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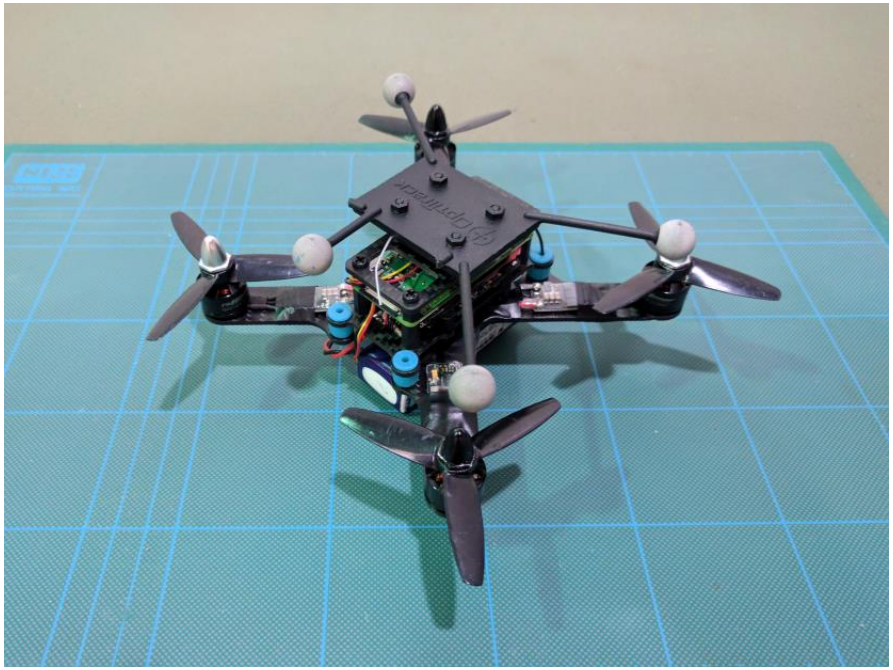

Attitude estimation for fixed-pitch quadrotor

Adaptive Adjustment of Process Noise Covariance in MEKF

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Estimation in Aerospace – Project – AY 2018-19 – Prof. Marco Lovera

Proposal

- **MEKF and process noise covariance update**

- Model the system
- Implement a Multiplicative Extended Kalman Filter (MEKF)
- Implement an adaptive adjustment of process noise covariance (matrix Q)
- Study implementation variations and their results

Error model: Kinematics

$$\dot{q} = \frac{1}{2}\Omega(\omega)q$$

$$\delta q = q \otimes \hat{q}^{-1}$$

$$\delta\omega = \omega - \hat{\omega}$$

$$\dot{\delta q} = -\begin{bmatrix} [\hat{\omega}\times]\delta\rho \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} \delta\omega \\ 0 \end{bmatrix} \otimes \delta q$$

Assumptions
- Small δq
- $\dot{q}_4 = 0, \quad q_4 \approx 1$

$$q = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} \rho \\ q_4 \end{bmatrix}$$

$$\alpha = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}$$

$$\dot{\delta\rho} = -[\hat{\omega}\times]\delta\rho + \frac{1}{2}\delta\omega$$

- Kinematics equation in terms of quaternion error (linearized)
- Need to compute estimated and angular rate error

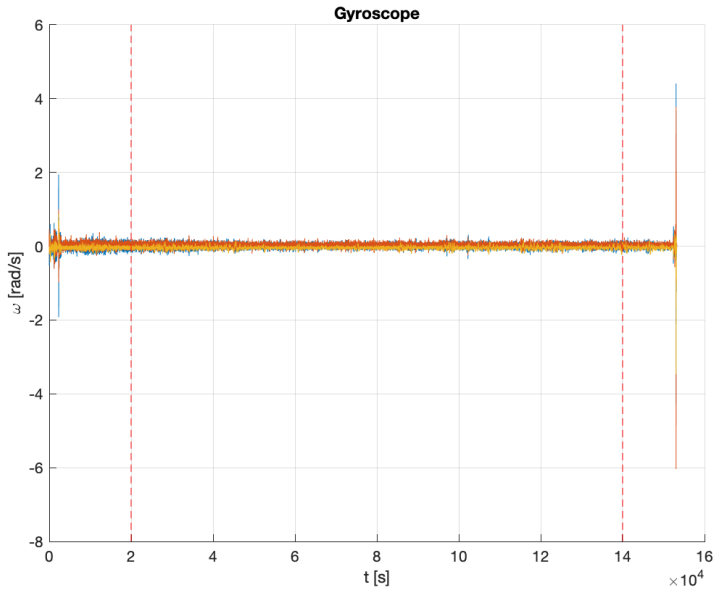
Gyroscope Noise

- **Typical model for rate gyro**

$$\tilde{\omega} = \omega + \beta + \eta_v$$

$$\dot{\beta} = \eta_w \,, \qquad \beta(0) = \beta_0$$

Fig 1: Gyroscope data.



- $\tilde{\omega}$ is the angular rate measurement
- ω is the true rate
- β is the drift (bias)
- $\eta_v \sim N(0, \sigma_v^2 \cdot I_3)$ is a white noise called Angular Random Walk (ARW)
- $\eta_w \sim N(0, \sigma_w^2 \cdot I_3)$ is a white noise called Rate Random Walk (RRW)

Error model: Bias estimation error

$$\left.\begin{aligned} \hat{\omega} &= \tilde{\omega} - \hat{\beta} \\ \dot{\hat{\beta}} &= 0 \\ \Delta\beta &= \beta - \hat{\beta} \end{aligned}\right\} \quad \delta\omega = -(\Delta\beta + \eta_v) \longrightarrow \delta\dot{\rho} = -[\hat{\omega}\times]\delta\rho - \frac{1}{2}(\Delta\beta + \eta_v)$$

- Small angles, $\delta\alpha \approx 2\delta\rho$

$$\longrightarrow \delta\dot{\alpha} = -[\hat{\omega}\times]\delta\alpha - (\Delta\beta + \eta_v)$$

- CT model: $\dot{x}(t) = A(t)x(t) + G\eta(t)$, where:

$$x(t) = \begin{bmatrix} \Delta\alpha(t) \\ \Delta\beta(t) \end{bmatrix}, \qquad A = \begin{bmatrix} -[\hat{\omega}\times] & I_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}, \qquad G = \begin{bmatrix} -I_3 & 0_3 \\ 0_3 & I_3 \end{bmatrix},$$

$$\eta(t) = \begin{bmatrix} \eta_v(t) \\ \eta_w(t) \end{bmatrix} \sim G(\underline{0}, Q), \qquad Q = \begin{bmatrix} \sigma_v^2 I_3 & 0_3 \\ 0_3 & \sigma_w^2 I_3 \end{bmatrix}$$

Error model: Attitude measurements

- Accelerometer and magnetometer measurements

$$b_{acc} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \qquad b_{mag} = \begin{bmatrix} m_x \\ m_y \\ m_z \end{bmatrix}, \quad \text{both } \underline{\text{normalized}}$$

- Noise v, $\quad v = \begin{bmatrix} v_{acc} \\ v_{mag} \end{bmatrix} \sim G(\underline{0}, R), \quad R = \begin{bmatrix} \sigma_{acc}^2 I_3 & 0_3 \\ 0_3 & \sigma_{mag}^2 I_3 \end{bmatrix}$

- Accelerometer and magnetometer reference vectors

$$r_{acc} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \qquad h_{mag} = \begin{bmatrix} h_x \\ h_y \\ h_z \end{bmatrix} = A(\hat{q}^-)^T b_{mag} \rightarrow r_{mag} = \begin{bmatrix} \sqrt{h_x^2 + h_y^2} \\ 0 \\ h_z \end{bmatrix}$$

Discretization

- Propagation step → high sampling rate
- Discrete time model:

$$F_k = \begin{bmatrix} I_3 - [\widehat{\omega}_k \times]\delta t & -I_3 \delta t \\ 0_3 & I_3 \end{bmatrix},$$

$$Q_k = \begin{bmatrix} (\sigma_v^2 \delta t + \frac{1}{3}\sigma_w^2 \delta t^3) & (\frac{1}{2}\sigma_w^2 \delta t^2) I_3 \\ (\frac{1}{2}\sigma_w^2 \delta t^2) I_3 & (\sigma_w^2 \delta t) I_3 \end{bmatrix}$$

