# The Magnetic Field of a Slinky

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**Abstract** The goal of this experiment is to explore and understand the magnetic field generated by the current in the solenoid and how the field varies in different parts of the solenoid by using the magnetic field sensor. The result we found is that there is no magnetic field outside of the solenoid, and there is a magnetic field inside of the solenoid. When the sensor is aligned with the center axis of the solenoid, the magnetic field reading is the largest. Our calculation to the permeability form varying current is: $\mu_0 = (1.433 \pm 0.039)10^{-6}\frac{N}{A^2}$, and the permeability in varying turn density is: $\mu_0 = (1.379 \pm 0.035)10^{-6}\frac{N}{A^2}$ which are both higher than the expected value: $1.257 \cdot 10^{-6}\frac{N}{A^2}$.

## Introduction

The purpose and goal for this experiment is to examine the magnetic field generated by current running through a solenoid, and what are the factors that will affect the magnetic field generated by the solenoid such as the turn density and current. Also, to explore how the field varies inside and outside of the solenoid. At last, we calculate the permeability constant $\mu_0$ with the data we collected. We conduct our experiment by controlling variables, varying the current and the turn density. By the reading from our magnetic field sensor to make two plots of magnetic field vs. current and magnetic field vs. turn density. The term solenoid is given by the French physicist André-Marie Ampère in 1823.[1] This characteristic of the solenoid is important in many real-world application such as transformer and MRI machines in medical field.

## Theory

As one of the applications of the Ampere's law, we have the following equation to describe the magnetic field in a solenoid with current:

$$\vec{B} = B_z\hat{z} \text{ where } B_z = \left\{ \begin{array}{ll} 0 & outside\ of\ solenoid \\ \mu_0\frac{N}{L}I & inside\ of\ solenoid \end{array} \right\} \qquad (1)^2$$

Where $\mu_0 = permeability\ constant\frac{C^2}{(N\cdot m)^2}, N = turns, L = length(m), I = current(A)$

The magnetic field in a solenoid with current running through is zero outside of the solenoid and $\mu_0\frac{N}{L}I$ inside of the solenoid.

[1] "Solenoid." Wikipedia, Wikimedia Foundation, 31 Oct. 2019, https://en.wikipedia.org/wiki/Solenoid.
[2] Moore, Thomas A. Six Ideas That Shaped Physics. McGraw-Hill Education, 2017.

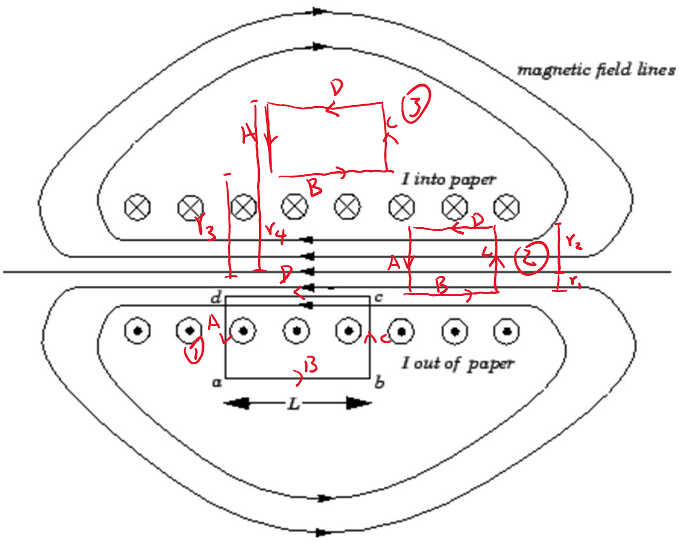



*Figure 1 Solenoid*[3]

The solenoid is wound from a single helical wire which carries a current $I$. The winding is sufficiently tight that each turn of the solenoid is well approximated as a circular wire loop, lying in the plane perpendicular to the axis of the solenoid, which carries a current $I$. Suppose that there are $n$ such turns per unit axial length of the solenoid.

