A Model of Language-Guided Concept Formation using a Common Framework for Unsupervised and Supervised Learning

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Abstract

A general learning rule, "BCM-δ", is proposed that subsumes both unsupervised learning as a form of the BCM rule (Bienenstock, Cooper, Munro, 1982; Munro, 1984) and the delta rule (Rosenblatt, 1958; Rumelhart, Hinton, and Williams, 1986). The "BCM-δ" unit is composed of two subunits, *T* and *L*, each integrating distinct input streams across distinct sets of synapses. The two subunits follow a common Hebb-like learning procedure that reduces to an unsupervised rule for the *T* subunit and a supervised rule for the *L* subunit in which the *T* response is the training signal. This model suggests a neurally plausible mechanism for the shaping of concepts by labels.

Keywords: concept learning, neural model, connectionism

Background

Supervised Learning using the Delta Rule

Error driven synaptic learning rules are typically written in a "Hebb-like" form with a postsynaptic factor that has a positive (target) term and a negative (response) term. The *delta rule* (eg., Rosenblatt, 1960; Widrow and Hoff 1960; Rumelhart, Hinton, and Williams, 1986) has this property (see Eq. 1). Here, w_{ij} is the weight of the synapse connecting unit *j* to unit *i*, and the postsynaptic factor δ_i is expressed as the difference between the desired response d_i and the actual response r_i. The step size or learning rate is signified by η.

$$\Delta w_{ij} = \eta \delta_i s_j \text{, where } \delta_i \equiv d_i - r_i \quad (1)$$

This rule is very well established as a procedure for training linear units or linear threshold units on labeled data, and can be extended to nonlinear units and to multilayered networks with modifications. Of primary interest here is the expression of the postsynaptic factor as a *difference* between two terms (d_i and r_i). This form of learning relies on the availability of the "correct" response to a set of training data; hence it is a form of *supervised learning*.

Unsupervised Learning using the BCM Rule

Bienenstock, Cooper, and Munro (1982) developed a synaptic modification rule to describe the development of ocular dominance and orientation selective cells in visual cortex. Like the delta rule, the BCM rule has a Hebb-like form (Equation 2.).

$$\begin{aligned} \Delta w_{ij} &= \eta_w \varphi_i s_j \text{, where } \varphi_i \equiv r_i\left(r_i - q_i\right) \\ \Delta q_i &= \eta_q\left(r_i^2 - q_i\right) \end{aligned} \quad (2)$$

A unit that samples input patterns from a set of patterns *S* using the BCM rule will develop a response profile that is highly selective with respect to the pattern set; specifically, the unit gives a near-zero response to most of the patterns in *S*, and responds significantly larger than zero to just a localized subset of *S* (Figure 1).

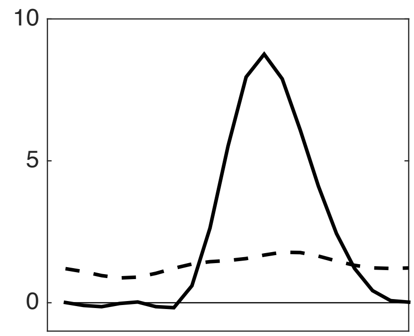

Figure 1: Selectivity maximization by BCM. Response to a set of 20 patterns in a 10 dimensional space. The initial response profile (dashed line) is roughly uniform. After learning, the response is selective.

Stability of the BCM rule critically depends on the form of the postsynaptic function $\varphi(r,q)$. Using the definition in Equation 2, φ is zero where r=0 and where r=q. As shown in Figure 2, φ <0 for 0<r<q, and φ >0 for r>q. The dynamic variable q tracks r^2, and thus changes over time until $\mathbf{w}(t)$ achieves equilibrium. Note that the dynamical system has distinct learning rates η_w and η_q.

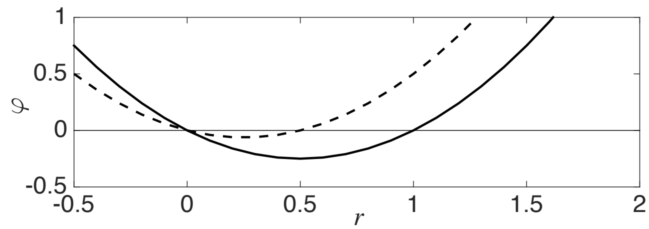

Figure 2: The postsynaptic function $\varphi(r)$. The function φ is plotted for q=0.5 (dashed line) and for q=1 (solid line).