- With the estimated variances, it corresponds to the estimated Q matrix

Gyroscope Noise

- **Allan Variance**

$$\theta(t) = \int^{t} \omega(t')\, dt'$$

$$\sigma^2(\tau) = \frac{1}{2\tau^2}\langle(\theta_{k+2m} - 2\theta_{k+m} + \theta_k)^2\rangle$$

$$\tau = m \cdot T_s$$

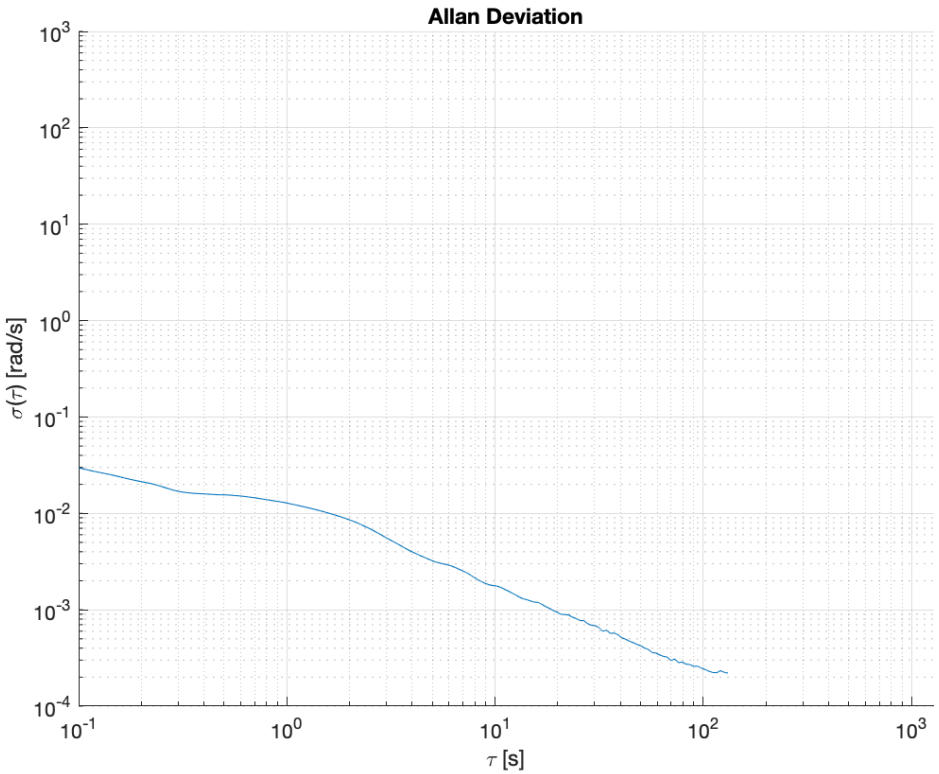


Fig 2: Allan deviation of the gyroscope data.

Gyroscope Noise

▪ ARW and RRW

- ARW estimation

$$S_\omega(f) = \sigma_v^2$$

$$\sigma^2(\tau) = \frac{\sigma_v^2}{\tau}$$

$$\sigma_v \simeq 9.600\times10^{-3}\ [rad/s]$$

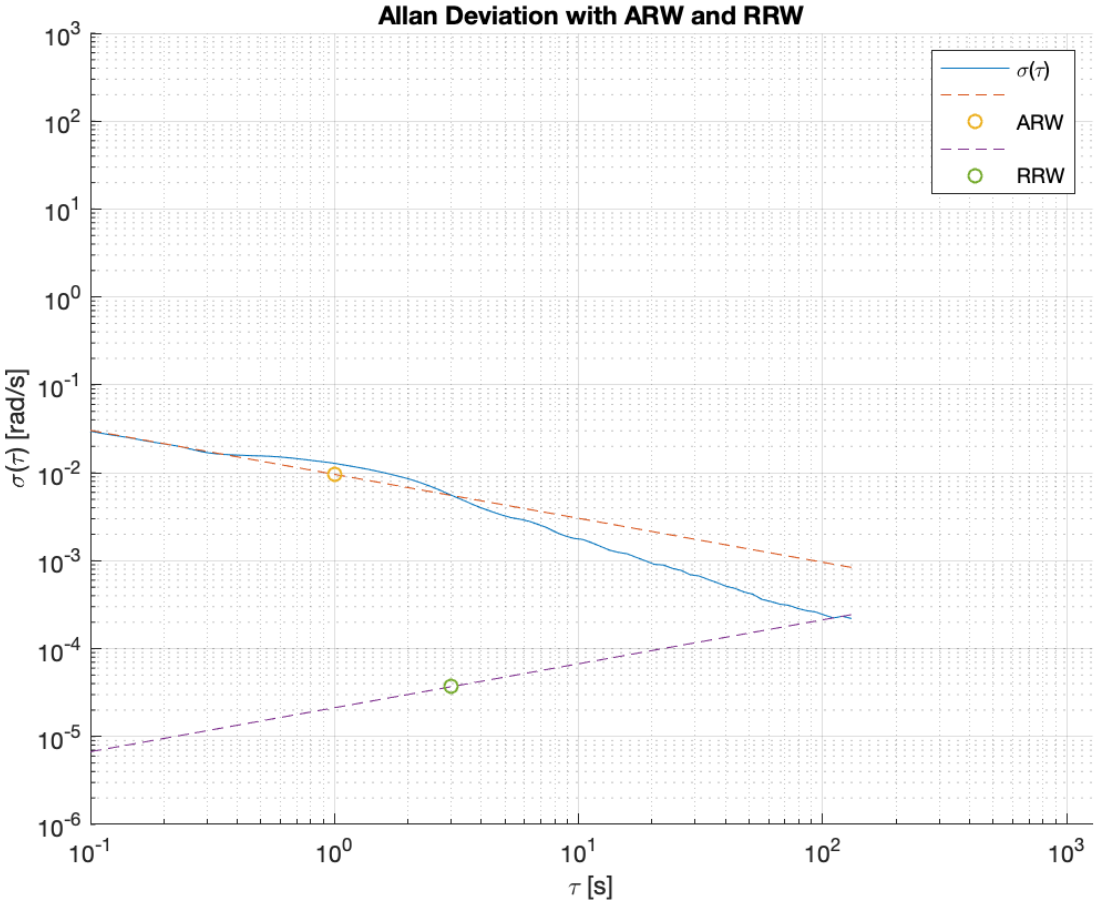
- RRW estimation

$$S_\omega(f) = \left(\frac{\sigma_w}{2\pi}\right)^2 \frac{1}{f^2}$$

$$\sigma^2(\tau) = \frac{\sigma_w^2 \cdot \tau}{3}$$

$$\sigma_w \simeq 3.684\times10^{-5}\ [rad/s^2]$$

Fig 3: Allan deviation, ARW and RRW.



Multiplicative Extended Kalman Filter

▪ Introduction

- The extended Kalman filter (EKF) is the nonlinear version of the Kalman filter.
- It linearizes around an estimate of the current mean and covariance.
- The Multiplicative EKF (MEKF) represents the attitude as the quaternion product $\hat{q} = \delta\hat{q}(\alpha) \otimes \hat{q}_{ref}$.

Multiplicative Extended Kalman Filter

- **Propagation step**

$$\widehat{\omega}_k = \widetilde{\omega}_k - \hat{\beta}_{k-1}^{+}$$

$$\hat{q}_k^{-} = \hat{q}_{k-1}^{+} + \dot{\hat{q}}_{k-1}^{+}\delta t = \left(I_4 + \frac{1}{2}\Omega(\widehat{\omega}_k)\delta t\right)\hat{q}_{k-1}^{+}$$

$$\hat{\beta}_k^{-} = \hat{\beta}_{k-1}^{+}$$

$$P_k^{-} = F_k P_{k-1}^{+} F_k^T + G Q_{k-1} G^T$$

Multiplicative Extended Kalman Filter

- **Update step (1/2)**

$$H_k = \begin{bmatrix} [A(\hat{q}_k^-) r_{acc} \times] & 0_3 \\ [A(\hat{q}_k^-) r_{mag} \times] & 0_3 \end{bmatrix}$$

$$K_k = P_k^- H_k^T \left(H_k P_k^- H_k^T + R \right)^{-1}$$

$$P_k^+ = (I_6 - K_k H_k) P_k^- (I_6 - K_k H_k)^T + K_k R K_k^T$$

Multiplicative Extended Kalman Filter

- **Update step (2/2)**

$$e_k = \begin{bmatrix} b_{acc} \\ b_{mag} \end{bmatrix} - \begin{bmatrix} A(\hat{q}_k^-) r_{acc} \\ A(\hat{q}_k^-) r_{mag} \end{bmatrix}$$

$$\Delta \hat{x}_{\mathrm{k}}^+ = K_k \cdot e_k$$

$$\hat{q}_k^+ = \delta \hat{q}_k^+ \otimes \hat{q}_k^-$$

$$\hat{\beta}_k^+ = \hat{\beta}_k^- + \Delta \hat{\beta}_k^+$$

Innovation Based Adaptive estimation of Q

- **Proposal**

$$w_{k-1} = x_k - f(x_{k-1}, u_{k-1})$$

$$\begin{aligned}\widehat{w}_{k-1} &= \hat{x}_k^+ - f(\hat{x}_{k-1}^+, u_{k-1}) \\ &= x_k^+ - \hat{x}_k^- = \overline{K}_k[y_k - h_k(\hat{x}_k^-)] \\ &= \overline{K}_k \cdot e_k\end{aligned}$$

$$\begin{aligned}G\hat{Q}_{k-1}G^T &= E\left[\widehat{w}_{k-1}\widehat{w}_{k-1}^T\right] = E\left[\overline{K}_k\left(e_k e_k^T\right)\overline{K}_k^T\right] \\ &= \overline{K}_k\, E\left[e_k e_k^T\right]\overline{K}_k^T\end{aligned}$$

Innovation Based Adaptive estimation of Q

▪ Implementation

- Forgetting factor α

$$Q_k = \alpha Q_{k-1} + (1-\alpha) G^T \left(K_k \, e_k \, e_k^T \, K_k^T \right) \mathrm{G}$$

- Mean value with window of size m

$$Q_k = G^T \, K_k \left(\frac{1}{m} \sum_{i=k-m+1}^{k} e_i \, e_i^T \right) K_k^T \, G$$

Filtering

▪ Parameters

- Log file of ~ 200 seconds of gyroscope, accelerometer and magnetometer.
- Selected time interval from 0 to 150 seconds and interpolated measurements for a sampling time of 0.004 seconds.
- Used estimated noise variances.
- Initial quaternion estimated through a constrained minimization using the initial measurements.
- Initial bias corresponds to the initial measurement from the gyroscope.
- $P_0 = I_6 \times 10^{-1}$ and $Q_0 = \begin{bmatrix} I_3 & I_3 \\ I_3 & I_3 \end{bmatrix} \times 10^{-8}$.

