An Introduction to OTFS Modulation:

Communications and Sensing Perspectives

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The OFDM Chronicle - A Legendary Success

- OFDM was introduced by Robert W. Chang of Bell Labs in 1966[1]
- FFT was proposed by Cooley and Tukey in 1965 [2] (Gauss was the first one in around 1805)
- OFDM was improved by Weinstein and Ebert in 1971 with the introduction of a guard interval and FFT[3]
- In 1980, the cyclic prefix was first proposed by Peled and Ruiz
- 1985: Cimini described use of OFDM for mobile communications
- Forward error correction (convolutional coding) and time/frequency interleaving are applied to the signal, i.e., coded OFDM (COFDM), by Alard in 1986 for Digital Audio Broadcasting for Eureka Project 147[4]
- Timing-frequency synchronization (Moose in 1994; Schmidl & Cox in 1997)
- Vector OFDM (VOFDM) by Xiang-Gen Xia in 2000[5]
- Scalable OFDM (SOFDM) in IEEE 802.16 standard (WiMax is based upon IEEE 802.16e-2005)

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[1] Chang, R. W. (1966). "Synthesis of band-limited orthogonal signals for multi-channel data transmission". Bell System Technical Journal. 45 (10): 1775-1796.

[2] Cooley, J.W., & Tukey, J.W. (1965). An algorithm for the machine calculation of complex Fourier series. Mathematics of Computation, 19, 297-301.

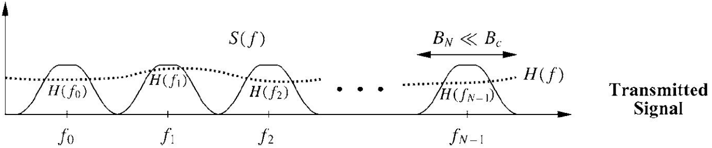
[3] Weinstein, S.; Ebert, P. (October 1971). "Data Transmission by Frequency-Division Multiplexing Using the Discrete Fourier Transform". *IEEE Transactions on Communication Technology*, 19 (5): 628-634.

[4] Pommier, Daniel & Alard, Michel, "Method and installation for digital communication, particularly between and toward moving vehicle", published 1988-01-14, assigned to Centre national detudes des telecommunications and Telediffusion de France

[5] X. -G. Xia, "Precoded and vector OFDM robust to channel spectral nulls and with reduced cyclic prefix length in single transmit antenna systems," in *IEEE Transactions on Communications*, vol. 49, no. 8, pp. 1363-1374, Aug. 2001.

Multi-Carrier Communications with Overlapping Sub-channels

- Multi-carrier Communications with Overlapping Sub-channels



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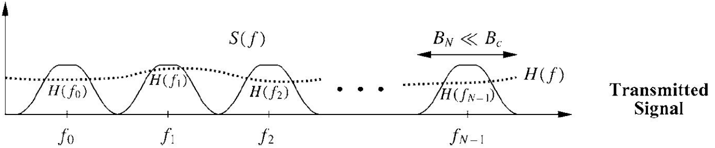

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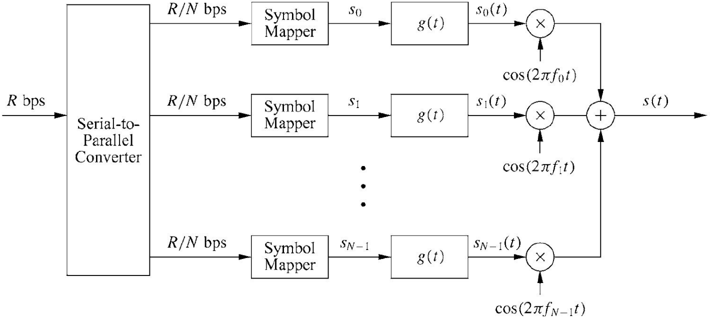
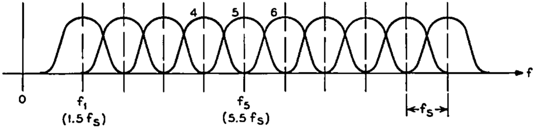


Overlapping Sub-channels

- Multi-carrier Communications with Overlapping Sub-channels in [1]



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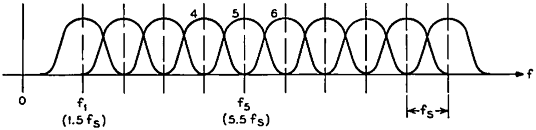
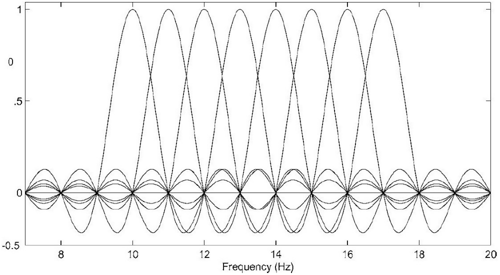


- OFDM through FFT/IFFT with CP



[1] Chang, R. W. (1966). "Synthesis of band-limited orthogonal signals for multi-channel data transmission". *Bell System Technical Journal*. 45 (10): 1775-1796.

OFDM Experiments and Protocols-I

- 1985: Telebit Trailblazer Modem introduced a 512 carrier Packet Ensemble Protocol (18 432 bit/s)
- 1988: In September TH-CSF LER, first experimental Digital TV link in OFDM, Paris area
- 1989: OFDM international patent application
- October 1990: TH-CSF LER, first OFDM equipment field test, 34 Mbit/s in an 8 MHz channel, experiments in Paris area
- December 1990: TH-CSF LER, first OFDM test bed comparison with VSB in Princeton USA
- September 1992: TH-CSF LER, second generation equipment field test, 70 Mbit/s in an 8 MHz channel, twin polarisations. Wuppertal, Germany
- October 1992: TH-CSF LER, second generation field test and test bed with BBC, near London, UK
- 1993: TH-CSF show in Montreux SW, 4 TV channel and one HDTV channel in a single 8 MHz channel
- 1993: Morris: Experimental 150 Mbit/s OFDM wireless LAN
- 1995: ETSI Digital Audio Broadcasting standard EUreka: first OFDM-based standard
- 1997: ETSI DVB-T standard
- 1998: Magic WAND project demonstrates OFDM modems for wireless LAN
- 1999: IEEE 802.11a wireless LAN standard (Wi-Fi)

OFDM Experiments and Protocols-II

- 2000: Proprietary fixed wireless access (V-OFDM, FLASH-OFDM, etc.)
- May 2001: The FCC allows OFDM in the 2.4 GHz license exempt band
- 2002: IEEE 802.11g standard for wireless LAN
- 2004: IEEE 802.16 standard for wireless MAN (WiMAX)
- 2004: ETSI DVB-H standard
- 2004: Candidate for IEEE 802.15.3a standard for wireless PAN (MB-OFDM)
- 2004: Candidate for IEEE 802.11n standard for next generation wireless LAN
- 2005: OFDMA is candidate for the 3GPP Long Term Evolution (LTE) air interface E-UTRA downlink
- 2007: The first complete LTE air interface implementation was demonstrated, including OFDM-MIMO, SC-FDMA and multi-user MIMO uplink

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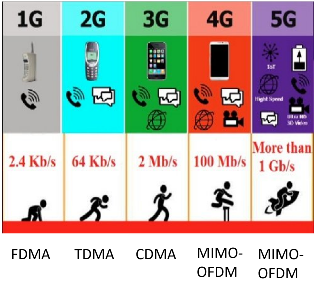
> "*OFDM is one of those ideas that had been building for a very long time, and became a practical reality when the appearance of mass market applications coincided with the availability of efficient software and electronic technologies.*"[1] - Stephen B. Weinstein

[1] Weinstein, S. B. "The history of orthogonal frequency-division multiplexing". *IEEE Communications Magazine*, vol. 47, no. 11, Nov. 2009, pp. 26-35.

Motivation

- OTFS was first proposed in [1] and developed in a series of papers [2-4]
- *History teaches us that every transition to a new generation of wireless network involves a disruption in the underlying air interface*[2] - Ronny Hadani & Anton Monk



[1] A. Monk, R. Hadani, M. Tsatsanis, and S. Rakib, "OTFS: Orthogonal Time Frequency Space," 2016. [Online]. Available: https://arxiv.org/abs/1608.02993

[2] Hadani, R., Monk, A.M., " OTFS: A New Generation of Modulation Addressing the Challenges of 5G". ArXiv, abs/1802.02623, 2018.

[3] R. Hadani, S. Rakib, M. Tsatsanis, etc., "Orthogonal time frequency space modulation," in *2017 IEEE Wireless Communications and Networking Conference (WCNC)*, 2017.

