Distributed Illumination Control With Local Sensing and Actuation in Networked Lighting Systems

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***Abstract*— We consider the problem of illumination control in a networked lighting system wherein luminaires have local sensing and actuation capabilities. Each luminaire: 1) consists of a light-emitting diode (LED) based light source dimmable by a local controller; 2) is actuated based on sensing information from a presence sensor, that determines occupant presence, and a light sensor, that measures illuminance, within their respective fields of view; and 3) a communication module to exchange control information within a local neighborhood. We consider distributed illumination control in such an intelligent lighting system to achieve presence-adaptive and daylight-integrated spatial illumination rendering. The rendering is specified as target values at the light sensors, and under these constraints, a local controller has to determine the optimum dimming levels of its associated LED luminaire so that the power consumed in rendering is minimized. The formulated optimization problem is a distributed linear programming problem with constraints on exchanging control information within a neighborhood. A distributed optimization algorithm is presented to solve this problem and its stability and convergence are studied. Sufficient conditions, in terms of parameter selection, under which the algorithm can achieve a feasible solution are provided. The performance of the algorithm is evaluated in an indoor office setting in terms of achieved illuminance rendering and power savings.**

***Index Terms*— Daylight and occupancy adaptive lighting, distributed lighting systems, distributed sensing and light actuation, lighting control.**

I. Introduction

ADVANCES in semiconductors have brought in a new generation of light sources in the form of light emitting diodes (LEDs). Minitiarization and rapid cost-downs from semiconductorization has also made it possible for greater integration of sensing, communication, computation and control functions into luminaires. This has made intelligent LED luminaires feasible, that (i) are dimmable by an associated local controller, (ii) can be actuated based on local sensing inputs such as presence detection and light intensity measurement within the sensor field of view, and (iii) can exchange control information within a local neighborhood using a communication module. We address the problem of illumination control in a system of such intelligent luminaires to achieve energy-efficient illumination rendering over the workspace.

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We consider illumination control for indoor office general lighting applications. Office lighting is one of the major constituents of electrical energy consumption in buildings [1]. As such, energy-efficient strategies for illumination rendering in lighting systems are of interest. We consider presence-adaptive, daylight-integrated illumination rendering in this paper. In particular, we want to achieve uniform illumination at an average illuminance of L^o_{WP} in an occupied zone and a lower average illuminance of L^u_{WP} otherwise over the workspace plane. Local occupancy in a zone is determined by a presence sensor. In office lighting, the illumination distribution achieved at the workspace plane, typically corresponding to desk level, is of most interest. This is a horizontal plane at a certain vertical height from the ceiling plane, in which the luminaires are located. Illuminance is however measured in practice using light sensors placed in the ceiling. As such, light measurement is in a plane different from the one where the spatial illumination rendering is desired. Also, a light sensor measures a spatial average of the light distribution in its field of view. A light sensor calibration step is thus employed to obtain target sensor values. These target light sensor illuminance values then specify the rendering constraints to be satisfied under distributed control. The specifics of the lighting system are described in Section II.

For a given realization of occupancy and daylight distribution in the office space, the local controller needs to determine the dimming level of its associated luminaire using local presence and light sensor measurements and communication with controllers in its neighborhood. This problem is mathematically a distributed linear programming problem with constraints on information exchange, and is formulated in Section III. We present a distributed optimization algorithm to obtain a suboptimum solution to this problem, and is described in Section IV. A bound is further obtained on the deviation of the resulting power consumption from that obtained under optimum dimming. In Section V, we analyze stability and convergence of the proposed algorithm. Sufficient conditions are provided for specifying the neighborhood for exchanging control information such that the algorithm can achieve the target sensor values, i.e. a feasible solution is obtained.

We evaluate the performance of the distributed illumination control algorithm with an example open office lighting simulation. The performance is evaluated based on the achieved illumination rendering and power savings. A comparison is made with an optimum centralized control algorithm. These numerical results are presented in Section VI.

Different lighting control approaches exist in literature [2]–[12], depending on the system architecture, connectivity

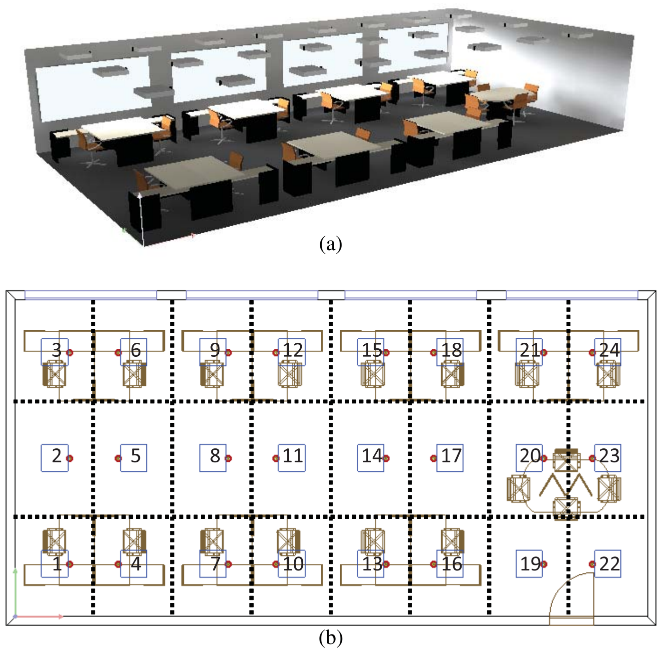

Fig. 1. (a) Lighting system in an example office room. (b) Depiction of logical zones in the workspace plane (blue rectangles depict luminaires and red circles depict light sensors).

and optimization algorithms employed. A numerical optimization approach to energy-efficient central control of polychromatic solid state lighting systems was presented in [2]. Under a centralized control system, a simplex algorithm was used in [3] to solve the resulting optimization problem for achieving illumination rendering adapted to occupancy, with [5] extending the control system to take daylight variations into account. In [6], user preferences for lighting conditions were additionally accounted for, assuming availability of rich light sensing information at the workspace plane, and centralized and distributed simplex algorithms were presented. The distributed simplex algorithm in [6] however requires information exchange between all controllers in the system. In [9], a heuristic distributed algorithm was proposed, again assuming availability of rich light sensing information at the workspace plane. Under different system considerations, linear programming and sequential quadratic programming approaches were used for centralized lighting control [11], [12]. A sensing solution to disaggregate daylight and artificial light was proposed in [13], which may be used to simplify daylight-adaptive lighting control.

II. System Model

Consider a lighting system in an indoor office with M LED luminaires arranged in the ceiling, an example illustration shown in Fig. 1(a) with $M = 24$. Parallel to the ceiling is the workspace plane, a plane over which spatial illumination rendering is of interest. The workspace plane is assumed to be divided into M logical zones, as shown in the example of Fig. 1(b). Each luminaire contains a local controller, an LED light source, a light sensor, a presence sensor and a communication module. Typically, a photodiode or a light dependent resistor may be used as a light sensor. Common modalities for presence sensing are passive infrared and ultrasound. We shall assume a simplified presence and light sensor model. We assume a light sensor with a well-defined sensing region defined by its opening angle. We assume that the presence sensing region is constrained to a zone, and that there are no missed detections or false alarms. In practice, the sensing region may extend further and an occupant outside the zone might be detected by the presence sensor. Furthermore, a missed detection over a zone would lead to specification of a lower average illuminance value to be achieved, while a false alarm would lead to higher average illuminance requirement. As a result, in the former case, the rendered illumination upon lighting control might lead to occupant dissatisfaction and in the latter case additional power consumption.

The local controller determines the dimming level of the associated LED luminaire, such that the resulting illumination rendered from the lighting system achieves target illuminance values at the light sensors. The target illuminance at the light sensor is the mapped value of the required average illuminance in the corresponding zone in the workspace plane, and is done in a dark room calibration step. Denote the target illuminance level at the m-th light sensor when the m-th zone is occupied (respectively, unoccupied) as L_m^o (respectively, L_m^u). We assume that after calibration, the measured illuminance at the m-th light sensor when the n-th luminaire is at maximum intensity is proportional to the average illuminance over the corresponding zone at the workspace plane.

We consider a distributed control scenario for illumination rendering with only neighborhood communication. Communication can be achieved using IEEE 802.15.4 radios [14] or visible light communications IEEE 802.15.7 [15]. Each controller only knows dimming level information of its own luminaire and that from its closest neighbors. Light sensing and presence information in a zone is only known locally to the corresponding controller. The local controllers also know the global occupancy/vacancy state in the office space so that the corresponding target average illuminance values in their associated zones may be set. Based on sensing information and coordination with neighboring controllers, each controller determines the dimming level of its luminaire such that the total power consumption is minimized while achieving the illumination rendering constraints as specified at the light sensors. We assume that each controller can communicate reliably with neighboring controllers.

A. Dimming of Light Sources

We assume that the LED light source is dimmed using pulse width modulation [16]. Let d_n, where $0 \leq d_n \leq 1$, be the dimming level of the n-th luminaire. The value $d_n = 0$ means that the LED luminaire is dimmed off completely, while $d_n = 1$ represents that the luminaire is at its maximum luminance level. Let $\boldsymbol{d}$ be the $M \times 1$ dimming vector for the luminaires of the lighting system given by

$$\boldsymbol{d} = \left[d_1, d_2, \ldots, d_M\right]^T,$$

indicating that the n-th LED luminaire is at dimming level d_n.

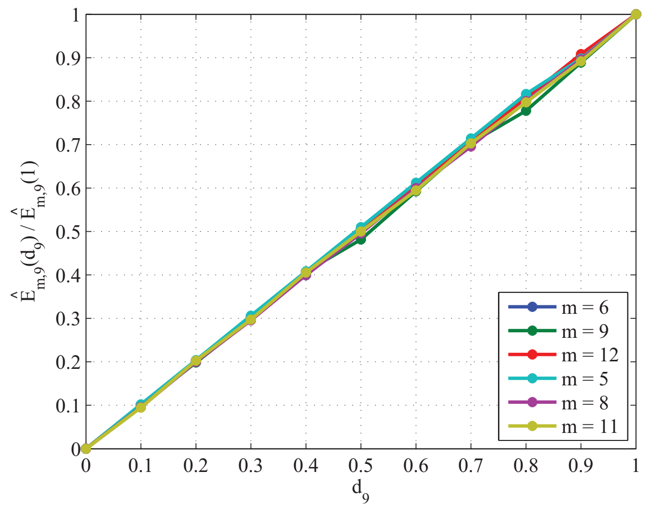

Fig. 2. Linearity of light sensor illuminance values with respect to the dimming level of luminaire 9.