MEKF results – Quaternions

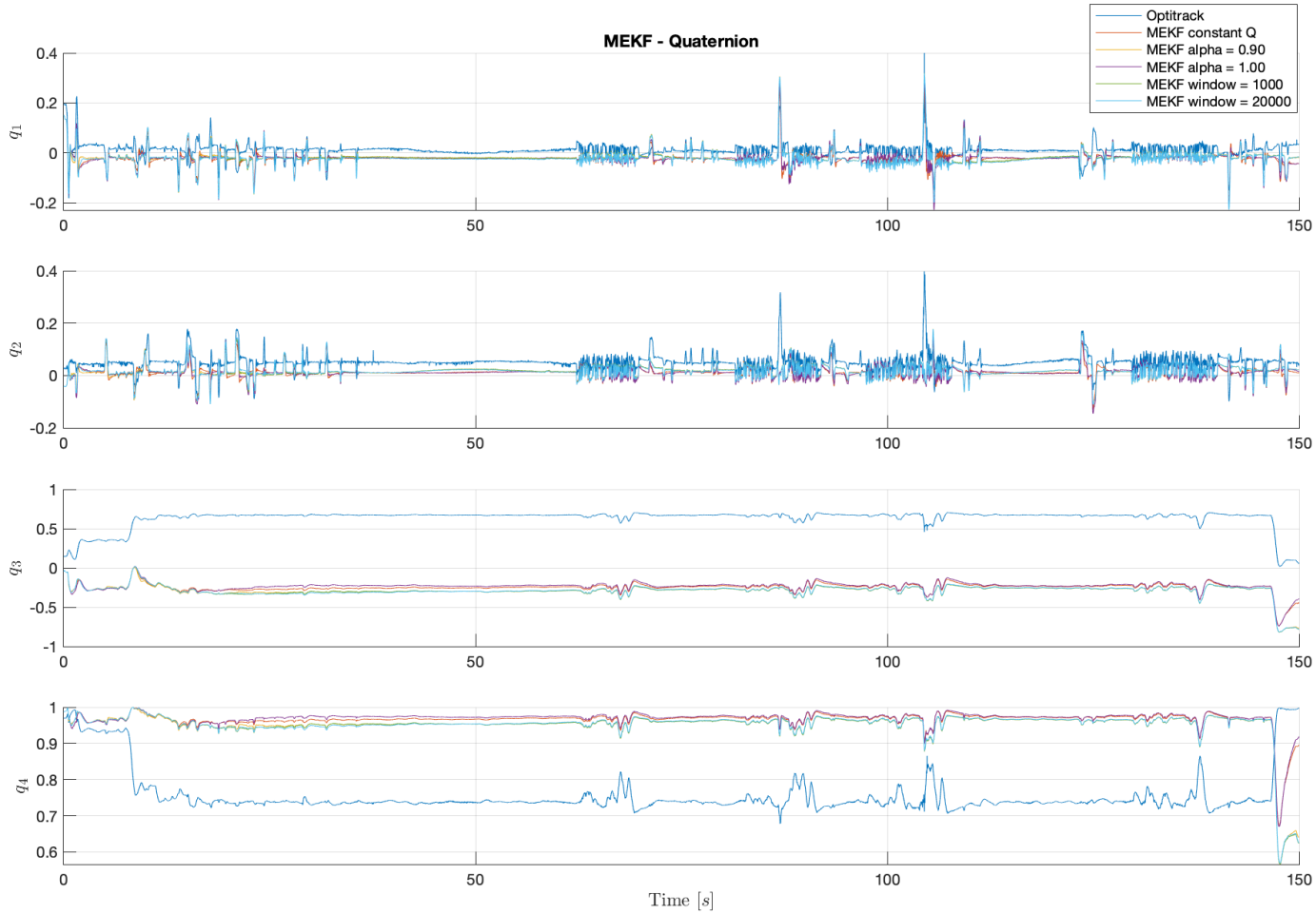


Fig 6: Attitude quaternions <u>for some values of alpha and window size</u>

MEKF results – Euler angles

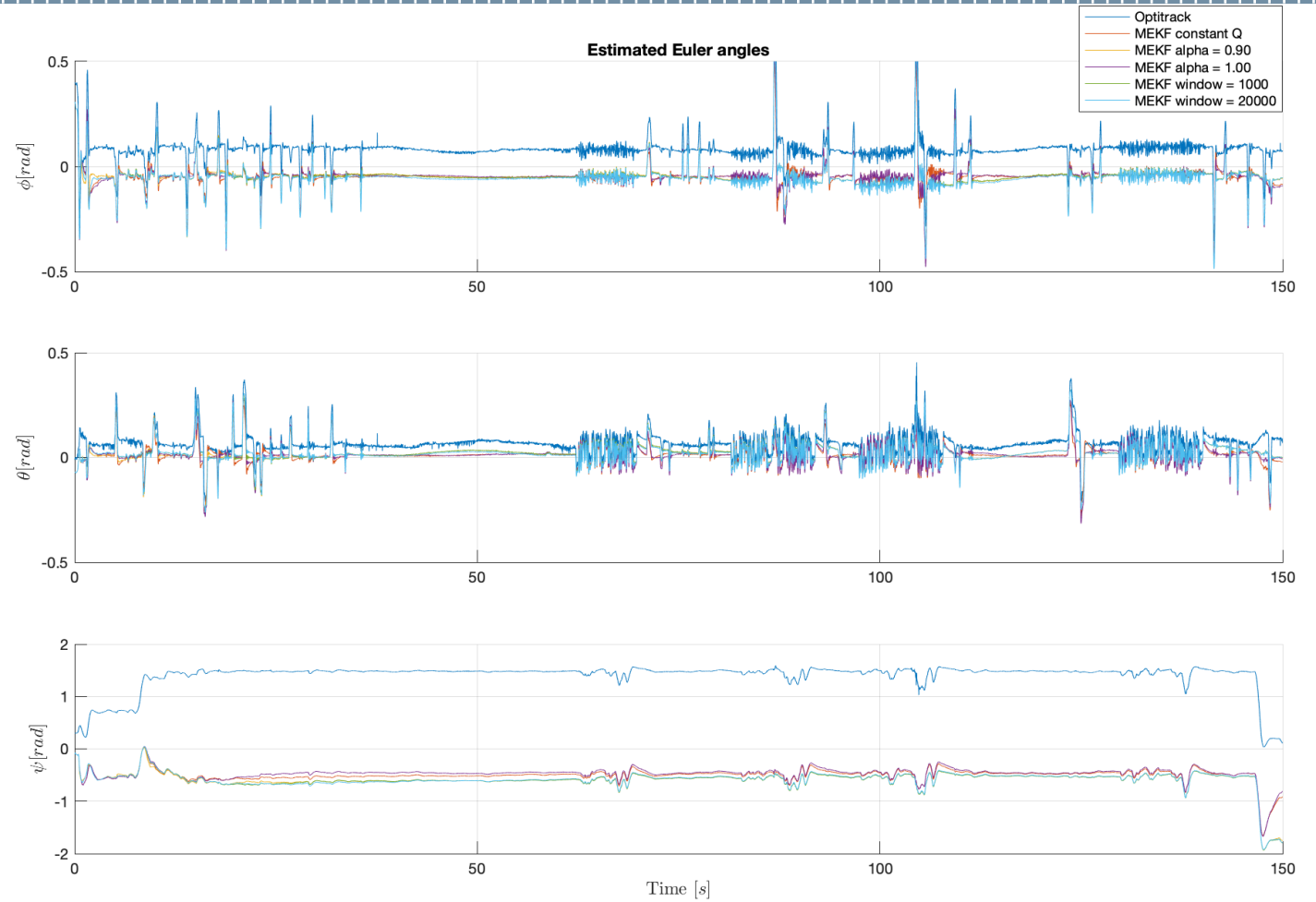


Fig 7: Attitude Euler angles <u>for some values of alpha and window size</u>

MEKF results – Estimated angular rate

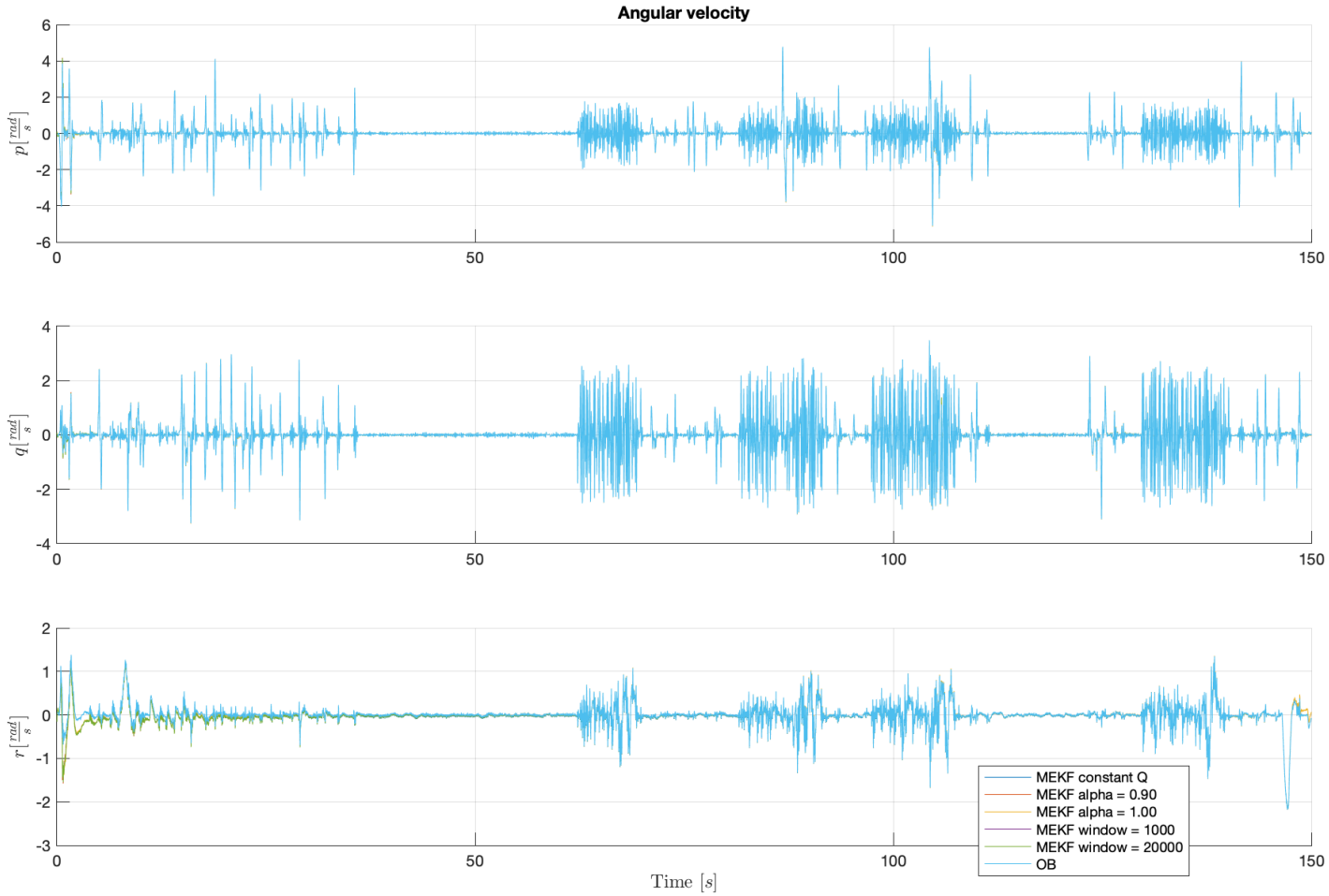


Fig 8: Estimated angular velocity <u>for some values of alpha and window size</u>