[4] R. Hadani, S. Rakib, S. Kons, etc., "Orthogonal time frequency space modulation," CoRR, vol. abs/1808.00519, 2018. [Online]. Available: http://arxiv.org/abs/1808.00519

The Wireless Channel

- Consider L paths between Tx and Rx, the impulse response of the channel is

$$h(\tau, t) = \sum_{l=0}^{L-1} a_l \delta(\tau - \tau_l(t)) \tag{1}$$

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- For a transmitted signal $s(t)$, the received signal is

$$r(t) = \int_0^{\infty} x(t-\tau) h(\tau, t) d\tau \tag{2}$$

  - The input signal τ ago has an impact on current output, i.e., $x(t-\tau)$
  - $h(t-\tau)$ quantifies the impact of input signal τ ago on the received signal at t

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- For a very short period of time (coherence time), the propagation delay is almost constant:

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- Basic Idea of OFDM
  - Divide the total bandwidth into small sub-channels
  - Each sub-channel has almost constant channel gain (flat-fading)
- Another way to understand OFDM
  - $h(\tau)$ describes a linear time-invariant (LTI) system
  - $e^{j\omega t}$ are the eigen-functions
  - Through FFT, the wideband signal is decomposed into a series of complex sinusoids
  - Each sinusoid goes through the channel without distortion

Resource Blocks in LTE

- One Physical Resource Block contains 12 sub-carriers $\times$ 6 or 7 symbols (depending on the length of the CP)
- Fixed sub-carrier spacing: 15 kHz
- Symbol duration: 66.7 us

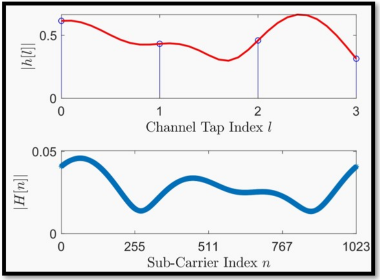

Figure 1: A typical LTI channel

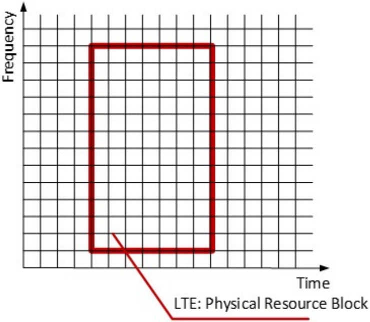

Figure 2: One physical resource block in LTE

A Fundamental Trade-off in OFDM

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Air-Interface Numerology

	LTE	802.11ad		B4G-MMW
Frequency Band	< 6 GHz	60 GHz		70 GHz
Supported Bandwidths	TBD	2160 MHz		2000 MHz
Maximum QAM	64	16	64	16
Modulation	OFDM	SC	OFDM	NullCP-SC
Channel Spacing (B)	20 MHz	2.16 GHz	2.16 GHz	2 GHz
FFT Size	2048	512	512	1024
Subcarrier Spacing	15 kHz	*4.2 kHz*	5.1 kHz	*2 MHz*
Sampling Frequency	3.072 MHz	1.76 GHz	2.46 GHz	1.54 GHz
Tsampling	32.6 ns	5.68 ps	406 fs	651 fs
Tsymbol	66.7 µs	245 ns	198 ns	666.7 ns
Tguard	4.7 µs	36.4 ns	52 ns	10.4 ns
T	71.4 µs	291 ns	250 ns	666.7 ns

CP and Pilot in OFDM

- Consider a wireless channel with delay spread τ_D, Doppler spread ν_D
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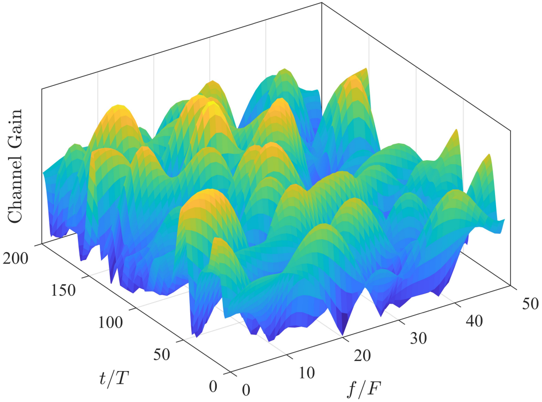
- We want to increase T as much as possible, but it is confined by coherence time in OFDM

An example of the Doubly-selective fading channel

- Carrier frequency 30 GHz
- Device speed 100 m/s
- $\tau_D = 1\mu$s, $\nu_D = 20\,$kHz
- $\tau_D \nu_D = 0.02$



Case Study: The Necessity of OTFS in LTE-R

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- High-speed train moves at 360 km/h, and the system works well with OFDM. Why is OTFS necessary?
- In [1], experiments for 5 Mhz bandwidth at 465 MHz, with a sub-carrier spacing of 15 kHz, trains moves at 371.1 km/h
- The LTE-R can work at 450 MHz carrier frequency, sub-carrier spacing is 15 kHz
- For a speed of 100 m/s, the Doppler spread is 300 Hz, comparable to that of LTE at 2 GHz
- However, if we want to move the carrier to mmWave band, the Doppler spread will increase by 10 to 100 times!
- To alleviate the impact of Doppler spread, we have to increase the sub-carrier spacing; but we cannot do so due to the limited *coherence bandwidth* resulting from *delay spread*!

[1] Y. Ma et al., “Characteristics of Channel Spreading Function and Performance of OTFS in High-Speed Railway,” in *IEEE Transactions on Wireless Communications*, doi: 10.1109/TWC.2023.3247736.

[2] Y. Ma, G. Ma, N. Wang, Z. Zhong and B. Ai, “OTFS-TSMA for Massive Internet of Things in High-Speed Railway,” in *IEEE Transactions on Wireless Communications*, vol. 21, no. 1, pp. 519-531, Jan. 2022, doi: 10.1109/TWC.2021.3098033.

[3] Gerald Matz, Franz Hlawatsch, “Chapter 1 - Fundamentals of Time-Varying Communication Channels,” Editor(s): Franz Hlawatsch, Gerald Matz, Wireless Communications Over Rapidly Time-Varying Channels, Academic Press, 2011, Pages 1-63.

Time Scaling

- The Doppler scaling factor:
  - Tx and Rx has a relative radial speed of v_r, Tx transmits a sinusoid at f Hz,
  - Rx receives a sinusoid at ρf with ρ given as

$$\rho = 1 - \frac{v_r}{c} \tag{5}$$

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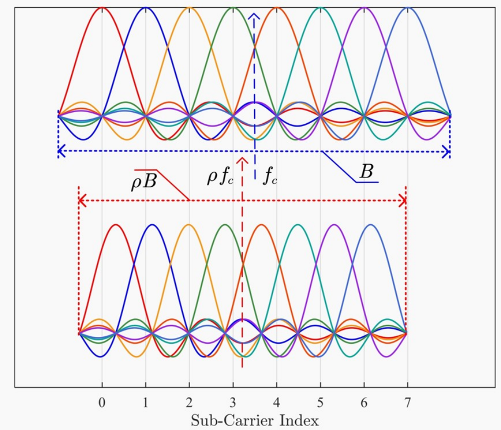
- Impulse channel response is

$$h(\tau, t) = \sum_{l=0}^{L-1} a_l \delta(\tau - (\tau_l + a_l t)) \tag{7}$$

- This model is based on the first-order Taylor expansion: $\tau_l(t) = \tau_l + a_l t$

Time Scaling

- The Doppler Effect is playing two roles here:
  - Spectrum shifting
  - Scaling of the baseband signal



When can we ignore the Doppler effect?

- Consider a bandwidth of B, divided into N sub-channels, with a carrier frequency is f_c
- The n-th sub-carrier has a frequency of $f_n = f_c - B/2 + nB/N$

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- The n-th sub-carrier has a frequency of $f_n = f_c - B/2 + nB/N$
- Due to Doppler effect, the n-th sub-channel is shifted to $\tilde{f}_n = (1-a)f_n$
- The frequency bias is $|\tilde{f}_n - f_n| = |a|f_n = |a|(f_c + nB/N - B/2)$

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- The frequency bias is $|\tilde{f}_n - f_n| = |a|f_n = |a|(f_c + nB/N - B/2)$
- The worst case happens for $n = N-1$, as

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When can we ignore the Doppler effect?

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- To ignore the Doppler effect, we must have $|\tilde{f}_n - f_n| \ll B/N \Rightarrow |a|(f_c + B/2) \ll B/N$
- For $f_c \gg B$, we have $|a| = \left|\frac{v_r}{c}\right| \ll \frac{B}{f_c N}$, i.e., we can reduce N to alleviate the Doppler effect, but to what extent (note: the sub-carrier spacing is confined by the coherence bandwidth)

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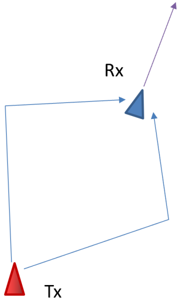
- Doppler shift is dependent on angle of arrival
- For signals coming from an angle of θ, the Doppler shift is

$$f_d = f_c \frac{v}{c} \cos\theta$$

- Different arriving paths have different Doppler shifts
- Multiple paths may add constructively or destructively



Terrestrial vs Underwater Comm.

- Terrestrial EM communications:
  - High-speed trains: 100 m/s
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  - $\frac{v_r}{c} \sim 10^{-6}$

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- 802.11n WiFi clock bias:
  - $\pm$ 20 ppm (5 GHz band), $\pm$ 25 ppm (2.4 GHz)
  - Equivalent to $\frac{v_r}{c} \sim 10^{-5}$

Wireless Channel in the Delay-Doppler Domain

Delay-Doppler Domain

- For a delay of τ_0 and a Doppler shift is ν_0, the impulse response is

$$h(\tau,\nu) = \delta(\tau - \tau_0)\delta(\nu - \nu_0)$$

- For baseband signal $s(t)$, we receive

$$r(t) = \int\int h(\tau,\nu)s(t-\tau)e^{j2\pi\nu(t-\tau)}d\tau d\nu$$

Delay-Time Domain

- For a delay of τ_0 and a Doppler scaling factor of a_0:

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- **Q**: Are these two models identical? If not, what's the fundamental difference?
- **A**: The DD domain model only considers the *shifting effect*, and ignores the *scaling effect*
- For underwater acoustic comm., we must use the delay-time domain model; or we can model the channel in delay-scale domain

An Example on DD Domain Channel

- Consider a wideband signal $s(t)$
- For the Delay-Doppler domain channel model $h(\tau, \nu) = \delta(\tau - \tau_0)\delta(\nu - \nu_0)$, the received signal is

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- When a_0 is large, we cannot ignore the scaling effect on baseband signal. Underwater acoustic communications is a typical example, and we need to model the channel in the delay-scale domain[1,2]

[1] Y. Jiang and A. Papandreou-Suppappola, "Discrete time-scale characterization of wideband time-varying systems," *IEEE Transactions on Signal Processing*, vol. 54, no. 4, pp. 1364-1375, 2006.

[2] K. P. Arunkumar and C. R. Murthy, "Orthogonal delay scale space modulation: A new technique for wideband time-varying channels," *IEEE Transactions on Signal Processing*, vol. 70, pp. 2625-2638, 2022.