In order to find the expression of the magnetic field of the solenoid, we drew three Amperian loops for our line integral for the Ampere's law.

$$\oint \vec{B} \cdot d\vec{S} = \mu_0 i_{enc} \qquad (2)$$

Where $i_{enc}$ is the current in the enclosed path.

For the second Amperian loop, it is located inside the solenoid, not perfectly center at the solenoid's axis. $B$ edge is closer to the center axis than the $D$ edge. According to Equation (2), we have the following equation:

$$\oint \vec{B} \cdot d\vec{S} = \int_A \vec{B} \cdot d\overrightarrow{S_A} + \int_B \vec{B} \cdot d\overrightarrow{S_B} + \int_C \vec{B} \cdot d\overrightarrow{S_C} + \int_D \vec{B} \cdot d\overrightarrow{S_D} \quad (2a)$$

Since $A, C$ edges are particular to the magnetic field, therefore the first term and third term in the right side of Equation (2a) are zero. So, we have the following:

$$\oint \vec{B} \cdot d\vec{S} = \int_B \vec{B} \cdot d\overrightarrow{S_B} + \int_D \vec{B} \cdot d\overrightarrow{S_D} \qquad (2b)$$

$$\oint \vec{B} \cdot d\vec{S} = B(r_2) \int d\overrightarrow{S_B} - B(r_1) \int d\overrightarrow{S_D} \qquad (2c)$$

[3] Magnetic Field of a Solenoid. Accessed November 30, 2019. http://farside.ph.utexas.edu/teaching/302l/lectures/node76.html.

Since $\int d\overrightarrow{S_B} = \int d\overrightarrow{S_D} = L$,

$$\oint \vec{B} \cdot d\vec{S} = B(r_2)L + B(r_1)L = L(B(r_2) - B(r_1)) \quad \text{(2d)}$$

According to Equation (2), we have:

$$L\big(B(r_2) - B(r_1)\big) = \mu_0 i_{enc} \quad \text{(2e)}$$

Since the $i_{enc} = 0$, no current within the solenoid, since $L$ doesn't equal zero, we have the following:

$$B(r_2) = B(r_1) \quad \text{(2f)}$$

Equation (2f) shows us that the magnetic field remains the same within the solenoid.

For the Amperian loop 3, which is outside of the solenoid. Same argument from loop 2 can be made in loop 3, we have:

$$B(r_3) = B(r_4) \quad \text{(2g)}$$

If we consider $r_4$ approaches to infinity, we know the magnetic field when it is infinitely away from the current should be zero, so therefore:

$$B(r_3) = B(r_4 \to \infty) = 0 \quad \text{(2h)}$$

This tells us that the magnetic field outside of the solenoid will be zero.

The same argument can also be made in loop 1, since the $D$ edge is outside of the solenoid, according to Equation (2i), we have:

$$\oint \vec{B} \cdot d\vec{S} = \int_B \vec{B} \cdot d\overrightarrow{S_B} = \vec{B}L = \mu_0 i_{enc} \quad \text{(2i)}$$

Since the length of the $dc$ edge is $L$, the loop intersects $n$L turns of the solenoid, each carrying a current $I$. Thus, the total current $i_{enc}$ which flows through the loop is $nLI$. We have:

$$\vec{B}L = \mu_0 nLI \quad \text{(2j)}$$

$$\vec{B} = \mu_0 nI \quad \text{(2k)}$$

Therefore, we conclude that the magnetic field inside of a solenoid has a linear relationship between the turn density and the current.

A major assumption we made is that our solenoid is an infinite solenoid. Since the diameter of our solenoid is about 5 cm, which is much shorter the length of the solenoid which is about 1 meter. Therefore, we can assume that our solenoid is an infinite solenoid and employ the infinite solenoid magnetic field equation.