B. Illuminance at Light Sensor

The measured illuminance at the light sensor at the ceiling is the combined illuminance contribution due to the luminaires and daylight reflected from the objects (e.g. furniture) in the office. Denote $\hat{E}_{m,n}(d_n)$ to be the measured illuminance at the m-th light sensor when the n-th luminaire is at dimming level d_n, in the absence of daylight and when the other luminaires are turned off. We assume that the illuminance scales linearly with the dimming level, i.e.

$$\hat{E}_{m,n}(d_n) = d_n \hat{E}_{m,n}(1). \tag{1}$$

This assumption holds well for LED luminaires. In particular in Fig. 2, we plot $\hat{E}_{m,n}(d_n)/\hat{E}_{m,n}(1)$ versus d_n for luminaire 9 w.r.t. light sensors $5, 6, 8, 9, 11, 12$ for the office lighting system in Fig. 1 to illustrate that linearity holds. These illuminance values are obtained from photometric data from an implementation model of the lighting system in DIALux [17].

The total illuminance at the m-th sensor at the ceiling, given that the lighting system is at dimming vector $\boldsymbol{d}$ and under daylight, can then be written as

$$\bar{L}_m(\boldsymbol{d}) = \sum_{n=1}^{M} d_n \hat{E}_{m,n}(1) + \hat{O}_m,$$

where $\hat{O}_m$ is the illuminance due to daylight measured at the m-th sensor. In practice, the mappings $\hat{E}_{m,n}(\cdot)$ may be computed a priori in a configuration phase by turning on the luminaires one at a time and measuring illuminance values at the light sensors.

III. Distributed Illumination Control With Local Sensing and Actuation: Problem Formulation

Our objective is to minimize the power consumption of the lighting system subject to illuminance constraints at the light sensors and limited neighbor communication across local controllers. Formally, the global optimization problem may be posed as follows.

$$\boldsymbol{d}^{\star} = \arg\min_{\boldsymbol{d}} \sum_{m=1}^{M} d_m$$
$$\text{s.t.} \begin{cases} \sum_{n=1}^{M} d_n \hat{E}_{m,n}(1) + \hat{O}_m \geq \hat{L}_m, \\ \qquad m = 1, \ldots, M, \\ 0 \leq d_m \leq 1, \quad m = 1, \ldots, M, \\ \{m\} \rightarrow \mathcal{N}_m, \quad m = 1, \ldots, M. \end{cases} \tag{2}$$

In the above,

$$\hat{L}_m = \begin{cases} L_m^o, & \text{if the } m\text{-th zone is occupied} \\ L_m^u, & \text{otherwise,} \end{cases}$$

is the target illuminance level at the m-th light sensor. In (2), the constraint $\{m\} \rightarrow \mathcal{N}_m$ means that the m-th controller can only communicate with controllers in a corresponding neighbor set, $\mathcal{N}_m$, where $\mathcal{N}_m$ is the set of indices of N_m neighbors from the m-th controller,

$$\mathcal{N}_m = \left[\nu_{m,1}, \nu_{m,2}, \ldots, \nu_{m,N_m}\right].$$

For simplicity, we assume reciprocity, i.e. if controller m is in the neighbor set of n, then controller n is also in the neighbor set of m. A local controller m has information on local occupancy in its associated zone and thus knows the appropriate target illuminance level $\hat{L}_m$, it has illuminance mappings $\{\hat{E}_{m,n}(1)\}$, $n \in \mathcal{N}_m \cup \{m\}$, and has the current measured illuminance value $\bar{L}_m$. Based on this information and exchange of control information, the local controllers seek a solution to (2).

Note that the objective function and the first two sets of constraints in (2) are linear in the variables d_m. Upon introduction of slack variables $\{s_m\}$ and $\{t_m\}$ to write (2) in standard form [18], the problem may be rewritten in matrix form as

$$\boldsymbol{d}^{\star} = \arg\min_{\boldsymbol{d},\boldsymbol{s},\boldsymbol{t}} \mathbf{1}^T \boldsymbol{d}$$
$$\text{s.t.} \begin{cases} \hat{\boldsymbol{E}}\boldsymbol{d} - \boldsymbol{s} = \hat{\boldsymbol{L}} - \hat{\boldsymbol{O}} \\ \boldsymbol{d} + \boldsymbol{t} = \mathbf{1} \\ d_m \geq 0, \quad m = 1, \ldots, M, \\ s_m \geq 0, \quad m = 1, \ldots, M, \\ t_m \geq 0, \quad m = 1, \ldots, M, \\ \{m\} \rightarrow \mathcal{N}_m \quad m = 1, \ldots, M, \end{cases} \tag{3}$$

where

$$\hat{\boldsymbol{E}} = \begin{bmatrix} \hat{E}_{1,1}(1) & \hat{E}_{1,2}(1) & \ldots & \hat{E}_{1,M}(1) \\ \hat{E}_{2,1}(1) & \hat{E}_{2,2}(1) & \ldots & \hat{E}_{2,M}(1) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{E}_{M,1}(1) & \hat{E}_{M,2}(1) & \ldots & \hat{E}_{M,M}(1) \end{bmatrix},$$

$$\hat{\boldsymbol{L}} = \left[\hat{L}_1, \hat{L}_2, \ldots, \hat{L}_M\right]^T,$$

$$\hat{\boldsymbol{O}} = \left[\hat{O}_1, \hat{O}_2, \ldots, \hat{O}_M\right]^T,$$

$$\boldsymbol{s} = \left[s_1, s_2, \ldots, s_M\right]^T,$$

$$\boldsymbol{t} = \left[t_1, t_2, \ldots, t_M\right]^T,$$

and $\mathbf{1}$ is the vector of 1s of size $M \times 1$. Also, $\hat{\boldsymbol{E}}$ is the illuminance transfer matrix relating the effect of the luminaires at the light sensors.

If we do not consider the neighbor communication constraints, problem (3) is a linear programming problem. A solution to this problem can be found by solving the primal problem

$$P_{min} = \min_{\boldsymbol{d},\boldsymbol{s},\boldsymbol{t}} \mathbf{1}^T \boldsymbol{d}$$
$$\text{s.t.} \begin{cases} \hat{\boldsymbol{E}}\boldsymbol{d} - \boldsymbol{s} = \hat{\boldsymbol{L}} - \hat{\boldsymbol{O}} \\ \boldsymbol{d} + \boldsymbol{t} = \mathbf{1} \\ d_m \geq 0, \quad m = 1, \ldots, M, \\ s_m \geq 0, \quad m = 1, \ldots, M, \\ t_m \geq 0, \quad m = 1, \ldots, M, \end{cases} \tag{4}$$

and the associated dual problem

$$D_{min} = \max_{\boldsymbol{u},\boldsymbol{v},\boldsymbol{w}} \left(\hat{\boldsymbol{L}} - \hat{\boldsymbol{O}}\right)^T \boldsymbol{u} - \boldsymbol{v}$$
$$\text{s.t.} \begin{cases} \hat{\boldsymbol{E}}^T \boldsymbol{u} - \boldsymbol{v} + \boldsymbol{w} = \mathbf{1} \\ u_m \geq 0, \quad m = 1, \ldots, M, \\ v_m \geq 0, \quad m = 1, \ldots, M, \\ w_m \geq 0, \quad m = 1, \ldots, M, \end{cases} \tag{5}$$

such that

$$\begin{aligned} d_m w_m &= 0, \quad m = 1, \ldots, M \\ s_m u_m &= 0, \quad m = 1, \ldots, M \\ t_m v_m &= 0, \quad m = 1, \ldots, M \end{aligned} \tag{6}$$

where

$$\begin{aligned} \boldsymbol{u} &= [u_1, u_2, \ldots, u_M]^T, \\ \boldsymbol{v} &= [v_1, v_2, \ldots, v_M]^T, \\ \boldsymbol{w} &= [w_1, w_2, \ldots, w_M]^T. \end{aligned}$$

For any feasible solution to the primal and dual problem that satisfies conditions in (6), the following condition holds [19, Chapter 5]

$$\left(\hat{\boldsymbol{L}} - \hat{\boldsymbol{O}}\right)^T \boldsymbol{u} - \boldsymbol{v} = \mathbf{1}^T \boldsymbol{d}. \tag{7}$$

Now, if we consider the neighbor communication constraint, then we need to limit all terms to the available information. It can be noted that we have all the required information for solving the primal problem in (4), given that

$$\begin{aligned} \hat{\boldsymbol{E}}\boldsymbol{d} + \boldsymbol{s} &= \hat{\boldsymbol{L}} - \hat{\boldsymbol{O}} \\ \hat{\boldsymbol{E}}\boldsymbol{d} + \hat{\boldsymbol{O}} + \boldsymbol{s} &= \hat{\boldsymbol{L}} \\ \bar{\boldsymbol{L}}(\boldsymbol{d}) + \boldsymbol{s} &= \hat{\boldsymbol{L}} \end{aligned}$$

where

$$\bar{\boldsymbol{L}}(\boldsymbol{d}) = \hat{\boldsymbol{E}}\boldsymbol{d} + \hat{\boldsymbol{O}}$$

and

$$\bar{\boldsymbol{L}}(\boldsymbol{d}) = \left[\bar{L}_1(\boldsymbol{d}) \ \bar{L}_2(\boldsymbol{d}) \ \cdots \ \bar{L}_M(\boldsymbol{d})\right]^T.$$

However, we do not have all the information to solve the dual problem in (5). Thus, the additional constraints implies a change into

$$D_{\max} = \max_{\tilde{\boldsymbol{u}},\tilde{\boldsymbol{v}},\tilde{\boldsymbol{w}}} \left(\hat{\boldsymbol{L}} - \hat{\boldsymbol{O}}\right)^T \tilde{\boldsymbol{u}} - \tilde{\boldsymbol{v}}$$
$$\text{s.t.} \begin{cases} \check{\boldsymbol{E}}^T \tilde{\boldsymbol{u}} - \tilde{\boldsymbol{v}} + \tilde{\boldsymbol{w}} = \mathbf{1} \\ \tilde{u}_m \geq 0, \quad m = 1, \ldots, M, \\ \tilde{v}_m \geq 0, \quad m = 1, \ldots, M, \\ \tilde{w}_m \geq 0, \quad m = 1, \ldots, M \end{cases} \tag{8}$$

where $\check{\boldsymbol{E}}$ is the matrix containing only neighbor information where each element is given by

$$\left[\check{\boldsymbol{E}}\right]_{m,n} = \begin{cases} \hat{E}_{m,n}, & \text{if } n \in \mathcal{N}_m \text{ or } m = n \\ 0, & \text{otherwise.} \end{cases}$$

The primal problem associated to the dual problem in (8) is given by

$$P_{max} = \min_{\tilde{\boldsymbol{d}},\tilde{\boldsymbol{s}},\tilde{\boldsymbol{t}}} \mathbf{1}^T \tilde{\boldsymbol{d}}$$
$$\text{s.t.} \begin{cases} \check{\boldsymbol{E}}\tilde{\boldsymbol{d}} - \tilde{\boldsymbol{s}} = \hat{\boldsymbol{L}} - \hat{\boldsymbol{O}} \\ \tilde{\boldsymbol{d}} + \tilde{\boldsymbol{t}} = \mathbf{1} \\ \tilde{d}_m \geq 0, \quad m = 1, \ldots, M, \\ \tilde{s}_m \geq 0, \quad m = 1, \ldots, M, \\ \tilde{t}_m \geq 0, \quad m = 1, \ldots, M. \end{cases} \tag{9}$$

We propose to find a suboptimal solution to (3) by seeking a vector

$$\left[\boldsymbol{d}^{\star T} \ \boldsymbol{s}^{\star T} \ \boldsymbol{t}^{\star T} \ \tilde{\boldsymbol{u}}^{\star T} \ \boldsymbol{v}^{\star T} \ \tilde{\boldsymbol{w}}^{\star T}\right]^T \tag{10}$$

that is a feasible solution for the primal problem (4) and dual problem (8), i.e.

$$\begin{cases} \check{\boldsymbol{E}}^T \tilde{\boldsymbol{u}}^\star - \tilde{\boldsymbol{v}}^\star + \tilde{\boldsymbol{w}}^\star = \mathbf{1} \\ \hat{\boldsymbol{E}}\boldsymbol{d}^\star - \boldsymbol{s}^\star = \hat{\boldsymbol{L}} - \hat{\boldsymbol{O}} \\ \boldsymbol{d}^\star + \boldsymbol{t}^\star = \mathbf{1} \end{cases} \tag{11}$$

and all the variables $\{d_m^\star\}$, $\{s_m^\star\}$, $\{t_m^\star\}$, $\{\tilde{u}_m^\star\}$, $\{\tilde{v}_m^\star\}$ and $\{\tilde{w}_m^\star\}$ are non-negative.

