

Rephrasing the claim as the contrapositive returns "For any integer k , if k is even, $3k + 1$ is odd".

2. **Prove** the contrapositive

We will now prove the contrapositive of the claim. That is, that for any integer k , if k is even, $3k + 1$ is odd.

An integer x is even if there is an integer y such that $x = 2y$.

An integer x is odd if there is an integer y such that $x = 2y + 1$.

Suppose that k is an integer and k is even, then $k = 2m$ where m is an integer. Simplifying the expression, $3k + 1 = 3(2m) + 1 = 2(3m) + 1$. Since m is an integer, so is $3m$.

Therefore, $3k + 1$ must be odd, proving the contrapositive so the original claim is true.

Chapter 4: Number Theory