ODSS and Wavelet-OFDM

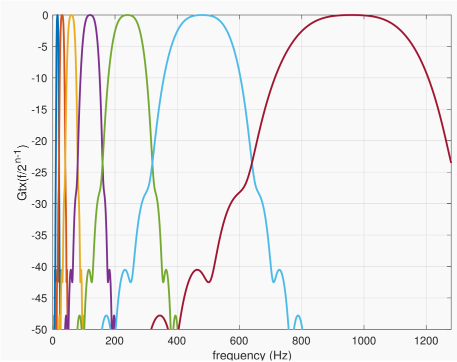

Figure 3: ODSS Modulation

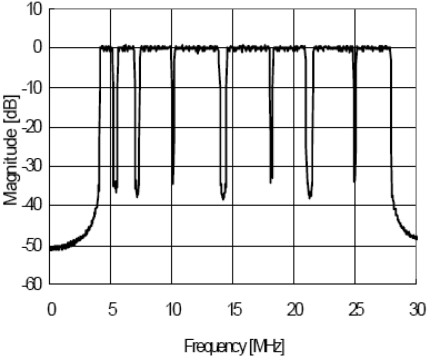

Figure 4: Wavelet-OFDM Modulation

[1] K. P. Arunkumar and C. R. Murthy, "Orthogonal delay scale space modulation: A new technique for wideband time-varying channels," *IEEE Transactions on Signal Processing*, vol. 70, pp. 2625-2638, 2022.

[2] S. Galli, H. Koga and N. Kodama, "Advanced signal processing for PLCs: Wavelet-OFDM," 2008 IEEE International Symposium on Power Line Communications and Its Applications, Jeju, Korea (South), 2008, pp. 187-192.

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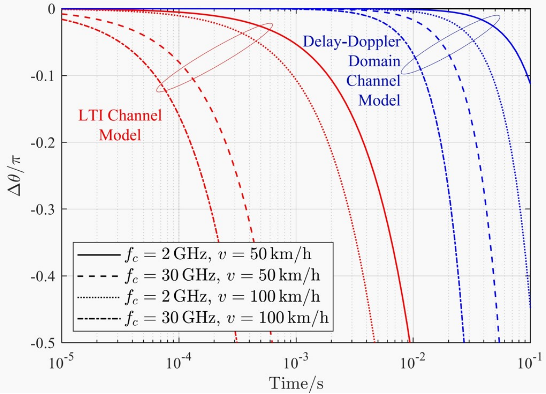
Time-Varying Channel

- Note that none of the above models are absolutely accurate
- When we increase the carrier frequency or device mobility, the modeling error accumulates over time faster



OTFS Modulation

- The modulated signal is

$$s(t) = \sum_{n,m} X_{n,m} g_t(t - nT) e^{j2\pi mF(t-nT)}$$

  - T is the symbol duration in time
  - F is the sub-carrier spacing, with $TF = 1$
  - $g_t(t)$ is the modulation waveform at the Tx side
  - There are totally M sub-carriers and N symbols in time, for one resource block
  - $X_{n,m}$ is the transmitted symbol on the n-th time slot and m-th sub-carrier
  - Referred to as the Heisenberg transform

Linear Time-Varying Channel in DD Domain

- The baseband equivalent channel model

$$h(\tau,\nu)=\sum_{l=0}^{L-1}\beta_l\delta(\tau-\tau_l)\delta(\nu-\nu_l)$$

  - There are totally L paths between Tx and Rx
  - β_l is the complex channel gain of the l-th path
  - τ_l is propagation delay, and ν_l is the Doppler shift
- Underlying idea: The channel changes very fast in time-frequency domain, but the way it changes in both domains is almost constant
- We can no longer use the conventional concept of *coherence time* to describe how fast the channel changes. The *Geometric Coherence Time* is proposed as a new metric[1,2]

[1] H. B. Mishra, P. Singh, A. K. Prasad, and R. Budhiraja, "OTFS channel estimation and data detection designs with superimposed pilots," *IEEE Transactions on Wireless Communications*, vol. 21, no. 4, pp. 2258-2274, Sept. 2022.

[2] M. K. Ramachandran, G. D. Surabhi, and A. Chockalingam, "OTFS: A new modulation scheme for high-mobility use cases," *Journal of the Indian Institute of Science*, vol. 100, pp. 315-336, April 2020.

Demmodulation

- The received signal is

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\begin{aligned}
r(t) &= \int\int s(t-\tau)h(\tau,\nu)d\tau d\nu \\
&= \sum_{n,m} X_{n,m} \int\int h(\tau,\nu)g_t(t-\tau-nT)e^{j2\pi(mF+\nu)(t-\tau)}d\tau d\nu
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- With a matched filter $g_r(t)$ (the Wigner transform), we can get

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- The cross-ambiguity function is

$$A_{g_t,g_r}(\tau,\nu) = \int_t g_t(t)g_r(t-\tau)e^{-j2\pi\nu(t-\tau)dt}$$

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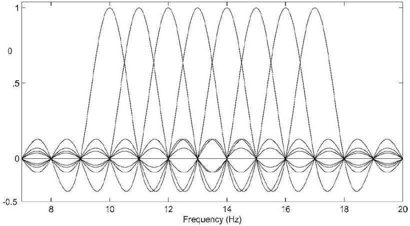
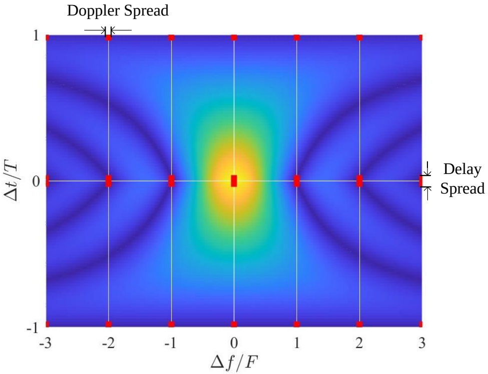
- We've already witnessed this in OFDM



Energy Leakage

- The cross-ambiguity function satisfies $A_{g_t,g_r}(\tau,\nu) = 0, \quad (\tau = nT,\ \nu = mF)$
- Energy leakage happens in both time and frequency domains (ICI & ISI)
- Doppler spread leads to ICI in OFDM (To suppress ICI to -30 dB, the Doppler spread should not surpass 1% of the sub-carrier spacing (15 kHz sub-carrier spacing in LTE, with a maximum Doppler spread of $2 \times 10^9 \times 30/c = 200$ Hz))



Bi-Orthogonality

- Sample the received signal in time frequency domain:

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\begin{aligned}
&Y_{n,m} \\
&= y(t,f)|_{t=nT,f=mF} \\
&= \sum_{\tilde{m},\tilde{n}} \mathbf{X}[\tilde{n},\tilde{m}] \int_{\tau}\int_{\nu} \underbrace{h(\tau,\nu)e^{-j2\pi(\nu-\delta_m F)\tau} A_{g_t,g_r}(\delta_n T-\tau,\delta_m F-\nu)}_{\kappa_{\delta_n,\delta_m}(\tau,\nu)} e^{j2\pi(nT\nu-mF\tau)} d\nu d\tau
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- $\delta_n = n-\tilde{n}$, $\delta_m = m-\tilde{m}$

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- Assume $A_{g_t,g_r}(\tau,\nu)=0, \quad (\tau \in (nT, nT+\tau_D),\ \nu \in (mF, mF+\nu_D), \forall n \neq 0, m \neq 0)$; both ICI and ISI are ignored

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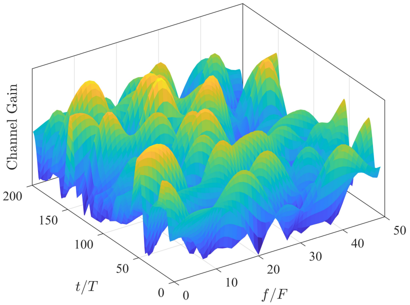
- A typical doubly-selective fading channel



OTFS Modulation and ISFFT

- Due to doubly-selective fading, information loss will be inevitable
- Hadani proposed to precode the symbols before transmission in [1]. The basic idea is to spread each symbol to the whole time-frequency field through the ISFFT (Inverse Sympletic Finite Fourier Transform), i.e., the OTFS modulation:

$$X_{k,l} = \sum_{n,m} X_{n,m} e^{-jnl\frac{2\pi}{N}} e^{jmk\frac{2\pi}{M}}$$

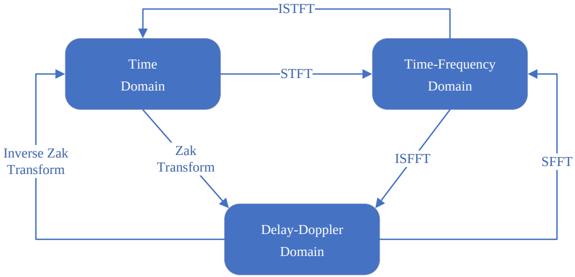
- In the matrix form, we have

$$\tilde{\mathbf{X}} = \mathbf{F}_M^H \mathbf{X}^T \mathbf{F}_N$$

- The two dimensions of $\tilde{\mathbf{X}}$ are now *Delay* (from Frequency through IDFT) and *Doppler* (from *Time* through DFT)

[1] R. Hadani, S. Rakib, M. Tsatsanis, etc., "Orthogonal time frequency space modulation," in *2017 IEEE Wireless Communications and Networking Conference (WCNC)*, 2017.

