COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence
Exercise 6

1. Are the following sets convex? Give a brief justification for each of the following cases:

   (a) $C = \left\{ \mathbf{x} \in \mathbb{R}^n | \mathbf{a}^\mathbf{T} \mathbf{x} \geq \mathbf{b} \text{ or } \|\mathbf{x} - \mathbf{a}\| \leq \varepsilon \right\}$

   (b) $C = \left\{ \mathbf{x} \in \mathbb{R}^n | \mathbf{x}^\mathbf{T} \mathbf{y} \geq 1 \text{ for all } \mathbf{y} \in S \right\}$

   (c) $C = \left\{ (x, y) \in \mathbb{R}^2 | y \geq x^2 \right\}$

2. Prove that any locally minimum point of a convex function is globally minimum.

3. Consider whether the following functions are convex function:

   (a) $f(x) = ax + b$, where $a, b \in \mathbf{R}$

   (b) $f(x) = x^p$, where $x > 0$, and $p \geq 1$ or $p \leq 0$

   (c) $f(x) = x^p$, where $x > 0$, and $0 < p < 1$.

   (d) $f(x) = x \log x$, for $x > 0$.

   (e) $f(x) = \log x$, for $x > 0$.