Deep residual learning with product units

Ziyuan Li^{1,2*}, Uwe Jaekel¹ and Babette Dellen¹

¹Faculty of Mathematics, Informatics, Technology,
 University of Applied Sciences Koblenz, Joseph-Rovan-Allee 2,
 Remagen, 53424, Rhineland-Palatinate, Germany.
 ²TUM School of Natural Sciences, Technical University of Munich,
 Boltzmannstr. 10, Garching, 85748, Bavaria, Germany.

*Corresponding author(s). E-mail(s): ziyuan.li@tum.de; Contributing authors: jaekel@hs-koblenz.de; dellen@hs-koblenz.de;

Abstract

We propose a deep product-unit residual neural network (PURe) that integrates product units into residual blocks to improve the expressiveness and parameter efficiency of deep convolutional networks. Unlike standard summation neurons, product units enable multiplicative feature interactions, potentially offering a more powerful representation of complex patterns. PURe replaces conventional convolutional layers with 2D product units in the second layer of each residual block, eliminating nonlinear activation functions to preserve structural information. We validate PURe on three benchmark datasets. On Galaxy10 DECaLS, PURe34 achieves the highest test accuracy of 84.89%, surpassing the much deeper ResNet152, while converging nearly five times faster and demonstrating strong robustness to Poisson noise. On ImageNet, PURe architectures outperform standard ResNet models at similar depths, with PURe34 achieving a top-1 accuracy of 80.27% and top-5 accuracy of 95.78%, surpassing deeper ResNet variants (ResNet50, ResNet101) while utilizing significantly fewer parameters and computational resources. On CIFAR-10, PURe consistently outperforms ResNet variants across varying depths, with PURe272 reaching 95.01% test accuracy, comparable to ResNet1001 but at less than half the model size. These results demonstrate that PURe achieves a favorable balance between accuracy, efficiency, and robustness. Compared to traditional residual networks, PURe not only achieves competitive classification performance with faster convergence and fewer parameters, but also demonstrates greater robustness to noise. Its effectiveness across diverse datasets highlights the potential of product-unit-based architectures for scalable and reliable deep learning in computer vision.

Keywords: product unit, residual network, image classification

1 Introduction

Deep learning has profoundly advanced the field of image processing and computer vision [21, 16]. A pivotal innovation in this progress is the residual network (ResNet) [11], which effectively addresses the degradation problem encountered in deep networks, where increasing depth can hinder training due to vanishing gradients. By introducing residual connections, ResNet enables stable training of deeper architectures and facilitates effective feature reuse [12]. These advantages have established ResNet as a preferred architecture for numerous applications, including object classification and detection [1, 18], as well as medical image processing [8, 17].

Despite its strengths, ResNet exhibits notable limitations, particularly regarding computational efficiency [15]. Residual connections, while beneficial, do not inherently enhance the model's capability to capture complex interactions or hierarchical relationships within data [33].

Recently, advantages of product-unit neural networks over standard multilayer perceptrons have been demonstrated for certain tasks, e.g., 3D shape completion of point clouds [24] or the prediction of nuclear masses from data [3]. Product units, first introduced by [7], allow a multiplicative combination of inputs, increasing the representation capabilities of single network units or layers. For example, a layer of product units can directly represent sparse high-dimensional polynomials and functions containing roots and fractions.

While product units offer enhanced representational capabilities, their integration into deep architectures poses significant challenges. In particular, the strong nonlinearity of product units makes network training more difficult and can exacerbate issues such as vanishing gradients and degradation in very deep models.

To address these challenges while leveraging the expressive power of product units, we propose a novel architecture that combines them with residual learning: the product-unit residual network (PURe). More precisely, PURe integrates product units [7, 22, 4, 3, 24] into the residual block framework, thereby enhancing computational efficiency and significantly improving the ability to model complex nonlinear interactions. An example of a nonlinear relation that can be implemented this way is a convolutional filter that compares ratios rather than differences of neighboring regions. A filter of this form can identify identical patterns in regions with different lighting conditions while a conventional linear convolution is only sensitive to absolute differences and works best only on structures that appear under the same condition in the whole data set. Moreover, residual connections within PURe effectively mitigate optimization challenges associated with product units [4], ensuring stable gradient flow. The effectiveness of PURe is validated through extensive experiments on multiple image classification datasets, including Galaxy10 DECaLS [23], ImageNet [5] and CIFAR-10 [19].

Our contributions can be summarized as follows: (1) We propose a novel image classification network that integrates product units into residual blocks to enhance nonlinear modeling, PURe. (2) We combine residual connections with product units to stabilize gradient flow. (3) We design a two-dimensional (2D) product unit layer for convolutional processing, enabling compact modeling of complex nonlinear feature

interactions without the need for explicit activation functions. (4) Extensive experiments on Galaxy10 DECaLS, ImageNet, and CIFAR-10, showing superior accuracy, robustness, and efficiency compared to standard ResNets.

2 Related Work

ResNet [11] significantly advanced deep network training by mitigating the degradation problem that occurs as network depth increases. Compared to earlier architectures like VGGNet [28], which rely on straightforward sequential stacking, ResNet achieves higher accuracy with fewer parameters and greater computational efficiency by introducing residual connections. However, while residual structures facilitate optimization, they do not inherently enhance the expressiveness of the network for complex feature modeling. Moreover, deeper ResNet variants tend to suffer from increasing computational costs with diminishing performance gains [33, 14, 29].

