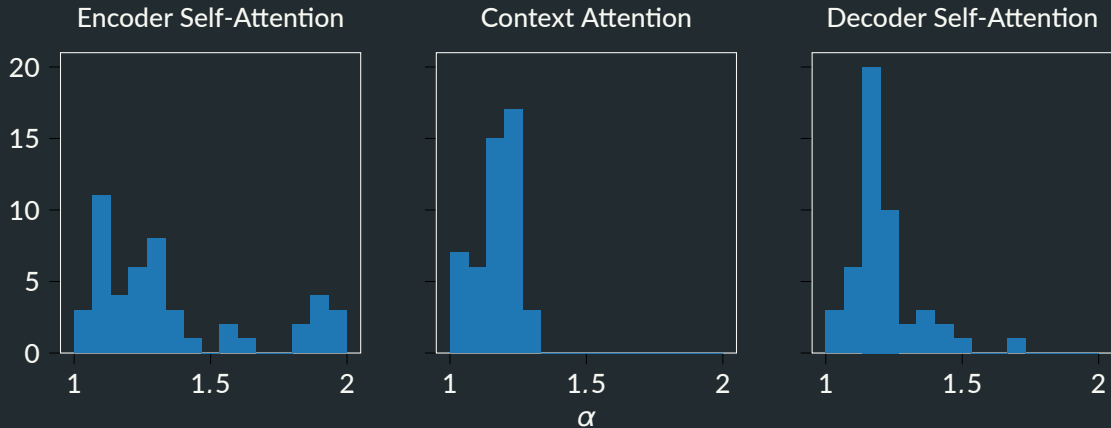
activation	de→en	ja→en	ro→en	en→de
softmax	29.79	21.57	32.70	26.02
1.5-entmax	29.83	22.13	33.10	25.89
$\alpha$ -entmax	29.90	21.74	32.89	26.93

## BLEU Scores

activation	de→en	ja→en	ro→en	en→de
softmax	29.79	21.57	32.70	26.02
1.5-entmax	29.83	22.13	33.10	25.89
$\alpha$ -entmax	29.90	21.74	32.89	26.93

For analysis for other language pairs, see Appendix A.

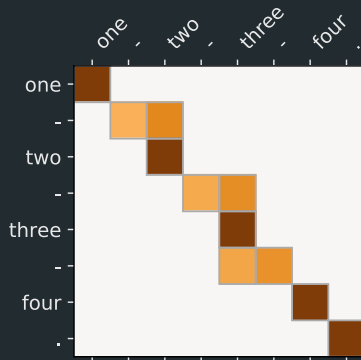
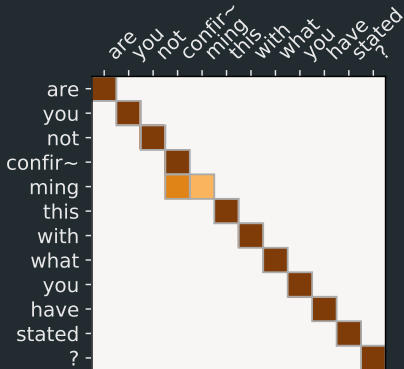
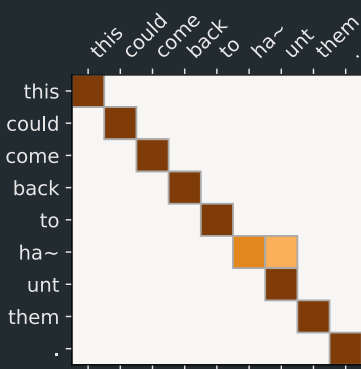
# Learned $\alpha$



Bimodal for the encoder, mostly unimodal for the decoder.



# Subword-Merging Head



Learned  $\alpha = 1.91$ .

# Semi-Supervised VAE

$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

- Semi-Supervised VAE on MNIST:  $z$  is one of 10 categories

# Semi-Supervised VAE

$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

Gaussian VAE

classification network

The diagram illustrates the components of the Semi-Supervised VAE loss function. The first line of the equation,  $\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z)$ , shows the loss as a sum over all possible latent classes  $z$ . The term  $\pi(z|x)$  is highlighted in red, and an arrow from the text 'Gaussian VAE' points to it, indicating it represents the latent distribution. The term  $\ell(x, z)$  is highlighted in yellow, and an arrow from the text 'classification network' points to it, indicating it represents the loss from the classification network. The second line,  $= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)$ , shows the loss as an expectation over the latent distribution, with the same red and yellow highlighting and arrows.