MEKF results – Estimated gyro bias

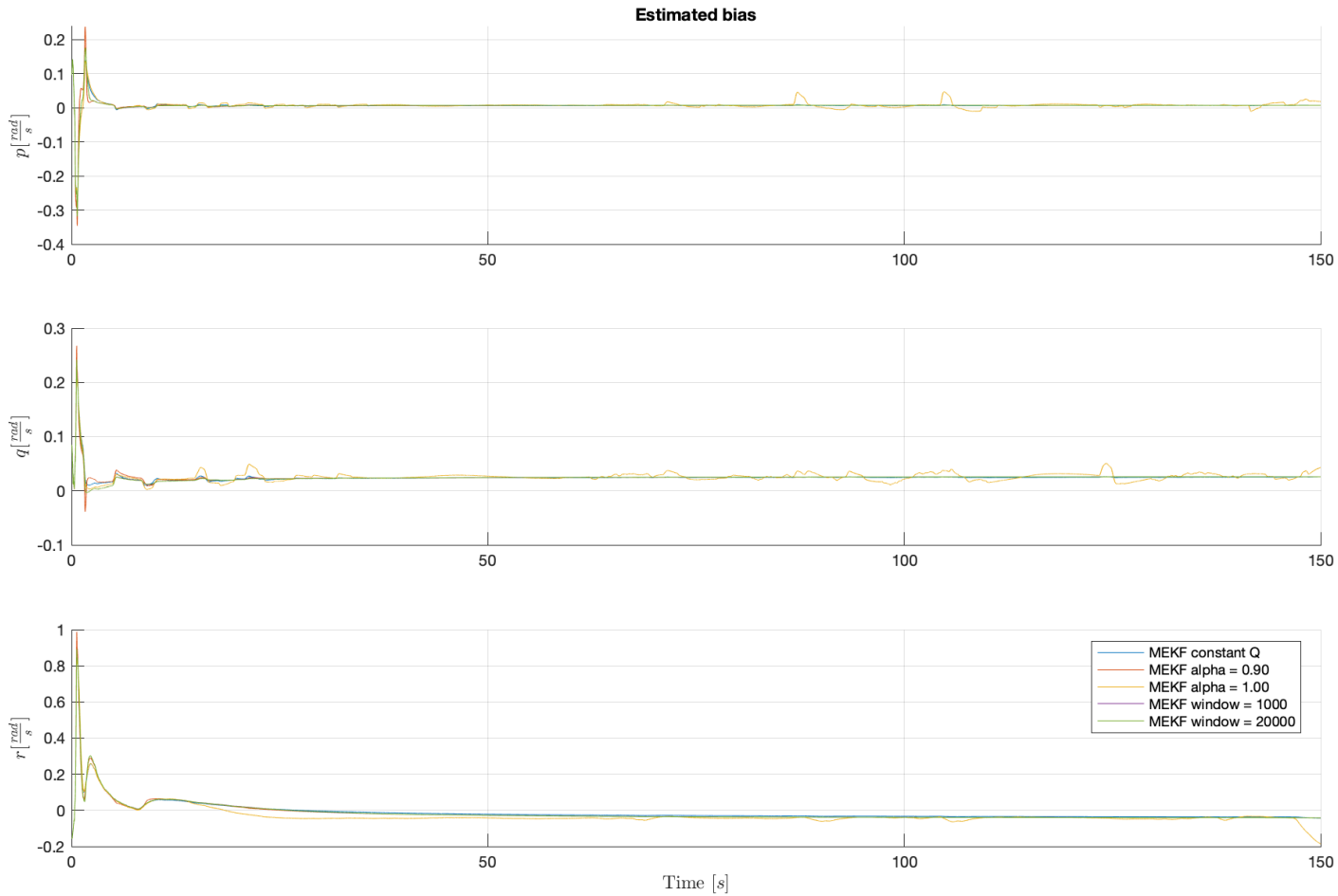


Fig 9: Estimated gyroscope drift <u>for some values of alpha and window size</u>

Adaptive estimation of Q – trace over time

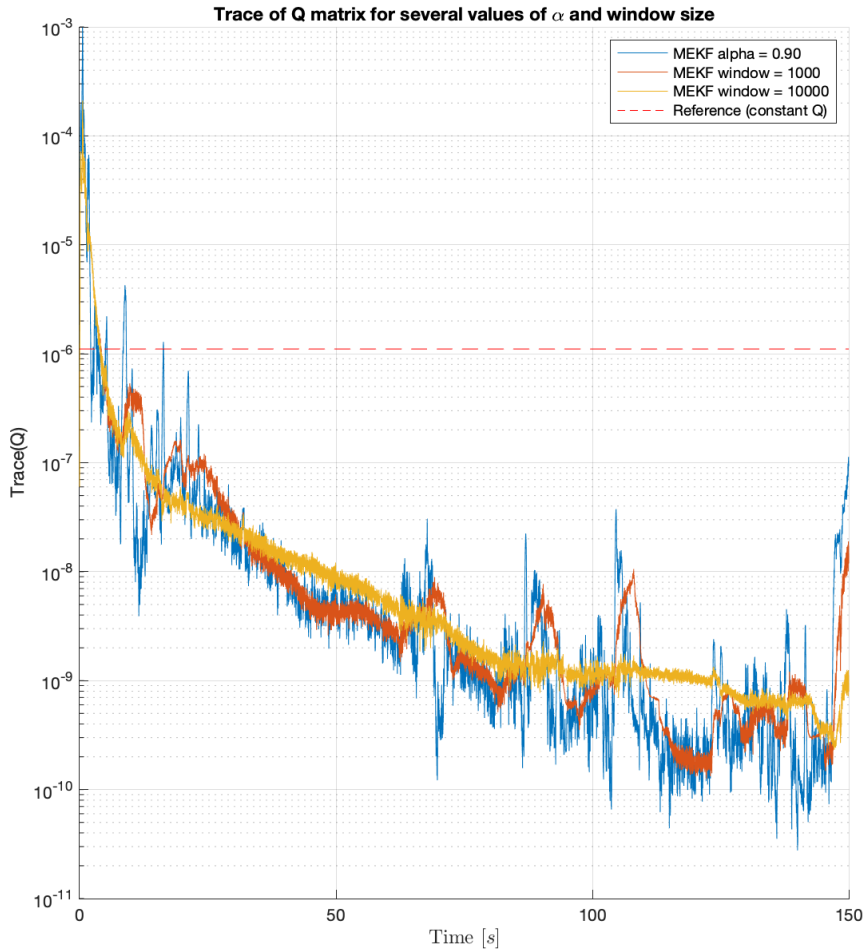


Fig 10: Trace of Q matrix over time