We verify our results with the universal constant, the permeability, which is $\mu_0 = 1.257 \cdot 10^{-6} \frac{C^2}{(N \cdot m)^2}$ by the following equation:

$For\ the\ fixed\ turn\ denesity$:

$$\mu_0 = \frac{B_z}{I}\frac{L}{N} = a_1\frac{L}{N} \qquad (3a)$$

$$where\ N\ is\ the\ number\ of\ turns, L\ is\ the\ length\ of\ the\ solenoid$$

$For\ the\ fixed\ current$:

$$\mu_0 = \frac{B_z}{N}\frac{L}{I} = \frac{a_1}{I} \qquad (3b)$$

$$where\ I\ is\ the\ current$$

**Experimental Procedure**[4]

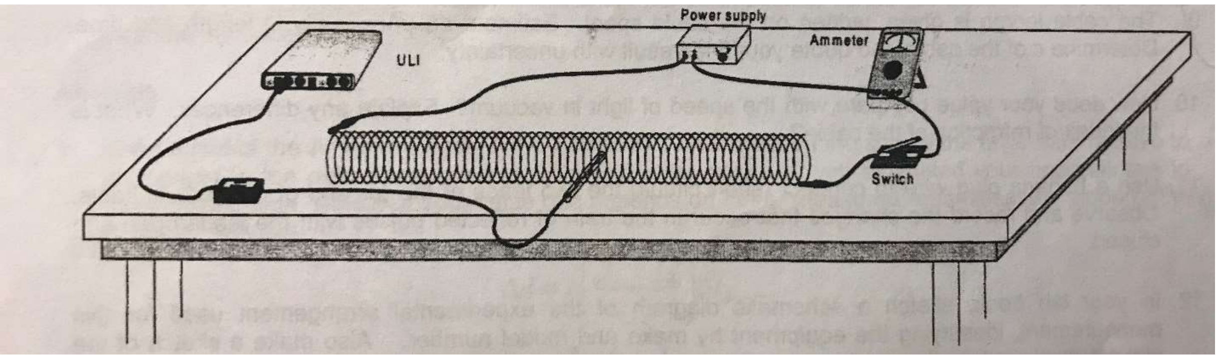



*Figure 2 The circuit*

The equipment is a PSU, logger pro, magnetic field sensor, ammeter, switch, meter stick and a metal slinky.

Preliminary observations

1. Close the circuit, place the sensor in the solenoid, rotate the sensor and find the placement where the magnetic field reading is the largest;

2. Rotate the sensor 180 degrees, observe the readings;

3. Place the sensor at different location along its axis, observe the readings;

4. Place the sensor outside the solenoid and observe the readings.

Part 1: Varying current

1. Place the magnetic field sensor along its center axis;

2. Zero the sensor when the circuit is open;

3. Close the circuit and collect the magnetic field sensor data;

[4] Lab Manual, 2019

4. Adjust the PSU and increase the current by 0.25A, repeat 2-4 until the current reach 1.9A;

5. Count the numbers of turns in the Slinky within the length of the wire connection, measure the length.

Part 2: Varying turn density

1. Adjust the PSU so that our current in the circuit is 1.9A;

2. With magnetic field sensor in place, open the circuit and zero the sensor;

3. Close the circuit and collect the magnetic field sensor data, find the average magnetic field reading;

4. Repeat step 2 – 3 after changing the length of the slinky from 1m to 0.4 m in roughly 0.1 m steps while maintain the current level to 1.9A.

**Data and Analysis**

Preliminary observations

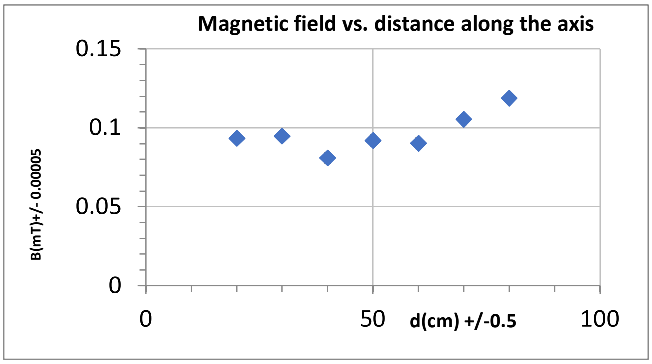
When the white dot is facing towards and on the axis of the solenoid, the reading of the magnetic field sensor is the largest. When we rotate the sensor 90 degrees from the previous location, we have a zero magnetic field reading. Continuing rotating the sensor 90 degrees, we have a negative reading of the magnetic field. Lastly, we move the sensor outside of the solenoid and the reading is 0.0019 mT which we consider a zero magnetic field reading because of the noise.

