TQ-Using IBM Open-Quantum-Computing Services to test Violation of Local Realism with Mermin Inequalities

Gonçalo Gouveia
2018277419|MEF
uc2018277419@student.uc.pt

2nd February 2022

1

a)

Testing the inequalities for the M_3 polynomial:

$$M_3 = a_1 a_2 a_3' + a_1 a_2' a_3 + a_1' a_2 a_3 - a_1' a_2' a_3' \tag{1}$$

Where a_i and a_i' will be Pauli operators.The maximum value $\langle M_3 \rangle$ can take for 3-qubits is $2^{n-1} = 2^{3-1} = 4$[3].

$\langle M_3 \rangle$ is maximized by the GHZ-like states [2]:

$$|\Phi\rangle_{GHZ} = \frac{1}{\sqrt{2}} \left(|000\rangle + e^{i\phi} |111\rangle \right)$$

Where ϕ is a relative phase factor which will depend on our choice for the a_i and a_i'. For $a_i = \sigma_x$ and $a_i' = \sigma_y, \phi = \pi/2$. For $a_i' = \sigma_x$ and $a_i = \sigma_y, \phi = \pi$, $|\Phi\rangle_{GHZ}$ maximizes $\langle M_3 \rangle$. If we take $\phi = \pi/2$, becomes:

$$\sigma_x^1 \sigma_x^2 \sigma_y^3 + \sigma_x^1 \sigma_y^2 \sigma_x^3 + \sigma_y^1 \sigma_x^2 \sigma_x^3 - \sigma_y^1 \sigma_y^2 \sigma_y^3 \tag{2}$$

Therefore:

$$\sigma_x^1 \sigma_x^2 \sigma_y^3 (|000\rangle + i\,|111\rangle) = |000\rangle + i\,|111\rangle = \sqrt{2}\,|\phi_{GHZ}\rangle$$
$$\sigma_x^1 \sigma_y^2 \sigma_x^3 (|000\rangle + i\,|111\rangle) = |000\rangle + i\,|111\rangle = \sqrt{2}\,|\phi_{GHZ}\rangle$$
$$\sigma_y^1 \sigma_x^2 \sigma_x^3 (|000\rangle + i\,|111\rangle) = |000\rangle + i\,|111\rangle = \sqrt{2}\,|\phi_{GHZ}\rangle$$
$$-\sigma_y^1 \sigma_y^2 \sigma_y^3 (|000\rangle + i\,|111\rangle) = |000\rangle + i\,|111\rangle = \sqrt{2}\,|\phi_{GHZ}\rangle$$

Therefore, summing all of the above, we get

$$M_3\,|\phi\rangle = 4\,|\phi_{GHZ}\rangle$$
$$\langle \phi_{GHZ} | \phi_{GHZ} \rangle = \frac{1}{2} (\langle 000| - i\,\langle 111|)(|000\rangle\, i\,|111\rangle) = 1$$

Thus,

$$\langle \phi_{GHZ}|\, M_3\,|\phi_{GHZ}\rangle = 4\,\langle \phi_{GHZ} | \phi_{GHZ} \rangle = 4$$

Proceeding the same way from $\phi = \pi$. In this case:

$$\sigma_y^1\sigma_y^2\sigma_x^3 + \sigma_y^1\sigma_x^2\sigma_y^3 + \sigma_x^1\sigma_y^2\sigma_y^3 - \sigma_x^1\sigma_x^2\sigma_x^3 \tag{3}$$

Therefore:

$$\sigma_y^1\sigma_y^2\sigma_x^3(|000\rangle - |111\rangle) = |000\rangle - |111\rangle$$
$$\sigma_y^1\sigma_x^2\sigma_y^3(|000\rangle - |111\rangle) = |000\rangle - |111\rangle$$
$$\sigma_y^1\sigma_y^2\sigma_x^3(|000\rangle - |111\rangle) = |000\rangle - |111\rangle$$
$$-\sigma_x^1\sigma_x^2\sigma_x^3(|000\rangle - |111\rangle) = |000\rangle - |111\rangle$$

Similarly, to before:

$$\langle\phi_{GHZ}| M_3 |\phi_{GHZ}\rangle = 4\langle\phi_{GHZ}|\phi_{GHZ}\rangle = 4$$

