COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence
Assignment 1

Note:

1. Instruction of assignment submission:

(a) Write all your answers clearly using Microsoft Word/Latex;

(b) For students who use Microsoft Word, please use "Insert → Equation" to write all the formulations;

(c) Name your document using the following format: COMP7180_Assignment_1_StudentID_StudentName.doc or COMP7180_Assignment_1_StudentID_StudentName.pdf;

(d) Submit the document on Moodle;

(e) Taking pictures/photos of handwritten manuscript wont be accepted and will be given Zero Mark!

2. The submission deadline is 2024 November 4 17:00

3. This is an individual work. Plagiarism is strictly forbidden. Students who plagiarized and who were plagiarized will be given Zero Mark.

1. **(10 Marks)**

(a) (4 Marks) Given vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 7 \\ 3 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 9 \\ 0 \\ 3 \end{bmatrix}$ and $\mathbf{x} = \begin{bmatrix} -28 \\ 35 \\ 22 \end{bmatrix}$. Please calculate a, b, c that satisfy equation $a\mathbf{u} + b\mathbf{v} + c\mathbf{w} = \mathbf{x}$, and write down the calculation details.

(b) (6 Marks) Construct 2 vectors $\mathbf{u}$ and $\mathbf{v}$ with the last four numbers of your student ID. $\mathbf{u} = \begin{bmatrix} a \\ b \end{bmatrix}$, where a, b are the fifth and sixth numbers of your ID. $\mathbf{v} = \begin{bmatrix} c \\ d \end{bmatrix}$, where c, d are the seventh and eighth numbers of your ID. (For student ID: 23456789, we have $\mathbf{u} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$). Calculate $cos\theta$, where θ is the angle between $\mathbf{u}$ and $\mathbf{v}$, and write down the calculation details.

2. **(14 Marks)** Given matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$. Proof the following multiplication laws of matrix:

   (a) (6 Marks) $\mathbf{AB} \neq \mathbf{BA}$.

   (b) (8 Marks) $(\mathbf{A} + \mathbf{B})\mathbf{C} = \mathbf{AC} + \mathbf{BC}$.

3. **(16 Marks)** Construct 2 vectors $\mathbf{u}$ and $\mathbf{v}$ with your student ID. $\mathbf{u} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}$, where a, b, c, d are the first four numbers of your ID. $\mathbf{v} = \begin{bmatrix} e \\ f \\ g \\ h \end{bmatrix}$, where e, f, g, h are the last four numbers of your ID. (For student ID: 23456789, we have $\mathbf{u} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 6 \\ 7 \\ 8 \\ 9 \end{bmatrix}$). We have vector $\mathbf{w} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

   (a) (6 Marks) Write down one vector $\mathbf{a}$, which is in the space that is spanned by vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$. And prove that vector $\mathbf{a}$ is in the space that is spanned by vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$.

   (b) (10 Marks) Write down one vector $\mathbf{b}$, which is **not** in the space that is spanned by vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$. Find the projection point p of vector $\mathbf{b}$ onto the space that is spanned by vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$, and write down the calculation details.

4. **(14 Marks)** Supposing 3 measurements b_1, b_2, b_3 are marked:

$$b = 0 \text{ at } t = 3 \text{ , } b = 2 \text{ at } t = 9 \text{ , } b = 5 \text{ at } t = 38 \tag{1}$$

   (a) (6 Marks) Find the closest straight line $b = Dt$, and write down the calculation details.

   (b) (8 Marks) Find the closest parabola $b = C + Dt + Et^2$, and write down the calculation details.

5. **(9 Marks)** Calculate the eigenvalue of following matrix.

   (a) (3 Marks) $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix}$.

   (b) (3 Marks) $\mathbf{B} = \begin{bmatrix} 0 & 1 \\ 2 & 0 \end{bmatrix}$.

   (c) (3 Marks) $\mathbf{C} = \begin{bmatrix} 5 & 5 \\ 5 & 5 \end{bmatrix}$.

6. **(14 Marks)** Consider a 3×3 matrix $\mathbf{A}$ with eigenvalues $0, 3, 8$. Calculate the following questions, and write down the calculation details.

   (a) (6 Marks) The rank of matrix $\mathbf{A}$.

   (b) (8 Marks) The eigenvalue of $(\mathbf{A}^3 + \mathbf{I})^{-1}$.

7. **(10 Marks) (10 Marks)** Performe SVD to matrix $\mathbf{A}$, and we have $\mathbf{A} = \mathbf{U}\Sigma\mathbf{V}^\top$. There are r singular values of matrix $\mathbf{A}$, which are $\sigma_1, \sigma_2, \cdots, \sigma_r$. Prove that: The eigenvalue of matrix $\mathbf{A}^\top\mathbf{A}$ is the square of singular value $\sigma_1^2, \sigma_2^2, \cdots, \sigma_r^2$.

8. **(13 Marks)** Construct 4 vectors $\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$, where the number $a_1, a_2, b_1, b_2, c_1, c_2, d_1, d_2$ are picked from your student ID. (For student ID: 23456789, we have $\mathbf{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 4 \\ 5 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 6 \\ 7 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 8 \\ 9 \end{bmatrix}$ or $\mathbf{a} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 3 \\ 7 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 4 \\ 8 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 5 \\ 9 \end{bmatrix}$.) And we have $\mathbf{e} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \mathbf{f} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

   (a) (6 Marks) Perform PCA with the data $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}, \mathbf{e}, \mathbf{f}$, write down the calculation details, and write down the largest principal component.

   (b) (2 Marks) Visualize these 6 vectors as data points. And divide these 6 vectors into 2 classes, each class contains 3 vectors. The vectors in each class are picked by yourself. (For example, we could have class 1 $(\mathbf{a}, \mathbf{b}, \mathbf{c})$, class 2 $(\mathbf{d}, \mathbf{e}, \mathbf{f})$, or class 1 $(\mathbf{a}, \mathbf{c}, \mathbf{e})$, class 2 $(\mathbf{b}, \mathbf{d}, \mathbf{f})$). Ensure that these two classes are linearly separable.

   (c) (5 Marks) Perform LDA with the data you obtain in question 8(b), write down the projection vector $\mathbf{w}$, project your data in the subspace, and write down the calculation details.