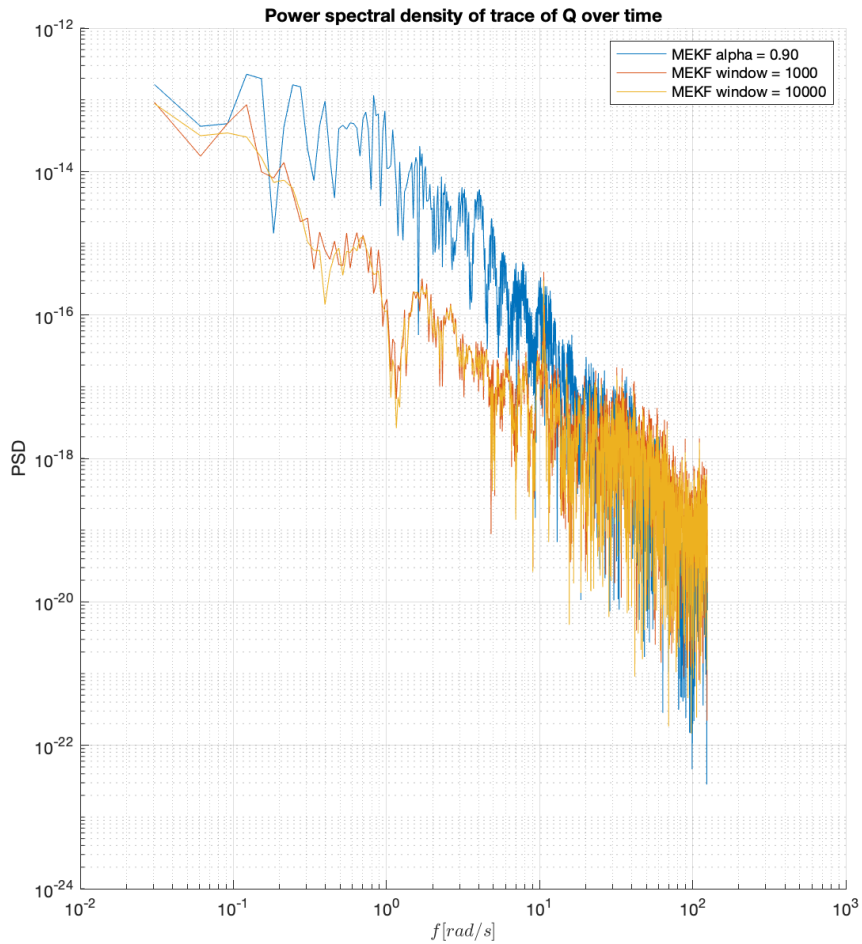


Fig 11: PSD of the trace of Q

Adaptive estimation of Q – mean and variance of the trace of Q

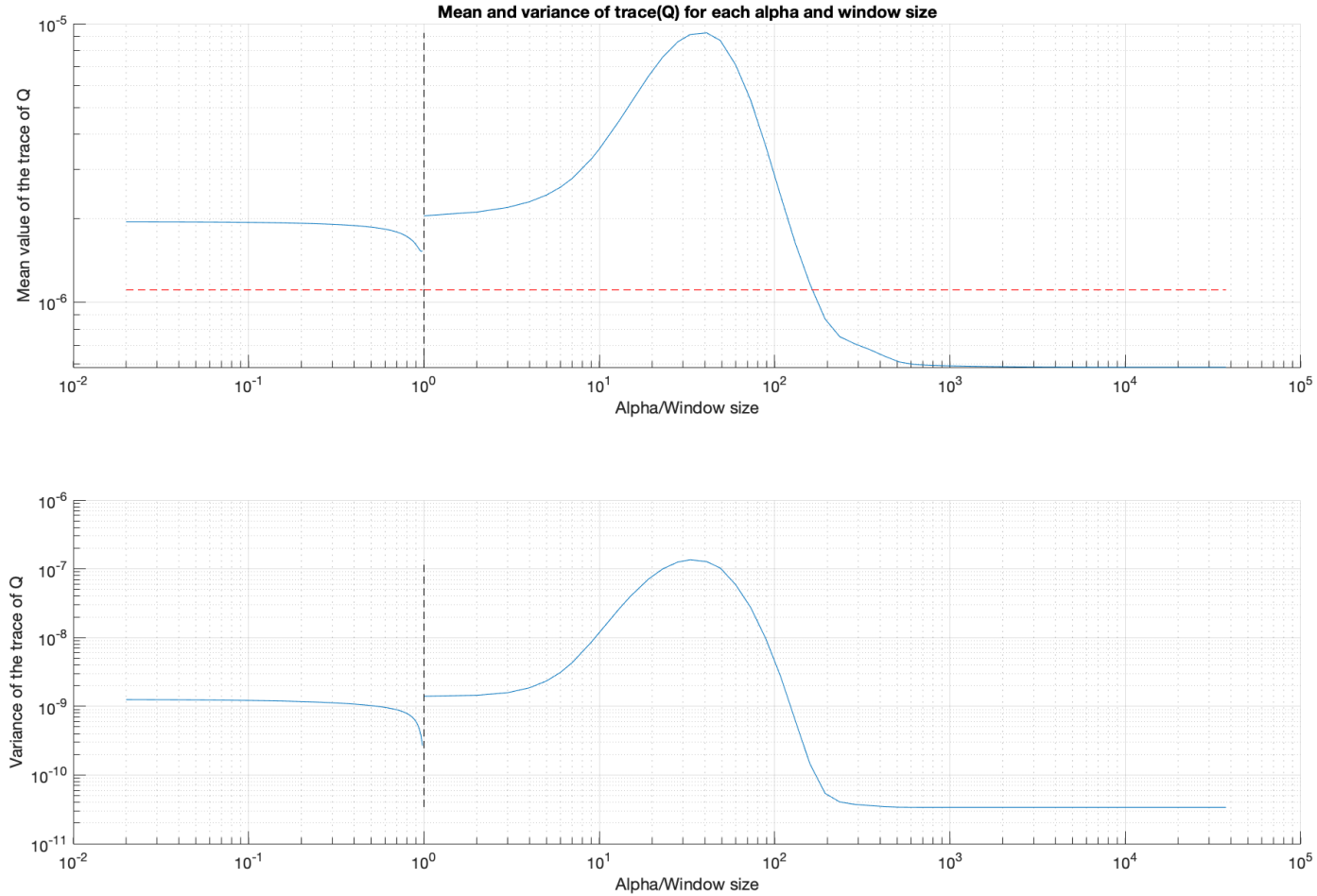


Fig 12: Mean and variance of the trace of Q as function of alpha and the window size

Attitude Error (MEKF vs OptiTrack)

