

# Final test

FU AI Club

March 22, 2021

## 1 Matrix Fundamentals (30pts)

- (20 points) Given matrix  $A = \begin{bmatrix} 2 & 1 \\ 5 & 3 \end{bmatrix}$  and  $f(x) = 3x^2 - 5x + 1$ 
  - (10 points) Calculate  $f(A)$
  - (10 points) Find matrix  $X$  so that  $(5A^2 - A^3)X = A^T$
- (10 points) Let  $a_1, a_2, \dots, a_n$  be numbers with  $n > 2$ . Prove that

$$\det \begin{bmatrix} 1 & a_1 & a_1^2 & \cdots & a_1^{n-1} \\ 1 & a_2 & a_2^2 & \cdots & a_2^{n-1} \\ 1 & a_3 & a_3^2 & \cdots & a_3^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & a_n & a_n^2 & \cdots & a_n^{n-1} \end{bmatrix} = \prod_{1 \leq j < i \leq n} (a_i - a_j) \quad (1)$$

You can use this theorem in your proof:

**Theorem 1.1 (Remainder theorem)** *If  $f(x)$  is a polynomial of degree  $n \geq 1$  and  $a$  is any number, then there exists a polynomial  $q(x)$  such that*

$$f(x) = (x - a)q(x) + f(a) \quad (2)$$

where  $\deg(q(x)) = n - 1$

## 2 Matrix Calculus (30pts)

- (15 points) Let  $y = 2x^3 - f(x)$  and suppose that  $f(-1) = -3$  and  $\frac{dy}{dx} = 4$  when  $x = -1$ . Find  $f'(-1)$
- (15 points) Given a matrix  $A \in R^{m \times n}$  and vector  $x \in R^{n \times 1}$ . Let  $y = Ax$ 
  - (7.5 points) Find derivative of  $y$  with respect to  $x$
  - (7.5 points) Find derivative of  $y$  with respect to  $A$

## 3 Probability (30pts)

- (10 points) In FPT University, 56% of students are men and 44% are women. A study on e-cigarette conducted here showed that 9.5% males have smoked e-cigarettes before, whereas only 1.9% of females have smoked e-cigarettes.
  - (5 points) Suppose we choose a random person within the university, find the probability of him/her having smoked e-cigarettes.
  - (5 points) If a random male is chosen, what is the probability of him having smoked e-cigarettes?

2. (10 points) Let  $X$  be a random variable with PDF given by

$$f_X(x) = \begin{cases} k(2x+1)^3 & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

- (a) (3 points) Find the constant  $k$
  - (b) (4 points) Find  $E(X)$  and  $Var(X)$
  - (c) (3 points) Find  $P(X \geq \frac{1}{3})$
3. (10 points) FAIC math training classes start at 19:30 every Monday and Thursday. Our observation shows that the arrival time of students approximately follows a normal (Gaussian) distribution with mean time of 19:30 and standard deviation of 7 minutes. What is the percentage of students arriving late by ***at least*** 10 minutes?