Delay-Doppler Domain



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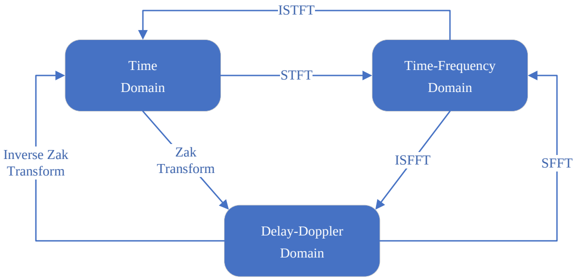

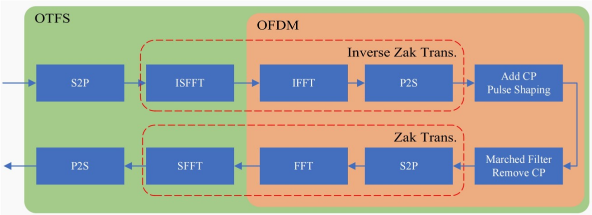

Delay-Doppler Domain-Unit Area

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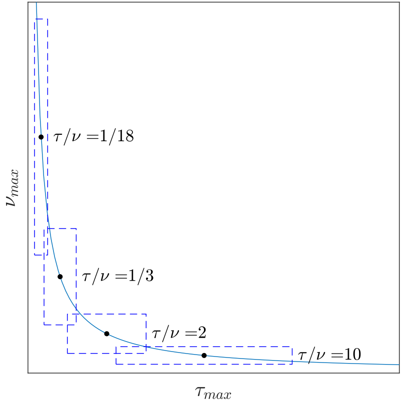
- From the DD domain representation to frequency domain:

$$\begin{aligned} S(f) =& \frac{1}{\sqrt{2\pi}} \int_0^{\tau_{max}} S_z(\tau, f) e^{-j2\pi\tau f} d\tau \\ =& \frac{1}{\sqrt{2\pi}} \int_0^{\tau_{max}} \sum_k s(\tau + k\tau_{max}) e^{-j2\pi(\tau + k\tau_{max}) f} d\tau \end{aligned}$$

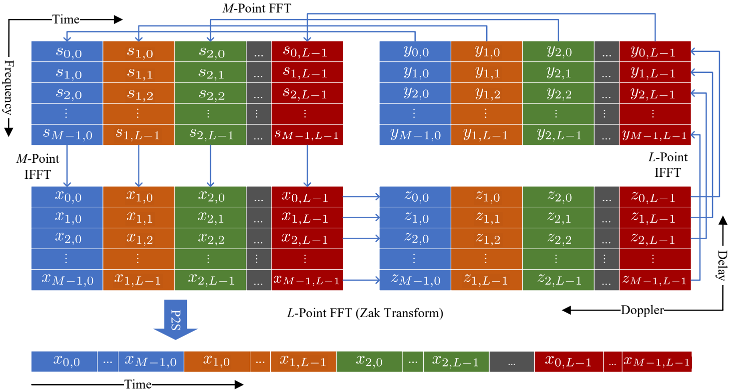
Delay-Doppler Invariance

- When we shift the signal in time by Δt, and frequency by Δf, the corresponding Zak transform will be

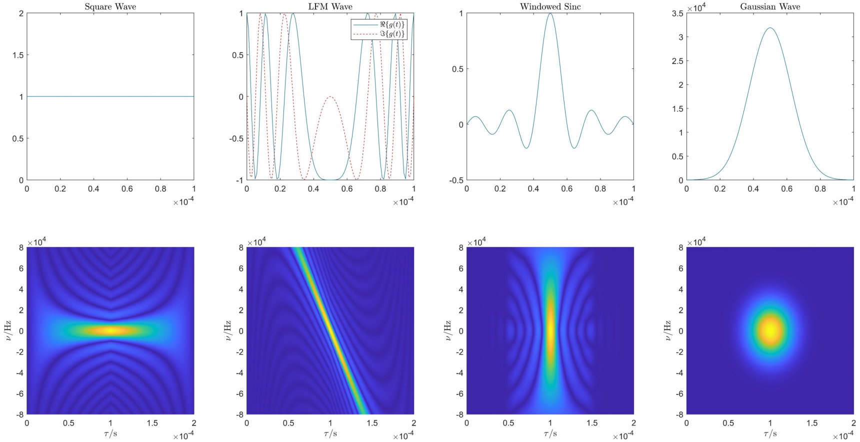
$$\mathcal{Z}\{s(t-\Delta t)e^{j2\pi \Delta f t}\} = S_z(\tau - \Delta t, \nu - \Delta f).$$



Zak Transform



The Ambiguity Function



Simulations in [1]

Parameter	Value
Carrier frequency (GHz)	4.0
Duplex mode	FDD
Subcarrier spacing (KHz)	15
Cyclic Prefix duration (us)	4.7
FFT Size	1024
Transmission Bandwidth (Resource Blocks)	50
Antenna configuration	1T1R, 2T2R
Rank	Fixed rank
MCS	Fixed: 4QAM, 16QAM, 64QAM rate ½
Control and pilot overhead	None
Channel estimation	Ideal
Channel model	TDL-C, DS = 300ns, Rural Macro, low correlation MIMO
UE speed (km/h)	30, 120, 500
Receiver	OTFS: Turbo Equalizer, OFDM: Sphere Decoder

- The channel is estimated for the (n, m)-th time-frequency slot as $\hat{H}_{n,m}$
- The estimated channel is used for data detection at the $(\tilde{n}, \tilde{m})$-th time-frequency slot
- The channel changes so fast in both time and frequency domains, and the modelling error will outweigh noise in high SNR regime (we will see error floor)

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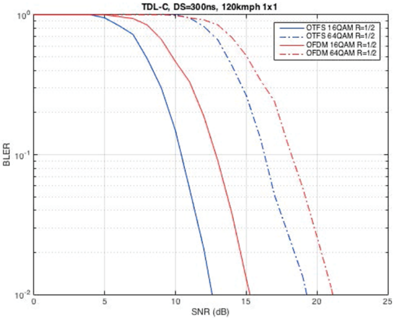
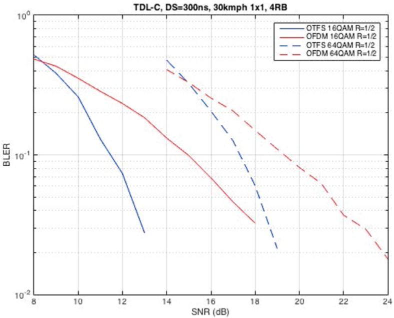
Simulations in [1]

- 120 km/h (444 Hz Doppler spread)



- 30 km/h (111 Hz Doppler spread)



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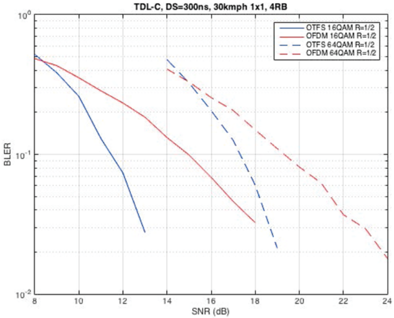

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- LTI model does not capture the time-varying characteristics of mobile wireless channels
- If we use a more accurate channel model, the modelling error accumulates over time slowly, and we can reduce the channel estimation frequency

An Example from My Course: *Wireless and Mobile Communications*

- March 2020 at Memorial University of Newfoundland, St. John's, NL, Canada

ENGI 8877/9878 Z. Gong

4. Consider an OFDM system, the carrier frequency is $f_c = 5\,\text{GHz}$, and the bandwidth is $B = 300\,\text{MHz}$.

   (a) Consider indoor communications, if the delay spread is $T_d = 100\,\text{ns}$, what is the minimum CP length? If we define coherence bandwidth as $B_c = 0.1/T_d$, what is the maximum bandwidth of sub-channels? The user's velocity is smaller than 3 meters per second, and suppose the coherence time is $T_c = 0.1/f_d$, where f_d is Doppler spread. On each sub-channel, suppose the frame length is at least 100 symbols, which means the channel must be constant over 100 symbols. What is the minimum bandwidth of sub-channels?

   (b) Consider outdoor communications, if the delay spread is $T_d = 1\,\mu\text{s}$, while the user's velocity is smaller than 60 meters per second. What is the maximum sub-channel bandwidth? It is possible to maintain the minimum frame length of 100 symbols? If not, what is the maximum frame length that can be supported?

   (c) Comment on the results in (a) and (b).

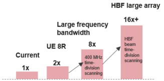
2020 Winter 2 of 3

Real Problems

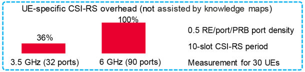
- Problem I:

Background

- Obtaining high-dimensional channel information has become increasingly complex due to increased system bandwidths, the proliferation of UE antennas, and heavier network loads. A shortage of air interface measurement resources leads to severe channel aging. The SRS pilot overhead is estimated to be increased by 16x.



- With the increase in frequency bands and the number of antennas, the CSI-RS overhead increases significantly. The UE-specific CSI-RS overhead occupies all time-frequency resources, making it impossible to use traditional channel measurement technologies.



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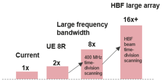

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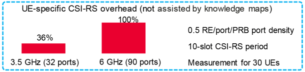
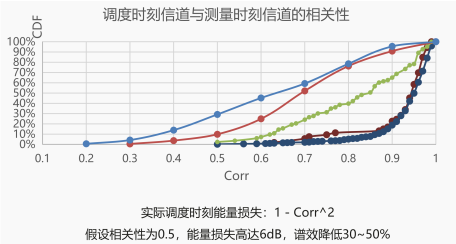


- Problem II:



Revisit the Channel

- The following questions are not answered yet
  - How much resources do we need for channel estimation?
  - Can we use OTFS for any bandwidth/frame length?
  - Do we need CP in OTFS (like we did in OFDM)?
  - What will happen if the bi-orthogonality does not hold?