Furthermore, this vector also satisfies the complementary conditions

$$d_m^\star \tilde{w}_m^\star = 0, \quad m = 1, \ldots, M \tag{12a}$$
$$s_m^\star \tilde{u}_m^\star = 0, \quad m = 1, \ldots, M \tag{12b}$$
$$t_m^\star \tilde{v}_m^\star = 0, \quad m = 1, \ldots, M. \tag{12c}$$

If the complementary conditions in (12b) are satisfied, then

$$\begin{aligned} \tilde{\boldsymbol{u}}^{\star T} \boldsymbol{s}^\star &= 0 \\ \tilde{\boldsymbol{u}}^{\star T}\left(\hat{\boldsymbol{E}}\boldsymbol{d}^\star - \hat{\boldsymbol{L}} + \hat{\boldsymbol{O}}\right) &= 0 \\ \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{E}}\boldsymbol{d}^\star - \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{L}} + \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{O}} &= 0 \\ \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{E}}\boldsymbol{d}^\star &= \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{L}} - \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{O}} \end{aligned}$$

and if (12a) and (12c) are satisfied, then

$$\begin{aligned} \tilde{\boldsymbol{w}}^{\star T} \boldsymbol{d}^\star &= 0 \\ \left(\mathbf{1} - \check{\boldsymbol{E}}^T \tilde{\boldsymbol{u}}^\star + \tilde{\boldsymbol{v}}^\star\right)^T \boldsymbol{d}^\star &= 0 \\ \mathbf{1}^T \boldsymbol{d}^\star - \tilde{\boldsymbol{u}}^{\star T}\check{\boldsymbol{E}}\boldsymbol{d}^\star + \tilde{\boldsymbol{v}}^{\star T}\boldsymbol{d}^\star &= 0 \\ \tilde{\boldsymbol{u}}^{\star T}\check{\boldsymbol{E}}\boldsymbol{d}^\star - \tilde{\boldsymbol{v}}^{\star T}\boldsymbol{d}^\star &= \mathbf{1}^T \boldsymbol{d}^\star \\ \tilde{\boldsymbol{u}}^{\star T}\check{\boldsymbol{E}}\boldsymbol{d}^\star - \tilde{\boldsymbol{v}}^{\star T}\left(\mathbf{1} - \boldsymbol{t}^\star\right) &= \mathbf{1}^T \boldsymbol{d}^\star. \end{aligned} \tag{13}$$

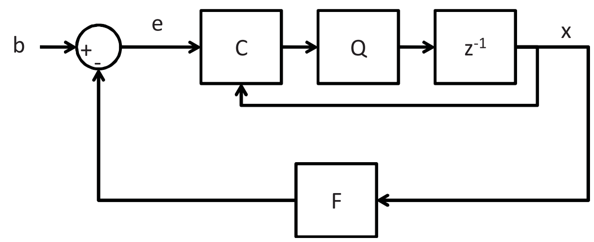

Fig. 3. Controller feedback loop.

Note that because $\{d_m^\star\}$ and $\{\tilde{u}_m^\star\}$ are non-negative, we have

$$\tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{E}}\boldsymbol{d}^\star \geq \tilde{\boldsymbol{u}}^{\star T}\check{\boldsymbol{E}}\boldsymbol{d}^\star$$

and so (13) becomes

$$\begin{aligned}\tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{E}}\boldsymbol{d}^\star - \tilde{\boldsymbol{v}}^{\star T}\left(\mathbf{1}-\boldsymbol{t}^\star\right) &\geq \mathbf{1}^T\boldsymbol{d}^\star\\ \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{L}} - \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{O}} - \tilde{\boldsymbol{v}}^{\star T}\mathbf{1} &\geq \mathbf{1}^T\boldsymbol{d}^\star\\ \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{L}} - \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{O}} - \tilde{\boldsymbol{v}}^{\star T}\mathbf{1} &\geq \mathbf{1}^T\boldsymbol{d}^\star.\end{aligned} \tag{14}$$

Using (4), (7)–(9),

$$\begin{aligned}P_{\max} = D_{\max} &\geq \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{L}} - \tilde{\boldsymbol{u}}^{\star T}\hat{\boldsymbol{O}} - \tilde{\boldsymbol{v}}^{\star T}\mathbf{1}\\ P_{\min} &\leq \mathbf{1}^T\boldsymbol{d}^\star\end{aligned}$$

and combining with (13), the suboptimal solution (10) to problem (3) is bounded as follows

$$P_{\min} \leq \mathbf{1}^T\boldsymbol{d}^\star \leq P_{\max}.$$

IV. Distributed Illumination Control Solution

We first provide a brief outline of the proposed control solution. The complementary conditions (12a)–(12c) are linearized around an initial point

$$\boldsymbol{x}^{(0)} = \left[\boldsymbol{d}^{(0)T}\ \boldsymbol{s}^{(0)T}\ \boldsymbol{t}^{(0)T}\ \tilde{\boldsymbol{w}}^{(0)T}\ \tilde{\boldsymbol{u}}^{(0)T}\ \tilde{\boldsymbol{v}}^{(0)T}\right]^T.$$

Then, we seek for an intermediate solution

$$\boldsymbol{x}^* = \left[\boldsymbol{d}^{*T}\ \boldsymbol{s}^{*T}\ \boldsymbol{t}^{*T}\ \tilde{\boldsymbol{w}}^{*T}\ \tilde{\boldsymbol{u}}^{*T}\ \tilde{\boldsymbol{v}}^{*T}\right]^T$$

that minimizes the violation of complementary conditions and that is in the neighborhood of $\boldsymbol{x}^{(0)}$ such that the system of equations in (11) are satisfied and all the variables are non-negative. The intermediate solution $\boldsymbol{x}^*$ is obtained by an iterative approach as detailed later in this section. Then, we repeat this procedure by linearizing the complementary conditions (12a) - (12c) around point $\boldsymbol{x}^*$. We repeat these steps till the difference between two consecutive solutions is within a predefined limit.

Let us model problem (3) as a control system as depicted in Fig. 3 with two loops. The inner loop ensures that the system of equations in (11) are satisfied and all variables are non-negative after convergence. Let us assume that the inner loop converges after K iterations (see Section V for convergence analysis). The outer loop moves further the solution after the inner loop has converged (i.e. iteration κK, $\kappa = 0, 1, \ldots$) towards satisfying complementary conditions in (12a) - (12c). In Fig. 3,

$$\begin{aligned}\boldsymbol{e}^{(k)} &= \boldsymbol{b}^{(k)} - \boldsymbol{F}\boldsymbol{x}^{(k-1)}\\ \boldsymbol{F} &= \begin{bmatrix}\mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{I} & \tilde{\boldsymbol{E}}^T & -\boldsymbol{I}\\ \boldsymbol{E} & -\boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0}\\ \boldsymbol{I} & \mathbf{0} & \boldsymbol{I} & \mathbf{0} & \mathbf{0} & \mathbf{0}\end{bmatrix},\\ \boldsymbol{x}^{(k)} &= \begin{bmatrix}\boldsymbol{d}^{(k)}\\ \boldsymbol{s}^{(k)}\\ \boldsymbol{t}^{(k)}\\ \tilde{\boldsymbol{w}}^{(k)}\\ \tilde{\boldsymbol{u}}^{(k)}\\ \tilde{\boldsymbol{v}}^{(k)}\end{bmatrix},\quad \boldsymbol{e}^{(k)} = \begin{bmatrix}\boldsymbol{e}_1^{(k)}\\ \boldsymbol{e}_2^{(k)}\\ \boldsymbol{e}_3^{(k)}\end{bmatrix},\\ \boldsymbol{b}^{(k)} &= \begin{bmatrix}\mathbf{1}\\ \boldsymbol{L}^{(k)} - \boldsymbol{O}^{(k)}\\ \mathbf{1}\end{bmatrix},\\ \boldsymbol{E} &= \boldsymbol{\Phi}\hat{\boldsymbol{E}},\qquad \tilde{\boldsymbol{E}} = \boldsymbol{\Phi}\check{\boldsymbol{E}},\\ \boldsymbol{L}^{(k)} &= \boldsymbol{\Phi}\hat{\boldsymbol{L}}^{(k)},\qquad \boldsymbol{O}^{(k)} = \boldsymbol{\Phi}\hat{\boldsymbol{O}}^{(k)},\\ \boldsymbol{\Phi} &= \operatorname{diag}\left\{\left[\phi_1,\ \phi_2,\ \cdots,\ \phi_M\right]\right\},\\ \boldsymbol{x}^{(k)} &= \left[x_1^{(k)},\ x_2^{(k)},\ \cdots,\ x_{4M}^{(k)}\right]^T,\\ \boldsymbol{e}_1^{(k)} &= \left[e_{1,1}^{(k)},\ e_{1,2}^{(k)},\ \cdots,\ e_{1,M}^{(k)}\right]^T,\\ \boldsymbol{e}_2^{(k)} &= \left[e_{2,1}^{(k)},\ e_{2,2}^{(k)},\ \cdots,\ e_{2,M}^{(k)}\right]^T,\\ \boldsymbol{e}_3^{(k)} &= \left[e_{3,1}^{(k)},\ e_{3,2}^{(k)},\ \cdots,\ e_{3,M}^{(k)}\right]^T,\end{aligned}$$

block $\boldsymbol{C}$ is the controller to be designed, z^{-1} is a unit delay and diag{˙} is a diagonal matrix with entries in the main diagonal equal to the corresponding entries of the vector in the argument. The vectors $\boldsymbol{L}^{(k)}$ and $\boldsymbol{O}^{(k)}$ are the normalized target illuminance and normalized daylight illuminance contribution at sensors during iteration k, respectively. Similarly, $\boldsymbol{E}$ and $\tilde{\boldsymbol{E}}$ are normalized versions of matrices $\hat{\boldsymbol{E}}$ and $\check{\boldsymbol{E}}$, respectively. Here, $\boldsymbol{\Phi}$ is a normalization matrix where

$$\phi_m = \frac{\xi_m}{\sum_{n\in\mathcal{N}_m}\hat{E}_{m,n}(1)}$$

and $\xi_m > 0$ is a design parameter. Also, we refer to the (m, n)-th element in matrix $\boldsymbol{E}$ and $\tilde{\boldsymbol{E}}$ as $E_{m,n}$ and $\tilde{E}_{m,n}$, respectively.

