Introduction to String Theory

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Introduction to String Theor**ies**

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Overview

1. Motivation
2. Bosonic String
   - Quantisation
   - Closed Spectrum
3. Superstring
   - Type IIA/B
   - Type I
   - Heterotic Strings

Why String Theory?

- It is fun

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- ~~It is fun~~
- UV-finite
- Provides a theory of quantum gravity
- Creates tools for other areas of physics and mathematics
- Has the right ingredients for producing standard model

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$$X_L^\mu(\tau+\sigma) = x^\mu + \frac{p^\mu l^2}{2}\cdot(\tau+\sigma) + il\sum_{n\neq 0}\frac{1}{n}\,\alpha_n^\mu\, e^{-in(\tau+\sigma)}$$

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- For the closed string, α_n^μ and $\widetilde{\alpha}_n^\mu$ are independent.

Quantisation

- Promote Fourier modes to operators with commutation relations

$$[x^\mu, p_\nu] = i\delta^\mu_\nu \qquad [\alpha^\mu_n, \alpha^\nu_m] = n\delta_{n+m}\eta^{\mu\nu} = [\widetilde{\alpha}^\mu_n, \widetilde{\alpha}^\nu_m]$$

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- For $n > 0$, α^μ_n and $\widetilde{\alpha}^\mu_n$ are annihilation operators
- For $n < 0$, α^μ_n and $\widetilde{\alpha}^\mu_n$ are creation operators

Pretty Pictures of String Excitations

Closed Spectrum

- There is a level matching condition $N = \widetilde{N}$
- Vacuum is a tachyon, with mass $M^2 < 0$!
- First excited state is $|\boldsymbol{\xi}\rangle = \xi_{ij}\alpha^i_{-1}\widetilde{\alpha}^j_{-1}|0\rangle$ with $M^2 = 0$
- Decompose ξ_{ij} in irreps

$$\xi_{ij} = \underbrace{\xi_{(ij)}}_{\text{symmetric}} + \underbrace{\xi_{[ij]}}_{\text{antisymmetric}} + \underbrace{\xi^{(0)}\delta_{ij}}_{\text{trace part}}$$

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- Symmetric part describes spin 2 gauge field $\implies$ graviton!
- Antisymmetric part is called *Kalb-Ramon Field*.
- Trace part describes the *dilaton*.

Superstring

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- By supersymmetry, fermions are related to bosons.
- Supersymmetry therefore forbids a tachyonic state.
- For left and right moving parts, we can choose either spin 1 bosons "B"
- Or spin 1/2 fermions "$\text{F}_{\pm}$" with two chiralities

Decomposition into Irreps

- (B,B) = Dilaton $\oplus$ 2-Form Field (Kalb-Ramon) $\oplus$ Graviton
- ($\mathrm{F}_{\pm}$, $\mathrm{F}_{\pm}$) = certain k-form Fields
- (B,F_{+}) = Dilatino $\oplus$ Gravitino
- (B,F_{-}) = Dilatino $\oplus$ Gravitino (with different chirality)

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  - (B,F_+): Dilatino, Gravitino
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  - (B,F_+): Dilatino, Gravitino
  - (F_+,B): Dilatino, Gravitino
- Type IIA consists of
  - (B,B): Dilaton, Kalb-Ramon, Graviton
  - (F_+,F_-): k-form Fields
  - (B,F_-): Dilatino, Gravitino (different chirality)
  - (F_+,B): Dilatino, Gravitino

Type I

- Type IIB consists of
  - (B,B): Dilaton, Kalb-Ramon, Graviton
  - (F_+,F_+): k-form Fields
  - (B,F_+): Dilatino, Gravitino
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- Type IIB theory is invariant under exchanging left and right movers.

Type I

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  - (F_+,B): Dilatino, Gravitino
- Type IIB theory is invariant under exchanging left and right movers.
- We call this symmetry Ω, then *closed spectrum* of type I theory is IIB$/\Omega$ i.e. it consists of
  - Dilaton, Graviton
  - Some of the k-form fields
  - Dilatino, Gravitino
- The closed spectrum of type I theory is inconsistent by itself
- Add open strings with gauge group $SO(32)$ to complete type I theory.
- Type I theory is in some sense "half" of Type IIB theory.

Heterotic Strings

- There are yet another consistent theories, which takes bosonic strings on left moving and supersymmetric strings on right moving sector.
- Bosonic part is compactified
- This gives two new theories with gauge groups $SO(32)$ and $E_8 \times E_8$

Bibliography

Weigand, Timo *Introduction to String Theory, Lecture Notes*

Ramon and Neveu-Schwarz Sectors

- If $\psi_{L,R}(\sigma) = +\psi_{L,R}(\sigma + 2\pi)$ then ψ is in *Ramon sector* with integer Fourier expansion

$$\psi_R^\mu = \sum_{n\in\mathbb{Z}} b_n^\mu e^{-in(\tau-\sigma)} \qquad \psi_L^\mu = \sum_{n\in\mathbb{Z}} \widetilde{b}_n^\mu e^{-in(\tau+\sigma)}$$

- If $\psi_{L,R}(\sigma) = -\psi_{L,R}(\sigma + 2\pi)$ then ψ is in *Neveu-Schwarz sector* with half integer Fourier expansion

$$\psi_R^\mu = \sum_{r\in\mathbb{Z}+\frac{1}{2}} b_r^\mu e^{-ir(\tau-\sigma)} \qquad \psi_L^\mu = \sum_{r\in\mathbb{Z}+\frac{1}{2}} \widetilde{b}_r^\mu e^{-ir(\tau+\sigma)}$$

- So there are total of 4 choices possible: (NS,NS) (NS,R) (R,NS) (R,R)

Quantisation

- Promote Fourier modes to operators with anti-commutation relations

$$\{b_n^\mu, b_m^\nu\} = \eta^{\mu\nu}\delta_{n+m} = \{\widetilde{b}_n^\mu, \widetilde{b}_m^\nu\}$$

- Define Vacua:

$$\alpha_n^\mu \left|0\right\rangle_{NS} = b_r^\mu \left|0\right\rangle_{NS} = \widetilde{b}_r^\mu \left|0\right\rangle_{NS} = 0 \quad \forall n, r > 0$$

$$\alpha_n^\mu \left|0\right\rangle_{R} = b_m^\mu \left|0\right\rangle_{R} = \widetilde{b}_m^\mu \left|0\right\rangle_{NS} = 0 \quad \forall n, m > 0$$

- Improtant: NS ground state is well defined and is a space-time scalar. However, R ground state is degenerate as

$$b_m^\mu b_0^\nu \left|0\right\rangle_R = -b_0^\nu b_m^\mu \left|0\right\rangle_R = 0$$

- Because of this degeneracy, one can show that $\left|0\right\rangle_R$ is a fermion.

Massless Spectrum

- Recall that $|0\rangle_R$ is a fermion and $|0\rangle_{NS}$ is a scalar.

Sector (L,R)	State	$SO(8)$ rep
(NS,NS)	$b^i_{-1/2}\,|0\rangle_{NS} \otimes \widetilde{b}^i_{-1/2}\,|0\rangle_{NS}$	$8_v \otimes 8_v$

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(R_+,R_+)	$|+\rangle_R \otimes |+\rangle_R$	$8_s \otimes 8_s$
(R_+,R_-)	$|+\rangle_R \otimes |-\rangle_R$	$8_s \otimes 8_c$
(R_-,R_-)	$|-\rangle_R \otimes |-\rangle_R$	$8_c \otimes 8_c$

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