Revisit the Channel

- Consider ICI and ISI, we actually have

$$
\begin{aligned}
&Y_{n,m}\\
&=\sum_{\tilde{m},\tilde{n}}\mathbf{X}[\tilde{n},\tilde{m}]\underbrace{\int_{\tau}\int_{\nu}\underbrace{h(\tau,\nu)e^{-j2\pi(\nu-\delta_m F)\tau}A_{g_t,g_r}(\delta_n T-\tau,\delta_m F-\nu)}_{\kappa_{\delta_n,\delta_m}(\tau,\nu)}e^{j2\pi(nT\nu-mF\tau)}d\nu d\tau}_{H_{\delta_n,\delta_m}[n,m]}
\end{aligned}
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- $\delta_n = n-\tilde{n}$, $\delta_m = m-\tilde{m}$

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- $\delta_n = n - \tilde{n}$, $\delta_m = m - \tilde{m}$
- $H_{\delta_n,\delta_m}[n,m]$ is sampled from

$$H_{\delta_n,\delta_m}(t,f) = W(t,f) \int_\tau \int_\nu \kappa_{\delta_n,\delta_m}(\tau,\nu) e^{j2\pi(t\nu - f\tau)} d\nu d\tau$$

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- $W(t,f)$ is the window function in time-frequency domain, given by

$$W(t,f) = \begin{cases} 1, & t\in[0,NT], f\in[0,MF] \\ 0, & \text{otherwise} \end{cases}$$

Revisit the Channel

- The ISFFT of the window function in Delay-Doppler domain is

$$w(\tau,\nu) = \int\int W(t,f)e^{-j2\pi(t\nu - f\tau)}d\tau d\nu$$
$$= \frac{\sin(\pi NT\nu)}{\pi\nu}\cdot\frac{\sin(\pi MF\tau)}{\pi\tau}e^{-j\pi(NT\nu - MF\tau)}.$$

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H_{\delta_n,\delta_m}(t,f) = & W(t,f)\,\mathcal{S}\{\kappa_{\delta_n,\delta_m}(\tau,\nu)\} \\
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- The equivalent channel response in delay-Doppler domain is

$$\begin{aligned} \tilde{h}_{\delta_n,\delta_m}(\tau,\nu) =& w(\tau,\nu)\circledast\kappa_{\delta_n,\delta_m}(\tau,\nu) \\ =& \sum_l \alpha_{l,\delta_n,\delta_m} w(\tau-\tau_l,\nu-\nu_l), \end{aligned}$$

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- The channel response has very limited spreading in both delay and Doppler ($\tau_D\nu_D \ll TF = 1$)

Delay-Doppler Domain Channel

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- One frame has $MF \times NT = MN$ slots in total; so the ratio of channel training overhead is

$$\tau_D\nu_D$$

Channel Reconstruction

- We sample the channel in DD domain as

$$\tilde{\mathbf{H}}_{\delta_m,\delta_n}[\tilde{m},\tilde{n}] = \tilde{h}_{\delta_m,\delta_n}\left(\frac{\tilde{m}}{MF},\frac{\tilde{n}}{NT}\right),$$

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$$\mathbf{H}_{\delta_m,\delta_n} = \underset{N,M}{\mathcal{S}}\{\tilde{\mathbf{H}}_{\delta_m,\delta_n}\}.$$

- Most of elements in $\tilde{\mathbf{H}}$ will be close to zero. Specifically, we have $|\tilde{\mathbf{H}}[\tilde{m},\tilde{n}]| \approx 0$ for $\tilde{m} > \lceil \tau_d MF \rceil$ in the delay domain and $\tilde{n} > \lceil \nu_d NT \rceil$ in the Doppler domain.

Channel Reconstruction

- Then we down-sample $\mathbf{H}$ by a factor of L_N and L_M in time and frequency domains, respectively. That is to say, we get

$$\begin{aligned}\breve{\mathbf{H}}[\breve{n},\breve{m}] =& \mathbf{H}[\breve{n}L_N,\breve{m}L_M] \\ =& \sum_{\tilde{n},\tilde{m}} \tilde{\mathbf{H}}[\tilde{m},\tilde{n}] e^{-j(\breve{n}\tilde{n}\omega_{\breve{N}} - \breve{m}\tilde{m}\omega_{\breve{M}})},\end{aligned} \tag{11}$$

  - $L_M\omega_M = 2\pi/(M/L_M) = \omega_{\breve{M}}$ and $L_N\omega_N = 2\pi/(N/L_N) = \omega_{\breve{N}}$
  - For $\breve{M} \geq \tilde{M}$ and $\breve{N} \geq \tilde{N}$, $\breve{\mathbf{H}}$ is the $\breve{N} \times \breve{M}$-point ISFFT of $\tilde{\mathbf{H}}$.

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  - For $\breve{M} \geq \tilde{M}$ and $\breve{N} \geq \tilde{N}$, $\breve{\mathbf{H}}$ is the $\breve{N} \times \breve{M}$-point ISFFT of $\tilde{\mathbf{H}}$.

- Consider confined spreading of the channel in DD domain: $\tilde{\mathbf{H}}[\tilde{m},\tilde{n}] = 0$ for $\tilde{n} \geq \tilde{N}$ and $\tilde{m} \geq \tilde{M}$, we have

$$\breve{\mathbf{H}} = \underset{\breve{N},\breve{M}}{\mathcal{S}}\{\tilde{\mathbf{H}}\}, \tag{12}$$

Channel Reconstruction

- Then we down-sample $\mathbf{H}$ by a factor of L_N and L_M in time and frequency domains, respectively. That is to say, we get

$$\begin{aligned}\breve{\mathbf{H}}[\breve{n},\breve{m}] =& \mathbf{H}[\breve{n}L_N,\breve{m}L_M] \\ =& \sum_{\tilde{n},\tilde{m}} \tilde{\mathbf{H}}[\tilde{m},\tilde{n}]e^{-j(\breve{n}\tilde{n}\omega_{\breve{N}}-\breve{m}\tilde{m}\omega_{\breve{M}})},\end{aligned} \tag{11}$$

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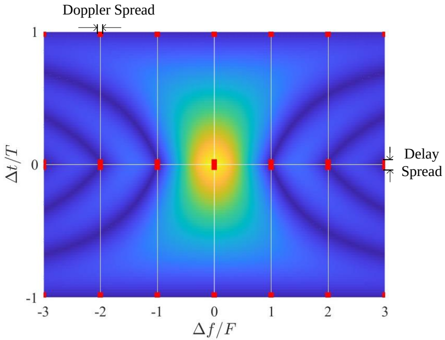
- We can recover $\tilde{\mathbf{H}}$ from $\breve{\mathbf{H}}$ first, and then reconstruct $\mathbf{H}$, given as

$$\mathbf{H} = \underset{N,M}{\mathcal{S}}\{\underset{\breve{N},\breve{M}}{\mathcal{S}^{-1}}\{\breve{\mathbf{H}}\}\}. \tag{13}$$

ICI and ISI

- We should choose T and F properly to minimize the overall ISI and ICI:[1,2]

$$\frac{\tau_D}{T} = \frac{\nu_D}{F}$$



[1] W. Kozek and A. Molisch, "Nonorthogonal pulseshapes for multicarrier communications in doubly dispersive channels," *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1579-1589, 1998.

[2] K. Liu, T. Kadous, and A. Sayeed, "Orthogonal time-frequency signalling over doubly dispersive channels," *IEEE Transactions on Information Theory*, vol. 50, no. 11, pp. 2583-2603, 2004.

CP and ZP in OTFS

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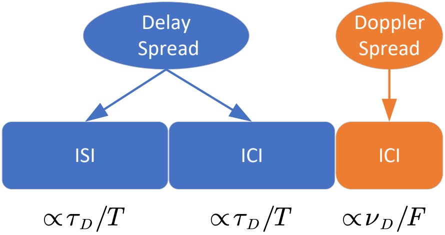

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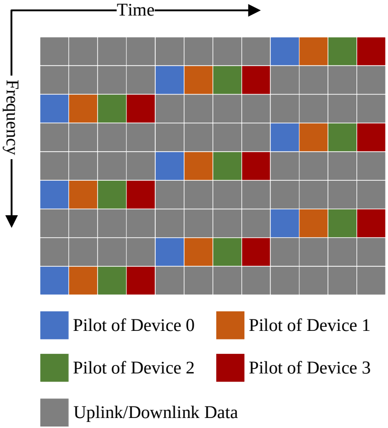
- In OFDM, sub-carrier spacing should be at least 100 times the Doppler spared, so that ICI can be ignored
- This means short symbol, leading to very large overhead! It eventually becomes impractical to do so!
- In the OFDM system design phisolophy, we are going to an extreme; we are trying too hard to suppress the ICI!

[1] B. Muquet, Z. Wang, G. Giannakis, M. de Courville, and P. Duhamel, "Cyclic prefixing or zero padding for wireless multicarrier transmissions?," *IEEE Transactions on Communications*, vol. 50, pp. 2136-2148, Dec. 2002.

Multiple Access

- Consider K single-antenna users, with multiple antennas at the BS
- The ratio of pilot overhead is at least $K\tau_D\nu_D$



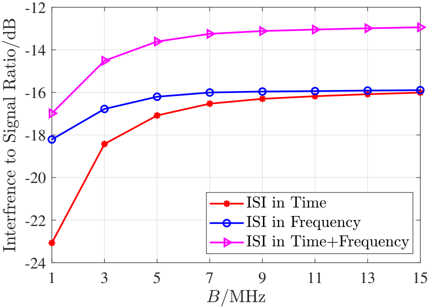
Simulation Setup

- Carrier frequency $f_c = 30\,\text{GHz}$, and a bandwidth of $B = 10\,\text{MHz}$
- The delay spread is $\tau_D = 300/c = 1\mu\text{s}$, and Doppler spread is $2 \times vf_c/c = 20\,\text{kHz}$
- Thus we choose a sub-carrier spacing of 200 kHz, or a symbol duration of 5 μs
- The reciprocal of Doppler spread is 50 μs, and the coherence time is at the level of 5μs, equal to one symbol duration
- We can for example choose the frame length as 5 OFDM symbols, equal to 25 μs
- We will thus evaluate the modelling error of this 10 MHz$\times$25 μs=250
- Wide-sense stationary uncorrelated scattering (WSSUS) channel model[1]

[1] P. Bello, "Characterization of randomly time-variant linear channels," *IEEE Transactions on Communications Systems*, vol. 11, no. 4, pp. 360-393, Dec. 1963.

