COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence
Exercise 11

1. Determine whether the following statement is Ture.
   (a) In MLE, the probability is calculated with the given parameter θ.
   (b) The likelihood function to determine the probability of flipping heads with a biased coin if we see 3 heads out of 5 flips is a polynomial of degree 5.
2. There is a bag with 12 red and blue balls. Each time, we take out one ball and put it back. After three times, we get "Blue, Blue, Red". What is the maximum likelihood for the number of blue balls in bag?
3. We have a coin that you think is biased. We flip it 4 times and get the sequence H H H T (H is heat, T is tail). What is the maximum likelihood estimate for the probability of getting heads?
4. We have a bag with 5 red and blue balls. We pull out a ball and it is red. We put it back and add 3 blue balls and pull out another ball, which is blue. What is the maximum likelihood for the original number of blue balls.
5. Consider the exponential distribution $f(x|\theta) = \theta e^{-\theta x}$ where $\theta > 0$. We have a random independent sample $x_1, x_2, \cdots, x_n$. The mean of this distribution is $\mu = \frac{1}{\theta}$.
   Find the maximum likelihood estimators of the mean μ and θ.
6. Consider the geometric density $f(x|p) = p(1-p)^x$ where $x = 0, 1, 2, \cdots, n$. We have a random independent samples $x_1, x_2, \cdots, x_n$. Find the maximum likelihood estimator of the mean and p.