Unlike the delta rule, the BCM rule is unsupervised; like many other unsupervised learning rules (eg. Zipser, 1986), it has exhibits a tendency to selectively respond to clusters in a nonuniform data set.

A Unified Framework

Reformulation of the BCM Postsynaptic Function

A different postsynaptic function $\psi(r,q)$ is proposed here (Equations 3 and 4). Simulations indicate that it brings the equilibrium states have the same stability properties as the function φ described in Equation 2 with very similar response profiles with respect to selectivity.

$$\sigma(r,h) \equiv \frac{r}{r+h} \tag{3}$$

$$\begin{aligned} \Delta w_{ij} &= \eta_w \psi_i s_j \\ \Delta q &= \eta_q (r_i - q_i) \end{aligned} \tag{4}$$

$$\text{where } \psi(r,q) \equiv \sigma(r,h_1) - q\sigma(r,h_2)$$

This version of BCM expresses the postsynaptic function as the difference between two bounded functions of r. It parallels the delta rule such that both rules can be formulated as special cases of a common general rule (next section). As in the original form of the rule, the homeostatic variable q depends on the history of r.

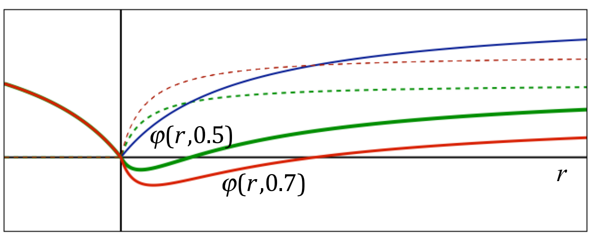

Figure 3: The postsynaptic function $\psi(r,q)$. The function is plotted using q=0.5 (green) and q=0.7 (red). The positive term is blue and the negative term for each function is plotted as a dashed line.

Oppositional Mechanisms

Both the unsupervised rule in Equation 4 and the delta rule are specific cases of a more general framework for synaptic learning (Equation 5), in which the postsynaptic factor is the difference between two terms.

$$\Delta w_{ij} = \eta\left(P_i - N_i\right)s_j \tag{5}$$

The function suggests that there are *separate oppositional associative mechanisms* for strengthening synapses and weakening synapses, resulting in LTP when $P_i > N_i$ and LTD otherwise. In the case of the delta rule, P_i is the training signal and N_i is the response, while in Equation 4, both terms are functions of the response r_i.

Subunits

Consider a hypothetical neuron with multiple loci for accumulating PSPs from different parts of the dendritic complex. Here, two subunits labeled T and L are stimulated by different sets of stimuli $\mathbf{s}^T$ and $\mathbf{s}^L$ which are incident on the cell on separate sets of afferents, with corresponding synaptic efficacy vectors $\mathbf{w}^T$ and $\mathbf{w}^L$.

The two subunits compute separated weighted sums r^T and r^L (Equation 6). The pyramidal cell type is a prime candidate for this kind of unit (Figure 4).

$$r_i^{(X)} = \sum_{j \in X} w_{ij}^{(X)} s_j^{(X)} \quad \text{where } X \in \{T, L\} \tag{6}$$

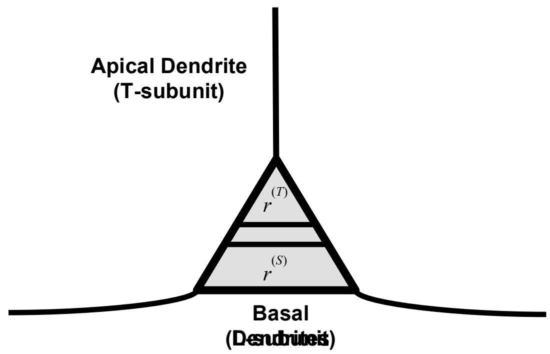

Figure 4: The pyramidal cell morphology as a model for the self-supervised framework. The two hypothetical partial responses could be integrated on mutually exclusive sets of afferents, such as those the apical dendrites and the basal dendrites.

One Learning Rule for Two Subunits

In this section, a *self-supervised* learning rule is presented. The T subunit synapses are trained according to a version of BCM similar to equation (4). The partial response from the T unit drives the P term for the L subunit learning procedure; thus, the L follows a form of the delta rule with r^T determining the training signal for L. The self-supervised procedure operates by selecting a specific region of the T stimulus space. Subsequently, the L subunit is trained to give a partial response that can be interpreted as predictive of the preferred T stimulus given the L stimulus.

