COMP7015 Artificial Intelligence (2024–25)
In-Class Quiz Sample Solutions

Notes: The In-class quiz accounts for 5% of the overall assessment. Please do not distribute the quiz sample solutions.

Question 1 (20 marks)

Navigation applications such as Google Maps is becoming increasingly important in modern day-to-day lives. AI methods play important roles in such applications. Answer the following questions about finding routes in navigation apps.

(a) Consider the problem of finding routes in a very large map, where we have over one million locations. For simplicity, consider a grid map, where there are four possible actions at each location (i.e., turning around, going straight, turning left, and turning right). In many cases, the optimal routes from one location to another need to go through more than 100 locations. Are the following statements correct? If not, explain the reason in one sentence.

i. (1 mark) If the optimal route from S to G goes through d other locations, the depth of the solution will be $d + 1$ (assuming the root node has depth 0).

Solution: Correct.

ii. (1 mark) BFS is not complete because it only finds the shallowest solution instead of all solutions.

Solution: Incorrect. Completeness means that the algorithm is guaranteed to find a solution if one exists. It does not require finding all solutions.

iii. (2 marks) DFS is suitable for this problem since it is more memory efficient.

Solution: Incorrect. DFS can get stuck in infinite loops and fail to find solutions.

iv. (2 marks) Iterative-deepening DFS cannot be used for this problem because we need to repeat upper levels multiple times, which takes more time.

Solution: Incorrect. Upper levels contain much fewer nodes than the bottom levels. IDS is suitable since it is memory efficient and can find a solution if it exists.

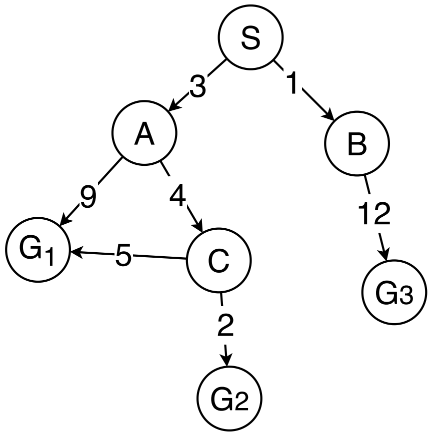
v. (2 marks) We can use a heuristic function and A* search will always find the optimal route.

Solution: Incorrect. A* is optimal only when the heuristic function is admissible.

vi. (2 marks) Greedy search always finds the optimal solution because a heuristic function is used.

Solution: Incorrect. Greedy search does not guarantee a cost-optimal solution.

(b) Following shows a simplified map where S is the initial state. G_1, G_2, and G_3 are three goal states. The table shows three heuristic functions.



	A	B	C	S
h_1	0	0	0	0
h_2	6	7	1	7
h_3	7	7	1	7

Select ***all*** of the goals that could be returned by each of the search algorithms below. For each algorithm, if there are multiple goals that are possible to be returned, select all of them.

i. (1 mark) Breadth-first search (BFS) ■ G_1 □ G_2 ■ G_3

ii. (1 mark) Depth-first search (DFS) ■ G_1 ■ G_2 ■ G_3

iii. (1 mark) Iterative-Deepening Depth-first search (IDS) ■ G_1 □ G_2 ■ G_3

iv. (1 mark) Greedy search with h_1 ■ G_1 ■ G_2 ■ G_3

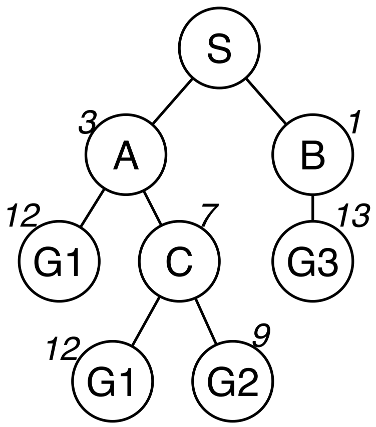
v. (1 mark) Greedy search with h_3 ■ G_1 □ G_2 ■ G_3

vi. (1 mark) A* search with h_1 □ G_1 ■ G_2 □ G_3

vii. (1 mark) A* search with h_2 □ G_1 ■ G_2 □ G_3

(c) (3 marks) Perform uniform-cost search (UCS) to find a solution given the map in the above sub-question (starting from S). Write down the solution found by UCS and draw out the final search tree constructed.

