

COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence

Exercise 5 Answer

1. Determine whether the following problems are optimization problems:

- a) A company wants to minimize production costs during production while meeting customer needs and resource constraints.
- b) Record the speed of chemical reaction at different temperatures and explore the effect of temperature on reaction rate.
- c) A company is assessing market demand for its product, collecting sales data for the past three years, in order to conduct market analysis and forecast future sales trends.
- d) A company is assessing market demand for its product, collecting sales data for the past three years, in order to develop a sales strategy to earn more profits, taking into account market competition and production costs.

Answer:

- a) It is an optimization problem.
Decision variables : product quality and quantity
Constraint: customer needs and resource
Objective : minimize production costs
- b) It is **NOT** an optimization problem.
There is no decision variables, and constraint.
- c) It is **NOT** an optimization problem.
There is no decision variables, and constraint.
- d) It is an optimization problem.
Decision variables : a sales strategy
Constraint: market competition and production costs
Objective : maximize profits

2. Solve the following optimization problem:

(a)

$$\max_{x,y} 2x + 3y$$

$$\text{s.t. } x \leq 15$$

$$2x + 5y \leq 50$$

$$x + y \leq 15 \quad (1)$$

$$3x + y \leq 35$$

$$x \geq 0$$

$$y \geq 0.$$

Answer:

$$\begin{aligned} x &= \frac{25}{3} \\ y &= \frac{20}{3} \end{aligned} \quad (2)$$

(b)

$$\min_{x,y} 4x + 5y$$

$$\text{s.t. } 2x + 3y \geq 30$$

$$x + 5y \geq 20 \quad (3)$$

$$2x - y \geq 0$$

$$x \geq 0$$

$$y \geq 0$$

Answer:

$$\begin{aligned} x &= \frac{15}{4} = 3.75 \\ y &= \frac{15}{2} = 7.5 \end{aligned} \quad (4)$$

3. A chicken farm has 10,000 chickens, feeding with mixed feed that is mixed with animal feed and grain feed. Each chicken eats 0.5kg mixed feed on average every day, of which animal feed shall not be less than $\frac{1}{5}$ of grain feed. Animal feed 0.9 dollar/kg, grain feed 0.28 dollar/kg. Feed companies can only supply 50000kg of grain feed per week. How to mix feed to minimize costs.

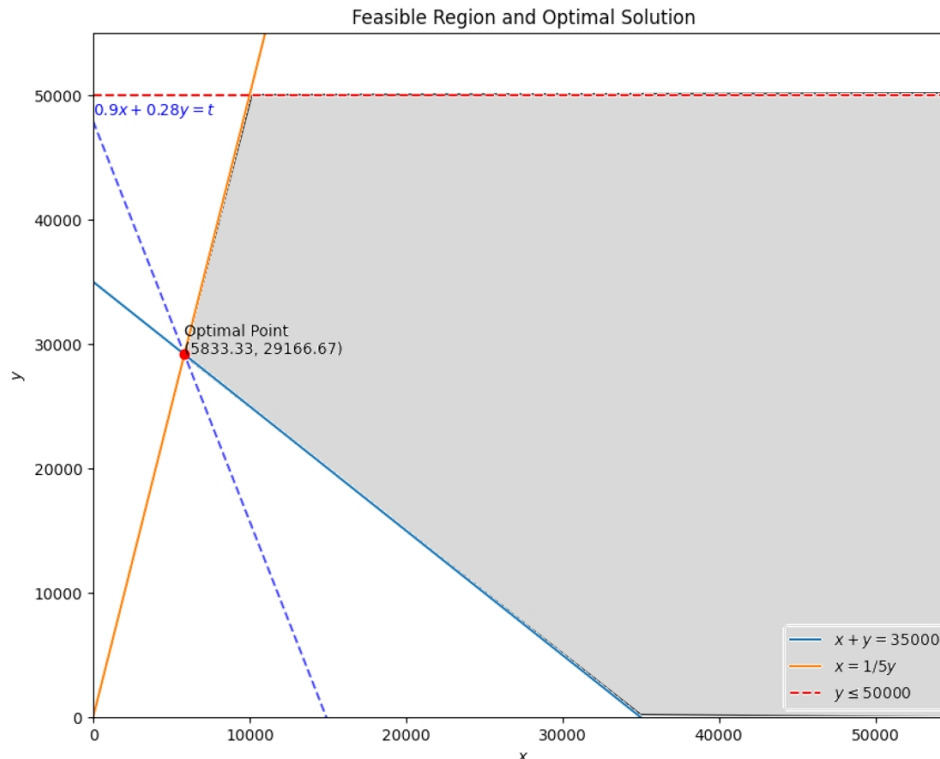
Answer:

We set that the farm need the animal feed x kg and grain feed y kg, per week.

Then, we write down the following optimization problem:

$$\begin{aligned}
 & \min_{x,y} 0.9x + 0.28y \\
 & \text{s.t. } x + y \geq 35000 \\
 & \quad x \geq \frac{1}{5}y \\
 & \quad 0 \leq y \leq 50000 \\
 & \quad 0 \leq x.
 \end{aligned} \tag{5}$$

Then, we solve the above linear programming problem. We plot the graph as follow, and find the intersection.



The solution:

$$\begin{aligned}
 x &= \frac{17500}{3} \\
 y &= \frac{87500}{3}
 \end{aligned} \tag{6}$$