数据科学与工程算法基础 习题13

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m = 11

$$k_2 = 2, k_3 = 2, k_4 = 4, k_5 = 4, k_6 = 3, k_7 = 3, k_8 = 2, k_9 = 2$$

$$\begin{split} Q &= \frac{1}{2m} \sum_{i,j} \left(A_{ij} - \frac{k_i k_j}{2m} \right) \delta(C_i, C_j) \\ &= \frac{1}{22} \left[\left(0 - \frac{k_2 k_2}{22} \right) + 2 \left(1 - \frac{k_2 k_3}{22} \right) + \left(0 - \frac{k_3 k_3}{22} \right) + \left(0 - \frac{k_4 k_4}{22} \right) \right. \\ &\quad + 2 \left(1 - \frac{k_4 k_5}{22} \right) + 2 \left(1 - \frac{k_4 k_6}{22} \right) + 2 \left(0 - \frac{k_4 k_7}{22} \right) + \left(0 - \frac{k_5 k_5}{22} \right) \\ &\quad + 2 \left(1 - \frac{k_5 k_6}{22} \right) + 2 \left(1 - \frac{k_5 k_7}{22} \right) + \left(0 - \frac{k_6 k_6}{22} \right) + 2 \left(1 - \frac{k_6 k_7}{22} \right) \\ &\quad + \left(0 - \frac{k_7 k_7}{22} \right) + \left(0 - \frac{k_8 k_8}{22} \right) + 2 \left(1 - \frac{k_8 k_9}{22} \right) + \left(0 - \frac{k_9 k_9}{22} \right) \\ &\quad = \frac{20}{121} \end{split}$$

3

1.
$$m = 2, k_1 = 2, k_2 = 1, k_3 = 1$$

$$egin{aligned} Q &= rac{1}{2m} \sum_{c \in C} \sum_{i \in C} \sum_{j \in C} \left(A_{ij} - rac{k_i k_j}{2m}
ight) \ &= rac{1}{4} igg[\left(0 - rac{k_1 k_1}{4}
ight) + 2 \left(1 - rac{k_1 k_3}{4}
ight) + \left(0 - rac{k_3 k_3}{4}
ight) + \left(0 - rac{k_2 k_2}{4}
ight) igg] \ &= -rac{1}{8} \end{aligned}$$

2.

$$egin{aligned} Q &= \sum_{c \in C} \left[rac{\sum_{in}^c}{2m} - \left(rac{\sum_{tot}^c}{2m}
ight)^2
ight] \ &= \left(rac{2}{4} - rac{3^2}{4^2}
ight) + \left(rac{0}{4} - rac{1^2}{4^2}
ight) \ &= -rac{1}{8} \end{aligned}$$

$$B = egin{pmatrix} -1 & rac{1}{2} & rac{1}{2} \ rac{1}{2} & -rac{1}{4} & -rac{1}{4} \ rac{1}{2} & -rac{1}{4} & -rac{1}{4} \end{pmatrix}, ec{S} = egin{pmatrix} 1 \ -1 \ 1 \end{pmatrix} \ Q = rac{1}{4m} ec{S}^T B ec{S} = -rac{1}{8} \ \end{pmatrix}$$

- 4. 能
- 5. 划分 I 个社区,即 $c_1 = \{1, 2, 3\}$

$$Q' = \sum_{c \in C} \left\lceil rac{\sum_{in}^c}{2m} - \left(rac{\sum_{tot}^c}{2m}
ight)^2
ight
ceil = \left(rac{4}{4} - rac{4^2}{4^2}
ight) = 0$$

因此Q' > Q

4

1.
$$m = 5, k_1 = 5, k_2 = 2, k_3 = 3$$

$$Q = rac{1}{2m} rac{1}{2m} \sum_{c \in C} \sum_{i \in C} \sum_{j \in C} \left(W_{ij} - rac{k_i k_j}{2m}
ight)$$

$$= -0.08$$

2.

$$Q = \sum_{c \in C} \left[rac{\sum_{in}^{c}}{2m} - \left(rac{\sum_{tot}^{c}}{2m}
ight)^{2}
ight]$$
 $= -0.08$

3.

$$B = \begin{pmatrix} -2.5 & 1 & 1.5 \\ 1 & -0.4 & -0.6 \\ 1.5 & -0.6 & -0.9 \end{pmatrix}, \vec{S} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$$
 $Q = \frac{1}{4m} \vec{S}^T B \vec{S} = -0.08$