W state

Author: Seung-Sup Lee

In quantum information theory, there are several special quantum states. One among them is the W state. The N-qubit W state is defined by the equal superposition of all possible pure states in which exactly one of the qubits is in the $|1\rangle$ state and the rest are in the $|0\rangle$ state:

$$|\mathrm{W}\rangle = \frac{1}{\sqrt{N}}(|100\cdots00\rangle + |010\cdots00\rangle + \cdots + |000\cdots01\rangle).$$

The W state is named after **W**olfgang Dür, an Austrian physicist who proposed the W state for three qubits, together with Guifré Vidal and J. Ignacio Cirac in [W. Dür, G. Vidal, and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000)]. You will see the latter two authors' names often during this lecture course, as they are early founders of tensor networks!

The N-qubit W state can be represented in terms of a rank-N tensor A,

$$|\mathrm{W}\rangle = |n_1 n_2 \cdots n_N\rangle A^{n_1+1,n_2+1,\cdots,n_N+1},$$

where $n_i = 0, 1$ indicate the state of the i-th qubit, associated with indices 1 and 2, respectively, along the i-th dimension of the tensor A. The repeated indices $n_1, \cdots, n_N$ are assumed to be summed over.

Exercise (a): Tensor representation of the W state

Write a script or function that generate the rank-n tensor A, taking a general input of N. Try to compose it in the most compact way, while keeping its computational efficiency.