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Improved Logical Error Rate via List Decoding of Quantum Polar Codes

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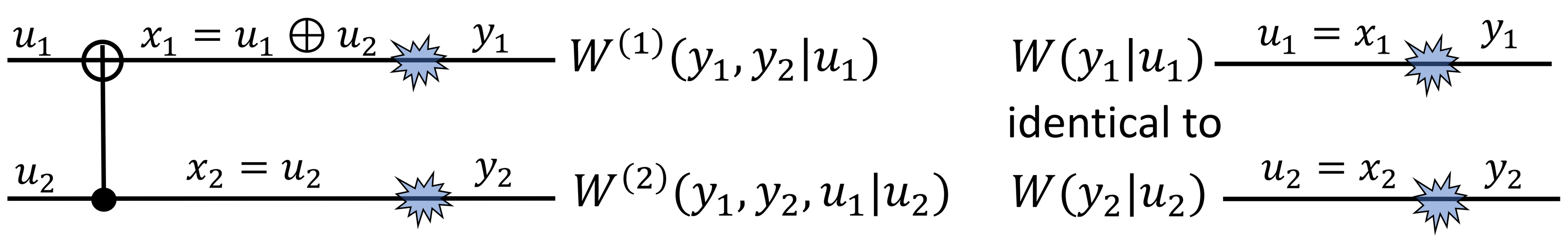
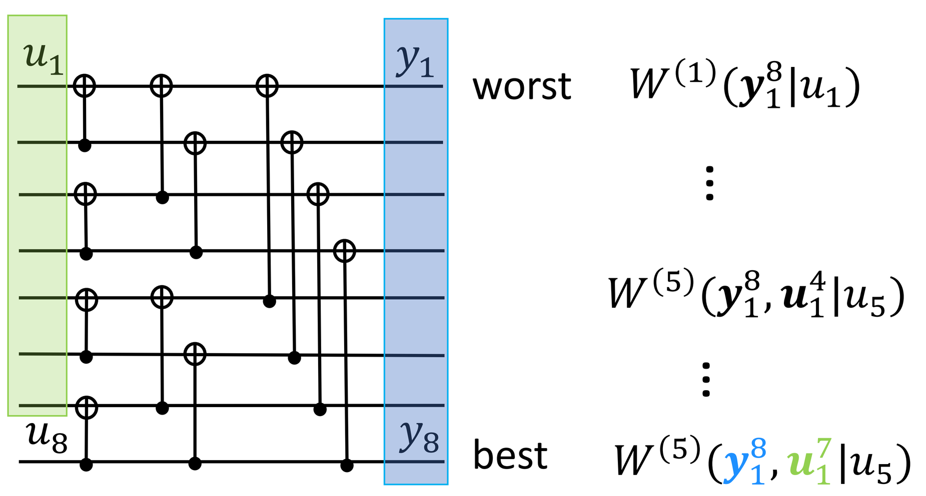
Polar Codes and Decoding [1,2]