The operator $\boldsymbol{Q}$ ensures that all variables $\{s_m^{(k)}\}$, $\{t_m^{(k)}\}$, $\{\tilde{u}_m^{(k)}\}$, $\{\tilde{v}_m^{(k)}\}$ and $\{\tilde{w}_m^{(k)}\}$ are non-negative and

$$0 \leq d_m^{(k)} \leq 1, \quad m = 1, \ldots, M.$$

We design the inner loop such that we solve the subproblem,

$$\begin{aligned}\boldsymbol{x}^* &= \arg\min_{\boldsymbol{x}} \|(\boldsymbol{Y}^{(\kappa)})^T\boldsymbol{x}\|^2 + \|\boldsymbol{x} - \boldsymbol{x}^{(\kappa K)}\|^2\\ &\text{s.t.} \begin{cases}\boldsymbol{F}\boldsymbol{x} = \boldsymbol{b}\\ x_p \geq 0, \quad p = 1, 2, \ldots, 6M,\end{cases}\end{aligned} \tag{15}$$

where

$$\boldsymbol{Y}^{(\kappa)} = \tilde{\boldsymbol{Y}}^{(\kappa)} \left(\tilde{\boldsymbol{Y}}^{(\kappa)} (\tilde{\boldsymbol{Y}}^{(\kappa)})^T\right)^{-\frac{1}{2}}$$

$$\tilde{\boldsymbol{Y}}^{(\kappa)} = \begin{bmatrix} \operatorname{diag}\left\{\tilde{\boldsymbol{w}}^{(\kappa K)}\right\} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \operatorname{diag}\left\{\tilde{\boldsymbol{u}}^{(\kappa K)}\right\} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \operatorname{diag}\left\{\tilde{\boldsymbol{v}}^{(\kappa K)}\right\} \\ \operatorname{diag}\left\{\boldsymbol{d}^{(\kappa K)}\right\} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \operatorname{diag}\left\{\boldsymbol{s}^{(\kappa K)}\right\} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \operatorname{diag}\left\{\boldsymbol{t}^{(\kappa K)}\right\} \end{bmatrix},$$

and $(\boldsymbol{Y}^{(\kappa)})^T \boldsymbol{x}$ is a linearization of complementary conditions (12a) - (12c) around vector $\boldsymbol{x}^{(\kappa K)}$,

$$\begin{aligned} 2d_m \tilde{w}_m &= d_m^{(\kappa K)} \tilde{w}_m + d_m \tilde{w}_m^{(\kappa K)}, \quad m = 1, \ldots, M \\ 2s_m \tilde{u}_m &= s_m^{(\kappa K)} \tilde{u}_m + s_m \tilde{u}_m^{(\kappa K)}, \quad m = 1, \ldots, M \\ 2t_m \tilde{v}_m &= t_m^{(\kappa K)} \tilde{v}_m + t_m \tilde{v}_m^{(\kappa K)}, \quad m = 1, \ldots, M. \end{aligned}$$

In the outer loop, we update the matrix $\boldsymbol{Y}^{(\kappa)}$ at each iteration κK.

The problem in (15) is solved iteratively in the inner loop by

$$\boldsymbol{x}^{(\kappa K+1)} = \boldsymbol{Q}\left\{\boldsymbol{\beta}^{(\kappa)} \boldsymbol{x}^{(\kappa K)} + \alpha \boldsymbol{\beta}^{(\kappa)} \Delta \boldsymbol{x}^{(\kappa K)}\right\}$$

and

$$\boldsymbol{x}^{(\kappa K+\tilde{k})} = \boldsymbol{Q}\left\{\boldsymbol{x}^{(\kappa K+\tilde{k}-1)} + \alpha \boldsymbol{\beta}^{(\kappa)} \Delta \boldsymbol{x}^{(\kappa K+\tilde{k})}\right\} \quad \tilde{k} = 2, \ldots, K,$$

where

$$\boldsymbol{\beta}^{(\kappa)} = \left(\boldsymbol{Y}^{(\kappa)} (\boldsymbol{Y}^{(\kappa)})^T + \boldsymbol{I}\right)^{-1},$$

$$\Delta \boldsymbol{x}^{(k)} = \begin{bmatrix} \Delta \boldsymbol{d}^{(k)} \\ \Delta \boldsymbol{s}^{(k)} \\ \Delta \boldsymbol{t}^{(k)} \\ \Delta \tilde{\boldsymbol{w}}^{(k)} \\ \Delta \tilde{\boldsymbol{u}}^{(k)} \\ \Delta \tilde{\boldsymbol{v}}^{(k)} \end{bmatrix} = \boldsymbol{G} \boldsymbol{e}^{(k)},$$

$$\boldsymbol{e}^{(k)} = \boldsymbol{b}^{(k)} - \boldsymbol{F} \boldsymbol{x}^{(k-1)},$$

$$\boldsymbol{G} = \begin{bmatrix} \boldsymbol{0} & \tilde{\boldsymbol{E}}^T & \boldsymbol{I} \\ \boldsymbol{0} & -\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{I} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \tilde{\boldsymbol{E}} & \boldsymbol{0} & \boldsymbol{0} \\ -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$$

and $\alpha \geq 0$ is a scaling factor.

Furthermore, $\boldsymbol{\beta}^{(\kappa)}$ can be rewritten as

$$\begin{bmatrix} \boldsymbol{H}_d^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{H}_{dw}^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{H}_s^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{H}_{su}^{(\kappa)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_t^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{H}_{tv}^{(\kappa)} \\ -\boldsymbol{H}_{dw}^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_w^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & -\boldsymbol{H}_{su}^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_u^{(\kappa)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{H}_{tv}^{(\kappa)} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{H}_v^{(\kappa)} \end{bmatrix}$$

where $\boldsymbol{H}_d^{(0)}$, $\boldsymbol{H}_s^{(0)}$, $\boldsymbol{H}_t^{(0)}$, $\boldsymbol{H}_w^{(0)}$, $\boldsymbol{H}_u^{(0)}$, $\boldsymbol{H}_v^{(0)}$, $\boldsymbol{H}_{dw}^{(0)}$, $\boldsymbol{H}_{su}^{(0)}$ and $\boldsymbol{H}_{tv}^{(0)}$ are diagonal matrices with entries in the main diagonal given by

$$\begin{aligned}
\left[\boldsymbol{H}_d^{(\kappa)}\right]_{m,m} &= \frac{2(d_m^{(\kappa K)})^2 + (\tilde{w}_m^{(\kappa K)})^2}{2(d_m^{(\kappa K)})^2 + 2(\tilde{w}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_s^{(\kappa)}\right]_{m,m} &= \frac{2(s_m^{(\kappa K)})^2 + (\tilde{u}_m^{(\kappa K)})^2}{2(s_m^{(\kappa K)})^2 + 2(\tilde{u}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_t^{(\kappa)}\right]_{m,m} &= \frac{2(t_m^{(\kappa K)})^2 + (\tilde{v}_m^{(\kappa K)})^2}{2(t_m^{(\kappa K)})^2 + 2(\tilde{v}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_w^{(\kappa)}\right]_{m,m} &= \frac{2(\tilde{w}_m^{(\kappa K)})^2 + (d_m^{(\kappa K)})^2}{2(d_m^{(\kappa K)})^2 + 2(\tilde{w}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_u^{(\kappa)}\right]_{m,m} &= \frac{2(\tilde{u}_m^{(\kappa K)})^2 + (\tilde{u}_m^{(\kappa K)})^2}{2(s_m^{(\kappa K)})^2 + 2(\tilde{u}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_v^{(\kappa)}\right]_{m,m} &= \frac{2(\tilde{v}_m^{(\kappa K)})^2 + (t_m^{(\kappa K)})^2}{2(t_m^{(\kappa K)})^2 + 2(\tilde{v}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_{dw}^{(\kappa)}\right]_{m,m} &= \frac{d_m^{(\kappa K)} \tilde{w}_m^{(\kappa K)}}{2(d_m^{(\kappa K)})^2 + 2(\tilde{w}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_{su}^{(\kappa)}\right]_{m,m} &= \frac{s_m^{(\kappa K)} \tilde{u}_m^{(\kappa K)}}{2(s_m^{(\kappa K)})^2 + 2(\tilde{u}_m^{(\kappa K)})^2}, \\
\left[\boldsymbol{H}_{tv}^{(\kappa)}\right]_{m,m} &= \frac{t_m^{(\kappa K)} \tilde{v}_m^{(\kappa K)}}{2(t_m^{(\kappa K)})^2 + 2(\tilde{v}_m^{(\kappa K)})^2}.
\end{aligned}$$

Hence we have, for the m-th controller

$$\Delta d_m^{(k)} = E_{m,m} e_{2,m}^{(k)} + \sum_{n \in \mathcal{N}_m} E_{n,m} e_{2,n}^{(k)} + e_{3,m}^{(k)}, \tag{16a}$$

$$\Delta s_m^{(k)} = -e_{2,m}^{(k)}, \tag{16b}$$

$$\Delta t_m^{(k)} = e_{3,m}^{(k)}, \tag{16c}$$

$$\Delta \tilde{u}_m^{(k)} = E_{m,m} e_{1,m}^{(k)} + \sum_{n \in \mathcal{N}_m} E_{m,n} e_{1,n}^{(k)}, \tag{16d}$$

$$\Delta \tilde{v}_m^{(k)} = -e_{1,m}^{(k)}, \tag{16e}$$

$$\Delta \tilde{w}_m^{(k)} = e_{1,m}^{(k)}, \tag{16f}$$

where

$$e_{1,m}^{(k)} = 1 - E_{m,m} \tilde{u}_m^{(k-1)} - \sum_{n \in \mathcal{N}_m} E_{n,m} \tilde{u}_n^{(k-1)} + \tilde{v}_m^{(k-1)} - \tilde{w}_m^{(k-1)}, \tag{17a}$$

$$e_{2,m}^{(k)} = L_m^{(k)} - \tilde{L}_m^{(k)}\left(\boldsymbol{d}^{(k)}\right) + s_m^{(k-1)}, \tag{17b}$$

$$e_{3,m}^{(k)} = 1 - d_m^{(k-1)} - t_m^{(k-1)} \tag{17c}$$

and

$$\begin{aligned} \tilde{L}_m^{(k)}\left(\boldsymbol{d}^{(k)}\right) &= \sum_{n=1}^{M} d_n^{(k)} E_{m,n} + O_m^{(k)} \\ &= \phi_m \left(\sum_{n=1}^{M} d_n^{(k)} \hat{E}_{m,n}(1) + \hat{O}_m^{(k)}\right) \\ &= \phi_m \bar{L}_m^{(k)}\left(\boldsymbol{d}^{(k)}\right). \end{aligned} \tag{18}$$

Here, $\bar{L}_m^{(k)}\left(\boldsymbol{d}^{(k)}\right)$ is the measured illuminance at the m-th light sensor during iteration k due to dimming vector $\boldsymbol{d}^{(k)}$ and daylight contribution $\hat{O}_m^{(k)}$, and $\tilde{L}_m^{(k)}\left(\boldsymbol{d}^{(k)}\right)$ is its normalized value. Note that the computation of (17b) does not require an estimation of daylight contribution, $\hat{O}_m^{(k)}$, but only the normalized measured illuminance at the m-th light sensor, $\tilde{L}_m^{(k)}\left(\boldsymbol{d}^{(k)}\right)$.

The updating term $\Delta d_m^{(k)}$ can be approximated as

$$\Delta d_m^{(k)} \approx \widehat{\Delta d}_{m,0}^{(k-1)} + \sum_{n=1}^{N_m} \widehat{\Delta d}_{m,n}^{(k-1)},$$

where

$$\widehat{\Delta d}_{m,0}^{(k)} = E_{m,m} e_{2,m}^{(k)} + e_{3,m}^{(k)} \tag{19}$$

is the proposed variation of dimming level of the m-th controller for its associated luminaire and

$$\widehat{\Delta d}_{m,n}^{(k)} = E_{\nu_{m,n},m} e_{2,\nu_{m,n}}^{(k)}, \quad n = 1, \ldots, N_m \tag{20}$$

is the variation of dimming level of the m-th controller proposed by the n-th neighbor.

Similarly, the error term $e_{1,m}^{(k-1)}$ can be rewritten as

$$e_{1,m}^{(k)} = \Delta e_{m,0}^{(k-1)} + \sum_{n=1}^{N_m} \Delta e_{m,n}^{(k-1)}$$

where

$$\Delta e_{m,0}^{(k)} = 1 - E_{m,m}\tilde{u}_m^{(k)} + \tilde{v}_m^{(k)} - \tilde{w}_m^{(k)} \tag{21}$$

is the own estimated error of the m-th controller and

$$\Delta e_{m,n}^{(k)} = -E_{\nu_{m,n},m}\tilde{u}_{\nu_{m,n}}^{(k)}, \quad n = 1, \ldots, N_m \tag{22}$$

is the error of the m-th controller estimated by the n-th neighbor.

Finally, by combining both loops we have the following updating formula for the m-th controller

$$\begin{bmatrix} d_m^{(k+1)} \\ s_m^{(k+1)} \\ t_m^{(k+1)} \\ \tilde{w}_m^{(k+1)} \\ \tilde{u}_m^{(k+1)} \\ \tilde{v}_m^{(k+1)} \end{bmatrix} = \boldsymbol{Q}\left\{ \boldsymbol{\gamma}_m^{(k)} \begin{bmatrix} d_m^{(k)} \\ s_m^{(k)} \\ t_m^{(k)} \\ \tilde{w}_m^{(k)} \\ \tilde{u}_m^{(k)} \\ \tilde{v}_m^{(k)} \end{bmatrix} + \alpha \boldsymbol{\beta}_m^{(\kappa)} \begin{bmatrix} \Delta d_m^{(k)} \\ \Delta s_m^{(k)} \\ \Delta t_m^{(k)} \\ \Delta \tilde{w}_m^{(k)} \\ \Delta \tilde{u}_m^{(k)} \\ \Delta \tilde{v}_m^{(k)} \end{bmatrix} \right\} \tag{23}$$

where

$$\boldsymbol{\gamma}_m^{(k)} = \begin{cases} \boldsymbol{\beta}_m^{(\kappa)}, & \text{if } k = \kappa K \\ \boldsymbol{I}_6, & \text{otherwise} \end{cases}$$

and

$$\boldsymbol{\beta}_m^{(\kappa)} = \left(\boldsymbol{Y}_m^{(\kappa)}\left(\boldsymbol{Y}_m^{(\kappa)}\right)^T + \boldsymbol{I}_6\right)^{-1},$$

$$\boldsymbol{Y}_m^{(\kappa)} = \tilde{\boldsymbol{Y}}_m^{(\kappa)}\left(\tilde{\boldsymbol{Y}}_m^{(\kappa)}(\tilde{\boldsymbol{Y}}_m^{(\kappa)})^T\right)^{-\frac{1}{2}}$$

$$\tilde{\boldsymbol{Y}}_m^{(\kappa)} = \begin{bmatrix} \tilde{w}_m^{(\kappa K)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \tilde{u}_m^{(\kappa K)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \tilde{v}_m^{(\kappa K)} \\ d_m^{(\kappa K)} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & s_m^{(\kappa K)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & t_m^{(\kappa K)} \end{bmatrix}$$

where $\boldsymbol{I}_6$ is the identity matrix of size 6×6.

A. Distributed Control Algorithm

We assume that the system initializes with all the luminaires off. That is

$$d_m^{(0)} = 0, \qquad m = 1, \ldots, M.$$

Additionally, all the variables $\{s_m^{(0)}\}$, $\{t_m^{(0)}\}$, $\{\tilde{u}_m^{(0)}\}$, $\{\tilde{v}_m^{(0)}\}$ and $\{\tilde{w}_m^{(0)}\}$ are initialized with zero.

Let the transmitted message at iteration k from the m-th controller be denoted by

$$\mu_m^{(k)} = \begin{bmatrix} \left\{d_m^{(k)}, e_{1,m}^{(k)}\right\} \\ \left\{\nu_{m,1}, \widehat{\Delta d}_{m,1}^{(k)}, \Delta e_{m,1}^{(k)}\right\} \\ \vdots \\ \left\{\nu_{m,N_m}, \widehat{\Delta d}_{m,N_m}^{(k)}, \Delta e_{m,N_m}^{(k)}\right\} \end{bmatrix},$$

where

- $d_m^{(k)}$: Current dimming level of the m-th luminaire,
- $e_{1,m}^{(k)}$: Current error of the m-th controller,
- $\nu_{m,n}$: Index of the n-th neighbor, $n = 1, \ldots, N_m$,
- $\widehat{\Delta d}_{m,n}^{(k)}$: Proposed variation for the dimming level of the luminaire of the n-th neighbor, $n = 1, \ldots, N_m$,
- $\Delta e_{m,n}^{(k)}$: Estimated error of the n-th neighbor, $n = 1, \ldots, N_m$.

The pseudo-code for the distributed lighting control algorithm is presented in *Algorithm 1*.

V. Analysis of the Algorithm

In this section, we establish conditions under which the controller is stable and converges to a feasible solution $\boldsymbol{x}^\star$ for both the primal problem (4) and the dual problem (8) such that the complementary conditions (12a)–(12c) are satisfied. We assume that $\boldsymbol{b}$ is constant during the inner and outer loop.

Let us establish conditions on matrix $\boldsymbol{G}$ for stability and convergence of the inner loop to a feasible intermediate solution $\boldsymbol{x}^*$ to exist.

First, we consider the inner loop without the operator $\boldsymbol{Q}$. Let $\boldsymbol{x}^{(0)}$ be the initial vector. It can be shown using classic control theory that the system is stable and converges to solution $\bar{\boldsymbol{x}}^*$ [20, Chapter 5],

$$\begin{aligned} \bar{\boldsymbol{x}}^* = \arg\min_{\boldsymbol{x}} \; & \|(\boldsymbol{Y}^{(0)})^T \boldsymbol{x}\|^2 + \|\boldsymbol{x} - \boldsymbol{x}^{(0)}\|^2 \\ \text{s.t. } & \boldsymbol{F}\boldsymbol{x} = \boldsymbol{b} \end{aligned}$$

when

$$\max\left\{\left|\operatorname{eig}\left(\boldsymbol{I} - \alpha \boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}\right)\right|\right\} < 1. \tag{24}$$

Choose parameters χ_i, χ and N such that the following holds

$$\sum_{n \in \mathcal{N}_i} E_{i,n} = \chi_i \sum_n E_{i,n}, \ \forall i, \tag{25a}$$

$$\sum_{n \notin \mathcal{N}_i} E_{i,n} = (1 - \chi_i) \sum_n E_{i,n}, \ \forall i, \tag{25b}$$

$$\chi_i \le \chi, \ \forall i, \tag{25c}$$

$$N_i \ge N, \ \forall i. \tag{25d}$$

Algorithm 1 Local Lighting Control in m-th Controller

Require: Set of received messages $\left\{\mu_n^{(k-1)} : n \in \mathcal{N}_m\right\}$
Scaling factor α
Normalization factor ϕ_m
Current light measurement, $\bar{L}_m^{(k)}(\boldsymbol{d}^{(k)})$
Current target illuminance, $\hat{L}_m^{(k)}$
Maximum number of iteration of inner loop, K
Threshold for changing dimming level, ϵ_d

loop

Normalization
$\tilde{L}_m^{(k)}(\boldsymbol{d}^{(k)}) \leftarrow \phi_m \bar{L}_m^{(k)}(\boldsymbol{d}^{(k)})$
$L_m^{(k)} \leftarrow \phi_m \hat{L}_m^{(k)}$
$E_{m,n} \leftarrow \phi_m \hat{E}_{m,n}(1),\ n \in \mathcal{N}_m$

Estimate errors terms
$e_{1,m}^{(k)} \leftarrow \Delta e_{m,0}^{(k-1)} +$
$\sum\left\{\Delta e_{i,j}^{(k-1)} : \nu_{i,j} = m, i \in \mathcal{N}_m \wedge j = 1, \ldots, N_m\right\}$
Use equations (17b) - (17c)

Estimate updating terms
$\Delta d_m^{(k)} \leftarrow \widehat{\Delta d}_{m,0}^{(k-1)} +$
$\sum\left\{\widehat{\Delta d}_{i,j}^{(k-1)} : \nu_{i,j} = m, i \in \mathcal{N}_m \wedge j = 1, \ldots, N_m\right\}$
Use equations (16b)-(16f)
$\kappa \leftarrow \left\lfloor \frac{k}{K} \right\rfloor$

if $\kappa K = k$ **then**
Update matrix $\boldsymbol{\beta}_m^{(\kappa)}$
$\boldsymbol{\gamma}_m^{(k)} \leftarrow \boldsymbol{\beta}_m^{(\kappa)}$
else
$\boldsymbol{\gamma}_m^{(k)} \leftarrow \boldsymbol{I}_6$
end if

Update terms
Use equation (23)

if $\|d_m^{(k)} - d_m^{(k-1)}\| > \epsilon_d$ **then**
Calculate propose terms
Use equations (19), (20), (21) and (22)

Transmit message $\mu_m^{(k)}$ to neighbors in $\mathcal{N}_m$
end if
end loop

Then sufficient conditions for (24) to hold are

$$0 < \xi \leq \frac{\sqrt{1.25^2 + \left(\frac{M-1}{\sqrt{2}}\right)\left(\frac{\chi}{1-\chi}\right)} - 1.25}{2\,(M-1)}, \tag{26}$$

with $\xi_i \leq \xi,\ \forall i$, and

$$0 < \alpha \leq \min\left\{\frac{1}{2.25 + (\xi + 0.25)\,\xi N}, \frac{\chi}{\chi + M\xi^2 + 1.25\xi}, \frac{1}{2.25 + N\xi}\right\}. \tag{27}$$

The proof is provided in the appendix.

