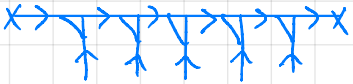
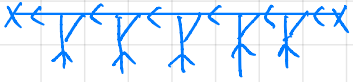
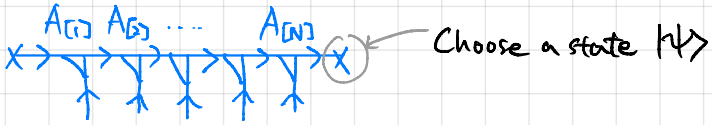
Canonical forms of MPSs (left, right, bond, and site)

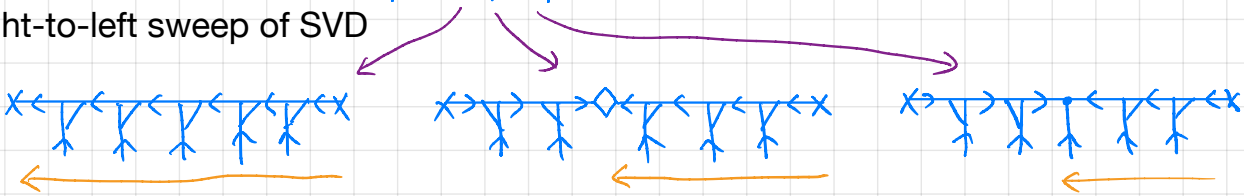
Left-canonical: 

Right-canonical: 

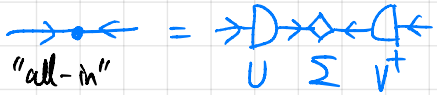
Bond-canonical: 

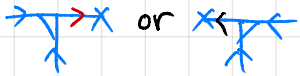
Site-canonical: 

After iterative diagonalization: 

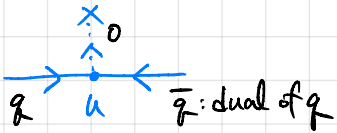
Right-to-left sweep of SVD 

How to revert leg directions only for some of them?

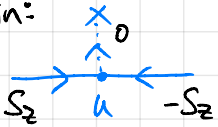
Recall SVD: 

But we will encounter: 

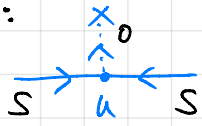
1j symbol:



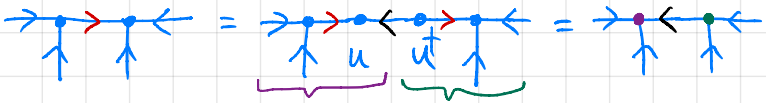
Ex) U(1) spin:

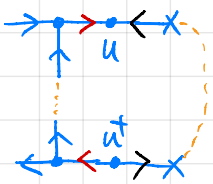


SU(2) spin:



1j symbols can carry sign factors, so should come in pair: $uu^\dagger = u^\dagger u = I$

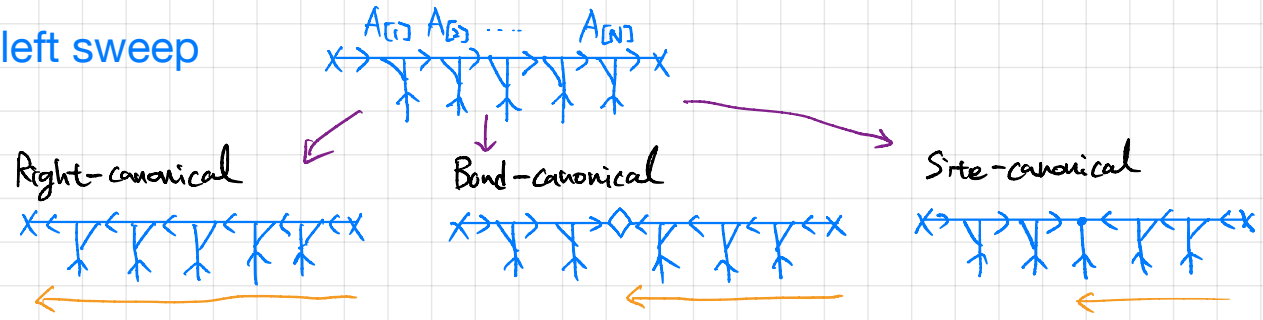




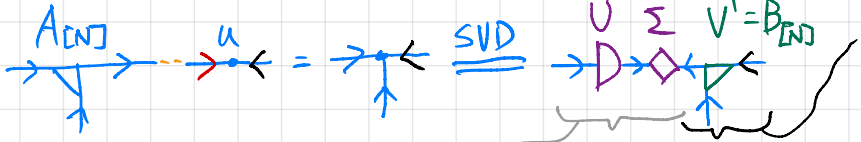
Physical observables require both ket and bra

When symmetry is not exploited, 1j symbols are simple identities

Right-to-left sweep

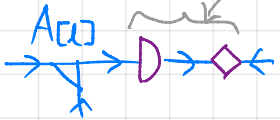


At the 1st iteration:

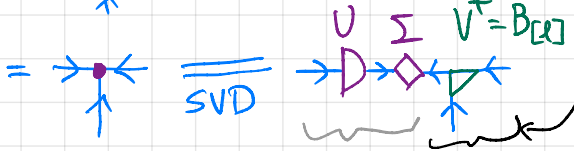


Right-normalized: replace the N-th tensor of the MPS

At each following iteration:

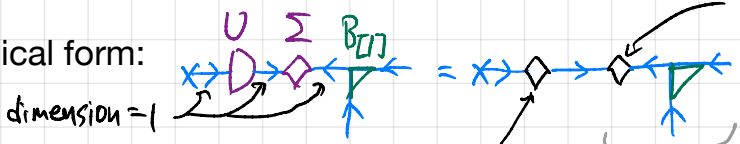


Contract U and Σ from the SVD of the $(l+1)$-th tensor



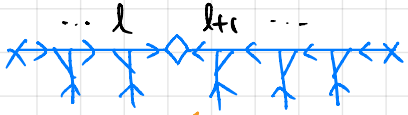
Replace the l-th tensor

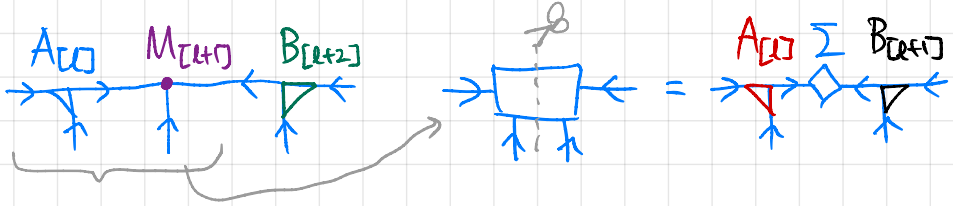
To get the right-canonical form:



Note: phase factor is important!
(Ex: time evolution)

To get the bond-canonical form:





To get the site-canonical form: