SOMA Cube Presentation

Aryaman Srikant, Geon Kim, Tri Do

University of Illinois at Urbana Champaign

May 8, 2023

Background Information

What is the SOMA cube puzzle

The SOMA cube puzzle is 7 separate pieces that when formed together it forms a 3-by-3-by-3 cube.

- The 7 pieces are pictured below:

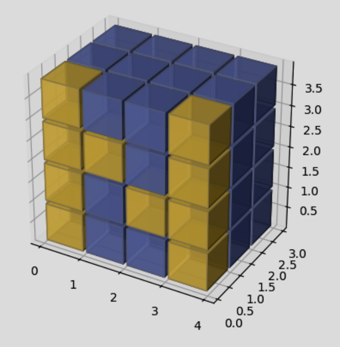
- Goal: Find all solutions (240 distinct) with step-by-step visualization of forming each solution

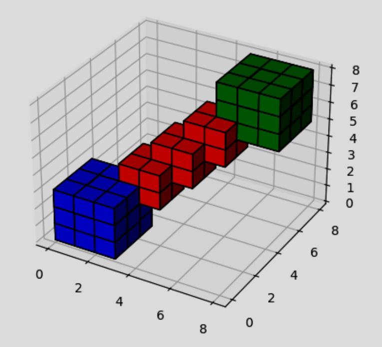
External Library for Visualization

For 3D graphing in Matplotlib, we use the Axes3D.voxels function

```
from mpl_toolkits.mplot3d import Axes3D
```

Example plots from matplotlib documentation:

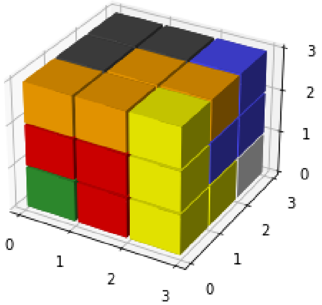




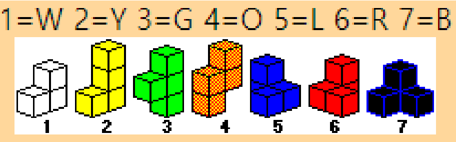
Visualization Steps

How to determine order?

- Given: A 3d-array of characters (represents solution)
- Each character represents different pieces by color



```
cube = np.array([[['B','B','L'],
                  ['B','O','O'],
                  ['O','O','Y']],
                 [['B','W','L'],
                  ['R','L','L'],
                  ['R','R','Y']],
                 [['G','W','W'],
                  ['G','G','Y'],
                  ['G','R','Y']]])
```



Visualization Steps

How to determine order?

- Goal: Find the order of pieces in a physically possible way
  1. Starting from the bottom layer, sort a set of characters by decreasing frequency
  2. Add characters in the output list in order
  3. Do the same for middle, top layer

```
from collections import Counter

def place_order(solution):
    order = []
    for layer in solution[::-1]:
        frequencies = Counter(layer.flatten())
        piece_sorted_by_freq = [elem for elem, freq in frequencies.most_common()]
        for piece in piece_sorted_by_freq:
            if piece not in order:
                order.append(piece)
    return order
```

- Returned list: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 1: Add 'Green' shape"

- Update the 'canvas' of 3x3x3 cube for each iteration in order
- Initialized with transparent canvas
- Iteration: Get index of characters in solution, update canvas at index with corresponding color

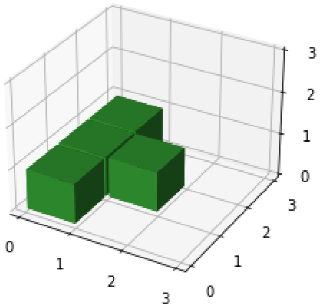


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 2: Add 'White' shape"

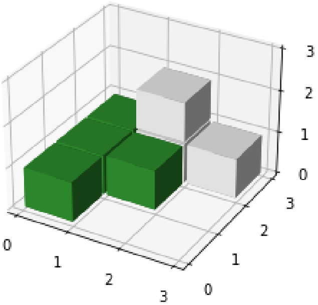


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 3: Add 'Yellow' shape"

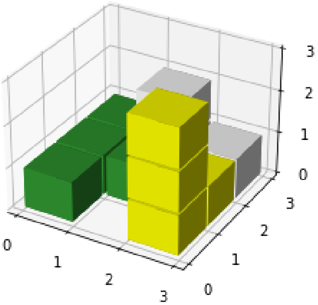


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 4: Add 'Red' shape"

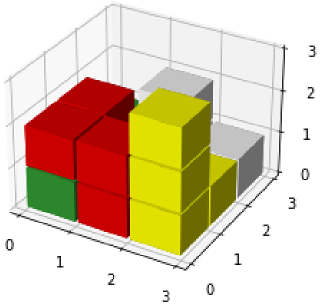


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 5: Add 'blue' shape"

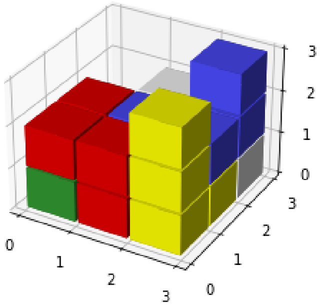


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 6: Add 'Black' shape"

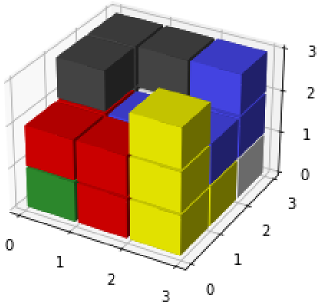


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"Step 7: Add 'Orange' shape"