ISI vs Bandwidth

- For a small block size of 250, we will see that the modelling error of OFDM is even larger than that of OTFS block ten times larger!
- For OFDM, the Doppler spread leads to ICI, and also time-domain channel variation
- For OTFS, the modelling error results from the time-frequency spreading of the cross-ambiguity function of the transmitting/receiving pulses
- We can see that the modelling error of OFDM can only be ignored over a very small block size, while that of OTFS is much more reliable



- In [1], the authors concluded there are no fixed eigen-functions for general linear time-varying channels

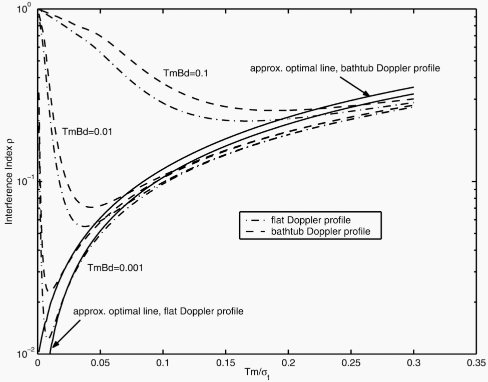


Fig. 3. Interference index for a rectangular pulse as a function of time scale for different channel spread factors. Flat multipath power profile is used. Both "bathtub" and flat Doppler profiles are used.

[1] Ke Liu, T. Kadous and A. M. Sayeed, "Orthogonal time-frequency signaling over doubly dispersive channels," in *IEEE Transactions on Information Theory*, vol. 50, no. 11, pp. 2583-2603, Nov. 2004.

- In [1], the authors investigated the optimal pulse design for minimized ISI and ICI in doubly-selective channels

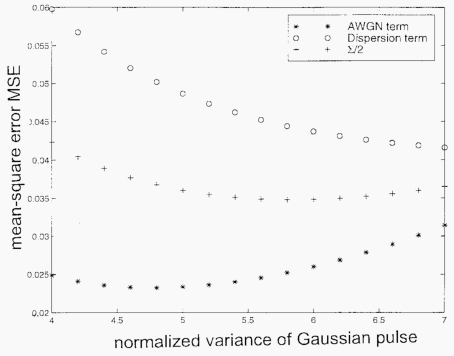
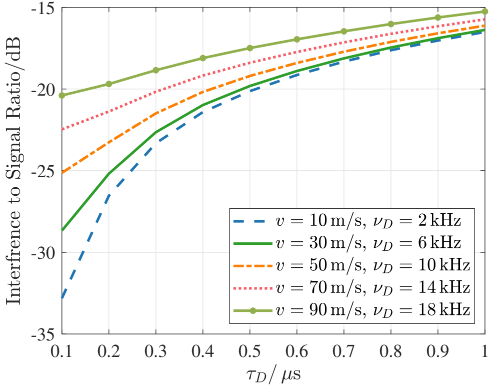


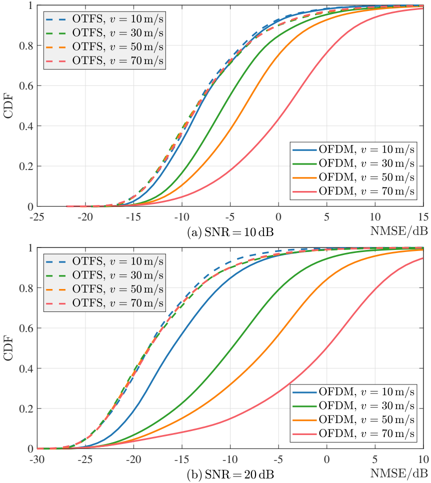
Fig. 7. Fading- and AWGN-caused coefficient error for Gaussian prototype with varying duration.

[1] W. Kozek and A. F. Molisch, "Nonorthogonal pulseshapes for multicarrier communications in doubly dispersive channels," in *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1579-1589, Oct. 1998.

ISI vs Delay Spread & Doppler Spread

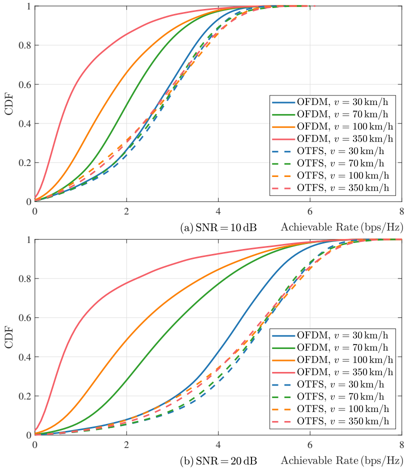


Channel Estimation Error for Same Overhead

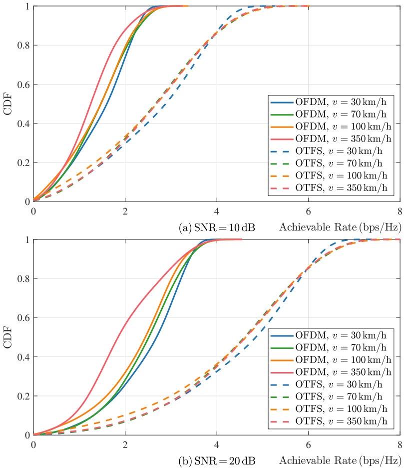


Achievable Rate

- Same overhead



- Same NMSE



Similarities of OFDM and OTFS

	OFDM	**OTFS**
Domain of Channel Response	Delay ($h(\tau)$)	Delay-Doppler ($h(\tau, \nu)$)
Domain of Signal	Time ($s(t)$)	Time-Frequency ($s(t, f)$)
Received Signal	$r(t) = s(t) * h(\tau)$	$r(t, f) = s(t, f) * h(\tau, \nu)$
Transforms	FFT/IFFT	SFFT/ISFFT

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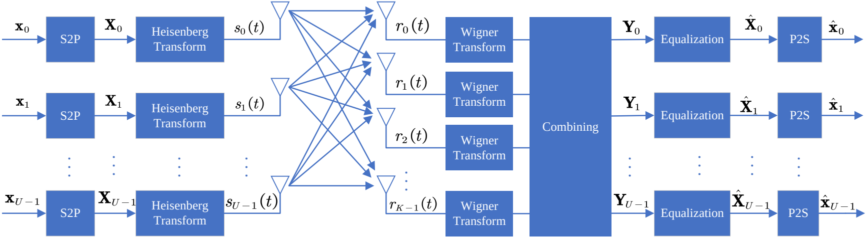
- **OFDM**: the channel gives the 1D convolution of the signal in time domain and the channel in delay domain; in frequency domain, it's point-wise multiplication
- **OTFS**: the channel gives 2D convolution of the signal in time-frequency domain and the channel in DD domain; in time-frequency domain, it's point-wise multiplication

Framework



[1] Zijun Gong, Fan Jiang, Cheng Li and Xuemin (Sherman) Shen, "Simultaneous localization and communications with massive MIMO-OTFS", *IEEE Journal on Selected Areas in Communications*, to appear in 2023.

Dilemma in Communications & Sensing

- For communications, the waveform should be robust to small channel variations
- For sensing, the waveform is expected to be extremely sensitive to channel variations
- The fundamental reason of the dilemma is: OFDM is based on an LTI channel; sensing is done in delay-Doppler domain, a time varying model

[1] Zijun Gong, Fan Jiang, Cheng Li and Xuemin (Sherman) Shen, "Simultaneous localization and communications with massive MIMO-OTFS", *IEEE Journal on Selected Areas in Communications*, to appear in 2023.

Multipath Channel Model

- Consider L path between Tx and Rx

$$h(t,\tau) = \sum_{l=0}^{L-1} a_l e^{-j2\pi f_c \tau_l(t)} \delta(\tau - \tau_l(t))$$

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- Suppose the 0-th path is the line-of-sight (LoS) one, we are interested in τ_0 and ν_0 (distance and velocity, respectively)

DD Domain Channel Model

- Define $S = NT$, given that $\rho_l t \ll 1/B$ for $t \in [0, S]$, we approximately have

$$\begin{aligned} r(t) &= \sum_l a_l e^{-j2\pi f_c(\tau_l + \rho_l t)} s(t - \tau_l - \rho_l t) \\ &\approx \sum_l a_l e^{-j2\pi f_c(\tau_l + \rho_l t)} s(t - \tau_l) \end{aligned}$$

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- To use this DD domain model, we must have $B \times S = MF \times NT = MN \ll |1/\rho_l|, \ \forall\, l$

MIMO and OTFS

- Consider a uniform linear array with K antennas at the BS, critically spaced with a distance of $D = \lambda_c/2$
- The received signal at the k-th antenna is

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$$\tau_{l,k} \approx \tau_l + kD\cos\theta_l/c$$

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- For $KD \ll 1/B$, we can take one more step

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- Different sub-channels have different spatial signatures
- The constraint is relaxed to $B/M \ll c/(KD)$ (original we need to guarantee that $B \ll c/(KD)$)

Summary of the Modelling

- The assumptions of the DD domain channel model are summarized below
  - (a) Suppose each resource block has a size of B Hz $\times$ S seconds. Given that the user device moves at v m/s, we have to guarantee that $BS \ll c/v$. For example, for a vehicle moving at 120 m/s, $c/v = 2.5 \times 10^6$
  - (b) For the covnentional MIMO model, we have $KD \ll c/B$ or $B \ll c/(KD)$
  - (c) Consdiering the spatial wideband effect, we can relax the contraint to $B/M \ll c/(KD)$
- From constraint (a), we can see that there is a tradeoff between bandwidth ($B = MF$) and frame length ($S = NT$), which indicates a tradeoff between sensing capabilities in delay and Doppler domains

Theoretical Foundations

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- With a ULA, 2D positioning can be easily done by combining AoA and ToA measurements. **By introducing Doppler shift, 3D positioning of a single-antenna device would be possible with a ULA at the BS.**

Time Frequency Spread of Signals

- For a function $g(t)$, suppose its Fourier transform is $G(f)$, we define the mean of $g(t)$ in time and frequency domains as

$$\mu_t = \int t|g(t)|dt,\ \mu_f = \int f|G(f)|df.$$

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- These two parameters quantify the 'centre' of $g(t)$'s energy in time and frequency domains, respectively. Similarly, the variance in time and frequency domains are defined as

$$\sigma_t^2 = \int (t-\mu_t)^2|g(t)|^2dt,\ \sigma_f^2 = \int (f-\mu_f)^2|G(f)|^2df,$$