- Semi-Supervised VAE on MNIST:  $z$  is one of 10 categories

# Semi-Supervised VAE

sum over the 10 digits

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z)$$

Gaussian VAE

classification network

$$= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)$$

The diagram illustrates the components of the Semi-Supervised VAE loss function. A curved arrow points from the text 'sum over the 10 digits' to the summation symbol in the equation. Another curved arrow points from the text 'Gaussian VAE' to the  $\pi(z|x)$  term. A third curved arrow points from the text 'classification network' to the  $\ell(x, z)$  term. A fourth curved arrow points from the text 'classification network' to the expectation operator  $\mathbb{E}_{z \sim \pi(z|x)}$  in the second equation.

- Semi-Supervised VAE on MNIST:  $z$  is one of 10 categories

# Semi-Supervised VAE

sum over the 10 digits

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z)$$

Gaussian VAE

classification network

$$= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)$$

The diagram illustrates the components of the Semi-Supervised VAE loss function. The loss  $\mathcal{L}_x(\theta)$  is calculated by summing over all possible digit classes  $z$  in the set  $\mathcal{Z}$ . The probability  $\pi(z|x)$  is derived from a Gaussian VAE, and the loss  $\ell(x, z)$  is provided by a classification network. The two forms of the equation are mathematically equivalent.

- Semi-Supervised VAE on MNIST:  $z$  is one of 10 categories
- Train this with 10% labeled data

# Semi-Supervised VAE

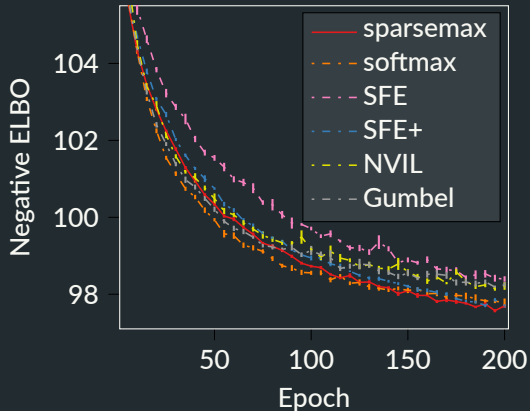
Method	Accuracy (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	94.75 $\pm$ .002	1
SFE+	96.53 $\pm$ .001	2
NVIL	96.01 $\pm$ .002	1
Gumbel	95.46 $\pm$ .001	1
<i>Marginalization</i>		
Dense	96.93 $\pm$ .001	10

# Semi-Supervised VAE

Method	Accuracy (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	94.75 $\pm$ .002	1
SFE+	96.53 $\pm$ .001	2
NVIL	96.01 $\pm$ .002	1
Gumbel	95.46 $\pm$ .001	1
<i>Marginalization</i>		
Dense	96.93 $\pm$ .001	10
<b>Sparse</b>	96.87 $\pm$ .001	1.01 $\pm$ 0.01

# Semi-Supervised VAE

Method	Accuracy (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	94.75 $\pm$ .002	1
SFE+	96.53 $\pm$ .001	2
NVIL	96.01 $\pm$ .002	1
Gumbel	95.46 $\pm$ .001	1
<i>Marginalization</i>		
Dense	96.93 $\pm$ .001	10
<b>Sparse</b>	96.87 $\pm$ .001	1.01 $\pm$ 0.01

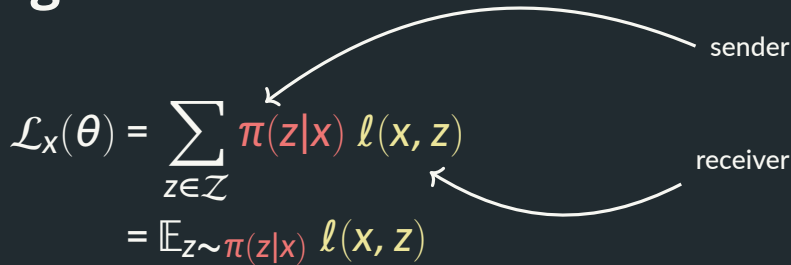




# Emergent communication

$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

# Emergent communication


$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

sender

receiver

- receiver picks image from a set  $\mathcal{V}$  based on message

# Emergent communication

sum over  
all possible messages  
in the vocabulary

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z)$$

=  $\mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)$

sender

receiver

- receiver picks image from a set  $\mathcal{V}$  based on message
- images come from ImageNet