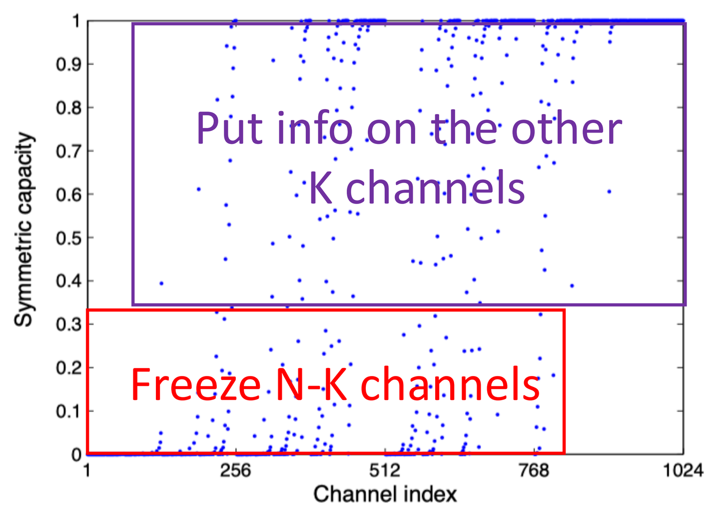


Channel Polarization: $I(W^{(1)}) < I(W) < I(W^{(2)})$

Successive Cancellation Decoding

$\mathcal{O}(N \log N)$ Time
$\mathcal{O}(N)$ Space

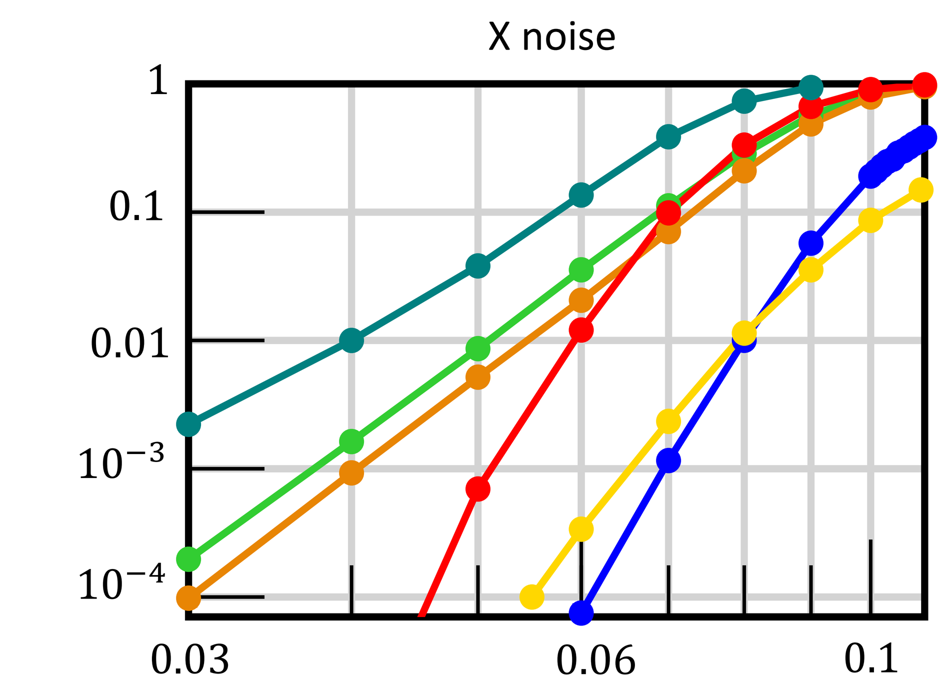




For i in $1, \dots, N$ do:
 if $i \in \mathcal{A}^c$: /* u_i is a frozen bit */
  $\hat{u}_i \leftarrow u_i$
 else: /* $i \in \mathcal{A}$, u_i is an info bit */
  $\hat{u}_i \leftarrow \mathrm{argmax}_{u_i \in \{0,1\}} W^{(i)}(\boldsymbol{y}, \hat{\boldsymbol{u}}_1^{i-1}|u_i)$

List decoding [2] always keeps a list of L decisions $\hat{\boldsymbol{u}}_1^i$, and their path metrics $\mathrm{PM}(\hat{\boldsymbol{u}}_1^i) = \Pr[\hat{\boldsymbol{u}}_1^i|\boldsymbol{y}]$ at each step.

