数据科学与工程算法基础 习题13

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1

$m=11$

$k_2=2,k_3=2,k_4=4,k_5=4,k_6=3,k_7=3,k_8=2,k_9=2$

$$
\begin{aligned}
Q&=\frac{1}{2m}\sum_{i,j}\left(A_{ij}-\frac{k_ik_j}{2m}\right)\delta(C_i,C_j)\\
&=\frac{1}{22}[\left(0-\frac{k_2k_2}{22}\right)+2\left(1-\frac{k_2k_3}{22}\right)+\left(0-\frac{k_3k_3}{22}\right)+\left(0-\frac{k_4k_4}{22}\right)\\
&\quad+2\left(1-\frac{k_4k_5}{22}\right)+2\left(1-\frac{k_4k_6}{22}\right)+2\left(0-\frac{k_4k_7}{22}\right)+\left(0-\frac{k_5k_5}{22}\right)\\
&\quad+2\left(1-\frac{k_5k_6}{22}\right)+2\left(1-\frac{k_5k_7}{22}\right)+\left(0-\frac{k_6k_6}{22}\right)+2\left(1-\frac{k_6k_7}{22}\right)\\
&\quad+\left(0-\frac{k_7k_7}{22}\right)+\left(0-\frac{k_8k_8}{22}\right)+2\left(1-\frac{k_8k_9}{22}\right)+\left(0-\frac{k_9k_9}{22}\right)\\
&=\frac{20}{121}
\end{aligned}
$$

3

1. $m=2,k_1=2,k_2=1,k_3=1$

$$
\begin{aligned}
Q&=\frac{1}{2m}\sum_{c\in C}\sum_{i\in C}\sum_{j\in C}\left(A_{ij}-\frac{k_ik_j}{2m}\right)\\
&=\frac{1}{4}\left[\left(0-\frac{k_1k_1}{4}\right)+2\left(1-\frac{k_1k_3}{4}\right)+\left(0-\frac{k_3k_3}{4}\right)+\left(0-\frac{k_2k_2}{4}\right)\right]\\
&=-\frac{1}{8}
\end{aligned}
$$

2.

$$
\begin{aligned}
Q&=\sum_{c\in C}\left[\frac{\sum_{in}^c}{2m}-\left(\frac{\sum_{tot}^c}{2m}\right)^2\right]\\
&=\left(\frac{2}{4}-\frac{3^2}{4^2}\right)+\left(\frac{0}{4}-\frac{1^2}{4^2}\right)\\
&=-\frac{1}{8}
\end{aligned}
$$

3.

$$B=\begin{pmatrix}-1 & \frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4}\\ \frac{1}{2} & -\frac{1}{4} & -\frac{1}{4}\end{pmatrix},\vec{S}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$Q=\frac{1}{4m}\vec{S}^TB\vec{S}=-\frac{1}{8}$$

4. 能
5. 划分 I 个社区，即 $c_1=\{1,2,3\}$

$$Q'=\sum_{c\in C}\left[\frac{\sum_{in}^c}{2m}-\left(\frac{\sum_{tot}^c}{2m}\right)^2\right]=\left(\frac{4}{4}-\frac{4^2}{4^2}\right)=0$$

因此 $Q'>Q$

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1. $m=5,k_1=5,k_2=2,k_3=3$

$$\begin{aligned}Q&=\frac{1}{2m}\frac{1}{2m}\sum_{c\in C}\sum_{i\in C}\sum_{j\in C}\left(W_{ij}-\frac{k_ik_j}{2m}\right)\\&=-0.08\end{aligned}$$

2.

$$\begin{aligned}Q&=\sum_{c\in C}\left[\frac{\sum_{in}^c}{2m}-\left(\frac{\sum_{tot}^c}{2m}\right)^2\right]\\&=-0.08\end{aligned}$$

3.

$$B=\begin{pmatrix}-2.5 & 1 & 1.5\\ 1 & -0.4 & -0.6\\ 1.5 & -0.6 & -0.9\end{pmatrix},\vec{S}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$$

$$Q=\frac{1}{4m}\vec{S}^TB\vec{S}=-0.08$$