# Emergent Communication

... but make it harder:  $|\mathcal{Z}| = 256$ ,  $|\mathcal{V}| = 16$

Method	success (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
NVIL	37.04 $\pm 1.61$	1
Gumbel	23.51 $\pm 16.19$	1
ST Gumbel	27.42 $\pm 13.36$	1
<i>Marginalization</i>		

# Emergent Communication

... but make it harder:  $|\mathcal{Z}| = 256$ ,  $|\mathcal{V}| = 16$

Method	success (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
NVIL	37.04 $\pm 1.61$	1
Gumbel	23.51 $\pm 16.19$	1
ST Gumbel	27.42 $\pm 13.36$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256

# Emergent Communication

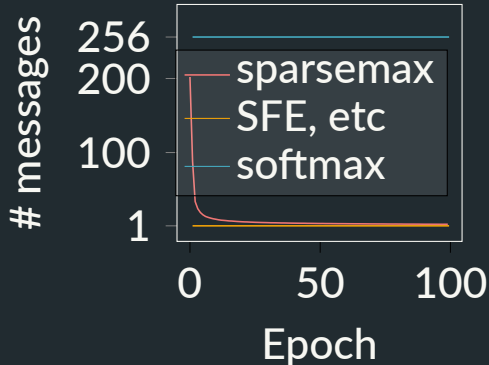
... but make it harder:  $|\mathcal{Z}| = 256$ ,  $|\mathcal{V}| = 16$

Method	success (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
NVIL	37.04 $\pm 1.61$	1
Gumbel	23.51 $\pm 16.19$	1
ST Gumbel	27.42 $\pm 13.36$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256
<b>Sparse</b>	93.35 $\pm 0.50$	3.13 $\pm 0.48$

# Emergent Communication

... but make it harder:  $|\mathcal{Z}| = 256$ ,  $|\mathcal{V}| = 16$

Method	success (%)	Dec. calls
<i>Monte Carlo</i>		
SFE	33.05 $\pm 2.84$	1
SFE+	44.32 $\pm 2.72$	2
NVIL	37.04 $\pm 1.61$	1
Gumbel	23.51 $\pm 16.19$	1
ST Gumbel	27.42 $\pm 13.36$	1
<i>Marginalization</i>		
Dense	93.37 $\pm 0.42$	256
<b>Sparse</b>	93.35 $\pm 0.50$	3.13 $\pm 0.48$



# Limitations

- Mostly (and eventually) very sparse.  
But fully dense worst case.
- For the same reason, sparsemax cannot handle structured  $z$ .



# Limitations

- Mostly (and eventually) very sparse.  
But fully dense worst case.
- For the same reason, sparsemax cannot handle structured  $z$ .

One solution: **top-k sparsemax**

$$k\text{-sparsemax}(s) = \arg \min_{p \in \Delta, \|p\|_0 \leq k} \|p - s\|_2^2$$

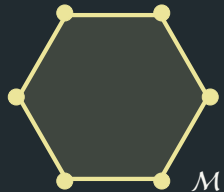
# Limitations

- Mostly (and eventually) very sparse.  
But fully dense worst case.
- For the same reason, sparsemax cannot handle structured  $z$ .

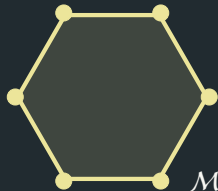
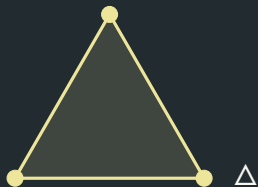
One solution: **top-k sparsemax**

$$k\text{-sparsemax}(s) = \arg \min_{p \in \Delta, \|p\|_0 \leq k} \|p - s\|_2^2$$

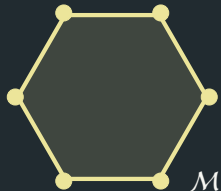
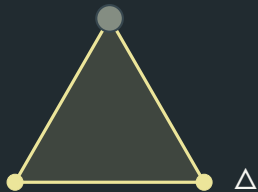
- Non-convex but easy: sparsemax over the  $k$  highest scores (Kyrillidis et al., 2013).
- Top-k oracle available for some structured problems.
- Certificate: if at least one of the top-k  $z$  gets  $p(z) = 0$ , **k-sparsemax = sparsemax!**  
thus, biased early on, but it goes away.