Solution: The solution found by UCS is: $S \to A \to C \to G_2$. The search tree constructed is (must match exactly):

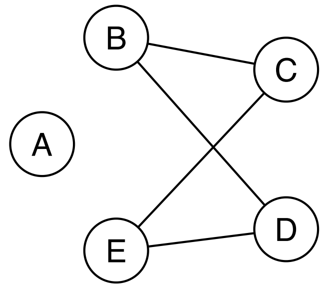


Question 2 (10 marks)

Consider a CSP with variables A, B, C, D, and E. All variables have the domain $\{1, 2, 3, 4\}$. The constraints are: $C > E$, $B > D$, $D > E$ and $B > C$.

(a) (2 marks) Draw the binary constraint graph for this CSP.

Solution:



(b) (8 marks) Perform AC3 algorithm on this CSP, write the domains for each variable after completing the AC3 algorithm. Show your steps.

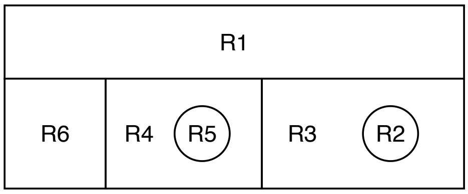
Solution: Initialize queue={(B,C), (C,B), (B,D), (D,B), (C,E), (E,C), (E,D), (D,E)}

1. EnforceArcConsistency(B, C): remove 1 from $\mathcal{D}_B$.
   queue={(C,B), (B,D), (D,B), (C,E), (E,C), (E,D), (D,E)}
2. EnforceArcConsistency(C, B): remove 4 from $\mathcal{D}_C$.
   queue={(B,D), (D,B), (C,E), (E,C), (E,D), (D,E)}
3. EnforceArcConsistency(B, D): consistent.
   queue={(D,B), (C,E), (E,C), (E,D), (D,E)}
4. EnforceArcConsistency(D, B): remove 4 from $\mathcal{D}_D$.
   queue={(C,E), (E,C), (E,D), (D,E)}
5. EnforceArcConsistency(C, E): remove 1 from $\mathcal{D}_C$, enqueue (B,C).
   queue={(E,C), (E,D), (D,E), (B, C)}
6. EnforceArcConsistency(E, C): remove 3 and 4 from $\mathcal{D}_E$.
   queue={(E,D), (D,E), (B, C)}
7. EnforceArcConsistency(E, D): consistent.
   queue={(D,E), (B, C)}
8. EnforceArcConsistency(D, E): remove 1 from $\mathcal{D}_D$, enqueue (B,D)
   queue={(B, C), (B,D)}
9. EnforceArcConsistency(B, C): remove 2 from $\mathcal{D}_B$, enqueu (D, B).
   queue={(B,D), (D, B)}
10. EnforceArcConsistency(B, D): consistent.
    queue={(D,B)}
11. EnforceArcConsistency(D, B): consistent.
    queue={}

Finally, the domains are: $A \in \{1, 2, 3, 4\}$, $B \in \{3, 4\}$, $C \in \{2, 3\}$, $D \in \{2, 3\}$, and $E \in \{1, 2\}$.

Question 3 (10 marks)

Consider the map below.



There are five regions and we want to color this map with the following constraints:

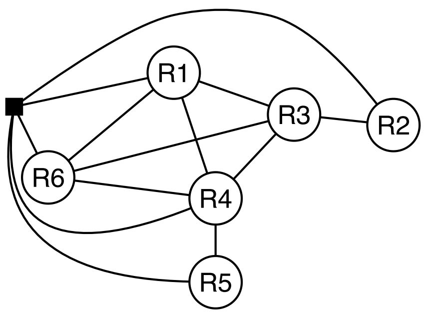
- R1 can only be blue or green.
- R2 can only be yellow or black.
- R3 can only be green or yellow.
- R4 can only be green or black.
- R5 can only be purple.
- R6 can only be green or yellow.
- R3 must have the same color as R6.
- Purple regions cannot be adjacent to any green regions.
- Adjacent regions must have different colors.
- Only one region can be colored black.

(a) (2 marks) Represent the problem as a CSP by specifying the variables and domains.

Solution:
- R1 $\in$ {blue, green}
- R2 $\in$ {yellow, black}
- R3 $\in$ {green, yellow}
- R4 $\in$ {green, black}
- R5 $\in$ {purple}
- R6 $\in$ {green, yellow}

(b) (2 marks) Draw the constraint hypergraph for this CSP.