Various advanced architectures have been proposed to further overcome these limitations and enhance network performance. DenseNet [14] strengthens feature reuse through densely connected layers, whereas EfficientNet [29] utilizes compound scaling to efficiently balance network depth, width, and resolution. Attention-based architectures, such as SENet [13] and Vision Transformers (ViTs) [6], enhance representational capacity by recalibrating feature maps and capturing long-range dependencies in image data, respectively. ResNeXt [31] introduces grouped convolutions to achieve an effective trade-off between accuracy and computational efficiency. MixNet [30] combines multiple kernel sizes within individual layers to better capture multiscale features. ConvNeXt [25], on the other hand, improves convolutional architectures by integrating transformer-inspired design elements, focusing primarily on optimizing architectural principles. Nevertheless, these methods often encounter challenges such as increased memory consumption, higher computational overhead [27], and limited effectiveness in modeling complex nonlinear relationships [9].

Other works aimed at improving the capability of neural networks in modeling nonlinear relationships by replacing the elementary summation units [26] of the neural network by product units [7, 22, 4, 3, 24]. Unlike traditional summation-based neurons, a product unit computes the product of its inputs, each increased to the power of a corresponding weight, which allows them to model high-dimensional sparse polynomial functions, but also roots and fractions. Product units have recently been embedded in feedforward neural networks with two hidden layers, one composed of summation units, one of product units [3, 24]. In [24], a complex-valued product-unit network was applied to the problem of 3D-shape completion of points clouds. In [3], product units were used to predict nuclear masses from data. In both works, it could be shown that neural networks containing product units showed better performance than standard neural networks for the selected tasks.

However, despite their potential, product units are currently mainly used in shallow networks that contain only a single product-unit layer and applied to low-dimensional data, limiting their utility in tasks involving higher-dimensional inputs, such as images and other structured data.

To overcome this limitation, we extend the product-unit concept into the image domain, enabling PURe to effectively process multidimensional 2D image data. By integrating product units into a residual-block framework, PURe benefits from stabilized training dynamics, as residual connections promote gradient flow across highly nonlinear transformations—such as those induced by exponentiated weights in product units. This integration not only retains the intrinsic advantages of product units but also significantly enhances their applicability to complex tasks involving structured, multidimensional data.

3 Methods

3.1 Product Unit

Mathematically, a product unit [7, 22, 4, 3, 24] is defined as

$$y = \prod_{i=1}^{n} x_i^{w_i},\tag{1}$$

where x_i represents the *i*-th input, w_i denotes the corresponding weight, and n is the total number of inputs. This formulation inherently captures complex nonlinear relationships, including power laws, roots, and higher-order polynomial interactions [4], which standard summation-based neurons struggle to model effectively without extensive architecture adjustments or increased depth, even when utilizing nonlinear activation functions. In contrast, the product unit naturally encodes these nonlinearities, providing more efficient parameter utilization and improved capability to represent intricate feature interactions.

To further enhance computational efficiency, the product operation can be reformulated using logarithmic and exponential transformations:

$$y = \exp\left(\sum_{i=1}^{n} w_i \log x_i\right). \tag{2}$$

This transformation leverages computationally efficient addition operations instead of direct multiplication, while preserving the expressive power and inherent advantages of the product unit formulation.

3.2 2D Product Unit

In convolutional neural networks (CNNs), each kernel performs localized operations by computing the weighted sum of spatially adjacent regions in the input image as it slides over the input. Mathematically, the convolution operation in CNNs is defined as

$$y(i,j) = \sum_{m=-k}^{k} \sum_{n=-k}^{k} w(m,n) \cdot x(i+m,j+n),$$
 (3)

where x(i,j) represents the input feature map, w(m,n) denotes the weight at position (m,n) in the kernel of size $(2k+1,n)\times(2k+1,n)$, and y(i,j) is the output at spatial position (i,j). Exactly as summation units, product units can also be adapted to data arranged in an 2D image grid. For a given kernel applied to a 2D input x, the 2D product unit computes the product of all inputs within the receptive field of the kernel, with weights determining the exponentiation of each input. This operation is expressed as

$$y(i,j) = \prod_{m=-k}^{k} \prod_{n=-k}^{k} x(i+m,j+n)^{w(m,n)}.$$
 (4)

The concept of 2D-dimensional arrangement of inputs is to be distinguished from the number of inputs or input dimensionality of each 2D product unit. The latter is determined by the resolution of the kernel, that is, $(2k+1)^2$, while the name 2D product unit refers to the former.

To enhance computational efficiency, the logarithmic transformation introduced in (2) can reformulate the product operation as a summation, that is,

$$y(i,j) = \exp\left(\sum_{m=-k}^{k} \sum_{n=-k}^{k} w(m,n) \cdot \log(x(i+m,j+n))\right).$$
 (5)

This formulation enables the computation of the 2D product unit in three steps. First, the inputs within the kernel's receptive field are transformed using a logarithmic function to linearize multiplicative interactions. Second, a standard weighted convolution is applied to the transformed inputs. Finally, the result is passed through an exponential function to revert to the original domain.