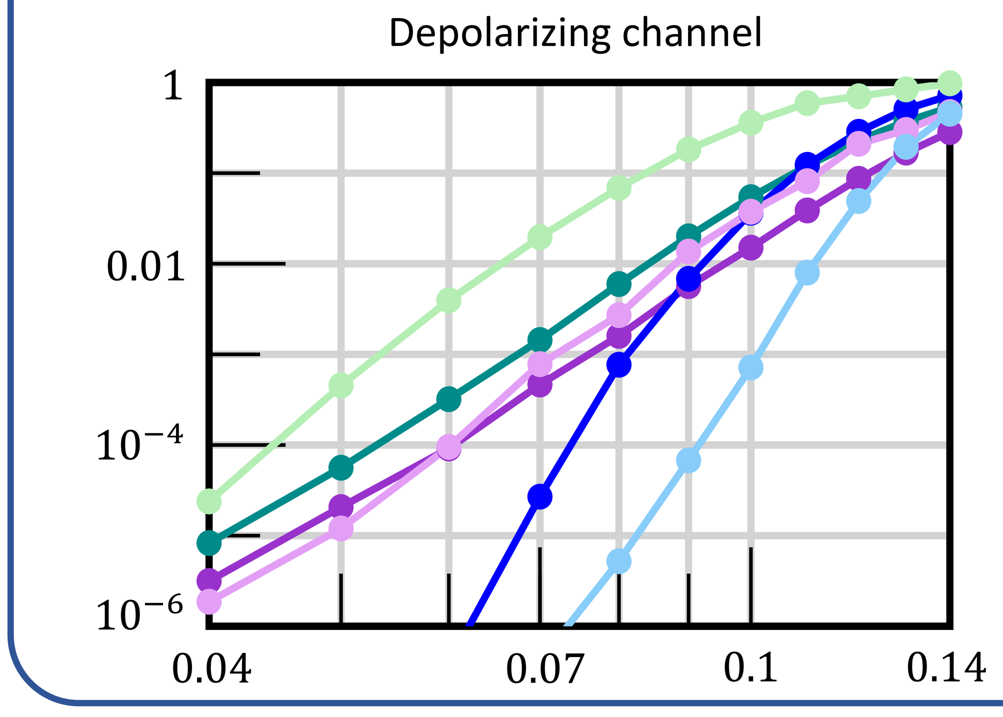
Comparison to other Codes



PW-QPC, $L = 16$
- $[[1024, 2, 32]]$, $\beta = 2^{1/4}$
- $[[1024, 36, 16]]$, $\beta = 2^{1/4}$
- $[[1024, 42, 16]]$, $\beta = 2^{1/4} - 0.2$
- $[[1024, 42, 32]]$, $\beta = 2^{1/4} - 0.12$

Random QLDPC, BP+OSD-CS [4]
- $[[900, 36, 10]]$

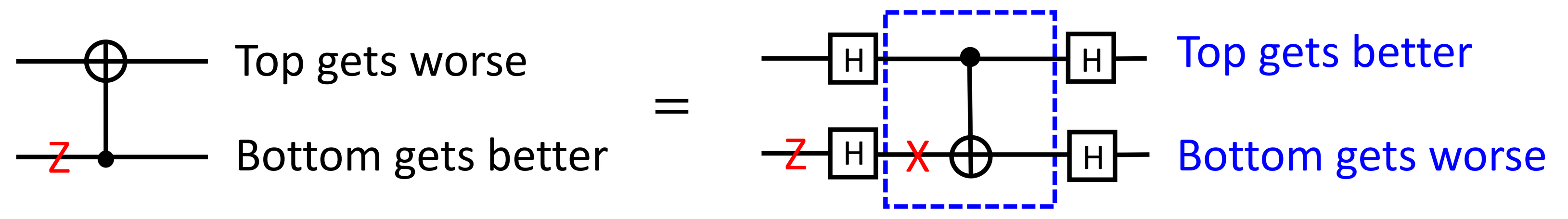
Surface Code, MLD [5]
- $[[1201, 1, 25]]$



PW-QPC, $L = 16$
- $[[256, 14, 16]]$, $\beta = 1.13$
- $[[256, 28, 16]]$, $\beta = 1.13$
- $[[1024, 28, 32]]$, $\beta = 1.08$

QLDPC, BP+OSD0 [6]
- $[[254, 14, 16]]$
- $[[254, 28, \le 20]]$
- $[[882, 24, \le 24]]$

Polarization Weight Construction



Protect against **bit** flips   Protect against **phase** flips

CSS constraint: no qubit is simultaneously frozen in both X and Z basis

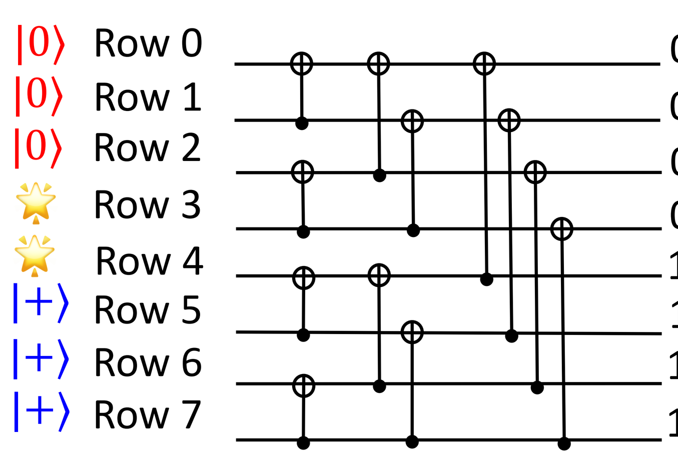
Polarization Weight (PW) Construction

Want to define Polarization Weight of row 6

$$6 = 110_2 = 1\times 2^2 + 1\times 2^1 + 0\times 2^0$$
$$\mathrm{PW}(6) = 110_\beta = 1\times\beta^2 + 1\times\beta^1 + 0\times\beta^0$$