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- For communications and sensing, we want to make this values as small as possible. However, this is not possible because the Heisenberg's uncertainty principle states that

$$\kappa \geq \frac{1}{4\pi}$$

Channel Estimation Error - Modelling

- Suppose we take P slots on time and Q slots on frequency for channel estimation and localization
- Insert pilots on (n_p, m_q)-th slots, with $n_p \in \{n_0, n_1, \cdots, n_{P-1}\}$ and $m_q \in \{m_0, m_1, \cdots, m_{Q-1}\}$
- Each path has four parameters, the complex gain, propagation delay, Doppler shift, and AoA (or spatial signature, equivalently), i.e., α_l, τ_l, ν_l, ω_l
- Further define ϕ_l as the phase of α_l, i.e., $\alpha_l = |\alpha_l| e^{j\phi_l}$ and $\boldsymbol{\theta}_l = [\tau_l, \nu_l, \omega_l, \phi_l]^T$. Then we can put all the parameters together and represent them more concisely as $\boldsymbol{\theta} = [\boldsymbol{\theta}_0^T, \boldsymbol{\theta}_1^T, \cdots, \boldsymbol{\theta}_{L-1}^T]^T$, $\mathbf{a} = [|\alpha_0|, |\alpha_1|, \cdots, |\alpha_{L-1}|]^T$ and $\tilde{\boldsymbol{\theta}} = [\boldsymbol{\theta}^T, \mathbf{a}^T]^T$.
- The estimated channel gains are $\hat{\mathbf{H}}[k, p, q] = \hat{H}_{k, n_p, m_q}$. Suppose the estimation errors of different channel gains are i.i.d. cyclically symmetric complex Gaussian random variables, with a variance of σ^2.

Channel Estimation Error-FIM

- The likelihood of $\hat{\mathbf{H}}$ is then given as

$$p(\hat{\mathbf{H}};\tilde{\boldsymbol{\theta}}) = (\pi\sigma^2)^{-KPQ}\exp\{-\|\hat{\mathbf{H}}-\mathbf{H}\|^2/\sigma^2\}, \tag{14}$$

with $\mathbf{H}$ being the perfect CSI on the chosen sub-channels, i.e., $\mathbf{H}[k,p,q] = H_{k,n_p,m_q}$, given as

$$H_{k,n_p,m_q} = \sum_l \alpha_l e^{j2\pi n_p T\nu_l} e^{-j2\pi m_q F\tau_l} e^{-jk\omega_l}. \tag{15}$$

The log-likelihood function is

$$l_g(\hat{\mathbf{H}};\tilde{\boldsymbol{\theta}}) = -KPQ\ln(\pi\sigma^2) - \|\hat{\mathbf{H}}-\mathbf{H}\|^2/\sigma^2. \tag{16}$$

Then the Fisher information matrix (FIM) concerning $\tilde{\boldsymbol{\theta}}$ is

$$\begin{aligned}\mathbf{F}_{\tilde{\boldsymbol{\theta}}} &= \mathbb{E}\left\{\nabla_{\tilde{\boldsymbol{\theta}}} l_g \nabla^T_{\tilde{\boldsymbol{\theta}}} l_g\right\}\\ &= \frac{2}{\sigma^2}\sum_{k,p,q}\Re\left\{\nabla_{\tilde{\boldsymbol{\theta}}} H_{k,n_p,m_q}\nabla^H_{\tilde{\boldsymbol{\theta}}} H_{k,n_p,m_q}\right\}.\end{aligned} \tag{17}$$

Channel Estimation Error-Multipath

- The FIM concerning $\boldsymbol{\theta}$ can be rewritten as

$$\mathbf{F}_{\boldsymbol{\theta}} = \begin{bmatrix} \mathbf{F}_{0,0} & \mathbf{F}_{0,1} & \cdots & \mathbf{F}_{0,L-1} \\ \mathbf{F}_{0,1} & \mathbf{F}_{1,1} & \cdots & \mathbf{F}_{1,L-1} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{F}_{0,L-1} & \mathbf{F}_{1,L-1} & \cdots & \mathbf{F}_{L-1,L-1} \end{bmatrix} \tag{18}$$

- For $l = \tilde{l}$ we have $\mathbf{F}_{l,l}$ as

$$\mathbf{F}_{l,l} = \frac{2|\alpha_l|^2}{\sigma^2} \sum_{k,p,q} \mathbf{A}_{k,p,q}. \tag{19}$$

- For $l \neq \tilde{l}$, we have

$$\mathbf{F}_{l,\tilde{l}} = \sum_{k,p,q} \Re\{a_{k,p,q,l} a^*_{k,p,q,\tilde{l}}\} \mathbf{A}_{k,p,q}. \tag{20}$$

- With $a_{k,p,q,l} = \alpha_l e^{j2\pi n_p T \nu_l} e^{-j2\pi m_q F \tau_l} e^{-jk\omega_l}$

Channel Estimation Error-Asymptotic Analysis

- For large B, S and K, the multipath components will be asymptotically orthogonal:

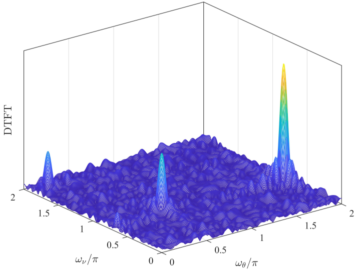
$$\mathbf{R}_{\boldsymbol{\theta}_0} \sim \mathbf{F}_{0,0}^{-1} \quad (21)$$

- The CRLB of τ_0 and ν_0 is thus simplified as

$$\mathbf{R}_{\tau_0,\nu_0} = \frac{\sigma^2}{2|\alpha_0|^2}\frac{3}{\pi^2}\frac{1}{KPQ}\begin{bmatrix} 1/B^2 & 0 \\ 0 & 1/S^2 \end{bmatrix} \quad (22)$$

- The CRLB of ω_0 is

$$R_{\omega_0} = \frac{\sigma^2}{2|\alpha_0|^2}\frac{12}{K^3PQ} = \frac{\sigma^2}{|\alpha_0|^2}\frac{6}{K^3PQ} \quad (23)$$



Channel Estimation Error-Remarks

- BS gives the product of bandwidth and frame length, while $PQ = PF \times QT$ gives the product of bandwidth and time assigned for channel estimation/localization

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- BS gives the product of bandwidth and frame length, while $PQ = PF \times QT$ gives the product of bandwidth and time assigned for channel estimation/localization
- The AoA estimation error is inversely proportional to the amount of time-frequency resources assigned for channel estimation/localization
- The CRLBs of propagation delay and Doppler shift, i.e., R_{τ_0} and R_{ν_0}, are dependent on the total bandwidth and frame length, respectively. The product of their CRLBs is inversely proportional to the product of bandwidth and frame length:

$$R_{\tau_0} R_{\nu_0} \propto \frac{3}{\pi^2 KPQ(BS)^2}. \tag{24}$$

Channel Estimation Error-Spatial Wideband Effect

- The FIM considering the spatial wideband effect is

$$\mathbf{F}_{\boldsymbol{\theta}}^{(SW)} \sim \frac{2|\alpha_0|^2}{\sigma^2} KQP \begin{bmatrix} 4\pi^2 B^2/3 & -\pi^2 BS & -(\frac{1}{2}+\frac{r}{3})\pi KB & -\pi B \\ -\pi^2 BS & \pi^2 S^2 & (1+r/2)\pi KS/2 & \pi S \\ -(\frac{1}{2}+\frac{r}{3})\pi KB & (\frac{1}{2}+\frac{r}{4})\pi KS & (1+r^2/3+r)K^2/3 & (\frac{1}{2}+\frac{r}{2})K \\ -\pi B & \pi S & (1+r)K/2 & 1 \end{bmatrix}$$

- Here $r = B/f_c$ is the ratio of bandwidth to carrier frequency
- When the spatial wideband effect is ignored, we have $r = 0$
- $\mathbf{F}_{\boldsymbol{\theta}}^{(SW)} \succeq \mathbf{F}_{\boldsymbol{\theta}}$

Position Information in Doppler Shift Measurements

Modelling

- Target located at $\mathbf{x}$, broadcasts at f_0 Hz
- The AUV is located at $\mathbf{x}_a$
- AUV moves at $\mathbf{v} = (v_x, v_y)$
- The radial velocity is

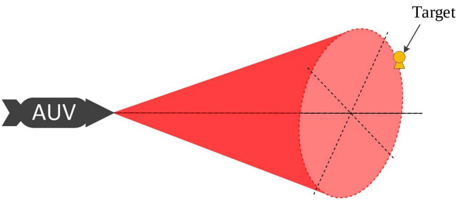
$$v_r = \frac{(\mathbf{x}_a - \mathbf{x})^T \mathbf{v}}{\|\mathbf{x}_a - \mathbf{x}\|}. \qquad (25)$$

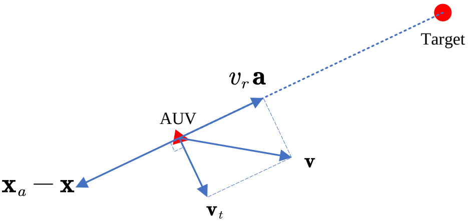
- The tangential velocity

$$\mathbf{v}_t = \mathbf{v} - v_r \mathbf{a} = (\mathbf{I} - \mathbf{a}\mathbf{a}^T)\mathbf{v}, \qquad (26)$$

- The Doppler shift is measured as

$$f_D = -f_0 \frac{v_r}{c}. \qquad (27)$$





Positioning Error

- Suppose $\mathbf{v}$ is the 3D velocity of the mobile device, and we have the following results:

$$\begin{aligned}\tau_0 =& \|\mathbf{x} - \mathbf{x}_{BS}\|/c, \\ \nu_0 =& \frac{(\mathbf{x} - \mathbf{x}_{BS})^T \mathbf{v}}{\|\mathbf{x} - \mathbf{x}_{BS}\| c} f_c, \\ \omega_0 =& \pi \cos \theta_0 = -\pi \frac{(\mathbf{x} - \mathbf{x}_{BS})^T \mathbf{a}}{\|\mathbf{x} - \mathbf{x}_{BS}\|},\end{aligned} \tag{28}$$

