COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence
Exercise 3

1. Calculate the eigenvector and eigenvalue of following matrices:
   (a) $\begin{bmatrix} 5 & -1 \\ 0 & 3 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & -1 \\ 2 & 3 \end{bmatrix}$
   (c) $\begin{bmatrix} 5 & -1 \\ 2 & 8 \end{bmatrix}$
2. Calculate the sum of eigenvalues of following matrix : $\begin{bmatrix} 4 & 3 & 3 \\ 6 & 10 & 2 \\ 7 & 7 & 1 \end{bmatrix}$
3. Prove "λ^{-1} is the eigenvalue of $\mathbf{A}^{-1}$", with equation $\mathbf{Ax} = \lambda\mathbf{x}$.
4. Given a matrix $\mathbf{A} = \begin{bmatrix} -3 & 1 \\ -5 & 3 \end{bmatrix}$, calculate the $\mathbf{A}^2, \mathbf{A}^5$ and $\mathbf{A}^{20}$, respectively.
5. Consider matrix $\mathbf{A} = \begin{bmatrix} 0.4 & 1-c \\ 0.6 & c \end{bmatrix}$. Find the eigenvalues of $\mathbf{A}$, which may represent by c. And calculate the rank of $\mathbf{A}$ when $c = 0.6$.
6. Find the condition of b that can ensure the symmetric matrix $\begin{bmatrix} 1 & b \\ b & 1 \end{bmatrix}$ has 1 negative eigenvalue. In this case, determine whether this matrix can have 2 negative eigenvalue.