For real-valued data, such as images, the network operates entirely within the real domain. To prevent the logarithmic transformation from introducing complex values due to negative inputs, a trainable threshold parameter is introduced. Specifically, the minimum magnitude is calculated as the softplus activation of the trainable parameter, shifted by a small constant (that is, 10^{-7}) to ensure strict positivity. Input values are then clamped from below by this threshold before applying the logarithm. This stabilizes the logarithmic transformation and prevents extremely small inputs from producing disproportionately large negative values, which could disrupt gradient flow. The computational flow for a kernel of size 3×3 is illustrated in Fig. 1.

3.3 Product-Unit Residual Block

The core component of ResNet, the standard residual block [11], is illustrated in Fig. 2(a). This block consists of two convolutional layers, each followed by a ReLU activation function. A skip connection directly links the block input to its output, bypassing intermediate transformations. When the input and output dimensions differ, a downsampling operation (denoted as $Conv^d$) is introduced within the skip connection to match dimensions.

To enhance the expressiveness of residual blocks, we propose the product-unit residual block, illustrated in Fig. 2(b). In this variant, the second convolutional layer is replaced by a 2D product unit, and ReLU activation functions are removed. This

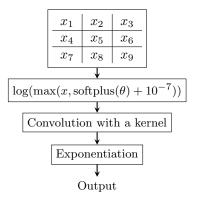


Fig. 1 Computational flow of the 2D product unit with a kernel of size 3×3 . θ is a trainable threshold parameter.

modification introduces multiplicative interactions within the block, enabling it to capture richer feature relationships while preserving detailed input information.

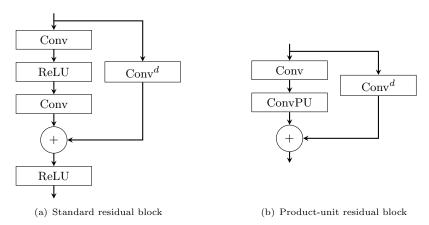


Fig. 2 Comparison between standard and product-unit residual blocks. $Conv^d$ denotes a convolutional layer used for dimension matching, and ConvPU represents a 2D product unit.

3.4 Network Architecture

The product-unit residual blocks can be further stacked to form PURe, as depicted in Fig. 3. The architecture begins with input layers tailored to the data format, followed by multiple stacked product-unit residual blocks for hierarchical and multiscale feature extraction, and finishes with task-specific output layers that translate learned representations into predictions for various applications such as classification, segmentation, or reconstruction.

Specifically, the architectural frameworks of PURe closely align with ResNet models that utilize standard two-layer convolutional residual blocks (Fig. 2(a)), such as

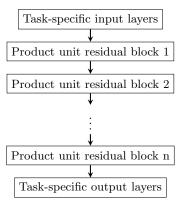


Fig. 3 Architecture of the proposed product-unit residual network.

ResNet18 and ResNet34 [11], which are commonly employed for benchmark datasets such as ImageNet. Notably, deeper ResNet variants, such as ResNet50 and ResNet101, adopt three-layer bottleneck residual blocks [11]. The primary modification in PURe involves replacing the standard convolutional residual blocks with product-unit residual blocks and removing all ReLU activation functions from the network. A detailed structural comparison between PURe18 and ResNet18 is provided in Table 1. It is important to note that, when comparing the parameter counts of the two architectures, each product-unit residual block introduces one additional trainable parameter compared to the standard residual block. Consequently, PURe18 contains eight more parameters than ResNet18.

The PURe architecture comprises four distinct residual stages, designated as Block A, B, C, and D, each progressively increasing channel dimensions and performing strategic downsampling. Specifically, downsampling with a stride of 2 is applied only at the initial residual blocks of stages B, C, and D. This design allows the network to transition smoothly from capturing low-level local features to high-level semantic features. Furthermore, the scale and capacity of PURe can be easily adjusted by varying the number of blocks within each stage like ResNet, for example, PURe18 employs the block configuration [2, 2, 2, 2] for Blocks A through D, respectively, whereas PURe34 adopts the configuration [3, 4, 6, 3].

4 Results

We train and test the PURe architecture for various depths on three different datasets, Galaxy10 DECaLS, ImageNet, and CIFAR-10. The implementation details and results for our network are presented separately for each dataset, including a comparative evaluation with ResNet [9].

Our network was implemented using PyTorch 2.6.0 with CUDA 12.4 support. All experiments were conducted on a workstation equipped with two NVIDIA Tesla V100-SXM2 GPUs (32 GB memory each).