And again, summing all of the above, we get:

$$\langle M_3\rangle_{GHZ} = 4\langle\Phi_{GHZ} \mid \Phi_{GHZ}\rangle = 4$$

Which proves that the GHZ-like states maximize $\langle M_3\rangle$.

b)

For the 3 Qubit case. Assuming $a_i = \sigma_x$, $a_i' = \sigma_y$, making $\phi = \pi$. This GHZ state can be simulated with the following simple circuit:

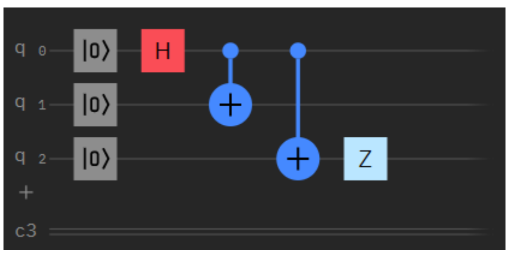


Figure 1: Circuit for the GHZ State simulation

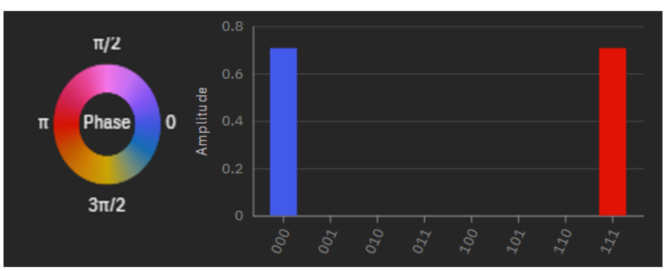


Figure 2: Computational basis states.

The final $\boxed{Z}$ gate on q_3 will give the state the desired ϕ phase.

However, since our computer is far from the theoretical idealism, we should pick only one of the qubits to be target of the CNOT operation. Thus, one does use the equivalence that $\text{CNOT}_{1\to 2} = (H_1 \otimes H_2)\,\text{CNOT}_{2\to 1}\,(H_1 \otimes H_2)$. In adiction, the phase gate $\boxed{Z}$ can be put anywhere, we place it in the most robust qubit, that is, the one with less gates.[1]

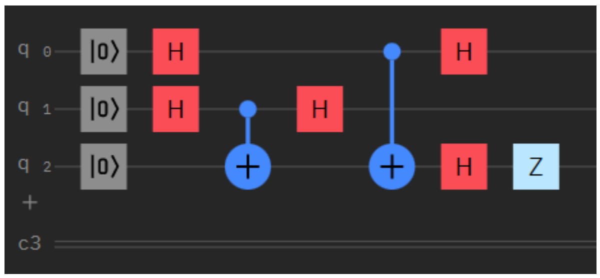


Figure 3: Used circuit for the GHZ State simulation

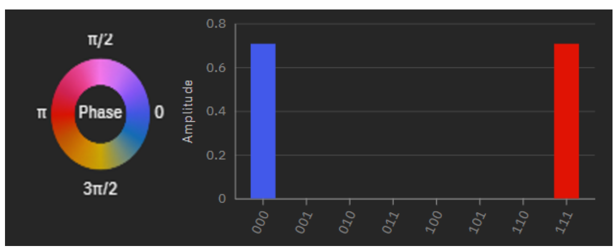


Figure 4: Computational basis states.

Acting accordingly to the problem we want to measure, Before each measurement q_i in x-basis, we put an Hadamard gate followed by a z-basis measurement in that qubit channel. If we want to measure in the y-basis, we apply a $S^\dagger$ and one follows as mentioned before.

The following circuits were ran 1024 times, not overloading the quantum computers with non research focused simulations, which yield in obtaining the following data:

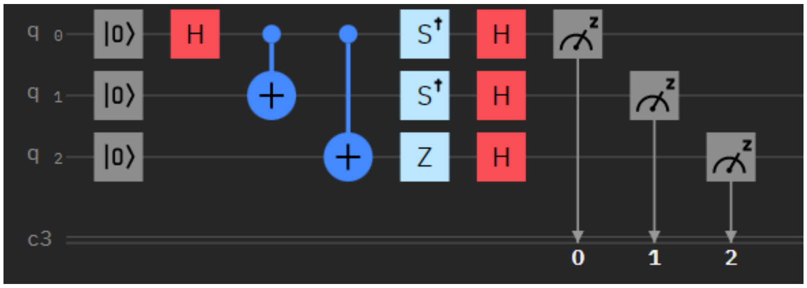


Figure 5: YYX ideal circuit.

