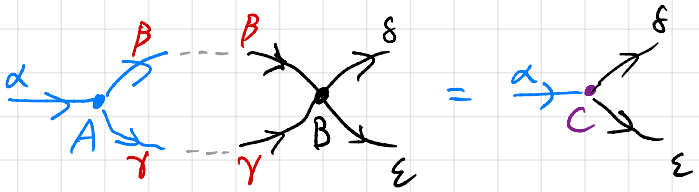
Tensor contractions

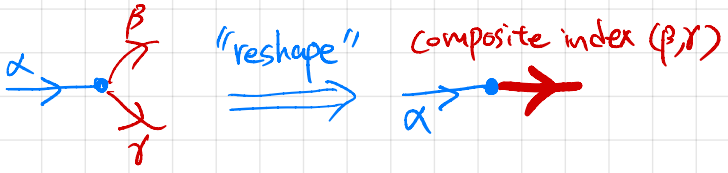
$$A^{\alpha}{}_{\beta\gamma}\, B^{\beta\gamma}{}_{\delta\varepsilon} = C^{\alpha}{}_{\delta\varepsilon}$$

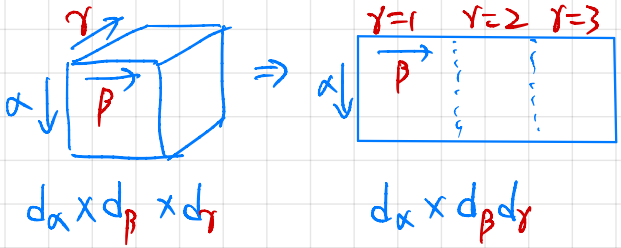


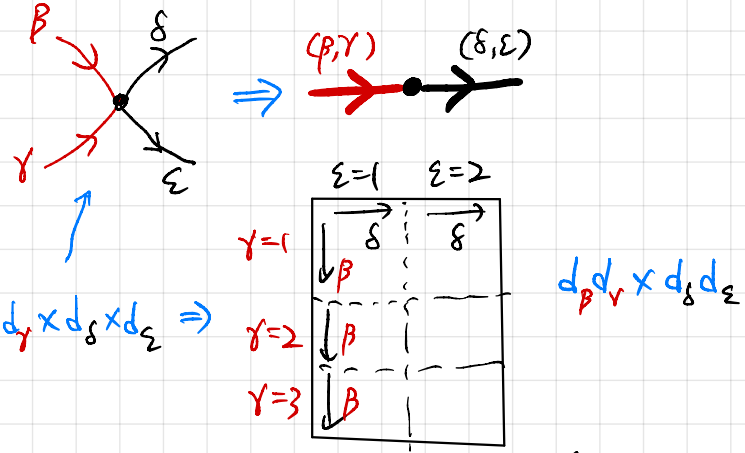
Leg directions are important: an outgoing leg (ket) contracts only to an incoming leg (bra).

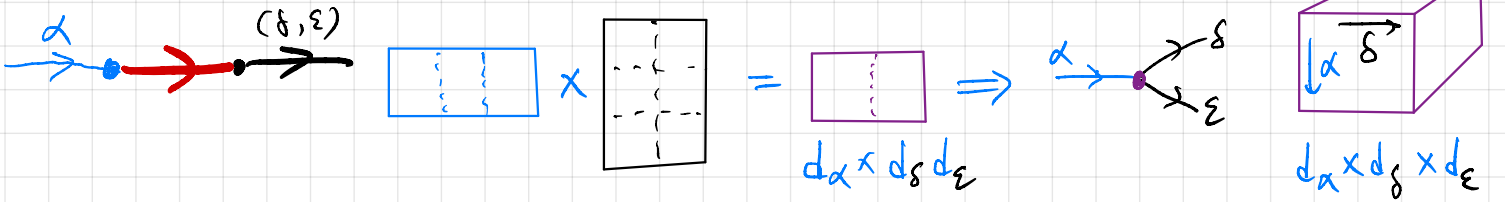
Dimensions of legs to contract should match.

Tensor contraction ~ matrix multiplication:

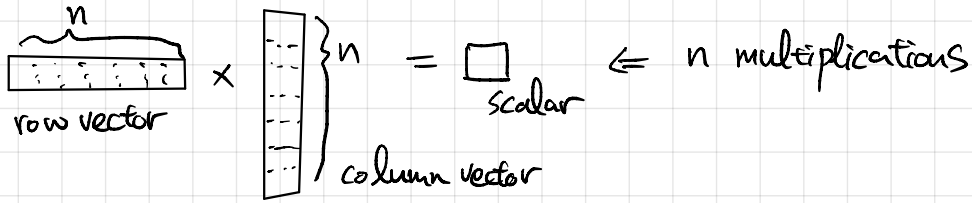




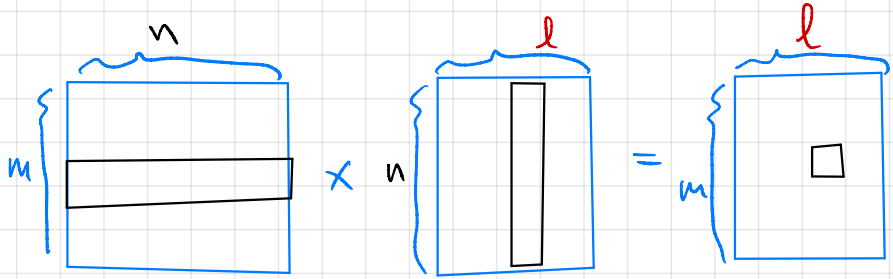




Computational costs

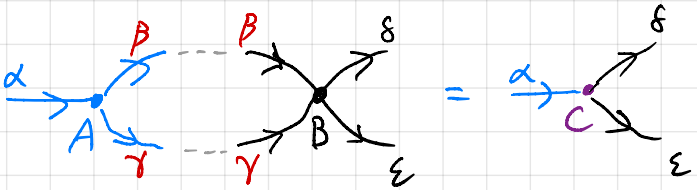


$\Leftarrow$ n multiplications



$\Leftarrow$ (n multiplications) $\times m \times l$

Total cost $= O(mnl)$



Cost $= O(d_\alpha d_\beta d_\gamma d_\delta d_\varepsilon)$

$d_\alpha \times d_\beta \times d_\gamma$ $\quad$ $d_\beta \times d_\gamma \times d_\delta \times d_\varepsilon$