Additionally if condition in (24) is satisfied, then we have the following property at each iteration k in the inner loop [20, Chapter 4]

$$\left\|\boldsymbol{x}^{(k)} - \bar{\boldsymbol{x}}^*\right\| < \left\|\boldsymbol{x}^{(k-1)} - \bar{\boldsymbol{x}}^*\right\|. \tag{28}$$

Now, let us consider the effect of the nonlinear operator $\boldsymbol{Q}$. Given that operator $\boldsymbol{Q}$ is projecting over a convex set which contains $\boldsymbol{x}^*$, where $\boldsymbol{x}^*$ is a solution to (15), then we have the following property [21]

$$\left\|\boldsymbol{Q}\left\{\boldsymbol{x}^{(k)}\right\} - \boldsymbol{x}^*\right\| \leq \left\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^*\right\|. \tag{29}$$

Note that the equality only holds when $\boldsymbol{Q}\left\{\boldsymbol{x}^{(k)}\right\} = \boldsymbol{x}^{(k)}$.

Let us consider that during the first K' iterations the intermediate solution vectors $\left\{\boldsymbol{x}^{(k)}\right\}$, $k = 0, 1, \ldots, K'$ satisfy

$$\boldsymbol{Q}\left\{\boldsymbol{x}^{(k)}\right\} = \boldsymbol{x}^{(k)}, \quad k = 0, 1, \ldots, K'.$$

Hence, we have for each iteration $k = 0, 1, \ldots, K'$,

$$\left\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^*\right\|^2 = \left\|\boldsymbol{x}^{(k)} - \bar{\boldsymbol{x}}^*\right\|^2 + \left\|\bar{\boldsymbol{x}}^* - \boldsymbol{x}^*\right\|^2 + 2(\boldsymbol{x}^{(k)} - \bar{\boldsymbol{x}}^*)^T(\bar{\boldsymbol{x}}^* - \boldsymbol{x}^*)$$

and using (28), we have

$$\left\|\boldsymbol{x}^{(k)} - \boldsymbol{x}^*\right\| < \left\|\boldsymbol{x}^{(k-1)} - \boldsymbol{x}^*\right\|. \tag{30}$$

Then, after combining with (29), we have at iteration $K'+1$,

$$\left\|\boldsymbol{Q}\left\{\boldsymbol{x}^{(K'+1)}\right\} - \boldsymbol{x}^*\right\| < \left\|\boldsymbol{x}^{(K'+1)} - \boldsymbol{x}^*\right\| < \left\|\boldsymbol{x}^{(K')} - \boldsymbol{x}^*\right\|$$

and also

$$\boldsymbol{e}^{(K'+1)} = \left\|\boldsymbol{F}\boldsymbol{Q}\left\{\boldsymbol{x}^{(K'+1)}\right\} - \boldsymbol{b}\right\| < \left\|\boldsymbol{F}\boldsymbol{x}^{(K')} - \boldsymbol{b}\right\|.$$

At iteration $K'+1$, the j-th variable in vector $\boldsymbol{x}^{(K)}$ reaches its lower bound (or upper bound) and cannot be further reduced (or increased). If the system is controllable (i.e. it is possible to transfer the system from any initial illuminance state to any desired illuminance state [20, pp. 379]), then there exists a control path from $\boldsymbol{x}^{(K')}$ to $\boldsymbol{x}^*$ which does not modify the j-th variable further. Moreover, this path reduces the error in each iteration and so

$$\lim_{k\to\infty}\left\|\boldsymbol{e}^{(k)}\right\| = 0.$$

Thus, the inner loop is stable and converges to a feasible solution $\boldsymbol{x}^*$.

In the outer loop the solution $\boldsymbol{x}^{((\kappa-1)K)}$ is moved towards a new intermediate solution $\boldsymbol{x}^{(\kappa K)}$ where

$$\|(\boldsymbol{Y}^{(\kappa)})^T\boldsymbol{x}^{(\kappa K)}\|^2 + \|\boldsymbol{x}^{(\kappa K)} - \boldsymbol{x}^{((\kappa-1)K)}\|^2, \leq \|(\boldsymbol{Y}^{(\kappa-1)})^T\boldsymbol{x}^{((\kappa-1)K)}\|^2$$

It can be noted that when the solution has converged (i.e. $\boldsymbol{x}^{(\kappa K)} = \boldsymbol{x}^{((\kappa-1)K)} = \boldsymbol{x}^{(\bar{\kappa}K)}$), then

$$\|(\boldsymbol{Y}^{(\kappa)})^T\boldsymbol{x}^{(\kappa K)}\|^2 = \|(\boldsymbol{Y}^{(\kappa-1)})^T\boldsymbol{x}^{((\kappa-1)K)}\|^2,$$

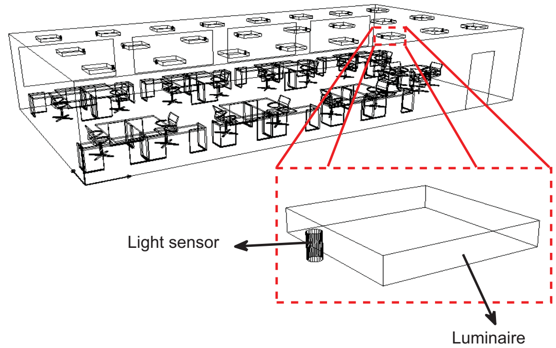

Fig. 4. Room configuration.

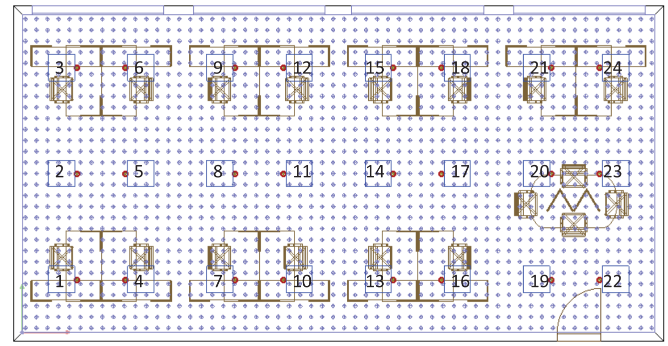

Fig. 5. Top view of office room.

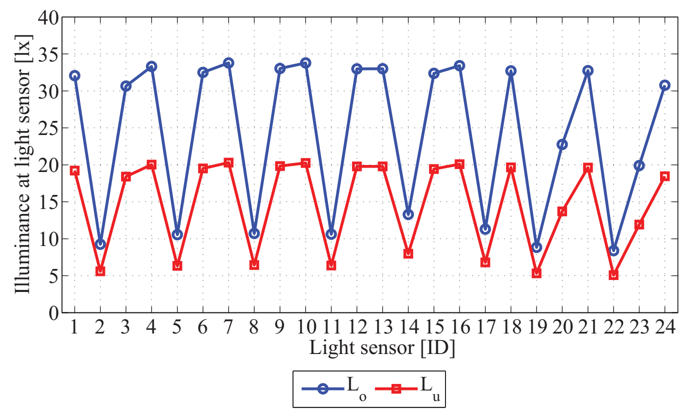

Fig. 6. Target illuminance values at light sensors.

where $\boldsymbol{x}^{(\tilde{\kappa}K)}$ is the solution that minimizes

$$\|(\boldsymbol{Y}^{(\tilde{\kappa})})^T \boldsymbol{x}^{(\tilde{\kappa}K)}\|^2 = \sum_m \frac{(d_m^{(\tilde{\kappa}K)}\tilde{w}^{(\tilde{\kappa}K)})^2}{(d_m^{(\tilde{\kappa}K)})^2 + (\tilde{w}^{(\tilde{\kappa}K)})^2} + \frac{(s_m^{(\tilde{\kappa}K)}\tilde{u}^{(\tilde{\kappa}K)})^2}{(s_m^{(\tilde{\kappa}K)})^2 + (\tilde{u}^{(\tilde{\kappa}K)})^2} + \frac{(t_m^{(\tilde{\kappa}K)}\tilde{v}^{(\tilde{\kappa}K)})^2}{(t_m^{(\tilde{\kappa}K)})^2 + (\tilde{v}^{(\tilde{\kappa}K)})^2}. \tag{31}$$

Furthermore, the minimum of (31) is achieved only when all the complementary conditions in (12a) - (12c) are satisfied. Hence, we have that $\boldsymbol{x}^{(\tilde{\kappa}K)} = \boldsymbol{x}^{\star}$.

A. Neighbor Selection

The selection of neighbors in the proposed control algorithm depends on the controllability of the system. A sufficient condition for the system to be controllable is that the m-th controller achieves the target illuminance at the m-th light sensor independently from other controllers outside its neighborhood, i.e.

$$\sum_{n \in \mathcal{N}_m} \hat{E}_{m,n}(1) > L_m^o. \tag{32}$$

VI. Simulation Results

In this section, we present simulation results to show the performance of the proposed control scheme. The office has length 14.4 m and width 7.4 m with height of the ceiling of 2.86 m. The distance of the ceiling to the workspace plane is 1.93 m. There are $M = 24$ luminaires arranged in a grid of 3 by 8, with light sensors co-located with the luminaires, as shown in Fig. 4. The beam profile of the luminaire was the same as used in [9]. The luminaire indexing in Fig. 5 also corresponds to that of the controllers and zones. The office has windows on one side of the room for daylight.

We consider a light sensor with a half opening angle of 20 degrees. Since there is no readily available model for a light sensor in DIALux, we created a model to emulate a light sensor. This was done by creating an opaque cylinder with an opening on the bottom end facing the workspace plane. The illuminance was measured at a point along the main axis so as to emulate a half opening angle of 20 degrees.