Table 1 Example network architectures of PURe and ResNet

Block	PURe18	ResNet18			
Task-specific input layers	7×7 Conv, 64, stride $2 + BN$ 3×3 Max Pool, stride 2	7×7 Conv, 64, stride $2 + BN + \mathbf{ReLU}$ 3×3 Max Pool, stride 2			
Block A-1	3×3 Conv, 64 + BN 3×3 ConvPU, 64 + BN Residual connection	3×3 Conv, $64 + BN + ReLU$ 3×3 Conv, $64 + BN$ Residual connection $+ ReLU$			
Block A-2	3×3 Conv, $64 + BN$ 3×3 ConvPU, $64 + BN$ Residual connection	3×3 Conv, $64 + BN + ReLU$ 3×3 Conv, $64 + BN$ Residual connection $+ ReLU$			
Block B-1	3×3 Conv, 128, stride 2 + BN 3×3 ConvPU, 128 + BN Residual connection	3×3 Conv, 128, stride $2 + BN + ReLU$ 3×3 Conv, 128 + BN Residual connection + $ReLU$			
Block B-2	3×3 Conv, 128 + BN 3×3 ConvPU, 128 + BN Residual connection	3×3 Conv, 128 + BN + ReLU 3×3 Conv , 128 + BN Residual connection + ReLU			
Block C-1	3×3 Conv, 256, stride 2 + BN 3×3 ConvPU, 256 + BN Residual connection	3×3 Conv, 256, stride $2 + BN + ReLU$ 3×3 Conv, 256 + BN Residual connection + $ReLU$			
Block C-2	3×3 Conv, 256 + BN 3×3 ConvPU, 256 + BN Residual connection	3×3 Conv, 256 + BN + ReLU 3×3 Conv , 256 + BN Residual connection + ReLU			
Block D-1	3×3 Conv, 512, stride 2 + BN 3×3 ConvPU, 512 + BN Residual connection	3×3 Conv, 512, stride $2 + BN + ReLU$ 3×3 Conv, $512 + BN$ Residual connection $+ ReLU$			
Block D-2	3×3 Conv, 512 + BN 3×3 ConvPU, 512 + BN Residual connection	3×3 Conv, $512 + BN + ReLU$ 3×3 Conv, $512 + BN$ Residual connection + ReLU			
Task-specific output layers	Adaptive Average Pool + Flatten layer Fully connected layer + Softmax				

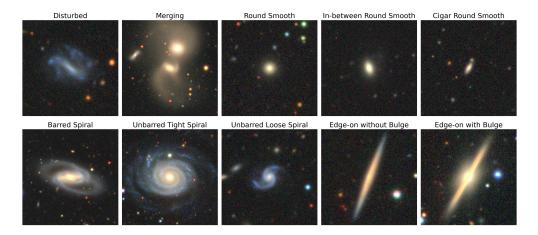
Note: Differences between the two architectures are highlighted in bold red. Units with a yellow background represent standard convolution operations, purple indicates 2D product units, and green corresponds to residual connections. Note that convolutions with a stride of 1 are not explicitly marked. Additionally, the downsampling operations in Block B-1, C-1, and D-1 with a kernel size of 3×3 and stride of 2 have been omitted for clarity.

4.1 Galaxy10 DECaLS

4.1.1 Dataset

The Galaxy10 DECaLS dataset [23] is derived from the Dark Energy Camera Legacy Survey (DECaLS), comprising approximately 17,736 RGB images categorized into 10 galaxy classes, and each image has dimensions of 256×256 pixels. As illustrated in Fig. 4, the dataset includes a diverse set of galaxy morphologies, such as Disturbed, Merging, Barred Spiral, and Unbarred Spiral types. Each class exhibits distinct structural characteristics, ranging from irregular or interacting forms to well-defined spiral

patterns and edge-on profiles. This morphological diversity presents a meaningful challenge for image classification models and serves as a valuable benchmark for evaluating the capacity of neural networks to learn complex visual patterns in astronomical imagery.



 ${\bf Fig.~4} \ \ {\bf Example~images~from~each~of~the~ten~classes~in~the~Galaxy10~DECaLS~dataset}.$

4.1.2 Implementation

For this classification task, we implemented two variants of our proposed architecture, PURe18 and PURe34, and compared them against five ResNet baselines: ResNet18, ResNet34, ResNet50, ResNet101, and ResNet152 [11].

Before training, the Galaxy10 DECaLS dataset was divided into training, validation, and test sets with a ratio of 0.90:0.05:0.05, using stratified sampling to maintain consistent class distributions across all subsets. To improve model generalization while ensuring fair comparisons, we employed static data augmentation, where augmented samples are generated and fixed before training, rather than applied dynamically during each epoch. Each training image was augmented once using random rotation ($\pm 20^{\circ}$), horizontal flipping, and random resized cropping, doubling the training-set size. No augmentation was applied to the validation or test sets.

In PURe, the parameters of the 2D product units are initialized using the Kaiming uniform initialization method [10] with a scaling factor of 5, while the additional trainable parameter within each product unit is initialized to zero. This initialization strategy provides stable and variance-controlled exponent weights for the multiplicative structure of the product unit, helping to prevent gradient instability during early training. All remaining layers, including convolutional and batch normalization layers, follow the standard ResNet initialization scheme: Kaiming normal initialization for convolutional weights, and constant initialization for batch normalization parameters, with weights set to 1 and biases to 0 [11, 10].