Solution:



For simplicity, only one square is drawn in the above figure. It is ok if two black squares are drawn as long as they connect to all variables.

(c) (6 marks) Apply backtracking search with forward checking to find the solution. Use the MRV heuristics. Show the steps.

Solution: The order of variable must be R5, R4, R2, R3, (R6, R1). Order of R6 and R1 can be swapped.

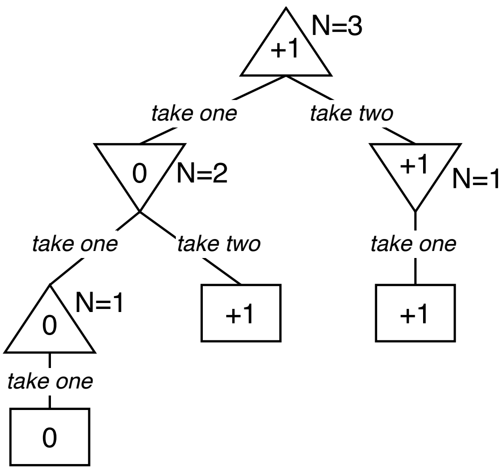
1. Assign R5=purple, remove purple from the domain of R4.
2. Assign R4=black, remove black from the domain of R2.
3. Assign R2=yellow, remove yellow from the domain of R3.
4. Assign R3=green, remove green from the domain of R1 and remove yellow from the domain of R6.
5. Assign R1=blue.
6. Assign R6=green.

The solution is {R1: blue, R2: yellow, R3: green, R4: black, R5: purple, R6: green}.

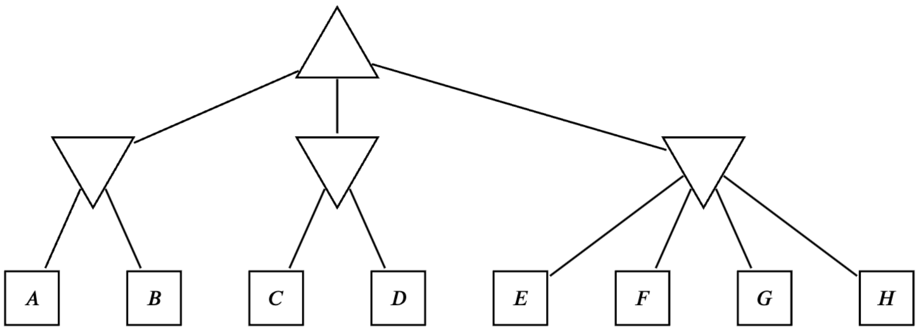
Question 4 (20 marks)

(a) (6 marks) Two players, MAX and MIN, are playing the Matchstick game. There are N matchsticks and player take turns to take away one or two matchsticks. The player who takes the last one loses. The MAX player starts the first move and the utility for winning is +1 and that for losing is 0. Consider $N = 3$, perform mini-max search to determine the move for MAX. Draw out the game tree generated.

Solution: The move for MAX is taking two matchsticks. The game tree generated is given by:



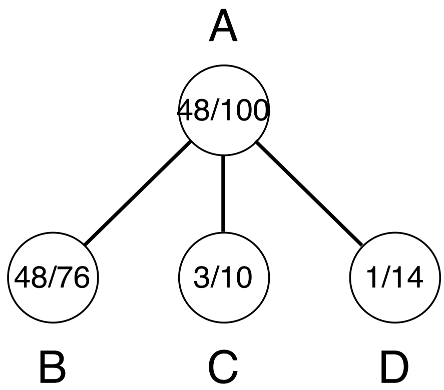
(b) (4 marks) In the following mini-max game tree, assuming that use alpha-beta pruning and visit nodes from left to right, select ***all*** nodes that are not possible to be pruned.