Counts for each result are expressed in probabilities computed out of 1024 runs.

State	$\|111\rangle$	$\|110\rangle$	$\|101\rangle$	$\|100\rangle$	$\|011\rangle$	$\|010\rangle$	$\|001\rangle$	$\|000\rangle$
Eigenvalue	-1	1	1	-1	1	-1	-1	1
Probability	0.015	0.193	0.198	0.023	0.225	0.038	0.055	0.250

$$\left\langle \sigma_y^1 \sigma_y^2 \sigma_x^3 \right\rangle_{\text{Perfect}} = 0.74 \pm 0.02$$

Noticing that we only need to simulate 2 circuits, since the cross terms of (3) yield the same probabilities [1].

$$\left\langle \sigma_y^1 \sigma_y^2 \sigma_x^3 \right\rangle_{\text{Perfect}} = \left\langle \sigma_y^1 \sigma_x^2 \sigma_y^3 \right\rangle_{\text{Perfect}} = \left\langle \sigma_x^1 \sigma_y^2 \sigma_y^3 \right\rangle_{\text{Perfect}}$$

Every uncertainty was calculated assuming the probabilities follow a multinomial distribution: $\delta_p = \sqrt{p(1-p)/N}$. [1]

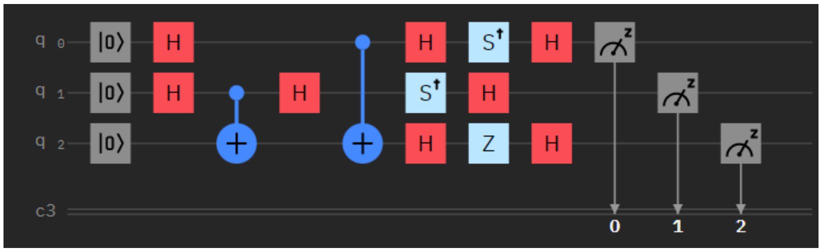


Figure 6: YYX measurement circuit.

State	$\|111\rangle$	$\|110\rangle$	$\|101\rangle$	$\|100\rangle$	$\|011\rangle$	$\|010\rangle$	$\|001\rangle$	$\|000\rangle$
Eigenvalue	-1	1	1	-1	1	-1	-1	1
Probability	0.033	0.164	0.173	0.046	0.2025	0.052	0.106	0.217

$$\langle \sigma_y^1 \sigma_y^2 \sigma_x^3 \rangle = 0.52 \pm 0.02$$

$$\langle \sigma_y^1 \sigma_y^2 \sigma_x^3 \rangle = \langle \sigma_y^1 \sigma_x^2 \sigma_y^3 \rangle = \langle \sigma_x^1 \sigma_y^2 \sigma_y^3 \rangle$$

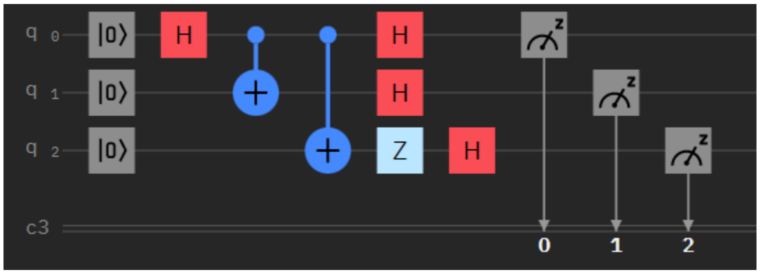


Figure 7: XXX ideal circuit.