Both architectures were trained using stochastic gradient descent (SGD) with a batch size of 64. The hyperparameters for both architectures were carefully tuned through preliminary experiments to ensure optimal performance, thereby enabling a fair and meaningful comparison. The ResNet models were trained with an initial learning rate of 0.1, a momentum of 0.9, and a weight decay of 0.001. Due to the class imbalance inherent in the Galaxy10 DECaLS dataset, ResNet required relatively stronger regularization, thus we adopted a higher weight decay compared to the commonly used value of 0.0001 for more balanced datasets, such as ImageNet [11]. PURe models, due to the higher sensitivity of product units to optimization hyperparameters [4], were trained with a lower initial learning rate of 0.01, the same momentum of 0.9, and a larger weight decay of 0.01. For both architectures, a learning rate scheduler was employed that reduced the learning rate by a factor of 0.1 if the validation loss did not improve over three consecutive epochs. The model achieving the lowest validation loss during training was selected as the final model for evaluation. All models were trained for 30 epochs. Cross-entropy loss was used as the objective function during training.

Each network was trained five times. During training, the following metrics were recorded for each run: validation accuracy at each epoch, the time required to reach 80% validation accuracy, the time to reach the best validation performance, and the total training time.

Finally, all trained models were evaluated on both the original test set and a noisy variant with Poisson noise to assess robustness under perturbations. The noisy test set was created by simulating photon noise commonly encountered in astronomical imaging. Specifically, each test image was corrupted using Poisson-distributed noise whose intensity was proportional to the pixel values, thereby preserving the signal-dependent nature of real-world noise. This process was applied statically prior to evaluation to ensure consistency across all model comparisons. In order to quantify robustness, we also computed the relative performance drop under noise using the following metric:

$$\Delta_{\text{noise}} = \frac{\text{Accuracy}_{\text{clean}} - \text{Accuracy}_{\text{noisy}}}{\text{Accuracy}_{\text{clean}}} \times 100\%, \tag{6}$$

where $Accuracy_{clean}$ and $Accuracy_{noisy}$ represent the classification accuracies on the original and noisy test sets, respectively.

4.1.3 Result and Analysis

The overall performance of all evaluated models, including PURe and ResNet variants, is presented in Table 2. This includes results on both the original and noisy test sets, as well as training-related metrics such as convergence speed and total training time.

Analyzing Table 2, it is evident that PURe models generally outperform their ResNet counterparts. Specifically, PURe34 achieved the highest accuracy for the original test set, with a best accuracy of 84.89% and an average accuracy of 84.28%, surpassing even the significantly deeper ResNet152, which recorded a best accuracy of 84.44% and an average accuracy of 83.86%. Similarly, PURe18 not only showed superior performance compared to the ResNet18 architecture but also exceeded the

Table 2 Model Performance on Galaxy10 DECaLS

Model	#Params	Orig.	Noisy	Perf.	T@80%	T@Best	Train
	(M)	Acc. (%)	Acc. (%)	Drop (%)	Val. (s)	Val. (s)	Time (s)
PURe18	11.18	$84.44 \ (83.63 \pm 0.65)$	83.04	0.70	444.22 541.93	946.71	1211.51
PURe34	21.29	$84.89 \ (84.28 \pm 0.56)$	83.83	0.54		841.13	1593.92
ResNet18	11.18	$83.43 (82.07 \pm 1.28) \\ 83.43 (82.84 \pm 0.65) \\ 84.55 (83.68 \pm 0.89) \\ 84.44 (83.20 \pm 0.79) \\ 84.44 (83.86 \pm 0.61)$	81.38	0.85	528.50	508.45	1148.91
ResNet34	21.29		82.01	1.01	635.38	727.11	1537.20
ResNet50	23.53		82.86	0.97	1523.09	1738.18	2456.60
ResNet101	42.52		82.59	0.73	2330.46	2386.12	3884.10
ResNet152	58.16		83.16	0.83	3817.27	3990.62	5351.32

Note: In the "Orig. Acc. (%)" column, results are reported in the format "best (mean \pm standard deviation)", based on five independent training runs. All other metrics represent the average values across the five runs. Bold values highlight the best result in each column.

accuracy of the deeper ResNet34 model. This indicates that the integration of product units effectively enhances feature representation with almost no increase in the number of parameters.

In terms of robustness to noise, the PURe models exhibited consistently strong performance, with PURe34 achieving the smallest relative performance drop of only 0.54%. Among the ResNet variants, ResNet101 demonstrated the highest noise robustness, with a relative drop of 0.73%, which nonetheless remains higher than that of PURe34. These results suggest that the PURe architecture provides improved stability under noisy conditions, likely due to its enhanced capacity for capturing complex feature interactions. This robustness indicates that PURe is more effective at retaining discriminative information even in the presence of signal-dependent noise.

Regarding training efficiency, PURe architectures reached 80% validation accuracy significantly faster than most ResNet variants, with PURe18 and PURe34 achieving this milestone in approximately 444.22 and 541.93 seconds, respectively. In contrast, deeper ResNet models such as ResNet152 required substantially more time, with nearly nine times longer duration compared to PURe18. While ResNet18 reached its best validation performance earlier than PURe18 (508.45s vs. 946.71s), this may be attributed to PURe18 achieving a higher final accuracy, thus requiring additional training to converge to its superior optimum. When comparing architectures with similar accuracy levels, such as PURe34 and ResNet152, PURe34 reached its best validation performance considerably faster (841.13s vs. 3990.62s), highlighting the efficiency benefits of incorporating product units in deeper networks.