- $\mathbf{a}$ is a unit vector aligned with the BS antenna array
- the BS is located at the origin and the FIM concerning $\mathbf{x}$ is

$$\mathbf{F}_{\mathbf{x}} = \nabla_{\mathbf{x}}^T \psi \mathbf{F}_{\psi} \nabla_{\mathbf{x}} \psi, \tag{29}$$

with $\mathbf{F}_{\psi}$ denoting the FIM concerning ψ. The Jacobian is given as

$$\nabla_{\mathbf{x}} \psi = \begin{bmatrix} \boldsymbol{\rho}_{\mathbf{x}}^T / c \\ \mathbf{v}^T (\mathbf{I} - \boldsymbol{\rho}_{\mathbf{x}} \boldsymbol{\rho}_{\mathbf{x}}^T) f_c / d / c \\ \pi \mathbf{a}^T (\mathbf{I} - \boldsymbol{\rho}_{\mathbf{x}} \boldsymbol{\rho}_{\mathbf{x}}^T) / d \end{bmatrix}. \tag{30}$$

with $d = \|\mathbf{x} - \mathbf{x}_{BS}\| = \|\mathbf{x}\|$, and $\boldsymbol{\rho}_{\mathbf{x}} = \mathbf{x}/d$

Positioning Error

- The CRLB of positioning error can be written as

$$\sigma_{\mathbf{x}}^2 = \frac{\sigma^2}{2|\alpha_0|^2}\frac{3}{KPQ\pi^2}\left(\frac{c^2}{B^2} + \frac{d^2c^2}{f_c^2S^2v_t^2}\frac{1}{(1-c_{v,a}^2)} + \frac{4d^2}{K^2}\frac{1}{a_t^2(1-c_{v,a}^2)}\right) \tag{31}$$

  - $\mathbf{v}_t = \mathbf{v}^T(\mathbf{I} - \boldsymbol{\rho}_{\mathbf{x}}\boldsymbol{\rho}_{\mathbf{x}}^T), \quad \mathbf{a}_t = \mathbf{a}^T(\mathbf{I} - \boldsymbol{\rho}_{\mathbf{x}}\boldsymbol{\rho}_{\mathbf{x}}^T)$
  - $c_{v,a}$ is defined as

$$c_{v,a} = \frac{\mathbf{v}_t^T\mathbf{a}_t}{\|\mathbf{v}_t\|\|\mathbf{a}_t\|} \tag{32}$$

  - As long as $\mathbf{v}_t$ and $\mathbf{a}_t$ are not aligned, i.e., $c_{v,a} \neq \pm 1$, we will be able to achieve 3D positioning with a ULA.
  - Here $Sv_t/(c/f_c/2)$ is the equivalent length of the synthesized array through the Doppler effect

Algorithm Design

System Description

- The sampled signal is

$$\mathbf{s}[n] = A\sin\left(2\pi(f_c + f_D)/f_s n + \theta\right) + \mathbf{n}_s[n]$$

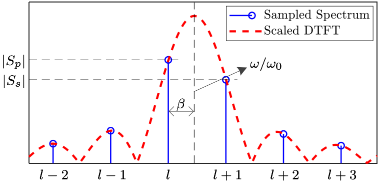
- Let $\omega = 2\pi(f_c + f_D)/f_s$, we have

$$\mathbf{s}[n] = A\sin\left(\omega n + \theta\right) + \mathbf{n}_s[n]$$

- The discrete spectrum of s is

$$\mathbf{s}_\omega[k] = \frac{1}{N}\sum_{n=0}^{N-1} \mathbf{s}[n]e^{-jkn\omega_0} \;\; (\omega_0 = 2\pi/N)$$

- There must exist $l \in \{0, 1, \cdots, N-1\}$ and $\beta \in [0, 1)$ that satisfy $\omega = (l+\beta)\omega_0$



For $\beta \in (0, 0.5)$

- $|S_p| = |\mathbf{s}_\omega[l]| \approx \frac{A\sin(\beta\pi)}{2\beta\pi}$
- $|S_s| = |\mathbf{s}_\omega[l+1]| \approx \frac{A\sin(\beta\pi)}{2(1-\beta)\pi}$
- β can be estimated as:

$$\hat{\beta} = \frac{|S_s|}{|S_s| + |S_p|}$$

WLS Localization

- Suppose the parameters are estimated as $\hat{\tau}_0$, $\hat{\nu}_0$ and $\hat{\omega}_0$, the non-linear weighted LS estimate of the mobile device's position is obtained through

$$\min_{\hat{\mathbf{x}}} \frac{(\hat{\tau}_0 - \tau_0(\hat{\mathbf{x}}))^2}{\sigma_\tau^2} + \frac{(\hat{\nu}_0 - \nu_0(\hat{\mathbf{x}}))^2}{\sigma_\nu^2} + \frac{(\hat{\omega}_0 - \omega_0(\hat{\mathbf{x}}))^2}{\sigma_\omega^2}, \quad (33)$$

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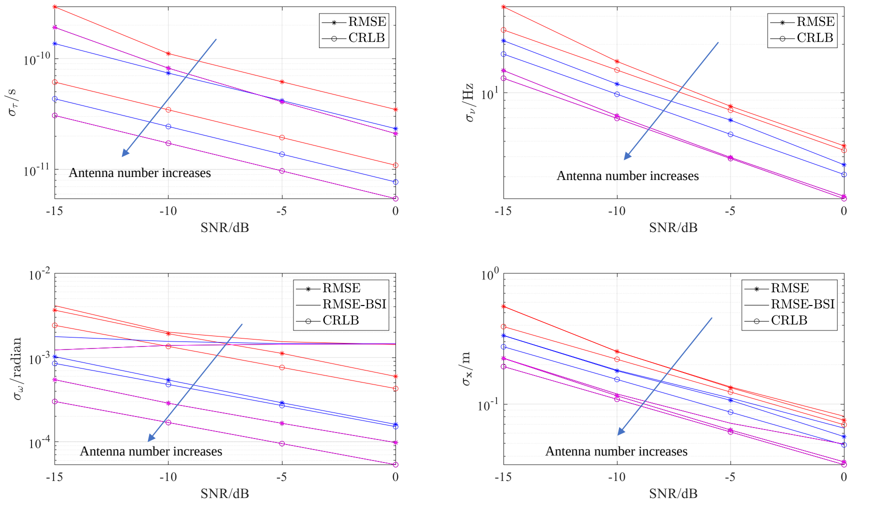
- The estimation errors are asymptotically uncorrelated
- The variances can be roughly obtained through CRLB analysis

Simulation Setup

- Consider a bandwidth of 30 GHz, a mobile device at 100 m/s, corresponding to a Doppler spread of 200 kHz
- Delay spread is 0.5μs, or 150 meters
- Consider $T = 5\mu$s and $F = 200$ kHz
- We can afford $MN = 1 \times 10^5$
- $P/N = Q/M = 0.1$

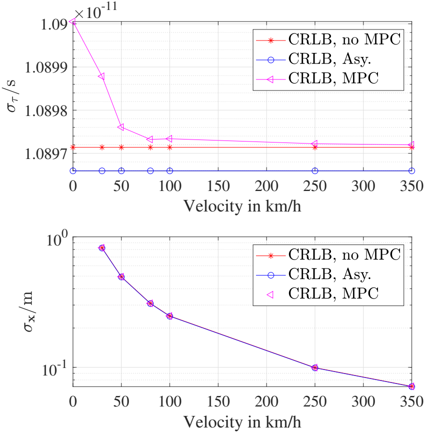
Simulations - Multi-path Effect

- Bandwidth is 200 MHz, frame length is 0.5 ms; No. of antennas at the BS is $\{32, 64, 128\}$



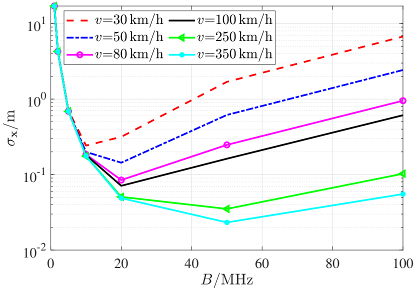
Doppler Effect

- $BS = 1 \times 10^5$
- Doppler effect helps to improve positioning accuracy



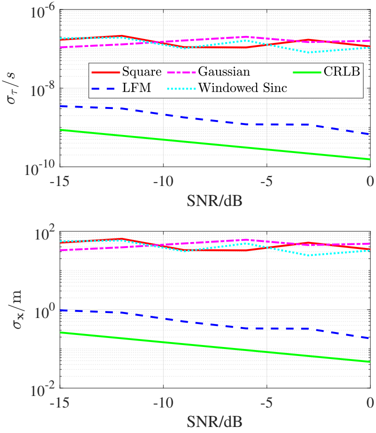
Frame Length-Bandwidth Tradeoff

- $BS = 1 \times 10^5$
- Optimal bandwidth-frame length is a function of mobile devices' speeds
- The optimal bandwidth is always around 20 MHz



Pulse Design

- For small IoT devices, large bandwidth would be impractical
- Consider a bandwidth of 2 MHz, and an array size of 32
- Can we still suppress the multipath interference through proper pulse design?



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- Multipath effect is not a big issue for large bandwidth/frame length/antenna array
- The LFM waveform can further suppress multipath interference

Future Work

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- Timing frequency synchronization
- OTFS multiple access (similar to OFDMA)
- Single-carrier time-frequency domain equalization (similar to SC-FED)

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- SLAC/ISAC for autonomous driving
  - GPS is too slow (50 Hz for high-end chips; 30 seconds for satellite orbital information update)
  - GPS coverage is poor in big cities due to the tall buildings, Viaduct, etc.
  - SLAC can refresh position information at 1000 Hz with much improved coverage

[1] J. B. Kenny, "Dedicated short-range communications (DSRC) standards in the United States," Proceedings of the IEEE, vol. 99, no. 7, pp.1162-1182, 2011.