The minimum desired average illuminance value L_{WP}^u in an unoccupied zone is chosen to be 300 lux, and in an occupied zone to be $L_{WP}^o = 500$ lux, following recommended norms for office lighting [22]. The target intensity when the m-th zone is empty, L_m^u, is calculated by setting all the luminaires such that 300 lx is achieved across the room, i.e. $d_m = 0.51$, $m = 1, \ldots, M$. That is equivalent to

$$L_m^u = 0.51 \sum_{n=1}^{N} \hat{E}_{m,n}(1), \qquad m = 1, \ldots, M.$$

Similarly, we calculate the target intensity when the m-th zone is occupied, L_m^o, with a dimming level such that 500 lx is achieved across the room. In this case, all the luminaires are at dimming level 0.85, and

$$L_m^o = 0.85 \sum_{n=1}^{N} \hat{E}_{m,n}(1), \qquad m = 1, \ldots, M.$$

The resulting target illuminance values at the light sensors are shown in Fig. 6. Note that the target illuminance at the light sensor are not uniform, due to differences in reflectance across zones in the workspace plane.

The daylight data is obtained from DIALux using clear sky settings for 11 November, 2011 in Amsterdam. The daylight distribution is simulated from 7:00 to 20:00 hours. An average occupancy level of 50% was used in each zone along the simulated day.

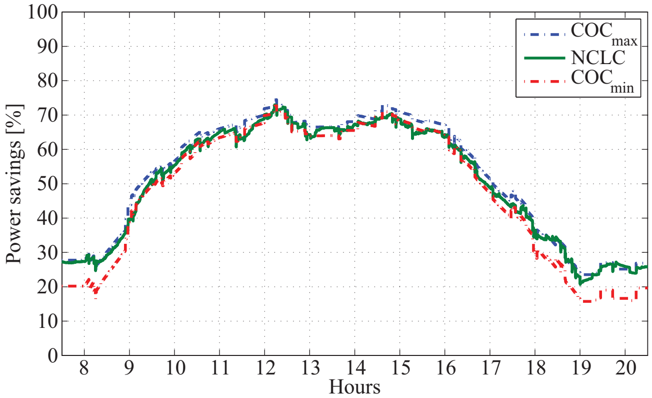

Fig. 7. Power savings NCLC with respect to COC$_{max}$ and COC$_{min}$.

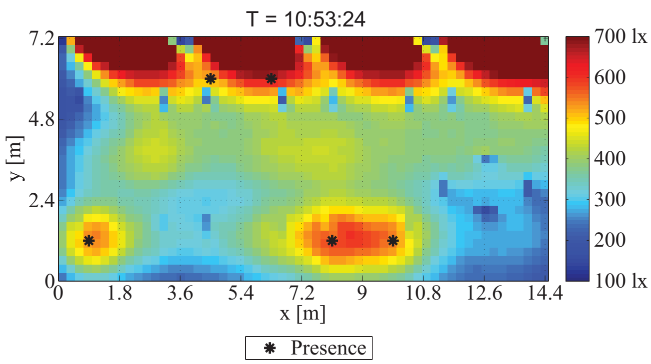

Fig. 8. Illuminance distribution at workspace plane at 10:53 hours.

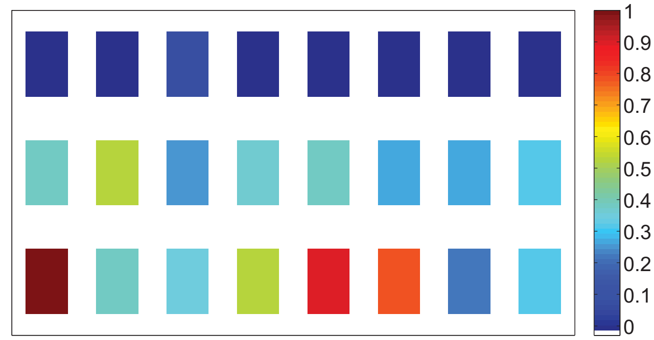

Fig. 9. Dimming levels at 10:53 hours.

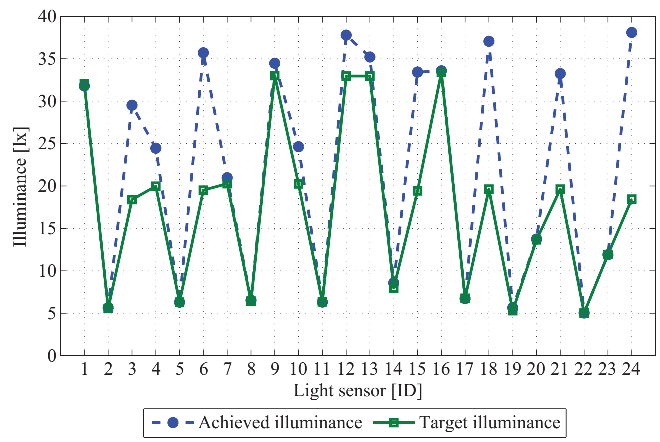

Fig. 10. Target versus achieved illuminance values at light sensors at 10:53 hours.

The updating time of the dimming levels per controller is set to 4 seconds. Also, each controller broadcasts its message to its neighbors every 4 seconds. We assume the neighborhood of the m-th controller to be the closest controllers to it (i.e. $N = 10$). Furthermore, this neighborhood corresponds to $\chi = 0.85$. Hence, we choose parameter $\xi = 0.1$ (satisfying (26)), normalization factor

$$\phi_m = \frac{0.1}{\sum_{n \in \mathcal{N}_m} \hat{E}_{m,n}(1)}, \qquad \forall m$$

and scaling factor $\alpha = 0.3$ (satisfying (27)). The threshold for transmitting a message is set to $\epsilon_d = 0.001$ and $K = 30$.

We calculate the power savings with respect to a reference static rendering, in which the dimming levels of the luminaires are set to a fixed level 0.85 such that an illuminance level of 500 lux is achieved over the workspace plane.

The power savings of our proposed scheme, which we will refer as NCLC (Neighbor Communication Lighting Control), are compared with a centralized implementation of the optimal control for (9) and (4), which we will refer as COC$_{max}$ and COC$_{min}$, respectively. The power savings achieved by NCLC along the simulated day are shown in Fig. 7. We note that the savings achieved by NCLC are within the limits COC$_{max}$ and COC$_{min}$.

The rendering solution at a given time instance (10:53 hours) is depicted in Figs. 8 and 9. We can note that the luminaires close to the windows are dimmed down due to larger daylight contribution. The differences between target illuminance and achieved illuminance at the sensors are shown in Fig. 10. As expected, the achieved illuminance is no smaller than the target illuminance. Note that the illuminance values at the light sensors that are closer to the windows are much higher than the target illuminance values due to the large contribution of daylight over the corresponding zones.

VII. Conclusion

We presented a distributed algorithm for illumination control under local presence and light sensing, with networking constraints on exchanging control information. Sufficient conditions were provided under which the algorithm can achieve the target light sensor values, calibrated to achieve a corresponding desired spatial illumination rendering in the workspace plane. The performance of the distributed control algorithm was evaluated in an open office room and shown to result in substantial power savings when compared to the luminaires being at a fixed dimming level, with no presence or daylight adaptation.

In this paper, we assumed a simplified presence and light sensor model. Improvements in presence sensor technology [24] minimize detection errors and as such the rendered illumination is close to the desired one. In our analysis, we had assumed perfect light sensor calibration. In practice, this may not be the case. Further, light sensor readings may show fluctuations. Considering such sensor imperfections and evaluating their impact on lighting control is a topic for further investigation.

The developed distributed sensing and control framework may be extended to other illumination applications, e.g. [25]. Developing appropriate analytical models and control solutions for such an application would be interesting future work.

APPENDIX
SUFFICIENT CONDITIONS FOR (24)

Sufficient conditions for (24) to hold are

$$\mathrm{Re}\left\{\mathrm{eig}\left(\boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}\right)\right\} > 0 \tag{33a}$$

$$\mathrm{Re}\left\{\mathrm{eig}\left(\boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}\right)\right\} > \mathrm{Im}\left\{\mathrm{eig}\left(\boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}\right)\right\} \tag{33b}$$

$$\alpha\left|\mathrm{eig}\left(\boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}\right)\right| < 1 \tag{33c}$$

for any eigenvalue of $\boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}$.

Note that $\boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G}$ can be rewritten as the summation of a symmetric matrix $\boldsymbol{P} = \boldsymbol{G}^T(\boldsymbol{\beta}^{(0)} - \tilde{\boldsymbol{\beta}}^{(0)})\boldsymbol{G}$,

$$\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{1,1} & \boldsymbol{P}_{1,2} & \boldsymbol{P}_{1,3} \\ \boldsymbol{P}_{2,1} & \boldsymbol{P}_{2,2} & \boldsymbol{P}_{2,3} \\ \boldsymbol{P}_{3,1} & \boldsymbol{P}_{3,2} & \boldsymbol{P}_{3,3} \end{bmatrix}.$$

where

$$\begin{aligned}
\boldsymbol{P}_{1,1} &= \boldsymbol{H}_w^{(0)} + \tilde{\boldsymbol{E}}^T\boldsymbol{H}_u^{(0)}\tilde{\boldsymbol{E}} + \boldsymbol{H}_v^{(0)}, \\
\boldsymbol{P}_{1,2} &= -\boldsymbol{H}_{dw}^{(0)}\tilde{\boldsymbol{E}}^T + \tilde{\boldsymbol{E}}^T\boldsymbol{H}_{su}^{(0)}, \\
\boldsymbol{P}_{1,3} &= -\boldsymbol{H}_{dw}^{(0)} + \boldsymbol{H}_{tv}^{(0)}, \\
\boldsymbol{P}_{2,1} &= -\tilde{\boldsymbol{E}}\boldsymbol{H}_{dw}^{(0)} + \boldsymbol{H}_{su}^{(0)}\tilde{\boldsymbol{E}}, \\
\boldsymbol{P}_{2,2} &= \tilde{\boldsymbol{E}}\boldsymbol{H}_d^{(0)}\tilde{\boldsymbol{E}}^T + 0.5\boldsymbol{H}_s^{(\kappa)}, \\
\boldsymbol{P}_{2,3} &= \tilde{\boldsymbol{E}}\boldsymbol{H}_d^{(0)}, \\
\boldsymbol{P}_{3,1} &= -\boldsymbol{H}_{dw}^{(0)} + \boldsymbol{H}_{tv}^{(0)}, \\
\boldsymbol{P}_{3,2} &= \boldsymbol{H}_d^{(0)}\tilde{\boldsymbol{E}}^T, \\
\boldsymbol{P}_{3,3} &= \boldsymbol{H}_d^{(0)} + \boldsymbol{H}_t^{(0)},
\end{aligned}$$

and

$$\tilde{\boldsymbol{\beta}}^{(0)} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} \\ \boldsymbol{0} & 0.5\boldsymbol{H}_s^{(0)} & \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0}\,\boldsymbol{0} \end{bmatrix},$$

and an asymmetric matrix $\bar{\boldsymbol{P}} = \boldsymbol{F}\boldsymbol{\beta}^{(0)}\boldsymbol{G} - \boldsymbol{P}$,

$$\bar{\boldsymbol{P}} = \begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ -\bar{\boldsymbol{E}}\boldsymbol{H}_{dw}^{(0)} & \bar{\boldsymbol{E}}\boldsymbol{H}_d^{(0)}\tilde{\boldsymbol{E}}^T + 0.5\boldsymbol{H}_s^{(0)} & \bar{\boldsymbol{E}}\boldsymbol{H}_d^{(0)} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}.$$

where $\bar{\boldsymbol{E}} = \boldsymbol{E} - \tilde{\boldsymbol{E}}$.