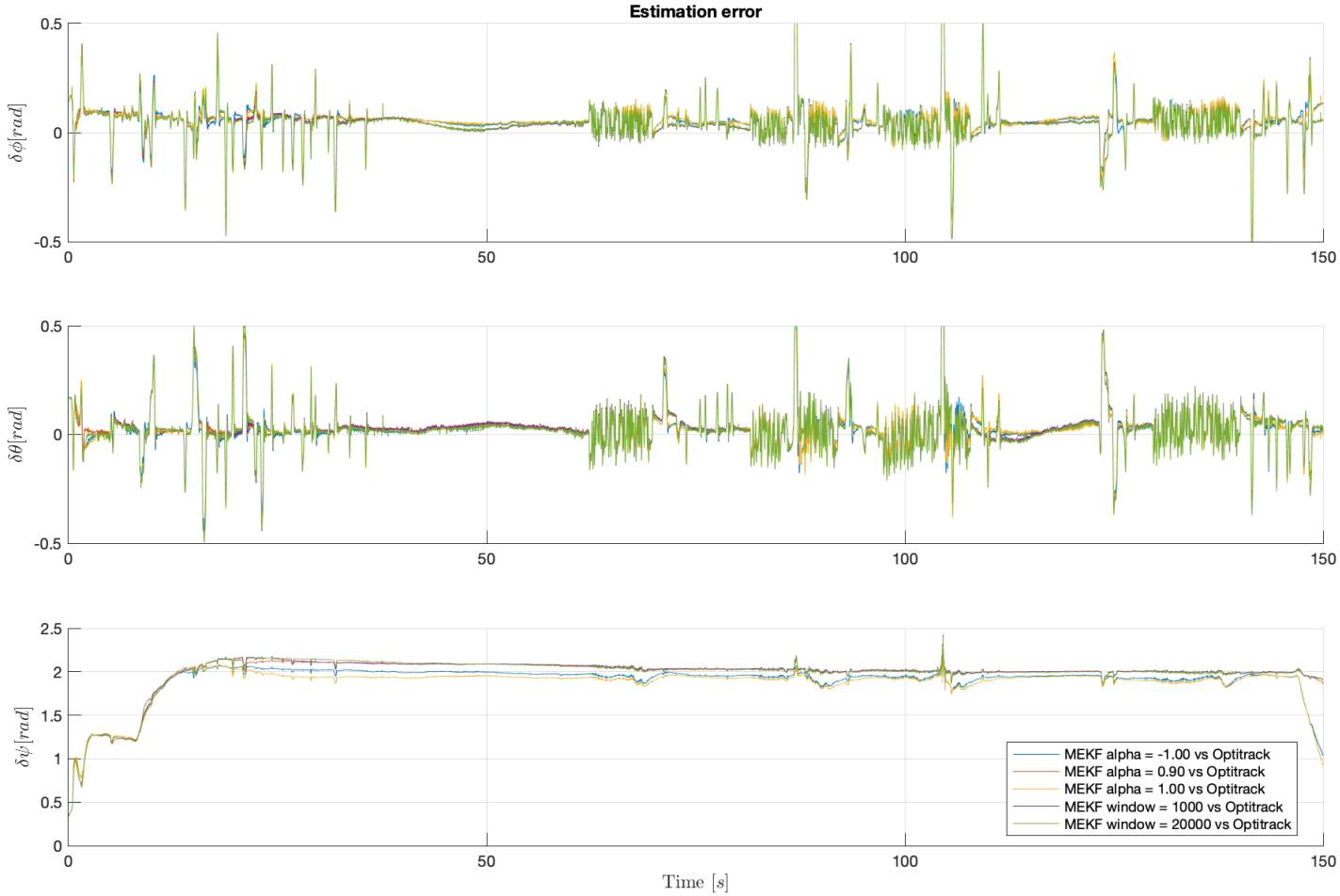


Fig 13: Estimated Euler angles error <u>for some values of alpha and window size</u>

Attitude Error (MEKF vs OptiTrack)

- **Forgetting factor α**

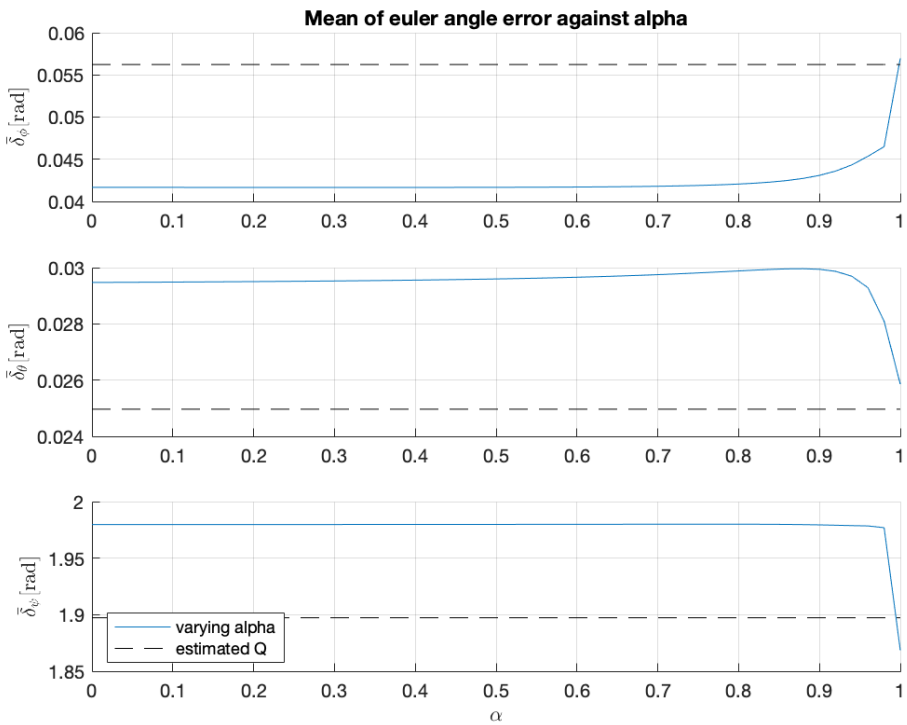


Fig 14: Mean value of Euler angles error in function of alpha.

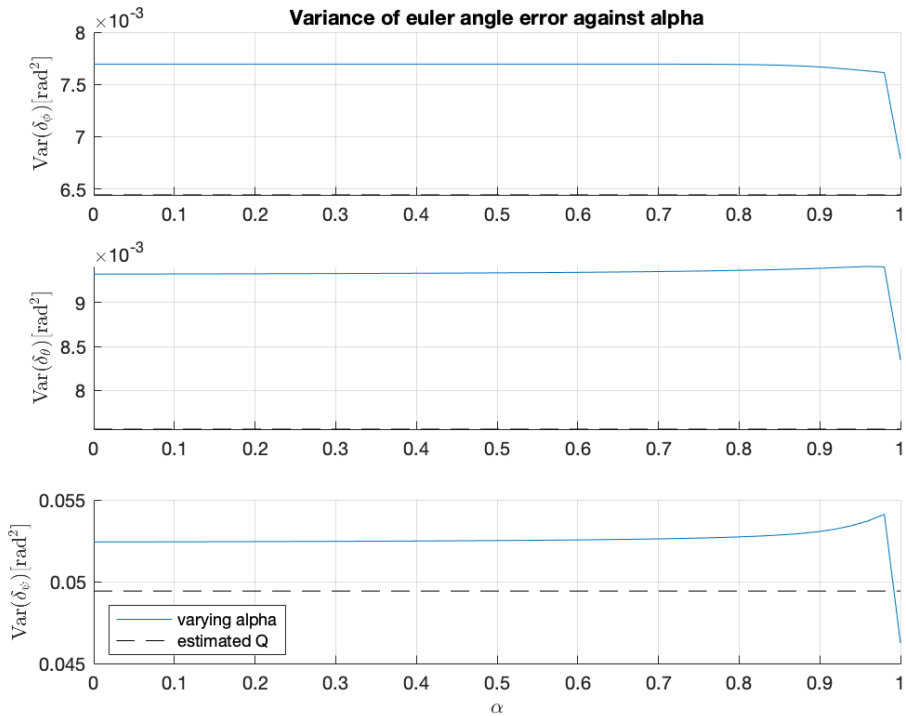


Fig 15: Variance of Euler angles error in function of alpha.

Angular Rate Error (MEKF vs Onboard)

- **Forgetting factor α**

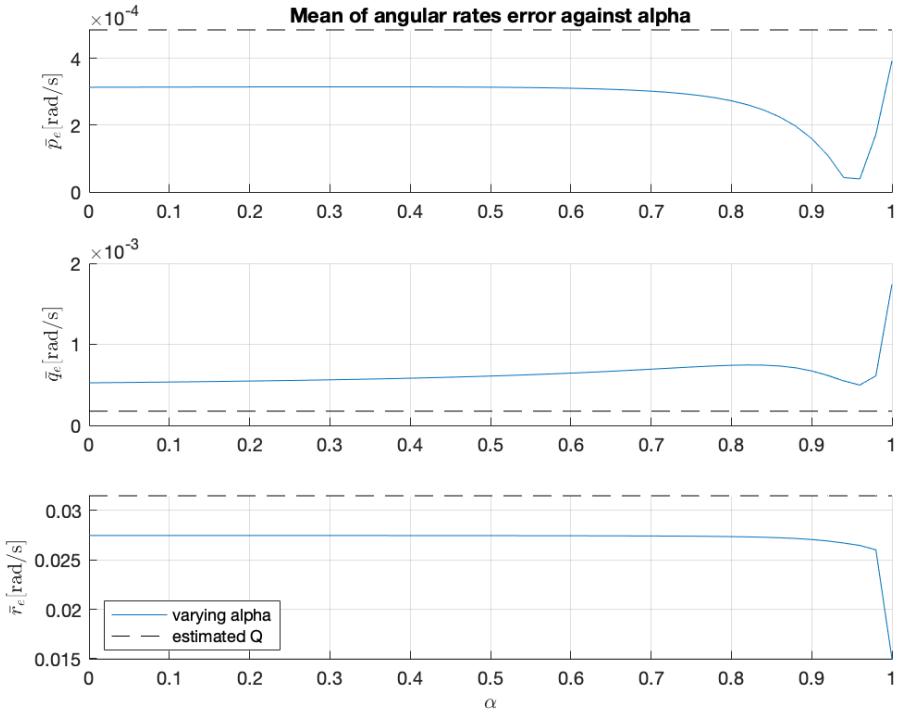


Fig 16: Mean value of angular rates error in function of alpha.