To further illustrate these observations, Fig. 5 presents the validation loss and accuracy curves during training. While PURe models tend to exhibit slightly higher initial validation loss, this behavior can be attributed to the nature of product units, which involve logarithmic and exponential transformations. These operations are more sensitive to input scale and weight initialization, especially in the early stages of training. However, the models converge more rapidly and achieve steady performance earlier than their ResNet counterparts.

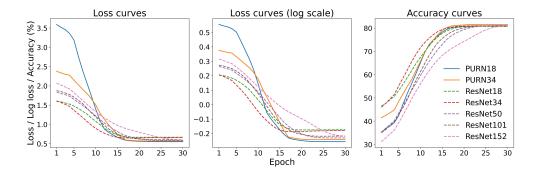


Fig. 5 Validation loss and accuracy curves during training on Galaxy10 DECaLS. All curves are smoothed for visual clarity using a moving average. Results reflect the average performance across five independent runs.

In addition to this, we also compare our results with those from previously published studies evaluating traditional ResNet architectures on the Galaxy10 DECaLS dataset. Table 3 summarizes the reported accuracies of ResNet models trained using different data augmentation strategies across three studies. Notably, the baseline ResNet results obtained in our experiments align well with these previously reported benchmarks, thereby confirming the reliability and comparability of our experimental setup. In comparison, the proposed PURe34 model achieves a substantially higher classification accuracy of 84.89%, significantly outperforming all traditional ResNet configurations evaluated.

Table 3 Classification accuracy of ResNet variants on Galaxy10 DECaLS under different data augmentation strategies

Data Augmentation Type	Model	Acc. (%)
FSL-GAN transformations [32]	ResNet [†]	76.81
Randomized single-image transformations [20]	ResNet18 ResNet50 ResNet101 ResNet152	78.51 78.88 79.99 80.61
Basic single-image transformations [2]	ResNet50	80.00

Note: † denotes that the specific ResNet variant used in conjunction with FSL-GAN augmentation was not specified in the original source.

4.2 ImageNet

4.2.1 Dataset

The ImageNet Large Scale Visual Recognition Challenge (ILSVRC) dataset [5] is one of the most widely used benchmarks in image classification and object recognition. It contains approximately 1.28 million training images and 50,000 validation images, spanning 1,000 object categories, which covers a wide variety of everyday objects, animals, and scenes.

4.2.2 Implementation

To evaluate the performance of the proposed PURe architecture on the ImageNet dataset, we implemented two network configurations: PURe18 and PURe34. These architectures structurally correspond to ResNet18 and ResNet34, respectively.

Data preprocessing and augmentation methods adhered to those outlined in [11], employing random resized cropping, aspect ratio variations, and random horizontal flips. All images were normalized using the standard ImageNet statistics, in accordance with established conventions [11]. Training procedures also followed [11] with minor modifications to accommodate the characteristics of PURe, specifically, the initial learning rate was set to a lower value of 0.01, and the weight decay was increased to a higher value of 0.001. Training utilized SGD with a batch size of 256 images per iteration and momentum of 0.9. Models were trained for 90 epochs using a step decay learning rate schedule, which decreased the learning rate by a factor of 0.1 at epochs 30 and 60. Cross-entropy loss served as the training objective.

Performance evaluation was conducted on the validation set, recording top-1 and top-5 accuracy metrics on the ImageNet dataset. The results of PURe were compared against the reported accuracies of ResNet34 to ResNet152 as presented in [11].

4.2.3 Result and Analysis

Table 4 presents the classification results of PURe and compares them against standard ResNet architectures on the ImageNet validation set.

PURe		ResNet				
Model	Top-1 (%) Acc.	Top-5 (%) Acc.	Model	Top-1 (%) Acc.	Top-1 (%) Acc.	#Params (M)
PURe18	78.21	95.04		-		11.69
PURe34	80.27	95.78	ResNet34 [11]	78.16	94.29	21.80
	-		ResNet50 [11]	79.26	94.75	25.56
	_		ResNet101 [11]	80.13	95.40	44.55
	_		ResNet152 [11]	80.62	95.51	60.19

Table 4 Classification Accuracy on ImageNet Validation Set

Note: Accuracy values represent the performance of a single trained model without ensemble averaging. The best-performing result across all models is highlighted in bold. As shown in Table 4, PURe18 achieves a top-1 accuracy of 78.21% and top-5 accuracy of 95.04%, outperforming ResNet34 which has a top-1 accuracy of 78.16% and top-5 accuracy of 94.29%, while utilizing roughly half the parameters (11.69M vs. 21.80M). The more substantial advantage of PURe is observed in the PURe34 configuration, achieving an impressive top-1 accuracy of 80.27% and a top-5 accuracy of 95.78%. This significantly exceeds the performance of ResNet50 (79.26% top-1 and 94.75% top-5), ResNet101 (80.13% top-1 and 95.40% top-5), and closely approaches ResNet152 (80.62% top-1 and 95.51% top-5). Remarkably, PURe34 achieves these results with fewer parameters compared to deeper ResNet variants (21.80M compared to 25.56M for ResNet50, 44.55M for ResNet101, and 60.19M for ResNet152). This highlights the remarkable efficiency of PURe, providing or superior performance with significantly fewer computational resources. In our experiments, increasing the number of layers beyond PURe34 did not lead to further performance improvements; instead, a saturation effect was observed (data not shown), possibly reflecting a limit of the network's representational capacity or of the information available in the dataset.

