COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence
Exercise 1 Answer

1. Let $\mathbf{v} = \begin{bmatrix} 4 \\ 2 \\ -3 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 7 \\ -2 \\ 9 \end{bmatrix}$, answer the following questions:
   (a) Calculate $4\mathbf{v} + 3\mathbf{w}$
   (b) Calculate $\mathbf{v} \cdot \mathbf{w}$
   (c) Calculate $\|\mathbf{v}\| \cdot \|\mathbf{w}\|$
   (d) Calculate $cos\theta$, where θ is the angle between $\mathbf{v}$ and $\mathbf{w}$.
2. Find the vector $\mathbf{v}$ which is orthogonal to $\mathbf{w} = \begin{bmatrix} 7 \\ \\ 2 \end{bmatrix}$ and $\|\mathbf{v}\| = 1$.
3. For two vector $\mathbf{v}$ and $\mathbf{w}$, determine if $\|\mathbf{v} + \mathbf{w}\| \leq \|\mathbf{v}\| + \|\mathbf{w}\|$ is true. If yes, prove it. If not, give examples.
4. For three martix $\mathbf{V} = \begin{bmatrix} 4 & 6 \\ & \\ 2 & 2 \end{bmatrix}$, $\mathbf{W} = \begin{bmatrix} 7 & 8 \\ & \\ -2 & 3 \end{bmatrix}$ and $\mathbf{U} = \begin{bmatrix} \mathbf{V} & \mathbf{W} \end{bmatrix}$, answer the following questions:
   (a) Calculate $2\mathbf{V} + 5\mathbf{W}$
   (b) Calculate $\mathbf{VU}$.
   (c) Calculate $2\mathbf{VU} + 5\mathbf{WU}$
5. For matrix $\mathbf{A} = \begin{bmatrix} 0.5 & 4 \\ & \\ -2 & 4 \end{bmatrix}$ vector $\mathbf{b} = \begin{bmatrix} 6 \\ \\ 2 \end{bmatrix}$, find vector $\mathbf{x}$ that satisfies $\mathbf{Ax} = \mathbf{b}$