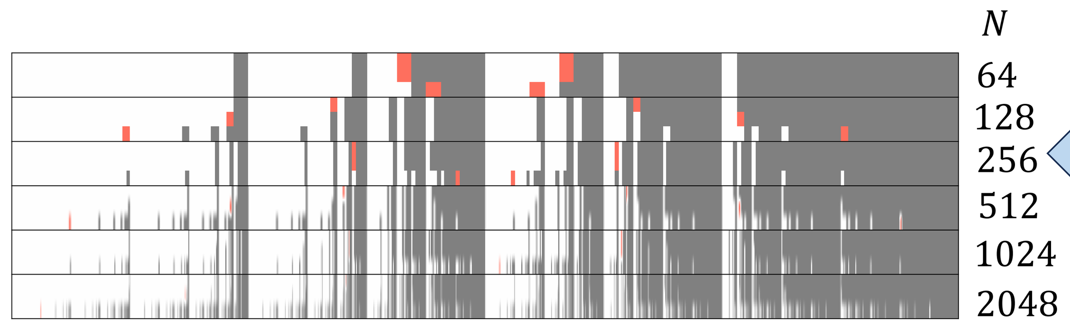
Satifies CSS constraint. No entanglement assistance needed. Gives good codes.

Ex: want an $[[8,2]]$ quantum code



		Binary	Against bit flips PW $\beta = 2^{1/4}$	Against phase flips PW $\beta = 2^{1/4}$
$\vert 0\rangle$	Row 0	000	0	3.603
$\vert 0\rangle$	Row 1	001	1	2.603
$\vert 0\rangle$	Row 2	010	1.189	2.414
	Row 3	011	2.189	1.414
	Row 4	100	1.414	2.189
$\vert +\rangle$	Row 5	101	2.414	1.189
$\vert +\rangle$	Row 6	110	2.603	1
$\vert +\rangle$	Row 7	111	3.603	0

freeze 3 best   =
freeze 3 worst   freeze 3 worst

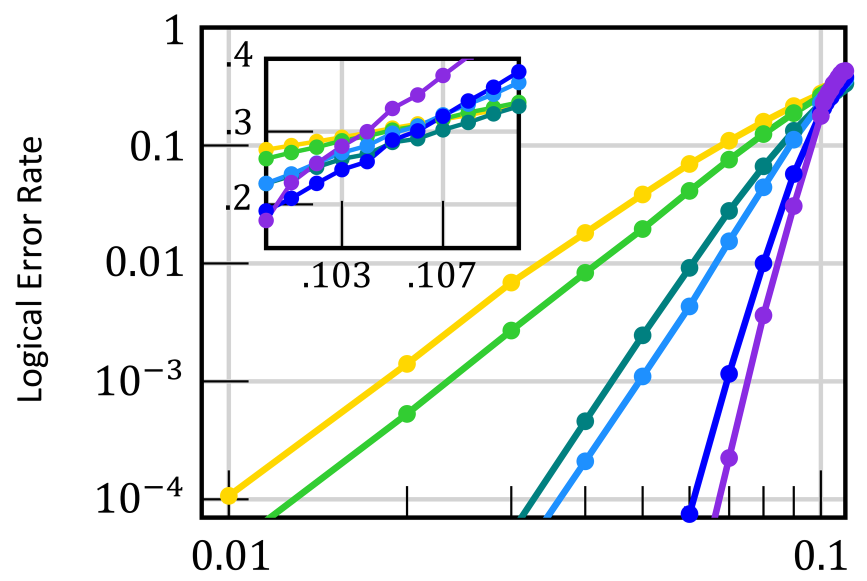


Mixing
- $\beta = 2^{1/4}$
- $\beta_1 = 2^{1/4} = \beta_2^4$ (High-order PW)
- $\beta = 1$ (Reed-Muller)