Overall, the consistent improvements in top-5 accuracy indicate that PURe architectures reliably capture meaningful hierarchical features, demonstrating competitiveness in recognizing intricate class relationships within the challenging ImageNet dataset.

4.3 CIFAR-10

4.3.1 Dataset

The CIFAR-10 dataset [19] consists of 60,000 color images of size 32×32 pixels, distributed evenly across 10 distinct object classes, including airplanes, automobiles, birds, cats, and dogs. The dataset is divided into 50,000 training images and 10,000 test images. It serves as a standard benchmark for evaluating model performance, efficiency, and scalability in small-data regimes.

4.3.2 Implementation

For the CIFAR-10 dataset, the original PURe architecture adopts the same structural modifications as its corresponding ResNet variants [11] to accommodate the smaller input resolution (32×32 pixels). Specifically, unlike ImageNet-scale models, which typically begin with a 7×7 convolution followed by a 3×3 max-pooling layer, the CIFAR-10 versions replace this initial stage with a single 3×3 convolution with stride 1 and no pooling. This modification preserves spatial resolution in the early layers, which is crucial for capturing fine-grained features in low-resolution images. In addition, both PURe and ResNet variants remove Block D from the architecture defined in Table 1, as the reduced image size makes further downsampling unnecessary. These adjustments ensure a fair and consistent comparison between the two architectures under dataset-specific constraints.

PURe variants for CIFAR-10 are defined by varying the number of residual blocks per stage, as shown in Table 5. Each model follows a symmetric structure with equal numbers of blocks in the three main residual stages (Blocks A, B, and C), corresponding to the ResNet architectures defined for CIFAR in [11], i.e., ResNet20 to ResNet110.

In addition, subsequent work such as, such as [12], proposed deeper ResNet variants for CIFAR, including ResNet164 and ResNet1001, which adopt a different design from [11]. In particular, these architectures are based on three-layer residual blocks, rather than the original two-layer basic block design used in ResNet20 to ResNet110. Moreover, [12] also proposed the pre-activation residual unit, in which the batch normalization and activation layers are moved before the convolutional layers within each residual block. This design improves optimization and generalization, particularly in deeper architectures. In this study, we compare these ResNet variants and the proposed PURe models under the same experimental conditions on CIFAR-10.

Table 5 PURe Architecture Configurations

Model	Block Configuration (A, B, C)
PURe20 PURe32 PURe44	[3, 3, 3] [5, 5, 5] [7, 7, 7]
PURe56 PURe110 PURe272	$ \begin{bmatrix} 9, 9, 9 \\ 18, 18, 18 \\ \hline 45, 45, 45 \end{bmatrix} $

The training procedure for PURe largely follows the methodologies adopted in [11] and [12] for training ResNet models on CIFAR-10. As previous tasks, PURe uses a lower learning rate and a larger weight decay. Specifically, all PURe variants are trained using SGD with with a batch size of 128, an initial learning rate of 0.01, momentum of 0.9, and a weight decay of 0.001. Training is conducted for 160 epochs. A multistep learning rate scheduler is employed to reduce the learning rate by a factor of 0.1 at epochs 80 and 120, consistent with the same used for the ResNet baselines. For deeper PURe models, such as PURe110 and PURe272, in addition to the standard 160-epoch training used, these deeper variants were also trained for 220 epochs with a modified learning rate decay strategy, where the learning rate was reduced at epochs 140 and 180, to ensure adequate convergence. As with ImageNet, cross-entropy loss was employed as the loss function.

Each PURe variant was trained five times, and the final model from each run was evaluated on the test set to obtain classification accuracy. For comparison, the results of ResNet models were obtained from [11] and [12].

4.3.3 Result and Analysis

Table 6 summarizes the classification accuracy of PURe and ResNet architectures on the CIFAR-10 dataset, presenting outcomes under both standard and extended training schedules, alongside comparative model sizes.

Under the standard 160-epoch training regime, all PURe variants consistently outperform their ResNet counterparts across varying depths. For instance, PURe20 achieves 91.80% accuracy compared to ResNet20's 91.25%, and PURe56 reaches 93.51%, exceeding ResNet56's 93.03%. In deeper configurations, PURe110 surpasses

Table 6 Classification Accuracy on CIFAR-10 Test Set

PURe		ResNet		
Model	Acc.	Model	Acc.	#Params (M)
PURe20	91.80 ± 0.17	ResNet20 [11]	91.25	0.27
PURe32	92.83 ± 0.18	ResNet32 [11]	92.49	0.46
PURe44	93.21 ± 0.16	ResNet44 [11]	92.83	0.66
PURe56	93.51 ± 0.12	ResNet56 [11]	93.03	0.85
PURe110 PURe110 [†]	93.93 ± 0.09 94.61 ± 0.11	ResNet110 [11] ResNet110 [12]	93.39 ± 0.16 93.63	1.73
	-	ResNet164 [12]	94.54	1.76
$\overline{\mathrm{PURe}272^\dagger}$	95.01 ± 0.09	-		4.36
	-	ResNet1001 [12]	95.11 ± 0.14	10.15