Hence, conditions (33a) and (33b) are satisfied when

$$\mathrm{Re}\{\mathrm{eig}(\boldsymbol{P})\} > 0 \tag{34a}$$

$$\mathrm{Re}\{\mathrm{eig}(\boldsymbol{P})\} > \mathrm{Im}\{\mathrm{eig}(\boldsymbol{P})\} \tag{34b}$$

$$\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\} > 0 \tag{34c}$$

$$\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\} > \mathrm{Im}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\} \tag{34d}$$

for any eigenvalue of $\boldsymbol{P}$ and $\bar{\boldsymbol{P}}$, respectively.

Note that all the entries in the main diagonal of $\boldsymbol{H}_d^{(0)}$, $\boldsymbol{H}_s^{(0)}$, $\boldsymbol{H}_t^{(0)}$, $\boldsymbol{H}_w^{(0)}$, $\boldsymbol{H}_u^{(0)}$ and $\boldsymbol{H}_v^{(0)}$ are within the range [0.5, 1]. Similarly, all the entries in the main diagonal of $\boldsymbol{H}_{dw}^{(0)}$, $\boldsymbol{H}_{su}^{(0)}$ and $\boldsymbol{H}_{tv}^{(0)}$ are within the range $[0, 0.25]$. Hence, $(\boldsymbol{\beta}^{(0)} - \tilde{\boldsymbol{\beta}}^{(0)})$ is diagonal dominant with non-negative elements in the main diagonal (i.e. positive semidefinite) and so condition (34a) is satisfied. Additionally, given that $\boldsymbol{P}$ is symmetric we have that its eigenvalues are real and so condition (34b) is satisfied.

Using Gershgorin circle theorem [23, Chapter 6], we have that any eigenvalue of $\bar{\boldsymbol{P}}$ satisfies for at least one i^*

$$\left(\mathrm{eig}\left(\bar{\boldsymbol{P}}\right) - \left[\bar{\boldsymbol{P}}\right]_{i^*,i^*}\right)^2 \leq \left(\sum_{j\neq i^*}\left|\left[\bar{\boldsymbol{P}}\right]_{i^*,j}\right|\right)^2$$

$$\left(\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\} - \left[\bar{\boldsymbol{P}}\right]_{i^*,i^*}\right)^2 + \left(\mathrm{Im}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\}\right)^2 \leq \left(\sum_{j\neq i^*}\left|\left[\bar{\boldsymbol{P}}\right]_{i^*,j}\right|\right)^2$$

and by using (34d) then

$$\begin{aligned}
&\left(\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\}\right)^2 \geq \left(\sum_{j\neq i^*}\left|\left[\bar{\boldsymbol{P}}\right]_{i^*,j}\right|\right)^2 \\
&\quad -\left(\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\} - \left[\bar{\boldsymbol{P}}\right]_{i^*,i^*}\right)^2 \\
&2\left(\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\}\right)^2 - 2\left[\bar{\boldsymbol{P}}\right]_{i^*,i^*}\mathrm{Re}\left\{\mathrm{eig}\left(\bar{\boldsymbol{P}}\right)\right\} \\
&\quad +\left(\left[\bar{\boldsymbol{P}}\right]_{i^*,i^*}\right)^2 - \left(\sum_{j\neq i^*}\left|\left[\bar{\boldsymbol{P}}\right]_{i^*,j}\right|\right)^2 \geq 0
\end{aligned} \tag{35}$$

We have that (35) is true if

$$\begin{aligned}
&\left(\left[\bar{\boldsymbol{P}}\right]_{i,i}\right)^2 - 2\left(\left(\left[\bar{\boldsymbol{P}}\right]_{i,i}\right)^2 - \left(\sum_{j\neq i}\left|\left[\bar{\boldsymbol{P}}\right]_{i,j}\right|\right)^2\right) \leq 0, \quad \forall i \\
&-\left(\left[\bar{\boldsymbol{P}}\right]_{i,i}\right)^2 + \left(\sum_{j\neq i}\left|\left[\bar{\boldsymbol{P}}\right]_{i,j}\right|\right)^2 \leq 0, \quad \forall i \\
&\left[\bar{\boldsymbol{P}}\right]_{i,i} \geq \sqrt{2}\sum_{j\neq i}\left|\left[\bar{\boldsymbol{P}}\right]_{i,j}\right|, \quad \forall i.
\end{aligned} \tag{36}$$

Hence, sufficient conditions to (34c) and (34d) are given by (36) and so

$$\begin{aligned}
0.5\left[\boldsymbol{H}_s^{(0)}\right]_{i,i} \geq \sqrt{2}\Bigg(&\sum_{j\neq i}\sum_{\substack{n\notin\mathcal{N}_i \\ n\in\mathcal{N}_j}} E_{i,n}\left[\boldsymbol{H}_d^{(0)}\right]_{n,n}E_{j,n} \\
&+\sum_{n\notin\mathcal{N}_i} E_{i,n}\left[\boldsymbol{H}_d^{(0)}\right]_{n,n} + \sum_{n\notin\mathcal{N}_i} E_{i,n}\left[\boldsymbol{H}_{dw}^{(0)}\right]_{n,n}\Bigg), \quad \forall i
\end{aligned} \tag{37}$$

Note that the following inequalities hold

$$\sum_{n\in\mathcal{N}_j} E_{i,n}E_{j,n} \leq \sum_m E_{i,m}\sum_{n\in\mathcal{N}_j} E_{j,n}, \quad \forall i, j,$$

$$\sum_{n\in\mathcal{N}_i} E_{i,n}^2 \leq \left(\sum_{n\in\mathcal{N}_i} E_{i,n}\right)^2, \quad \forall i.$$

and due to reciprocity between neighbors we have the following equality

$$\sum_{n:i\in\mathcal{N}_n} E_{n,i} = \sum_{n\in\mathcal{N}_i} E_{n,i}, \quad \forall i.$$

Let

$$\xi_i \le \xi, \quad \forall i$$

and so the following inequalities hold

$$\sum_{n\in\mathcal{N}_i} E_{i,n} \le \xi, \quad \forall i,$$
$$\sum_{n\in\mathcal{N}_i} E_{n,i} \le N\xi, \quad \forall i.$$

Now suppose conditions (25a–25d) hold. Hence, (37) becomes

$$0.25 \ge \sqrt{2}\left(\sum_{j\neq i}\sum_{\substack{n\notin\mathcal{N}_j\\ n\in\mathcal{N}_j}} E_{i,n}E_{j,n} + 1.25\sum_{n\notin\mathcal{N}_i} E_{i,n}\right),$$

$$0.25 \ge \sqrt{2}\left(\sum_{j\neq i}\sum_{m\notin\mathcal{N}_i} E_{i,m}\sum_{n\in\mathcal{N}_j} E_{j,n} + 1.25\left(\frac{1-\chi}{\chi}\right)\sum_{n\in\mathcal{N}_i} E_{i,n}\right),$$

$$0.25 \ge \sqrt{2}\left(\sum_{j\neq i}\left(\frac{1-\chi}{\chi}\right)\sum_{m\in\mathcal{N}_i} E_{i,m}\xi + 1.25\left(\frac{1-\chi}{\chi}\right)\xi\right),$$

$$0.25 \ge \sqrt{2}\left(\sum_{j\neq i}\left(\frac{1-\chi}{\chi}\right)\xi^2 + 1.25\left(\frac{1-\chi}{\chi}\right)\xi\right),$$

$$0.25 \ge \sqrt{2}\left((M-1)\left(\frac{1-\chi}{\chi}\right)\xi^2 + 1.25\left(\frac{1-\chi}{\chi}\right)\xi\right),$$

$$(M-1)\,\xi^2 + 1.25\xi - \frac{\chi}{4\sqrt{2}\,(1-\chi)} \le 0,$$

thus

$$\xi \le \frac{\sqrt{1.25^2 + \left(\frac{M-1}{\sqrt{2}}\right)\left(\frac{\chi}{1-\chi}\right)} - 1.25}{2\,(M-1)}.$$

A sufficient condition to (33c) is

$$\alpha\sum_j\left(\left|[\boldsymbol{P}]_{i,j}\right| + \left|[\bar{\boldsymbol{P}}]_{i,j}\right|\right) \le 1, \quad \forall i$$

and so

$$\alpha\left(2.25 + \sum_j\sum_{n:\substack{i\in\mathcal{N}_n\\ j\in\mathcal{N}_n}} E_{n,i}E_{n,j} + 0.25\sum_{n:i\in\mathcal{N}_n} E_{n,i}\right) \le 1 \tag{38a}$$

$$\alpha\left(1 + \sum_j\sum_{n\in\mathcal{N}_j} E_{i,n}E_{j,n} + 1.25\sum_n E_{i,n}\right) \le 1 \tag{38b}$$

$$\alpha\left(2.25 + \sum_{n:i\in\mathcal{N}_n} E_{n,i}\right) \le 1. \tag{38c}$$

Using (38a) we have

$$\alpha\left(2.25 + \sum_{n:i\in\mathcal{N}_n} E_{n,i}\sum_{j\in\mathcal{N}_n} E_{n,j} + 0.25\sum_{n:i\in\mathcal{N}_n} E_{n,i}\right) \le 1$$

$$\alpha\left(2.25 + \xi\sum_{n\in\mathcal{N}_i} E_{n,i} + 0.25\sum_{n\in\mathcal{N}_i} E_{n,i}\right) \le 1$$

$$\alpha\left(2.25 + \left(\xi + 0.25\right)\sum_{n\in\mathcal{N}_i} E_{n,i}\right) \le 1$$

$$\alpha\left(2.25 + \left(\xi + 0.25\right)N\xi\right) \le 1$$

$$\alpha \le \frac{1}{2.25 + (\xi + 0.25)\,\xi N}.$$

Similarly, from (38b) and (38c) we obtain respectively

$$\alpha\left(1 + \sum_j\sum_m E_{i,m}\sum_{n\in\mathcal{N}_j} E_{j,n} + 1.25\sum_n E_{i,n}\right) \le 1$$

$$\alpha\left(1 + \sum_j \frac{\xi}{\chi}\xi + 1.25\frac{\xi}{\chi}\right) \le 1$$

$$\alpha\left(1 + \frac{M\xi^2}{\chi} + 1.25\frac{\xi}{\chi}\right) \le 1$$

$$\alpha \le \frac{\chi}{\chi + M\xi^2 + 1.25\xi}$$

and

$$\alpha\left(2.25 + \sum_{n\in\mathcal{N}_i} E_{n,i}\right) \le 1$$

$$\alpha\,(2.25 + N\xi) \le 1$$

$$\alpha \le \frac{1}{2.25 + N\xi}.$$

Summing up, sufficient conditions to (24) are given by (26) and (27).

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