Both sets of weights have the same P term driven by the T subunit and have N terms that are functions of their respective partial responses (Equation 7).

$$\begin{aligned} \Delta w_{ij}^{(X)} &= \eta_w \left(\sigma(r_i^{(T)}, h_1) - q_i^{(T)} \sigma(r_i^{(X)}, h_2)\right) s_j^{(X)} \\ \Delta q_i^{(T)} &= \eta_q (r_i^{(T)} - q_i^{(T)}) \end{aligned} \tag{7}$$

This dynamical system suggests that a single synaptic modification based on oppositional mechanisms can subsume both unsupervised selectivity across a set of stimuli which can in turn drive a supervised learning procedure.

Simulations

The simulations explore a situation where the *T* inputs are linearly independent and well separated and the stimulus space for the *L* subunit consists of 500 patterns in 200 dimensions clustered about 10 "prototype" vectors. This is meant to simulate word-like inputs to the *T* subunit, which act like labels for patterns in the *L* stimulus space, which is presumably has less structure than the word-like *T* space.

Two scenarios are simulated: without *T* inputs and with *T* inputs. The *L* stimulus space is meant to simulate some primitive sensory space like visual space. The scenarios are therefore meant to simulate concept learning without language compared to concept learning with language.

Pattern Sets

The five patterns to the *T* subunit are nonorthogonal but linearly independent -- see Table 1.

Table 1: The *T* pattern set

A	B	C	D	E
1	0	0	0	1
1	1	0	0	0
0	1	1	0	0
0	0	1	1	0
0	0	0	1	1

Input patterns to the *L* subunit are drawn from a set of 500 patterns in 10 clusters of 50 patterns (Figure 5). The prototype vectors are randomly generated, with small random numbers added to each component.

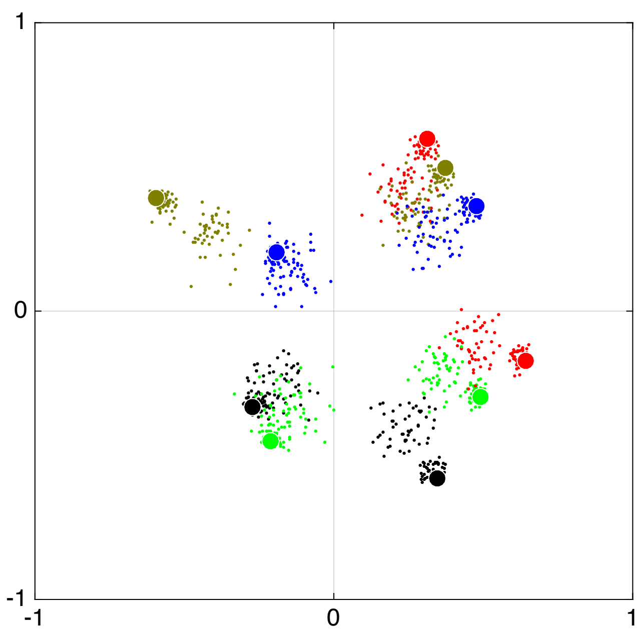

Figure 5. A PCA plot of the 10 clusters. The large spots are the prototype vectors. The color coding refers to the pairing with the five patterns from the *T* stimulus space.

Scenario 1. *L* inputs alone.

In the first set of simulations the *L* subunit is trained without input to the *T* subunit. In every case the subunit selects one of the 10 clusters. Within cluster responses are fairly tight, and responses to the selected cluster are well separated front the other clusters.