Note: Accuracy values are reported as "mean \pm standard deviation" over five independent runs. † denotes PURe models trained with an extended 220-epoch schedule. ResNet results from [11] correspond to standard ResNet architectures, while those from [12] are based on the pre-activation residual unit. Some results obtained from [11] and [12] do not include standard deviation values and are therefore reported without error margins. For each group of models with comparable architectural depth, the best-performing result is highlighted in bold.

both the standard and pre-activation ResNet110 variants with an accuracy of 93.93% versus 93.39% and 93.63%, respectively. When trained under an extended 220-epoch schedule, PURe110 further improves to 94.61%, matching the performance of the deeper ResNet164 (94.54%) despite using fewer parameters. Most notably, PURe272 achieves 95.01% accuracy, comparable to the substantially deeper ResNet1001 (95.11%), while requiring less than half the number of parameters (4.36M vs. 10.15M). In our experiments, increasing the number of layers beyond PURe272 did not improve performance further, instead, a saturation was observed (data not shown), perhaps reflecting a limit in either the network's representational capability or the dataset's information content.

Overall, our findings highlight the superior parameter efficiency and scalability of the PURe architecture, particularly at greater network depths, making it well-suited for high-performance image classification tasks.

5 Discussion

In this work, we proposed a novel residual product-unit network for image classification. The challenge of learning in deep networks composed of product units, implementing highly nonlinear functions with nonlinear input coupling, is facilitated by residual connections within each processing block. Introducing 2D product-unit convolutional product-unit layers allows handling 2D images, representing high-dimensional inputs.

The experimental results presented across three diverse datasets, that is, Galaxy10 DECaLS, ImageNet, and CIFAR-10, consistently demonstrate the advantages of incorporating product units into the residual block framework. The proposed PURe achieves superior classification accuracy compared to conventional ResNet architectures while maintaining significantly lower parameter counts and faster convergence times. These findings validate the effectiveness of multiplicative feature interactions in enhancing representational capacity and model efficiency.

On Galaxy10 DECaLS, PURe34 achieved the highest test accuracy among all evaluated models, surpassing even the much deeper ResNet152. This result is particularly noteworthy considering the substantial reduction in model complexity and convergence time. Moreover, the minimal performance drop under Poisson noise confirms the improved robustness of PURe, which may stem from the product unit's ability to encode nonlinear interactions more compactly and resiliently than additive layers, thereby reducing noise accumulation and enhancing feature selectivity.

In the large-scale ImageNet benchmark, PURe34 attained a top-1 accuracy of 80.27% and a top-5 accuracy of 95.78%, outperforming ResNet50 and ResNet101, and approaching the performance of ResNet152 while using approximately one-third of the parameters. Performance on CIFAR-10 further corroborates these trends. Across multiple depths, PURe models consistently outperformed their ResNet counterparts. The deepest variant, PURe272, achieved a test accuracy of 95.01%, closely matching that of ResNet1001 while utilizing less than half the parameters. This efficiency advantage is especially valuable in resource-constrained scenarios where model size and inference time are critical considerations. However, further increasing the number of layers did not always improve PURe's performance on CIFAR-10 and ImageNet; instead, a saturation effect was observed, possibly indicating limitations in the network architecture or in the information content of the dataset.

The use of product units still faces challenges. First, the logarithmic and exponential transformations inherent are sensitive to input magnitudes and weight initialization [4]. This necessitates the introduction of a trainable threshold to stabilize the log transformation, as well as careful hyperparameter tuning, particularly during early training. The threshold problem could in the future be resolved by working with complex product units without thresholds [3]. We also observed that PURe models tend to exhibit higher initial loss values compared to standard residual networks. This is primarily due to the sensitivity of product units to improperly scaled inputs in the logarithmic domain, which can amplify numerical instability during the first few optimization steps. When training deeper variants such as PURe486 on CIFAR-10, we occasionally encountered gradient explosion and non-convergence, indicating that deeper PURes require further stabilization strategies such as advanced normalization techniques or gradient clipping.

Finally, while the current study demonstrates the effectiveness of PURe in image classification tasks. In the future, we plan to extend the PURe architecture to support complex-valued representations, enabling applications in domains such as medical imaging and signal processing where phase information is critical. Additionally, we

will explore the integration of product units into object detection and image segmentation frameworks, where their ability to model high-order feature interactions may yield further benefits.

In summary, PURe represents a competitive alternative to conventional residual architectures, offering a favorable balance between accuracy, parameter efficiency, convergence speed, and noise robustness. Continued investigation into its scalability, stability, and adaptability will potentially help establishing product-unit-based networks as a viable direction for future deep-learning research.

Data Availability Statement

The datasets used in this study are publicly available:

- 1. Galaxy10 DECaLS: https://zenodo.org/records/10845026
- 2. ImageNet: http://www.image-net.org
- 3. CIFAR-10: https://www.cs.toronto.edu/~kriz/cifar.html

Conflict of Interest

The authors declare that they have no conflict of interest.

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