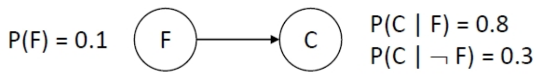
COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence
Exercise 9-10 Answer

1. There are 2 white balls and 1 black ball in the bag. Each time to take one ball out of the bag, and put it back to the bag. Event A: "the first time to take the white ball", Event B: "the second time to take the black ball". Calculate $P(AB)$.

   Answer:

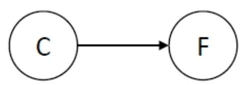
   It is clear that events A and B are independence. Therefore. $P(AB) = P(A)P(B) = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

2. Consider the following Bayesian network, where F = "having the flu" and C = "coughing":



   (a) Write down the joint probability table specified by the Bayesian network.

   (b) Determine the probabilities for the following Bayesian network, so that it specifies the same joint probabilities as the given one.



Answer:

(a) The joint probability table is shown as follows.

F	C	
t	t	$0.1 \times 0.8 = 0.08$
t	f	$0.1 \times 0.2 = 0.02$
f	t	$0.9 \times 0.3 = 0.27$
f	f	$0.9 \times 0.7 = 0.63$

(b)

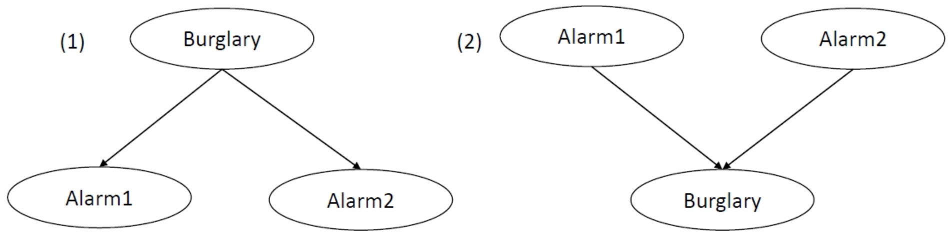
$$\begin{aligned} P(C) &= 0.08 + 0.27 = 0.35 \\ P(F|C) &= \frac{P(F,C)}{P(C)} = \frac{0.08}{0.35} \approx 0.23 \\ P(F|\neg C) &= \frac{P(F,\neg C)}{P(\neg C)} = \frac{0.02}{0.65} \approx 0.03 \end{aligned} \tag{1}$$

3. A circuit system consists of 2 components and both of them work independently, let the probability of each component working properly be 0.9, try to find the probability of the following system working properly.
   (a) Series circuits system S_1.
   (b) Parallel circuits system S_2.

   Answer:

   (a) For series circuits system S_1, system working properly means both components works properly. $S_1 = A_1A_2$, where A_i means the i^{th} component works properly. Therefore, $P(S_1) = P(A_1)P(A_2) = 0.9 \times 0.9 = 0.81$.
   (b) For parallel circuits system S_2, system working properly means at least one component works properly. $S_2 = A_1 \cup A_2$. Therefore, $P(S_1) = P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1A_2) = 0.9 + 0.9 - 0.9^2 = 0.99$.

4. To safeguard your house, you recently installed two different alarm systems by two different reputable manufacturers that use completely different sensors for their alarm systems.



(a) Consider the first Bayesian network. How many variables are required to describe this joint distribution?

(b) Consider the second Bayesian network. Assume that:

$$\begin{aligned} &P(Alarm1) = 0.1 \\ &P(Alarm2) = 0.2 \\ &P(Burglary|Alarm1, Alarm2) = 0.8 \\ &P(Burglary|Alarm1, : Alarm2) = 0.7 \\ &P(Burglary|\neg Alarm1, Alarm2) = 0.6 \\ &P(Burglary|\neg Alarm1, \neg Alarm2) = 0.5 \end{aligned} \tag{2}$$

Calculate $P(Alarm2|Burglary, Alarm1)$. Show all of your reasoning.

Answer:

(a) We need to specify 5 variables (probabilities), namely $P(Burglary)$, $P(Alarm1|Burglary)$, $P(Alarm1|\neg Burglary)$, $P(Alarm2|Burglary)$ and $P(Alarm2|\neg Burglary)$.

4 variables is acceptable. As the total probability is 1, 4 variables is also enough (the last variable can be obtained by $1 - \sum_{i=1}^{4} P_i$).

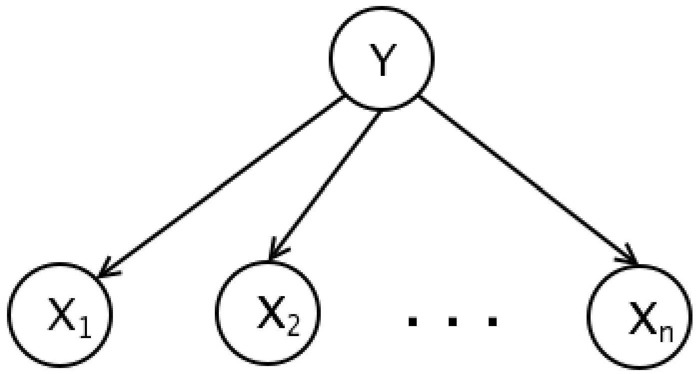
(b)

$$P(Alarm2|Burglary, Alarm1) = \frac{P(Alarm1, Alarm2, Burglary)}{P(Burglary, Alarm1)} = \frac{0.016}{0.072} \approx 0.22 \tag{3}$$

where

$$\begin{aligned} P(Alarm1, Alarm2, Burglary) &= P(Alarm1)P(Alarm2)P(Burglary|Alarm1, Alarm2) \\ &= 0.1 \times 0.2 \times 0.8 = 0.016 \\ P(Alarm1, \neg Alarm2, Burglary) &= P(Alarm1)P(\neg Alarm2)P(Burglary|Alarm1, \neg Alarm2) \\ &= 0.1 \times 0.8 \times 0.7 = 0.056 \\ P(Burglary, Alarm1) &= P(Alarm1, Alarm2, Burglary) + P(Alarm1, \neg Alarm2, Burglary) \\ &= 0.016 + 0.056 = 0.072 \end{aligned} \tag{4}$$

5. Suppose we have a boolean data X. To complete describe the distribution $P(X)$, we need to specify one value: $P(X = 0)$(since $P(X = 1) = 1 - P(X = 0)$). Thus, we say, this distribution can be characterized with one variable. Now, consider $N + 1$ binary random data $X_1 \cdots X_N$, Y that factorize according to following figure.



(a) Suppose you wish to store the joint probability distribution of these $N + 1$ data as a single table. How many variables will you need to represent this table?

(b) Now, suppose you were to utilize the fact the joint distribution factorizes according to the Bayes Network. How many variables will you need to completely describe the distribution if you use the Bayesian Network representation? In other words, how many variables will you need to fully specify the values of all the conditional probability tables in this Bayesian Network.

Answer:

(a) There are $N+1$ boolean data. If we have a variable for every possible instantiation of the variables, there will be 2^{N+1} variables. But these variables need to sum to one, so we can drop one variable of the 2^{N+1}. The answer is 2^{N+1} or $2^{N+1} - 1$.

(b) The CPT of Y will need one variable(its a boolean data without any parents). For each X, we'll need 2 variables (for example, $P(X|Y)$, $P(X|\neg Y)$). Therefore, we have $2N+1$ variables in total.