(a) Encontre todas as soluções de mínimos quadrados do sistema linear

$$\begin{aligned} x_1 - x_2 &= 4 \\ 3x_1 + 2x_2 &= 1 \\ -2x_1 + 4x_2 &= 3 \end{aligned}$$

(b) Encontre o vetor erro e o erro.

a) $A\vec{x} = B$

$$\begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

Por MQO:

$$\vec{X} = (A^TA)^{-1}(A^TB)$$

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$$(A^TA)^{-1} = \left(\begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 3 & 2 \\ -2 & 4 \end{bmatrix} \right)^{-1}$$

$$(A^TA)^{-1} = \left(\begin{bmatrix} 1(1) + 3(3) + (-2)(-2) & 1(-1) + 3(2) + (-2)4 \\ (-1)1 + 2(3) + 4(-2) & 1 + 2(2) + 4(4) \end{bmatrix} \right)^{-1}$$

$$(A^TA)^{-1} = \left(\begin{bmatrix} 14 & -3 \\ -3 & 21 \end{bmatrix} \right)^{-1}$$

$$\det(A^TA) = 294 - 9 = 285$$

$$(A^TA)^{-1} \Rightarrow (A^TA)(A^TA)^{-1} = I$$

$$\begin{bmatrix} 14 & -3 \\ -3 & 19 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 14a - 3c & 14b - 3d \\ -3a + 19c & -3b + 19d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

① $\begin{cases} 14a - 3c = 1 \\ -3a + 19c = 0 \end{cases}$

② $\begin{cases} 14b - 3d = 0 \\ -3b + 19d = 1 \end{cases}$

① $\begin{cases} 14a - 3c = 1 \\ -3a + 19c = 0 \end{cases}$

$$a = -\frac{19}{3}c$$

$$14\left(-\frac{19}{3}c\right) - 3c = 1$$

$$-\frac{266}{3}c - 3\left(\frac{3}{3}\right)c = 1$$

$$-\frac{275}{3}c = 1$$

$$\boxed{c = -\frac{3}{275}}$$

$$a = -\frac{19}{3}\left(\frac{-3}{275}\right)$$

$$\boxed{a = \frac{57}{825}}$$

② $\begin{cases} 14b - 3d = 0 \\ -3b + 19d = 1 \end{cases}$

$$14b = 3d$$

$$b = \frac{3d}{14}$$

$$-3b + 19d = 1$$

$$-3\left(\frac{3}{14}d\right) + 19d = 1$$

$$-\frac{9}{14}d + 19\left(\frac{14}{14}\right)d = 1$$

$$-\frac{9}{14}d + \frac{266}{14}d = 1$$

$$\frac{257}{14}d = 1 \therefore \boxed{d = 14/257}$$

$$b = \frac{3}{\cancel{14}} \cdot \frac{\cancel{14}}{257}$$

$$\boxed{b = 3/257}$$

$$(A^TA)^{-1} = \begin{bmatrix} 57/825 & 3/257 \\ -3/275 & 14/257 \end{bmatrix}$$

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$$(A^TB) = \begin{bmatrix} 1 & 3 & -2 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix}$$

$$(A^TB) = \begin{bmatrix} 1(4) + 3(1) + (-2)3 \\ (-1)4 + 2(1) + 4(3) \end{bmatrix}$$

$$(A^TB) = \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

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$$\vec{X} = (A^TA)^{-1} \cdot (A^TB)$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{19}{275} & \frac{3}{257} \\ -\frac{3}{275} & \frac{14}{257} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{19}{275}(1) + \frac{3}{257}(10) \\ \frac{-3}{275}(1) + \frac{14}{257}(10) \end{bmatrix}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0{,}185822427 \\ 0{,}533837991 \end{bmatrix}$$