■ **A** ■ **B** ■ **C** ☐ D ■ **E** ☐ F ☐ G ☐ H

☐ None of the above

(c) Tom and Jerry are playing a game and they are using Monte Carlo Tree Search to help them find the best move. Below shows a part of the tree constructed. The number in node A represents the win percentage for Tom and numbers in B, C and D represent the win percentages for Jerry corresponding to three different actions. For simplicity, consider B, C, and D leaf nodes and continue the MCTS.



i. (5 marks) Use UCT to determine the next leaf node to expand, use $C = 1.4$. You can use the approximate values in the tables below.

x	100
$\log x$	4.6

x	0.06	0.17	0.25	0.33	0.46	0.52
$\sqrt{x}$	0.25	0.41	0.50	0.57	0.68	0.72

Solution: We compute the UCB1 scores for B, C, and D as follows:

$\mathrm{UCB1}(B) = \frac{48}{76} + 1.4 \times \sqrt{\frac{\log 100}{76}} \approx 0.976,$

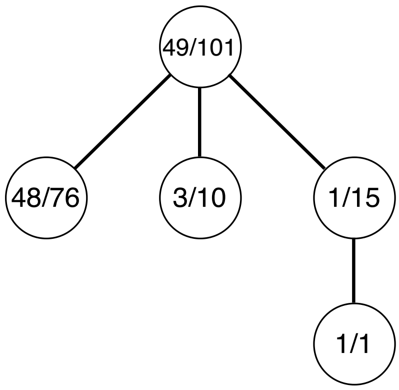
$\mathrm{UCB1}(C) = \frac{3}{10} + 1.4 \times \sqrt{\frac{\log 100}{10}} \approx 1.250,$

$\mathrm{UCB1}(D) = \frac{1}{14} + 1.4 \times \sqrt{\frac{\log 100}{14}} \approx 0.874$

Node C has the highest UCB1 score, so we expand C next.

ii. (5 marks) Suppose we use a different approach and the leaf node D is selected. The simulation result is that Tom wins. Show the updated tree.

Solution: The updated tree is as below.



Question 5 (20 marks)

(a) Suppose A is false, B is true, and C is false. Determine the truth value for the following propositional logic.

i. (2 marks) $(A \lor \neg A) \Rightarrow B \land \neg C \land A$ ____**false**____

ii. (2 marks) $(\neg A \lor \neg B) \Rightarrow \neg A \lor C$ ____**true**____

iii. (2 marks) $A \lor (B \land C) \Leftrightarrow (C \lor B) \land (A \lor C)$ ____**true**____

(b) Consider propositional atoms P, Q, and R, determine if the following statements are correct. If not, explain the reasons in one sentence.

i. (1 mark) $\neg P\&Q$ is true in propositional logic.

Solution: Incorrect. $\neg P\&Q$ is not a valid formula in propositional logic.

ii. (2 marks) $\neg P \land (Q \lor R)$ entails P.

Solution: Incorrect.
$M(\neg P \land (Q \lor R)) =$\{\{P=0,Q=1,R=1\}, \{P=0,Q=1,R=0\}, \{P=0,Q=0,R=1\}\}.
Clearly, $M(\neg P \land (Q \lor R)) \bigcap M(P) = \varnothing$.

iii. (2 marks) Given two sentences $R \lor \neg P \lor Q$ and $\neg R \lor P$, we can apply the resolution inference rule and obtain Q.

Solution: Incorrect. When applying resolution inference rule, we can only resolve one pair of complementary symbol at a time. We can obtain either $\neg R \lor R \lor Q$ or $\neg P \lor P \lor Q$, but not Q.

(c) Convert the following propositional logic sentences into conjunctive normal form (CNF).

i. (2 marks) $(P \Rightarrow Q) \Rightarrow R$

Solution:
1. Eliminate "$\Rightarrow$": $\neg(\neg P \lor Q) \lor R$
2. Move "$\neg$" inwards and eliminate double negation: $(P \land \neg Q) \lor R$
3. Distribute "$\lor$" over "$\land$": $(R \lor P) \land (R \lor \neg Q)$

ii. (2 marks) $(P \Rightarrow Q) \Leftrightarrow (P \Rightarrow R)$

Solution:
1. Eliminate "$\Leftrightarrow$": $((P \Rightarrow Q) \Rightarrow (P \Rightarrow R)) \land ((P \Rightarrow R) \Rightarrow (P \Rightarrow Q))$

For the part $((P \Rightarrow Q) \Rightarrow (P \Rightarrow R))$:
1. Eliminate "$\Rightarrow$": $\neg(\neg P \lor Q) \lor (\neg P \lor R)$
2. Move "$\neg$" inwards and eliminate double negation: $(P \land \neg Q) \lor (\neg P \lor R)$
3. Distribute "$\lor$" over "$\land$": $(\neg P \lor R \lor P) \land (\neg P \lor R \lor \neg Q)$

Since $\neg P \lor P$ is always true, we can also simplify this as $(\neg P \lor R \lor \neg Q)$.
Similarly, the second part is converted into $(\neg P \lor Q \lor P) \land (\neg P \lor Q \lor \neg R)$, or $(\neg P \lor Q \lor \neg R)$.
Therefore, the final solution is: $(\neg P \lor R \lor \neg Q) \land (\neg P \lor Q \lor \neg R)$.