For different location on the axis of the solenoid:

| D (cm) +/-(0.5cm) | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
|---|---|---|---|---|---|---|---|
| B (mT) +/-(0.00005mT) | 0.0934 | 0.0947 | 0.0809 | 0.0920 | 0.0902 | 0.1053 | 0.1187 |

We observed that at both end of the solenoid, the magnetic field reading is larger than the readings in the middle.

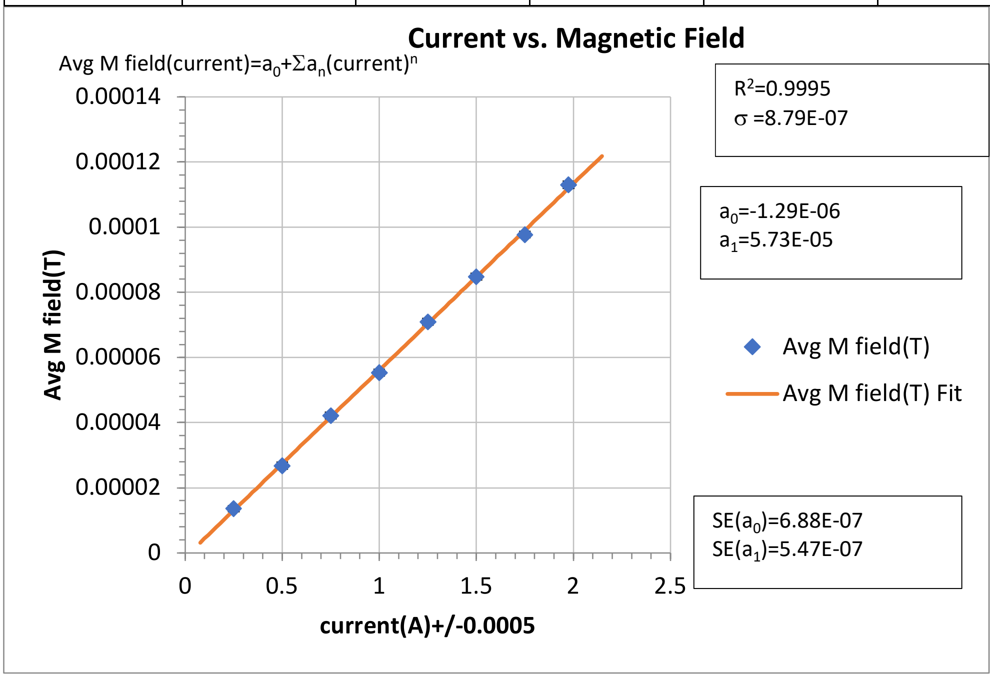
Therefore, we can draw a conclusion that the magnetic field is zero outside of the solenoid. The magnetic field is pointing along the axis of the solenoid.




*The error bar is very small to visualize on the plot

For varying current

| current(A)+/-0.0005 | Average M field(mT) | Avg M field(T) | stdev(mT) | stdev(T) | M field @ the end(mT)(1m) | $\mu_0$ (a1 / n) |
|---|---|---|---|---|---|---|
| 0.25 | 0.013603632 | 1.36036E-05 | 0.000936409 | 9.36409E-07 | 0.0784 | 1.4325E-06 |
| 0.5 | 0.026759775 | 2.67598E-05 | 0.001065886 | 1.06589E-06 | | |
| 0.75 | 0.042141234 | 4.21412E-05 | 0.001016 | 0.000001016 | | |
| 1 | 0.05531 | 0.00005531 | 0.001017 | 0.000001017 | | |
| 1.25 | 0.07094 | 0.00007094 | 0.001041 | 0.000001041 | turn density (1/m) | |
| 1.5 | 0.0848 | 0.0000848 | 0.001029 | 0.000001029 | 40 | |
| 1.75 | 0.097721691 | 9.77217E-05 | 0.00099 | 0.00000099 | | |
| 1.975 | 0.113014488 | 0.000113014 | 0.001129472 | 1.12947E-06 | | |
| | turns +/- 1 | length(m) +/-0.005m | Error_ density | | | |
| | 40 | 1 | 1.019803903 | | | |




Our R value is 0.9995, which is very close to 1, so the line fit is a good fit for our graph. Our $a_0$ = -1.29E-06 ± 6.88E-07, which means that it does not cross through the origin. We suspect there is some degree of systematic error in this set of data.