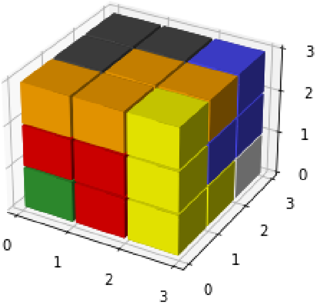
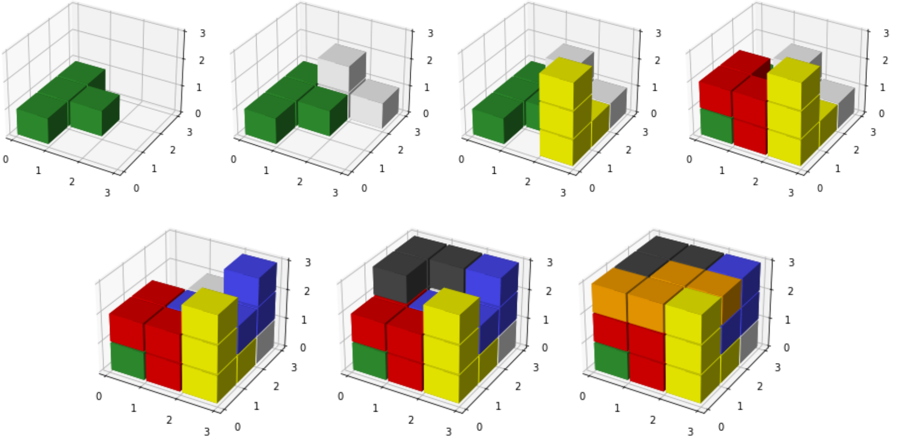


Figure: ['G', 'W', 'Y', 'R', 'L', 'B', 'O']

Plotting Steps in Order

"All steps at once"

- Step-by-step solution plots altogether:



- We counted the number of solutions of the SOMA cube using backtracking
- To begin counting the solutions to the soma cube, it helps to classify the cubes in the 3x3x3 cube into vertex(V), face(F), edge(E), and central(C) cubes
- This allows us to classify the types of solutions of the SOMA puzzle

- There is obviously only one piece of the SOMA cube that covers the central cube, which we will call the central piece
- As the respective pieces can occupy at most

W	Y	G	O	L	R	B
1	2	2	1	1	1	1

  of the vertices, there must exist one deficient piece of the SOMA puzzle that does not cover a vertex of the SOMA cube
- This implies that the green piece's spine lies on the edge of the 3x3x3 cube

Counting SOMA Cube solutions
classifying SOMA pieces

	W	Y	G	O	L	R	B
V & F	[1, 2]	2	3	2	2	2	[1, 3]
E & C	[2, 1]	2	1	2	2	2	[3, 1]

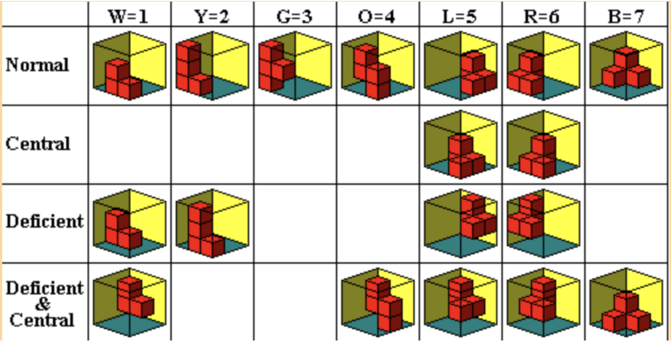
We observe that Y, G, O, L, and R pieces have fixed number of 'V & F' cells and 'E & C' cells that they can cover, but there are two ways for W and B pieces.

However, there must be total of 14 'V' + 'F' cells and 13 'E' + 'C' cells, which is only possible when we choose W piece to cover 2 'V' or 'F' cells and B piece to cover 1 'V' or 'F' cell.

Therefore, the table now becomes as below:

	W	Y	G	O	L	R	B
V & F	2	2	3	2	2	2	1
E & C	1	2	1	2	2	2	3

Counting SOMA Cube Solutions



Counting SOMA Cube Solutions

	Branch										
Piece	1	2	3	4	5	6	7	8	9	10	11
1	FEEV	FEEV	FEEV	FEEV	CFEV	FEEV	CFEV	FEEV	CFFE	FFEE	CFEV
2	CFEE	EEEV	EEEV	EEEV	EEEV	EEEV	EEEV	EEEV	EEEV	EEEV	EEEV
3	FEEV	CFFE	FEEV	FEEV	FEEV	FEEV	FEEV	FEEV	FEEV	FEEV	FEEV
4	FEVV	FEVV	FEVV	FEVV	FEVV	FEVV	FEVV	FEVV	FEVV	FEVV	FEVV
5	FEEV	FEEV	FEEV	CFEV	FEEV	CFEV	FEEV	CFFE	FEEV	CFEV	FFEE
6	EEVV	EEVV	EEVV	EEVV	EEVV	FEEV	FEEV	EEVV	EEVV	EEVV	EEVV
7	FEV	FEV	CFF	FFE	FFE	FEV	FEV	FEV	FEV	FEV	FEV
a	74	66	38	14	14	21	21	51	51	65	65
l	37	33	19	6	8	9	12	26	25	22	43
r	37	33	19	8	6	12	9	25	26	43	22

Thank You for Your Attention!