

数据科学与工程算法基础 习题6

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(1)

$$\begin{aligned} L(p) &= \prod_{i=1}^n P(X_i = x_i) \\ &= \prod_{i=1}^n p(1-p)^{x_i-1} \\ &= p^n (1-p)^{\sum_{i=1}^n x_i - n} \end{aligned}$$

(2)

$$l(p) = \ln L(p) = n \ln p + \left(\sum_{i=1}^n x_i - n \right) \ln (1-p)$$

令

$$\frac{\partial l}{\partial p} = \frac{n}{p} + \frac{1}{p-1} \left(\sum_{i=1}^n x_i - n \right) = 0$$

解得 p 的极大似然估计

$$\hat{p} = \frac{n}{\sum_{i=1}^n x_i}$$

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似然函数

$$\begin{aligned} L &= \prod_{i=1}^n \left(\frac{\beta^\alpha}{\Gamma(\alpha)} x_i^{\alpha-1} e^{-\beta x_i} \right) \\ &= \left(\frac{\beta^\alpha}{\Gamma(\alpha)} \right)^n \cdot \left(\prod_{i=1}^n x_i \right)^{\alpha-1} \cdot \exp \left\{ -\beta \left(\sum_{i=1}^n x_i \right) \right\} \end{aligned}$$

因此

$$l = \ln L = \alpha n \ln \beta - n \ln (\Gamma(\alpha)) + (\alpha - 1) \ln \left(\prod_{i=1}^n x_i \right) - \beta \left(\sum_{i=1}^n x_i \right)$$

令

$$\frac{\partial l}{\partial \beta} = \frac{\alpha n}{\beta} - \sum_{i=1}^n x_i = 0$$

解得 β 的极大似然估计

$$\hat{\beta} = \frac{\alpha n}{\sum_{i=1}^n x_i}$$