When we were varying the current, we maintained the turn density the same, which is 40 turns per meter; also, we kept the length at a meter.

The uncertainty in the permeability is the following:

$$\delta\mu_0 = \mu_0\sqrt{(\frac{\delta a_1}{a_1})^2 + (\frac{\delta I}{I})^2} \qquad (4a)$$

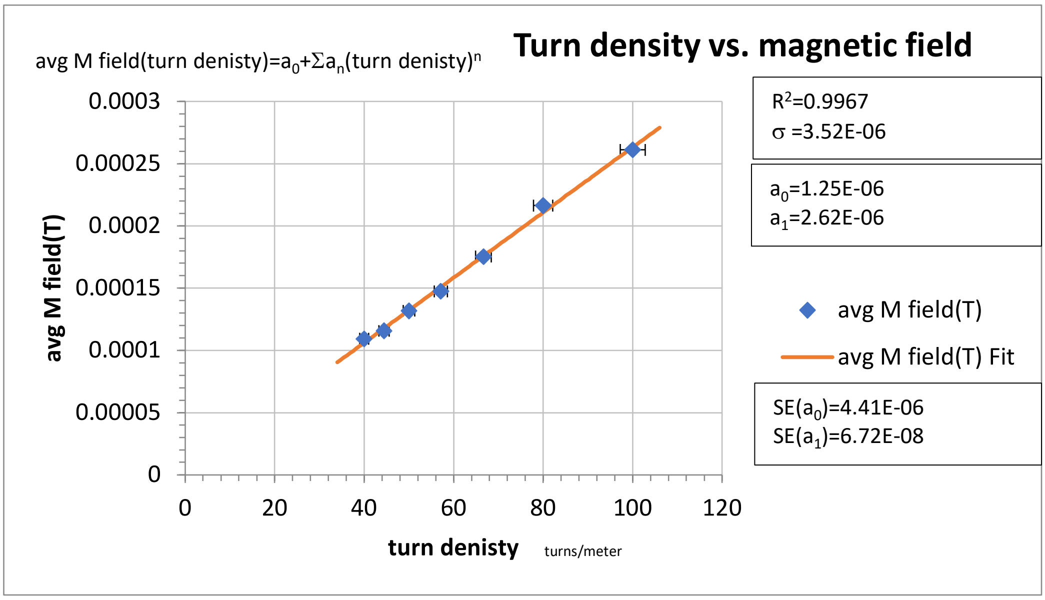
We can verify our result by calculate the permeability using Equation (3a). Our calculated permeability is: $(1.433 \pm 0.039) * 10^{-6}\frac{N}{A^2}$. Which doesn't agree with the actual permeability, which is $1.257 \cdot 10^{-6}\frac{N}{A^2}$.

There are degrees of systematic error and random error. As we zero the magnetic field sensor every time before we close the switch, the magnetic field readings were still not zero, there was some noise in our readings. Therefore, the sensor introduced a degree of random error. The reason we conclude there is a systematic error is because when we were varying the turn density, we also had a higher result than the expected values. Since both of our measurements appear to be higher than we expected, therefore there is certainly a degree of systematic error.

Our curve did present a linear relationship between the current and magnetic field. As the equation (1) shows, as the turn density remains constant, magnetic field strength increases as the current increases.