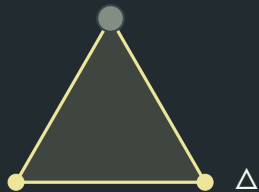
$$\begin{aligned}\mathcal{M} &:= \text{conv} \{ \mathbf{a}_z : z \in \mathcal{Z} \} \\ &= \{ \mathbf{A} \mathbf{p} : \mathbf{p} \in \Delta \} \\ &= \{ \mathbb{E}_{Z \sim p} \mathbf{a}_Z : \mathbf{p} \in \Delta \}\end{aligned}$$



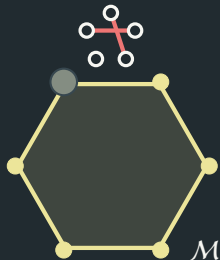
- $\operatorname{argmax}_{p \in \Delta} p^T s$



●  $\mathbf{argmax}_{p \in \Delta} \mathbf{p}^T \mathbf{s}$



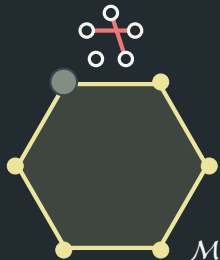
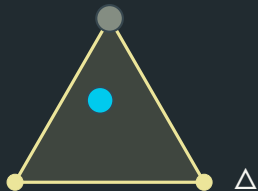
●  $\mathbf{MAP} \mathbf{argmax}_{\mu \in \mathcal{M}} \boldsymbol{\mu}^T \mathbf{t}$



- **argmax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^\top \mathbf{s}$

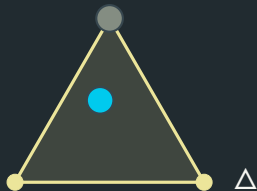
- **softmax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^\top \mathbf{s} + H(\mathbf{p})$

- **MAP**  $\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^\top \mathbf{t}$



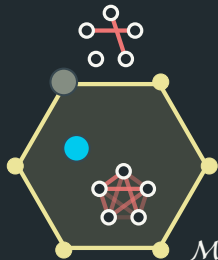
- **argmax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^T \mathbf{s}$

- **softmax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^T \mathbf{s} + H(\mathbf{p})$



- **MAP**  $\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^T \mathbf{t}$

- **marginals**  $\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^T \mathbf{t} + \tilde{H}(\boldsymbol{\mu})$

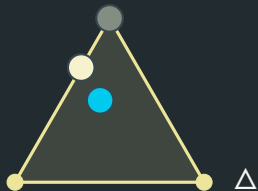




- **argmax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^T \mathbf{s}$

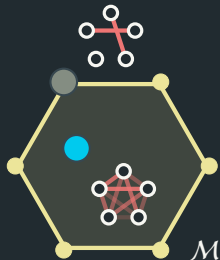
- **softmax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^T \mathbf{s} + H(\mathbf{p})$

- **sparsemax**  $\arg \max_{\mathbf{p} \in \Delta} \mathbf{p}^T \mathbf{s} - 1/2 \|\mathbf{p}\|^2$

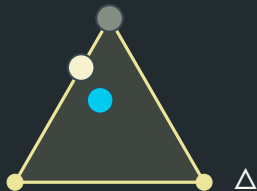


- **MAP**  $\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^T \mathbf{t}$

- **marginals**  $\arg \max_{\boldsymbol{\mu} \in \mathcal{M}} \boldsymbol{\mu}^T \mathbf{t} + \tilde{H}(\boldsymbol{\mu})$



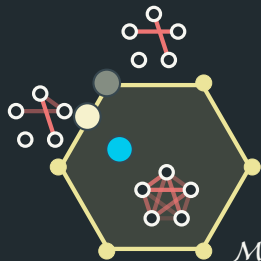
- **argmax**  $\arg \max_{p \in \Delta} p^T s$
- **softmax**  $\arg \max_{p \in \Delta} p^T s + H(p)$
- **sparsemax**  $\arg \max_{p \in \Delta} p^T s - 1/2 \|p\|^2$



● **MAP**  $\arg \max_{\mu \in \mathcal{M}} \mu^T t$

● **marginals**  $\arg \max_{\mu \in \mathcal{M}} \mu^T t + \tilde{H}(\mu)$

● **SparseMAP**  $\arg \max_{\mu \in \mathcal{M}} \mu^T t - 1/2 \|\mu\|^2$



# Bit-vector VAE

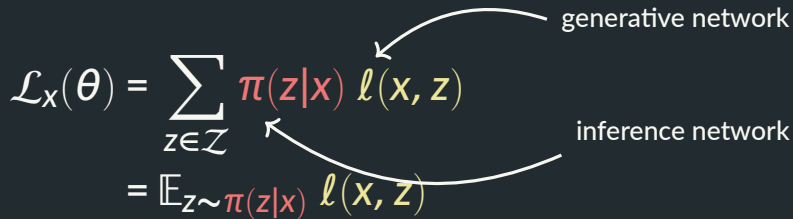
$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