(d) Interpret the following first-order logic using English sentences. $\text{Has}(x, y)$ means that object x has object y:

i. (1 mark) $\exists x \ \text{Has}(\text{Tom}, x) \land \text{Cat}(x) \land \text{Cute}(x)$.

Solution: Tom has a cute cat.

ii. (2 marks) $\exists x \ \text{Dog}(x) \land \neg\, \text{Scary}(x)$.

Solution: There are some dogs that are not scary.

iii. (2 marks) $\neg\,(\exists x \ (\text{Has}(\text{Jerry}, x) \land \text{Cat}(x))) \Rightarrow \text{Lonely}(\text{Jerry})$.

Solution: Unless Jerry has a cat, he is lonely. (Equivalently: if Jerry does not have any cats, he is lonely.)

Question 6 (20 marks)

Given a dataset with the following validation set:

	x_1	x_2	x_3	x_4	x_5	x_6	Label
1	0	0	0.3	0	1.1	0.6	$+1$
2	1	0	0.1	0	1.9	0.7	$+1$
3	0	0.1	0.4	1	1.3	0.6	-1
4	1	0	0.2	0	1.7	0.8	-1
5	1	0.1	0.5	0	1.1	0.7	-1

Suppose we have a linear classifier, with its coefficients and the bias given by:

$$\mathbf{w} = [0.4, 0.8, -2, 0.4, 0.9, -1]^\top \ \text{ and } \ b = -0.1$$

(a) (3 marks) Compute the logistic loss for the first data sample.

Solution: The logistic loss is given by $\ell = \log(1 + \exp(-(\mathbf{w}^\top \mathbf{x})y))$. Therefore, the loss values for the first data samples is 0.86.

(b) (3 marks) Compute the hinge loss for the second data sample.

Solution: The margin of the second data point is $(\mathbf{w}^\top \mathbf{x})y = 1.11 > 1$. Therefore, the hinge loss is zero.

(c) (6 marks) For the five data samples given above, suppose we use $\text{sign}(\mathbf{w}^\top \mathbf{x} + b)$ as the prediction function. Construct the confusion matrix for the predictions and compute the precision, recall, and F1 scores.

Solution: The prediction of the five samples are -1, $+1$, $+1$, $+1$, and -1. The confusion matrix is given by:

	Predicted Positive	Predicted Negative
Ground-truth Positive	TP=1	FN=1
Ground-truth Negative	FP=2	TN=1

The precision is $\text{Precision} = \frac{TP}{TP+FP} = 0.5$.

The recall is $\text{Recall} = \frac{TP}{TP+FN} = 0.33$.
The F1 score is $\text{F1} = \frac{2\times\text{Precision} \times \text{Recall}}{\text{Precision} + \text{Recall}} = 0.4$.

(d) (8 marks) Consider the following training sample and logistic loss. If we update the model parameters using gradient descent with the learning rate of $\eta = 0.1$, what are the updated $\mathbf{w}$ and b after one iteration?

x_1	x_2	x_3	x_4	x_5	x_6	Label
0.5	0.2	0	0.3	1.4	0	-1

Solution: First, we absorb the bias into the weight:
Let $\overline{\mathbf{w}} = [0.4, 0.8, -2, 0.4, 0.9, -1, -0.1]^\top$ and let $\overline{\mathbf{x}} = [0.5, 0.2, 0, 0.3, 1.4, 0, 1]$.
The gradient of the logistic loss is given by:$\nabla_{\overline{\mathbf{w}}} = -y\frac{\exp(-(\overline{\mathbf{w}}^\top\overline{\mathbf{x}})y)}{1+\exp(-(\overline{\mathbf{w}}^\top\overline{\mathbf{x}})y)}\overline{\mathbf{x}} = 0.84\overline{\mathbf{x}}$.
The update rule of gradient descent algorithm is: $\overline{\mathbf{w}} \leftarrow \overline{\mathbf{w}} - \eta\nabla_{\overline{\mathbf{w}}}$.
Therefore, the updated parameters are:
$\mathbf{w} = [0.358, 0.7832, -2, 0.3748, 0.7824, -1]^\top$ and $b = -0.184$.