For varying length (turn density)

| Current (A)+/- 0.0005 A | 1.9 | | | | | | $\mu_0$ (a1 / I) | |
|---|---|---|---|---|---|---|---|---|
| Length (m) +/- 0.005m | avg M field (mT) | Stdev (mT) | turns | turn density (turns / m) | avg M field(T) | stdev (T) | 1.3789E-06 | Error Density (turn/m) |
| 1 | 0.1093 | 0.001124 | 40 | 40 | 0.0001093 | 0.000001124 | | 1.019803903 |
| 0.9 | 0.1157 | 0.001011 | 40 | 44.44444444 | 0.0001157 | 0.000001011 | | 1.138215365 |
| 0.8 | 0.1318 | 0.001033 | 40 | 50 | 0.0001318 | 0.000001033 | | 1.288470508 |
| 0.7 | 0.1474 | 0.001087 | 40 | 57.14285714 | 0.0001474 | 0.000001087 | | 1.485736712 |
| 0.6 | 0.1754 | 0.001 | 40 | 66.66666667 | 0.0001754 | 0.000001 | | 1.756820922 |
| 0.5 | 0.2163 | 0.001006 | 40 | 80 | 0.0002163 | 0.000001006 | | 2.154065923 |
| 0.4 | 0.2611 | 0.000965 | 40 | 100 | 0.0002611 | 9.645E-07 | | 2.795084972 |




* The error bar in avg M-field is small that it's hard to read on the graph

When we were varying the turn density, we maintained the current the same, which is 1.9 A; also we fixed the number of turns, which was 40 turns.

Our R value is 0.9967, which is very close to 1, so the line fit is a good fit for our graph. Our $a_0$=1.25E-06 ± 4.41E-06, which means that it does cross through the origin as we expected.

The uncertainty in the permeability is the following:

$$\delta\mu_0 = \mu_0\sqrt{(\frac{\delta a_1}{a_1})^2 + (\frac{\delta n}{n})^2} \qquad (4b)$$

The error in turn density can be calculated by the following:

$$\delta n = n\sqrt{(\frac{\delta N}{N})^2 + (\frac{\delta L}{L})^2} \qquad (4c)$$

Our calculated permeability using Equation (3b) is: $(1.357 \pm 0.035) * 10^{-6}\frac{N}{A^2}$. Which doesn't agree with the actual permeability, which is $1.257 \cdot 10^{-6}\frac{N}{A^2}$.

Once again, our result doesn't match with the expected values. The same random error and systematic error from the previous experiment as explained above exist in this experiment. In changing the turn density, we faced another challenge: keeping the turn density constant. At the higher turn density, the spacing between the turns become smaller than the magnetic field sensor's width. Therefore, inserting the sensor into the solenoid will change the spacing between the turns, and it introduces an ununiformed turn density. Also, we change the turn density by changing the length of the solenoid. It is hard to determine the actual numbers of turns in the length at the end points. Since we're varying lengths into 7 readings, therefore another random error is introduced to our data.

Our curve did present a linear relationship between the turn density and magnetic field. As the equation (1) shows, as the current remains constant, magnetic field strength increases as the turn density increases.

**Result and Conclusion**

The purpose of our lab is to explore the magnetic field generated by sending current through the solenoid. What are the factors that determine the strength of the magnetic field, such as location, current and the turn density. We determined that there is no magnetic field outside of solenoid, and magnetic field within the solenoid is the same no matter the distance from the center axis. The magnetic field has a linear relationship with the current and the turn density.

Our calculation to the permeability form varying current is: $\mu_0 = (1.433 \pm 0.039)10^{-6}\frac{N}{A^2}$, and the calculated permeability in varying turn density is: $\mu_0 = (1.379 \pm 0.035)10^{-6}\frac{N}{A^2}$which are both higher than the expected value: $1.257 \cdot 10^{-6}\frac{N}{A^2}$. The errors exist in this lab includes both random error and the systematic error. The noise in our magnetic field sensor, the ununiformed charge density and the uncertain length are factors for our errors.

**References**

1. “Solenoid.” Wikipedia, Wikimedia Foundation, 31 Oct. 2019, https://en.wikipedia.org/wiki/Solenoid.

2. Moore, Thomas A. Six Ideas That Shaped Physics. McGraw-Hill Education, 2017.

3. Magnetic Field of a Solenoid. Accessed November 30, 2019.

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4. Lab Manual, 2019