# Bit-vector VAE

$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

generative network

inference network



- VAE where  $z$  is a collection of  $D$  bits

# Bit-vector VAE

$$\begin{aligned}\mathcal{L}_x(\theta) &= \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z) \\ &= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)\end{aligned}$$

generative network

inference network

- VAE where  $z$  is a collection of  $D$  bits
- Minimize the negative ELBO

# Bit-vector VAE

sum over  
an exponentially large  
set of structures

$$\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z)$$

generative network

inference network

$$= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)$$

The diagram illustrates the components of the Bit-vector VAE loss function. The first equation,  $\mathcal{L}_x(\theta) = \sum_{z \in \mathcal{Z}} \pi(z|x) \ell(x, z)$ , shows a sum over an exponentially large set of structures  $\mathcal{Z}$ . The second equation,  $= \mathbb{E}_{z \sim \pi(z|x)} \ell(x, z)$ , shows the same loss as an expectation over the generative network  $\pi(z|x)$ . The inference network is shown as the process of sampling  $z$  from the generative network to compute the loss.

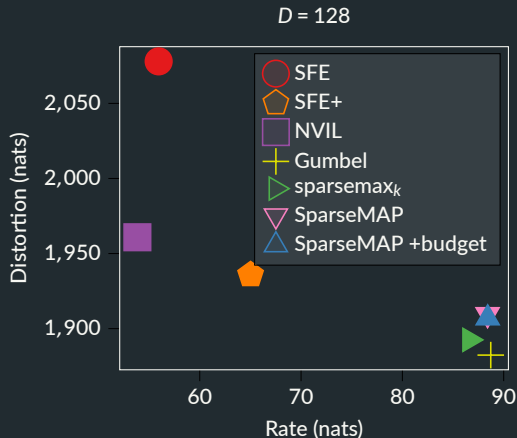
- VAE where  $z$  is a collection of  $D$  bits
- Minimize the negative ELBO

# Bit-vector VAE

Method	$D = 32$	$D = 128$
<i>Monte Carlo</i>		
SFE	3.74	3.77
SFE+	3.61	3.59
NVIL	3.65	3.60
Gumbel	3.57	3.49
<i>Marginalization</i>		
Top-k sparsemax	3.62	3.61
SparseMAP	3.72	3.67
SparseMAP (w/ budget)	3.64	3.66

# Bit-vector VAE

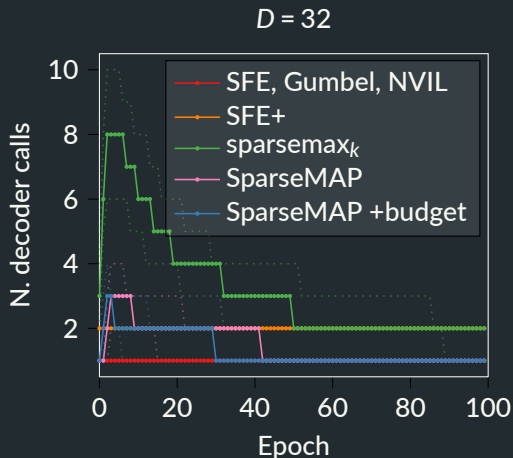
Method	$D = 32$	$D = 128$
<i>Monte Carlo</i>		
SFE	3.74	3.77
SFE+	3.61	3.59
NVIL	3.65	3.60
Gumbel	3.57	3.49
<i>Marginalization</i>		
Top-k sparsemax	3.62	3.61
SparseMAP	3.72	3.67
SparseMAP (w/ budget)	3.64	3.66





# Bit-vector VAE

Method	$D = 32$	$D = 128$
<i>Monte Carlo</i>		
SFE	3.74	3.77
SFE+	3.61	3.59
NVIL	3.65	3.60
Gumbel	3.57	3.49
<i>Marginalization</i>		
Top-k sparsemax	3.62	3.61
SparseMAP	3.72	3.67
SparseMAP (w/ budget)	3.64	3.66



# Bit-vector VAE

Method	$D = 32$	$D = 128$
<i>Monte Carlo</i>		
SFE	3.74	3.77
SFE+	3.61	3.59
NVIL	3.65	3.60
Gumbel	3.57	3.49
<i>Marginalization</i>		
Top-k sparsemax	3.62	3.61
SparseMAP	3.72	3.67
SparseMAP (w/ budget)	3.64	3.66