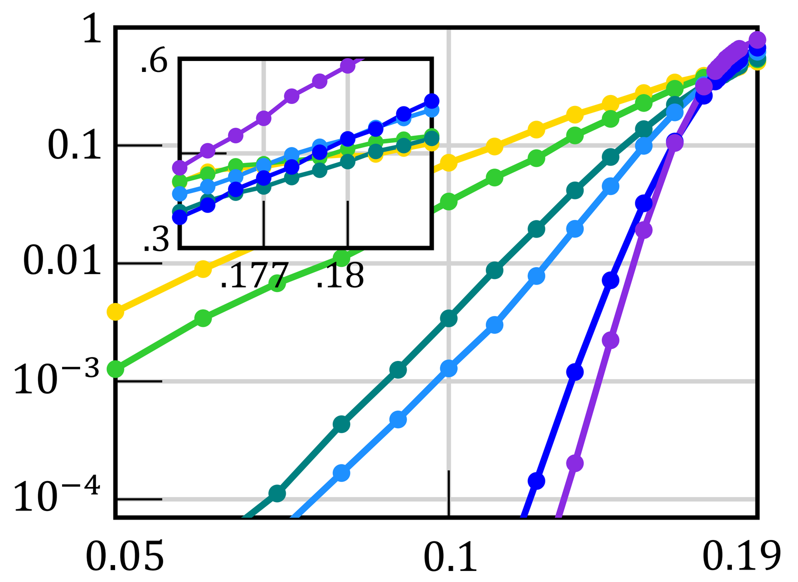
A family of $[[N, 2]]$ PW-QPC ($\beta = 2^{1/4}$) using list decoder of size L

N, L: 64, 4; 128, 8; 256, 8; 512, 16; 1024, 16; 2048, 32

Independent bit- and phase-flip channel, X noise only



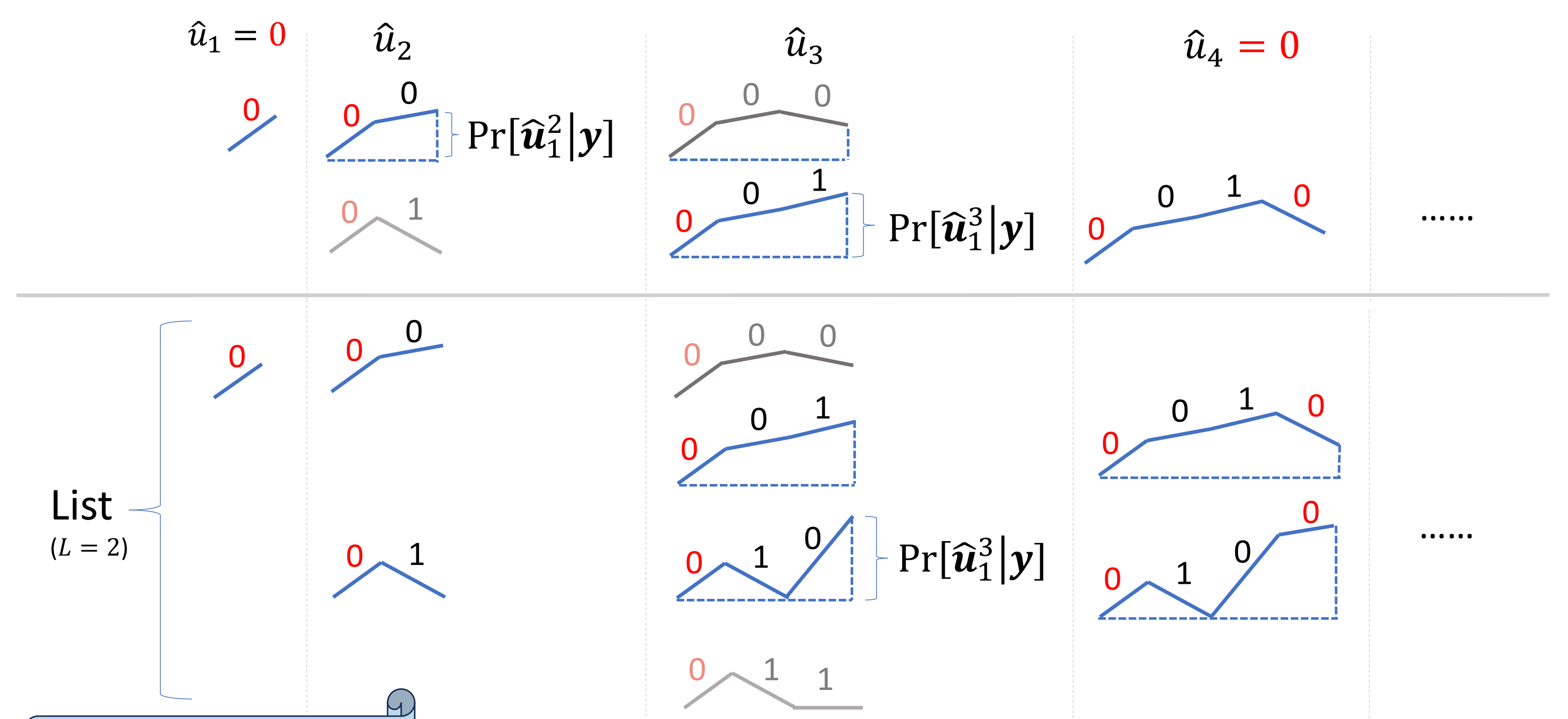
Depolarizing channel



List Decoding and Degeneracy

Following a local minimum each time does not necessarily lead to global minimum.