State	$\|111\rangle$	$\|110\rangle$	$\|101\rangle$	$\|100\rangle$	$\|011\rangle$	$\|010\rangle$	$\|001\rangle$	$\|000\rangle$
Eigenvalue	-1	1	1	-1	1	-1	-1	1
Probability	0.1845	0.0254	0.0264	0.254	0.024	0.177	0.250	0.056

$$\langle \sigma_x^1 \sigma_x^2 \sigma_x^3 \rangle_{ideal} = -0.73 \pm 0.02$$

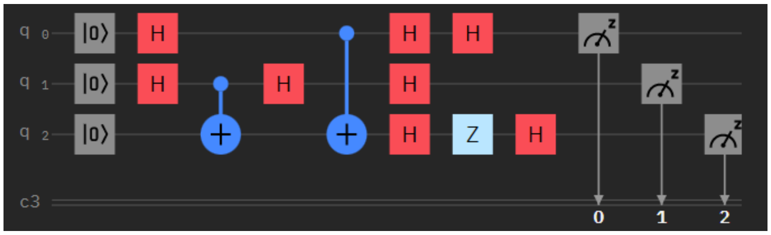


Figure 8: XXX measurement circuit.

State	$\|111\rangle$	$\|110\rangle$	$\|101\rangle$	$\|100\rangle$	$\|011\rangle$	$\|010\rangle$	$\|001\rangle$	$\|000\rangle$
Eigenvalue	-1	1	1	-1	1	-1	-1	1
Probability	0.178	0.015	0.043	0.2543	0.019	0.188	0.240	0.068

$$\langle \sigma_x^1 \sigma_x^2 \sigma_x^3 \rangle = -0.71 \pm 0.02$$

2 Results & Discussion

According to [1] :

$$\langle M_3 \rangle_{GHZ} = \sigma_y^1 \sigma_y^2 \sigma_x^3 + \sigma_y^1 \sigma_x^2 \sigma_y^3 + \sigma_x^1 \sigma_y^2 \sigma_y^3 - \sigma_x^1 \sigma_x^2 \sigma_x^3$$

Noticing again that the first 3 terms do yield the same probabilities.

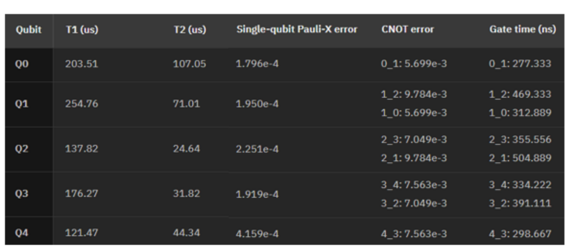
$$\langle M_3 \rangle_{GHZ} = 3 \langle \sigma_y^1 \sigma_y^2 \sigma_x^3 \rangle - \sigma_x^1 \sigma_x^2 \sigma_x^3$$

We obtain the final results of :

$$\langle M_3 \rangle_{ideal} = 2.95 \pm 0.06$$

$$\langle M_3 \rangle = 2.27 \pm 0.06$$

Which violate the Mermin Inequality for local realism $\langle M_3 \rangle_{LR} \leq 2.0$.
The difference between $\langle M_3 \rangle_{ideal}$ and $\langle M_3 \rangle$ are mostly due to statistical fluctuations and gate errors. Mostly due to the CNOT gate errors, Both teh $\boxed{Z}$ and $\boxed{H}$ gates can be described with CNOT gates
This circuits were implemented in the IBM quantum computer in the system *ibmq_manila*, that at the given moment of the runs were with the following characteristics:



Qubit	T1 (us)	T2 (us)	Single-qubit Pauli-X error	CNOT error	Gate time (ns)
Q0	203.51	107.05	1.796e-4	0_1: 5.699e-3	0_1: 277.333
Q1	254.76	71.01	1.950e-4	1_2: 9.784e-3 1_0: 5.699e-3	1_2: 469.333 1_0: 312.889
Q2	137.82	24.64	2.251e-4	2_3: 7.049e-3 2_1: 9.784e-3	2_3: 355.556 2_1: 504.889
Q3	176.27	31.82	1.919e-4	3_4: 7.563e-3 3_2: 7.049e-3	3_4: 334.222 3_2: 391.111
Q4	121.47	44.34	4.159e-4	4_3: 7.563e-3	4_3: 298.667

Figure 9: Calibration data in *ibmq _ manila*

References

[1] Alsina, D. Latorre, J. I. 2016, Phys. Rev. A, 94, 012314

[2] Greenberger, D. M., Horne, M. A., Zeilinger, A. 2007 [arXiv:0712.0921]

[3] Mermin, N. D. 1990, Phys. Rev. Lett., 65, 1838