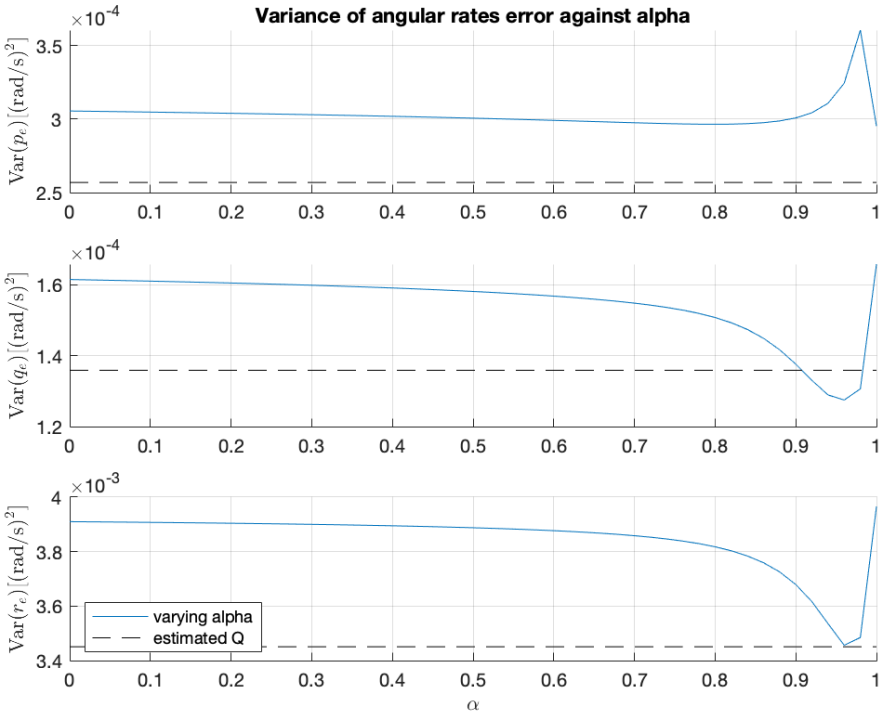


Fig 17: Variance of angular rates error in function of alpha.

Attitude Error (MEKF vs OptiTrack)

- **Mean value with window of size m**

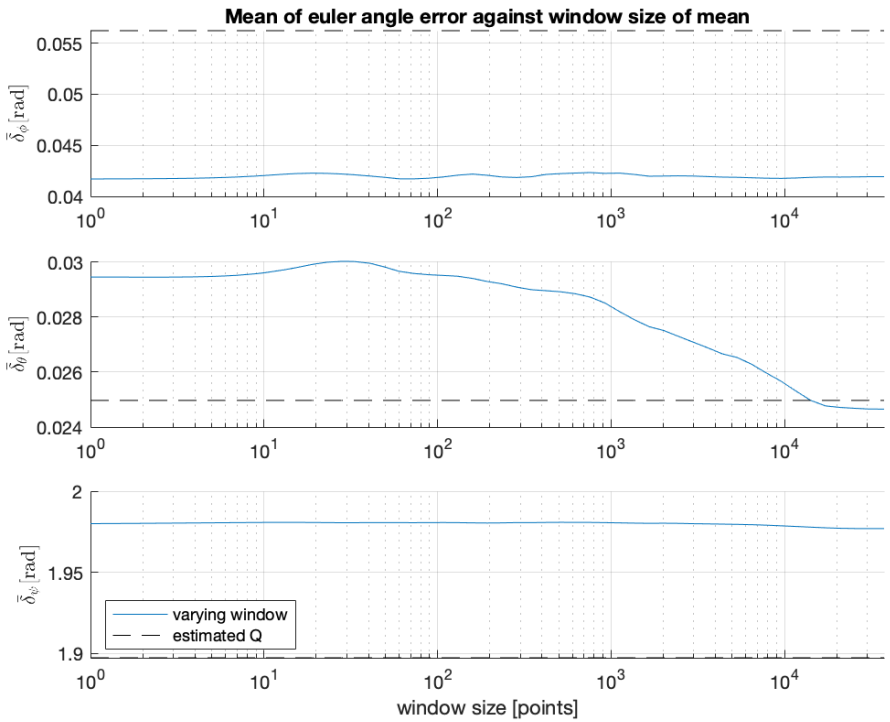


Fig 18: Mean value of Euler angles error in function of window.

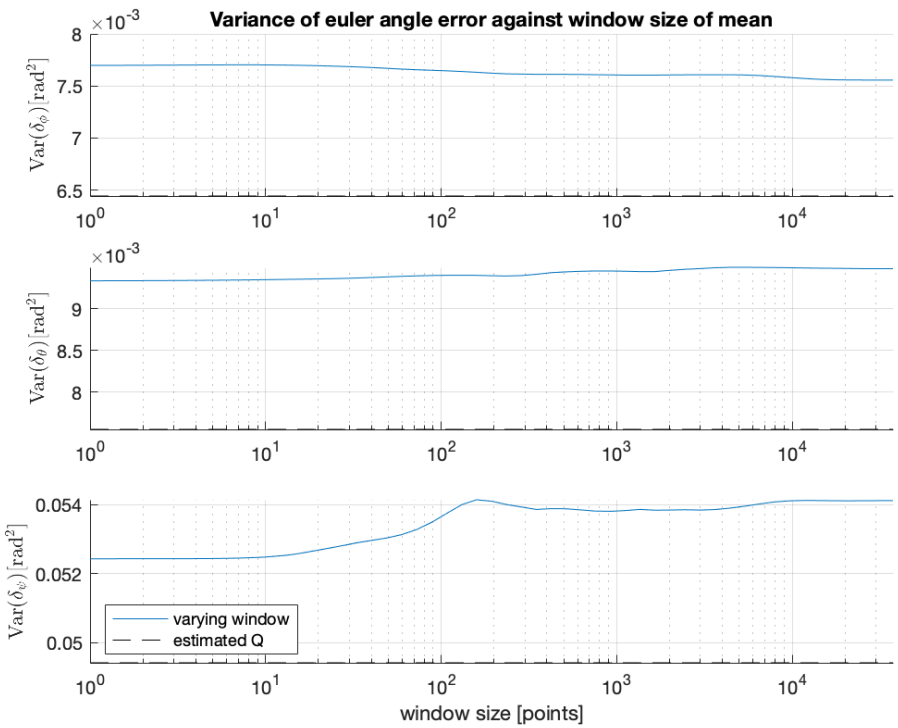


Fig 19: Variance of Euler angles error in function of window.

Angular Rate Error (MEKF vs Onboard)

- **Mean value with window of size m**

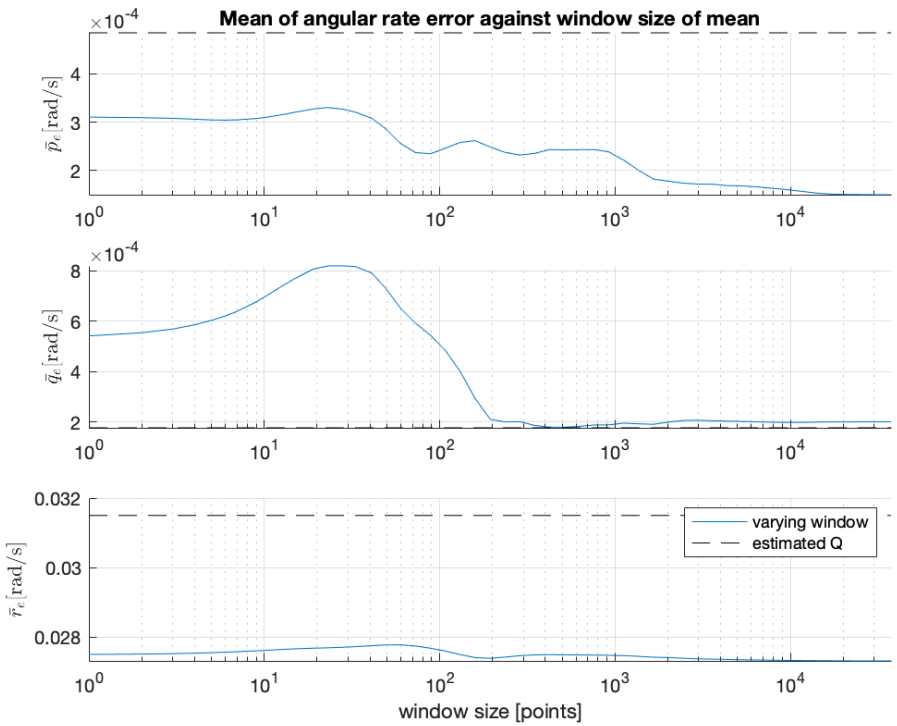


Fig 20: Mean value of angular rates error in function of window.