Successive Cancellation Decoding ($L = 1$)



Time $\mathcal{O}(LN \log N)$
Space $\mathcal{O}(LN)$

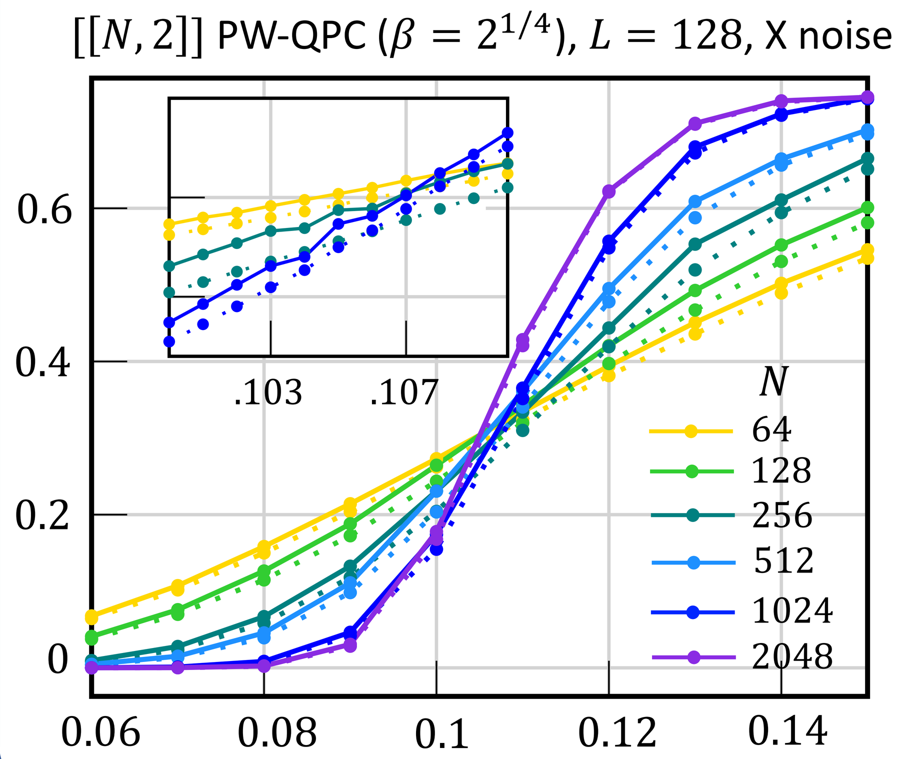
Conversion
- Put syndrome $\boldsymbol{s}$ at $\boldsymbol{u}_{\mathcal{A}^c}$ (both size $N - K$)
- Treat all-zero as the noisy codeword
- Decode $\boldsymbol{u}_{\mathcal{A}}$

Application: find quantum distance for QPC

Codeword list decoder   Syndrome list decoder

What's on the list? (with high probability)
- Classical: $\hat{\boldsymbol{u}}_1^n$'s such that $\Pr[\hat{\boldsymbol{u}}_1^n|\,\boldsymbol{y}]$ are the largest
- Quantum: Given syndrome $\boldsymbol{s}$, most likely error strings compatible with $\boldsymbol{s}$

$[[N, 2]]$ PW-QPC ($\beta = 2^{1/4}$), $L = 128$, X noise



Classical MLD: Pick most likely error string
Quantum MLD: Pick most likely error class

—— Classical MLD
- - - - Approximated Quantum MLD via List Decoder

Not a highly degenerate code, improvement ~10%
Using practical list size, at $p \approx 0.1$, essentially comparing how many lowest weight errors in each class.

References

[1] E. Arıkan, "Channel Polarization: A Method for Constructing Capacity-Achieving Codes for Symmetric Binary-Input Memoryless Channels", IEEE Transactions on Information Theory, 2009.
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[3] G. He et al, "Beta-Expansion: A Theoretical Framework for Fast and Recursive Construction of Polar Codes", IEEE Global Communications Conference, 2017.
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[6] P. Panteleev and G. Kalachev, "Degenerate Quantum LDPC Codes With Good Finite Length Performance", Quantum, 2021.