数据科学与工程算法基础 习题6

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(1)

$$egin{aligned} L(p) &= \prod_{i=1}^n P(X_i = x_i) \ &= \prod_{i=1}^n p(1-p)^{x_i-1} \ &= p^n (1-p)^{\sum\limits_{i=1}^n x_i - n} \end{aligned}$$

(2)

$$l(p) = \ln L(p) = n \ln p + \left(\sum_{i=1}^n x_i - n
ight) \ln \left(1 - p
ight)$$

令

$$rac{\partial l}{\partial p} = rac{n}{p} + rac{1}{p-1} \Biggl(\sum_{i=1}^n x_i - n \Biggr) = 0$$

解得p的极大似然估计

$$\hat{p} = \frac{n}{\sum_{i=1}^{n} x_i}$$

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似然函数

$$egin{aligned} L &= \prod_{i=1}^n \left(rac{eta^lpha}{\Gamma(lpha)} x_i^{lpha-1} e^{-eta x_i}
ight) \ &= \left(rac{eta^lpha}{\Gamma(lpha)}
ight)^n \cdot \left(\prod_{i=1}^n x_i
ight)^{lpha-1} \cdot \exp\left\{-eta\left(\sum_{i=1}^n x_i
ight)
ight\} \end{aligned}$$

因此

$$l = \ln L = lpha n \ln eta - n \ln \left(\Gamma(lpha)
ight) + (lpha - 1) \ln \left(\prod_{i=1}^n x_i
ight) - eta \left(\sum_{i=1}^n x_i
ight)$$

令

$$\frac{\partial l}{\partial \beta} = \frac{\alpha n}{\beta} - \sum_{i=1}^{n} x_i = 0$$

$$\hat{\beta} = \frac{\alpha n}{\sum_{i=1}^{n} x_i}$$