A proof of correctness for our parallelization

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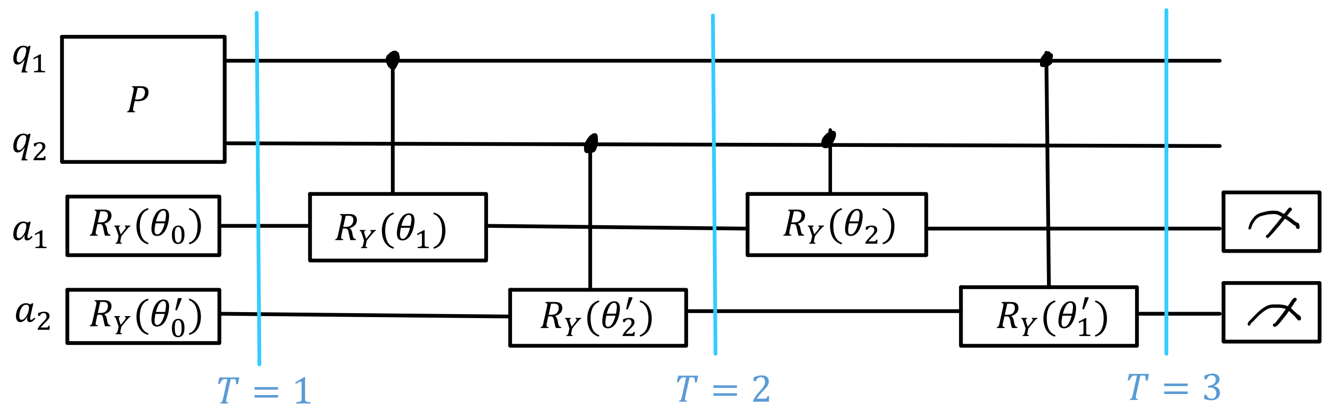

Write the state of q_1q_2 at $T = 1$ as $c_1|00\rangle + c_2|01\rangle + c_3|10\rangle + c_4|11\rangle$.

Write the state of a_1 and a_2 at $T = 1$ as $|\phi\rangle \coloneqq R_Y(\theta_0)|0\rangle$ and $|\phi'\rangle \coloneqq R_Y(\theta_0')|0\rangle$ respectively.

The Ctrl-R_Y gates between $T = 1$ and $T = 3$ can be written as a unitary:
$|00\rangle\langle 00|_{q_1q_2} \otimes I_{a_1a_2} + |01\rangle\langle 01|_{q_1q_2} \otimes R_Y(\theta_2)_{a_1} \otimes R_Y(\theta_2)_{a_2} + |10\rangle\langle 10|_{q_1q_2} \otimes R_Y(\theta_1)_{a_1} \otimes R_Y(\theta_1)_{a_2} + |11\rangle\langle 11|_{q_1q_2} \otimes R_Y(\theta_1+\theta_2)_{a_1} \otimes R_Y(\theta_1+\theta_2)_{a_2}$

Denote $|\phi_1\rangle = R_Y(\theta_1)|\phi\rangle$, $|\phi_2\rangle = R_Y(\theta_2)|\phi\rangle$, $|\phi_{12}\rangle = R_Y(\theta_1+\theta_2)|\phi\rangle$;
$|\phi_1'\rangle = R_Y(\theta_1')|\phi'\rangle$, $|\phi_2'\rangle = R_Y(\theta_2')|\phi'\rangle$, $|\phi_{12}'\rangle = R_Y(\theta_1'+\theta_2')|\phi'\rangle$.

The state at $T = 3$ is:
$c_1|00\rangle_{q_1q_2} \otimes |\phi\rangle_{a_1} \otimes |\phi'\rangle_{a_2} + c_2|01\rangle_{q_1q_2} \otimes |\phi_2\rangle_{a_1} \otimes |\phi_2'\rangle_{a_2} +$
$c_3|10\rangle_{q_1q_2} \otimes |\phi_1\rangle_{a_1} \otimes |\phi_1'\rangle_{a_2} + c_4|11\rangle_{q_1q_2} \otimes |\phi_{12}\rangle_{a_1} \otimes |\phi_{12}'\rangle_{a_2}$.

The probability of measuring ancilla a_1 in state $|1\rangle_{a_1}$ is:
$c_1 \cdot |\langle\phi|1\rangle|^2 + c_2 \cdot |\langle\phi_2|1\rangle|^2 + c_3 \cdot |\langle\phi_1|1\rangle|^2 + c_4 \cdot |\langle\phi_{12}|1\rangle|^2$. This probability is independent of the rotation angles on ancilla a_2.