Non-interacting fermions on a tight-binding chain

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Here we consider non-interacting spinless fermions (or spin-polarized fermions, equivalently) on a tight-binding chain. Its Hamiltonian is given by

$$\hat{H} = \sum_{\ell=1}^{L-1} (-t_\ell \hat{c}_{\ell+1}^\dagger \hat{c}_\ell - t_\ell^* \hat{c}_\ell^\dagger \hat{c}_{\ell+1}),$$

where the chain has L sites, t_ℓ indicates the hopping amplitute between sites ℓ and $\ell+1$, and $\hat{c}_\ell^\dagger$ creates a particle at a site $\ell \in [1, L]$.

Exercise (a): Compute the energy and degeneracy of the many-body ground states

Write a script or function that computes the ground-state energy and degeneracy of this non-interacting tight-binding chain. The script or function takes the following input and output:

< Input >

- `t` : [numeric vector] Each element `t(l)` indicates a hopping amplitude t_ℓ. The length of the vector `numel(t)` equals to the number of chain sites minus 1.

< Output >

- `E_G` : [numeric scalar] Ground-state energy.
- `d_G` : [numeric scalar] Ground-state degeneracy.

Once you complete a script or function, compute the ground-state energies and degeneracies for the following three cases:

(1) $L = 10$, $t_\ell = 1$ for all ℓ's.

(2) $L = 11$, $t_\ell = 1$ for all ℓ's.

(3) $L = 11$, $t_\ell = e^{i\ell}$.