

COMP 7180 Quantitative Methods for Data Analytics and Artificial Intelligence

Exercise 2

1. Given matrices $\mathbf{A} = \begin{bmatrix} 5 & 2 \\ 7 & -9 \\ -2 & 6 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 4 & 4 \\ 6 & 8 \\ -8 & 3 \end{bmatrix}$, answer the following questions:

(a) Calculate \mathbf{A}^T

(b) Calculate $\mathbf{A}^T \mathbf{B}$

(c) Calculate $\mathbf{B}^T \mathbf{A}$

2. Determine whether the following column vectors are linearly independent. If not, calculate the rank r and give an example of one vector that can be represented by other vectors.

(a) $\begin{bmatrix} 2 & 9 & 5 & 4 \\ 3 & 2 & 7 & 6 \\ -4 & 8 & 3 & -8 \end{bmatrix}$

(b) $\begin{bmatrix} 3 & 8 & 8 & 9 & 2 \\ 0 & 3 & 1 & 1 & 3 \\ 0 & 0 & 5 & 2 & 8 \\ 0 & 0 & 0 & 0 & 8 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 7 & 0 & 1 & 3 \\ 8 & 1 & 3 & 9 & 10 \\ 4 & 7 & 0 & 7 & 0 \end{bmatrix}$

3. Given matrix $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3]$ and vectors \mathbf{a}_1 , \mathbf{a}_2 and \mathbf{a}_3 are independent vectors. Let vectors $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$,

$$\mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}, \|\mathbf{b}\| = \|\mathbf{c}\| \text{ and } \mathbf{b} \cdot \mathbf{c} = -\|\mathbf{b}\|^2. \text{ Are vectors } \mathbf{A}\mathbf{b} \text{ and } \mathbf{A}\mathbf{c} \text{ independent?}$$

4. Project the vector \mathbf{b} on to the line through vector \mathbf{a} :

$$(a) \mathbf{b} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \mathbf{a} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$(b) \mathbf{b} = \begin{bmatrix} -4 \\ 4 \end{bmatrix}, \mathbf{a} = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

$$5. \text{ Given matrix } \mathbf{A} = \begin{bmatrix} -6 & 1 \\ 6 & -6 \end{bmatrix} \text{ and vector } \mathbf{b} = \begin{bmatrix} -20 \\ -21 \end{bmatrix}. \text{ Calculate vector } \mathbf{x} \text{ that minimize } \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2.$$