

数据科学与工程算法基础 习题10

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(1) 易见 $\nabla f(x, y) = (50x, 2y)$

因此 $\nabla f_c = (30, 8)$

(2) $\|\nabla f_c\|_2 = \sqrt{30^2 + 8^2} = 2\sqrt{241}$

因此标准化梯度向量 $\nabla f'_c = (\frac{15}{\sqrt{241}}, \frac{4}{\sqrt{241}}) \approx (0.9962, 0.2577)$

(3) $c^{(1)} = c^{(0)} - \varepsilon \cdot g = (0.1169, 3.8712)$

(4) $c^{(1)} = c^{(0)} - \varepsilon \cdot (1, 0) = (0.1, 4)$

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(1) 记 $e_{ui} = r_{ui} - p_u^T q_i$

因此

$$\begin{aligned} J &= \frac{1}{2} \sum_{(u,i) \in K} e_{ui}^2 \\ &= \frac{1}{2} \|R - PQ\|_F^2 \\ &= \frac{1}{2} \text{Tr} [(R - PQ)^T (R - PQ)] \end{aligned}$$

是关于 P, Q 的二次函数且二次项系数为正

也即 $J(R; P, Q)$ 是关于 P, Q 的凸函数

(2)

$$\begin{aligned} \frac{\partial J}{\partial p_{uj}} &= - \sum_{i: (u,i) \in K} e_{ui} q_{ji} \\ \frac{\partial J}{\partial q_{ji}} &= - \sum_{u: (u,i) \in K} e_{ui} p_{uj} \end{aligned}$$

(3)

$$\begin{aligned} p_{uj}^{(t+1)} &\leftarrow p_{uj}^{(t)} + \varepsilon e_{ui}^{(t)} q_{ji}^{(t)} \\ q_{ji}^{(t+1)} &\leftarrow q_{ji}^{(t)} + \varepsilon e_{ui}^{(t)} p_{uj}^{(t)} \end{aligned}$$

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$$P_{uj}^{(t+1)} \leftarrow P_{uj}^{(t)} + \varepsilon \left(e_{ui}^{(t)} q_{ji}^{(t)} - \lambda P_{uj}^{(t)} \right)$$

$$Q_{ji}^{(t+1)} \leftarrow Q_{ji}^{(t)} + \varepsilon \left(e_{ui}^{(t)} q_{uj}^{(t)} - \lambda Q_{ji}^{(t)} \right)$$

$$b_u^{(t+1)} \leftarrow b_u^{(t)} + \varepsilon \left(e_{ui}^{(t)} - \lambda b_u^{(t)} \right)$$

$$d_i^{(t+1)} \leftarrow d_i^{(t)} + \varepsilon \left(e_{ui}^{(t)} - \lambda d_u^{(t)} \right)$$