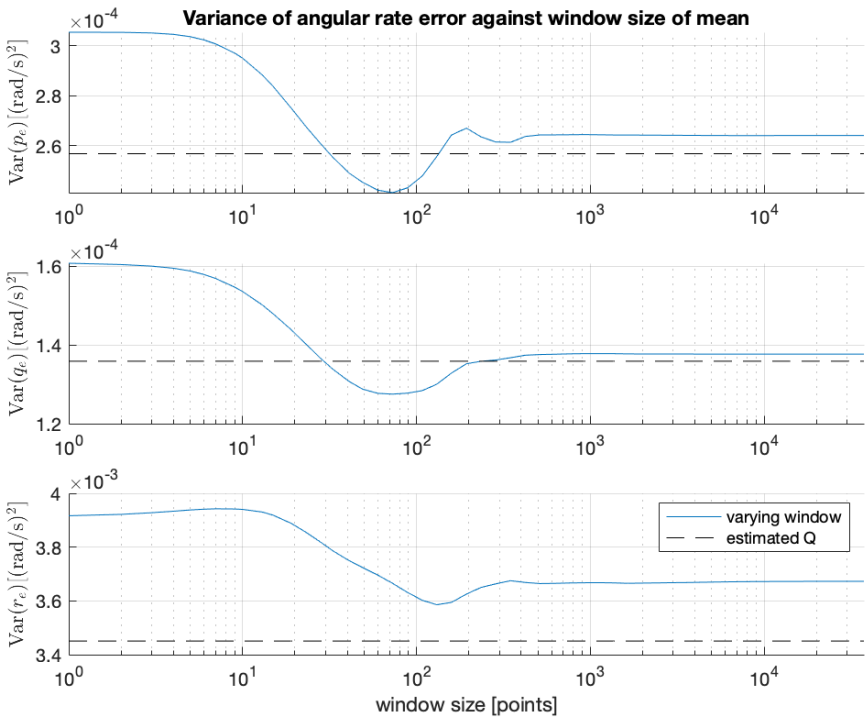


Fig 21: Variance of angular rates error in function of window.

Comparison: forgetting factor vs mean value

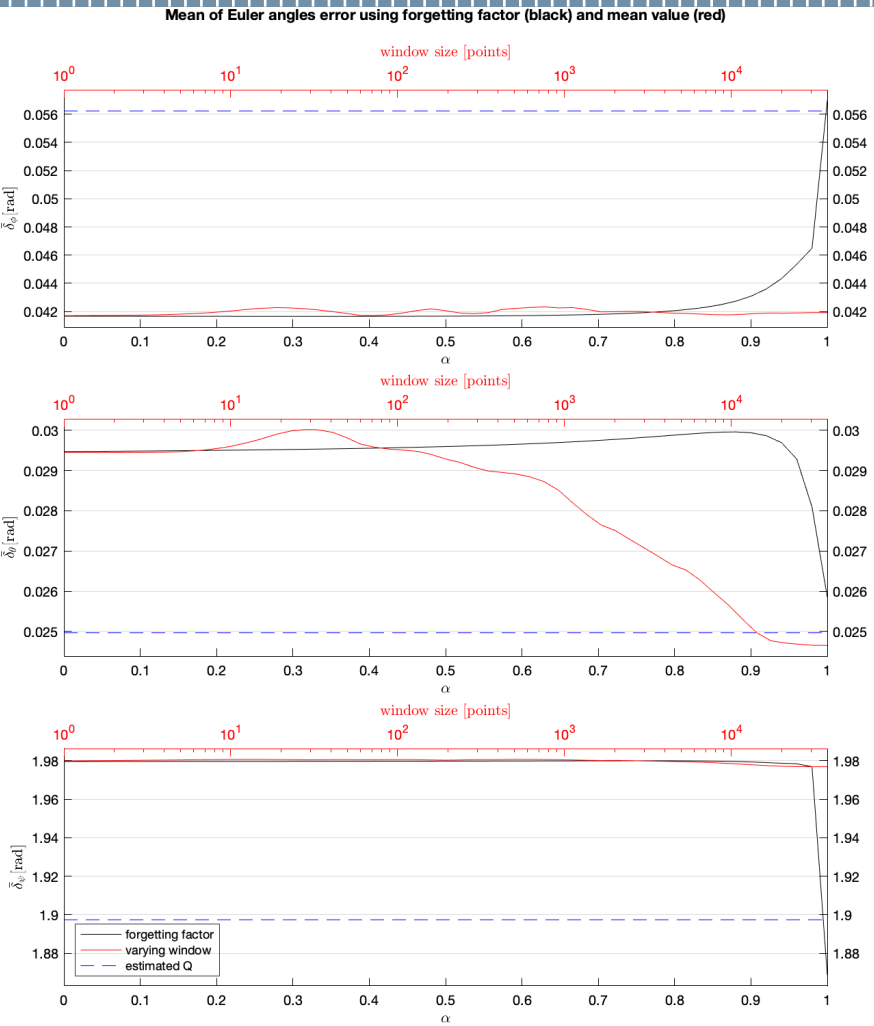


Fig 22: Mean value of Euler angles error.

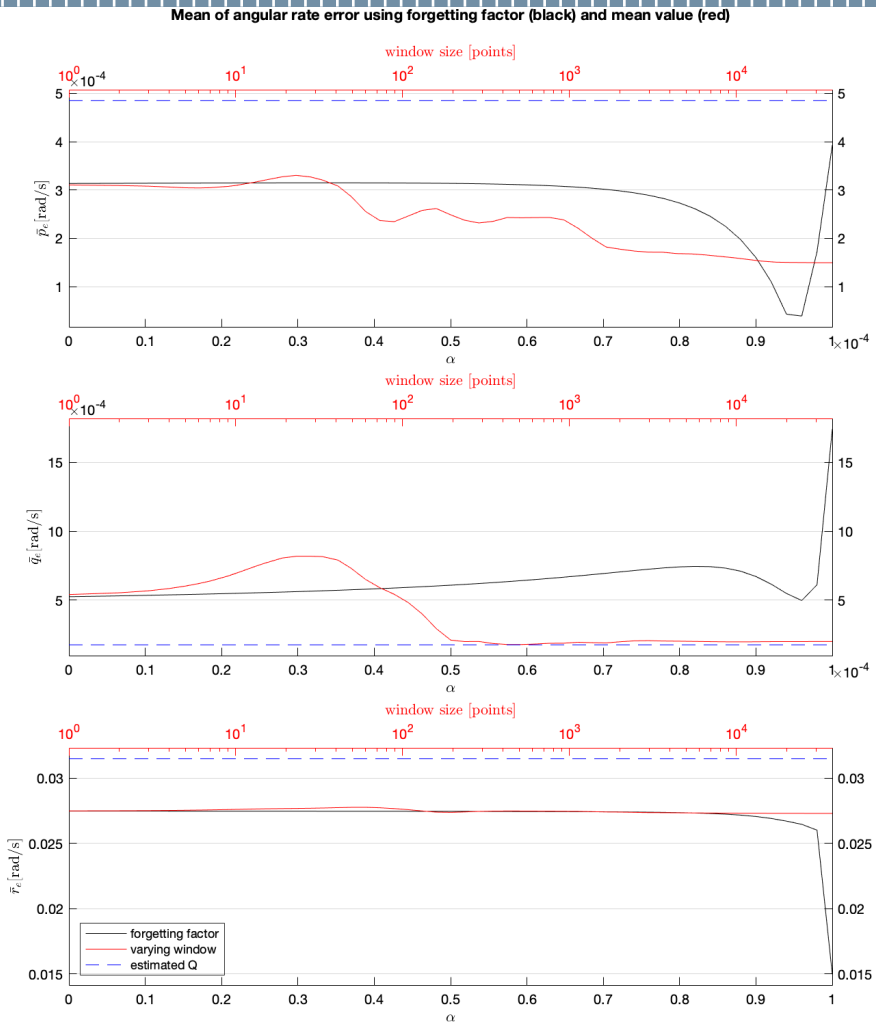


Fig 23: Mean value of angular rates error.

Conclusions

- Adaptive estimation of matrix Q is possible, although the results weren't as good as in the paper.
- The implementation using mean value with a window gave, generally, better results than using forgetting factor.
- Proposals to improve the validation of the method: longer dataset, better magnetometer model, update matrix R, …