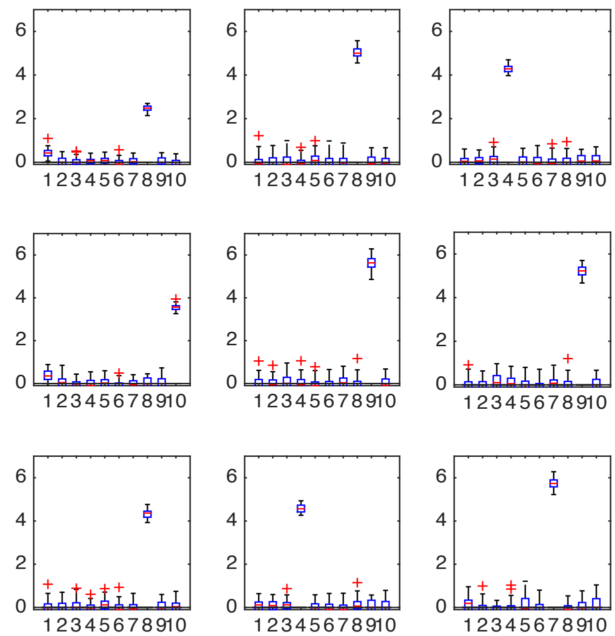

Figure 6. *L* subunit response profiles of 9 separately trained units to 200 units in 10 clusters without input to the *T* subunit.

Scenario 2. Joint input to both *T* and *L*

Here, inputs are presented to the *T* pattern set. The learning rule drives the *T*-subunit to choose one of the five patterns. Patterns in the *T*-environment is presented with an exemplar from pairs of clusters from the *L*-environment according to Table 2. The shading indicates which patterns from the *T* environment are presented simultaneously with exemplars from which clusters. The color coding corresponds to the PCA projections plotted in Figure 5. These are the same clusters used in Scenario 1. Note that the cluster prototypes were randomly generated and so the ordering of the clusters is not at all related to their similarity. The pairing in Table 2 is essentially arbitrary. It is done pairwise in sequence in order to facilitate the results displayed in Figure 7.

Table 2: Joint distribution table.

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}
A										
B										
C										
D										
E										

The *T*-subunit develops highly specific response properties over the 5 patterns A-E, by converging to a weight state that is nonresponsive to 4 of the 5. This is typical of the BCM rule and its variants. The green bar graphs in Figure 7 indicate the selected pattern.

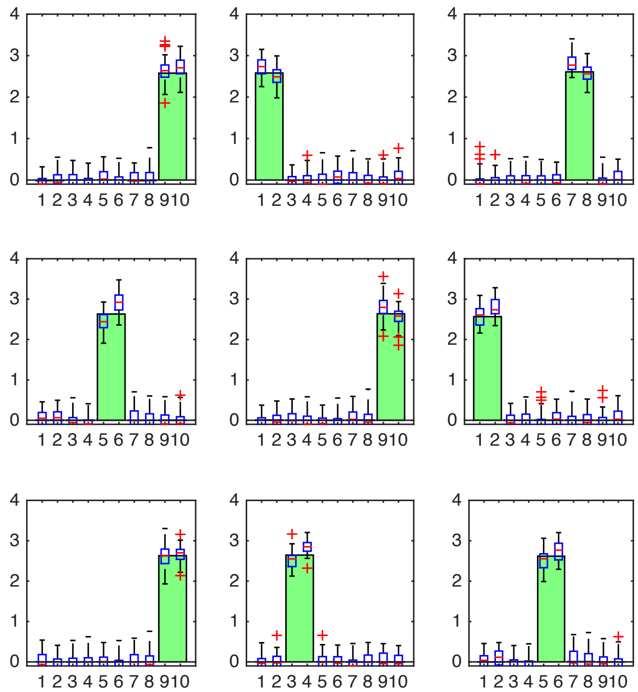

Figure 7. The boxplots show *L* subunit response profiles of 9 separately trained units to 200 units in 10 clusters. The bar graph (with only one non-zero bar) in green shows the response profile of the *T* subunit.

The selected pattern from the *T*-subunit acts as a training signal to the *L*-subunit, and is presented jointly with two clusters which are not generally closer to each other in the *L* stimulus space than any pair of arbitrary clusters. Thus the *T* patterns act as "labels" that help define the response properties of the "concept regions" in the *L* pattern space.

Discussion

An important insight from these rules is that the mechanisms for potentiating and depressing synaptic efficacies act simultaneously over the entire range of postsynaptic firing, resulting in a net LTP effect when the former outweighs the latter, and resulting in LTD in the opposite case.

This paper has demonstrated a system by which stimuli from one modality can shape the response properties of a unit to another modality using a framework that is biologically plausible and gives clues to the source of a teaching signal for supervised learning.

Furthermore, this model invites the interpretation that language can be viewed as an environment of patterned stimuli that compete for recognition by neural entities (neurons or groups of neurons) against stimuli from other modalities, and may fare ell in this competition due to the relatively discrete nature of linguistic stimuli.

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