Derivation of the Normal Vector to a 2D Electric Potential Surface

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March 20, 2020

The electric potential field is three dimensional and scalar. A two dimensional electric potential field, concerned only with charges in the same XY-plane, can be visualised by a surface, **S**. The Z-dimension of the surface is equal to the electric potential at the corresponding XY-coordinate.

The surface can be parameterised by a bivariate function $z = f(x, y)$. This function, $f(x, y)$, must calculate the electric potential at the point (x, y) given the locations and charge of all known charges.

The electric potential at position (x, y) due to charge i is given by

$$V = -k\frac{q_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}$$

where the charge has location (x_i, y_i) and charge q_i.

Due to the principle of super position, the total electric potential can be calculated by summing over all charges.

$$f(x, y) = -k\sum_i \frac{q_i}{\sqrt{(x - x_i)^2 + (y - y_i)^2}}$$

This describes **S**, allowing it to be easily plotted using computer software by iterating over a grid of (x, y) locations and generating the height of the surface using this formula. The surface can then be triangulated and rendered. This can be seen in Figure .

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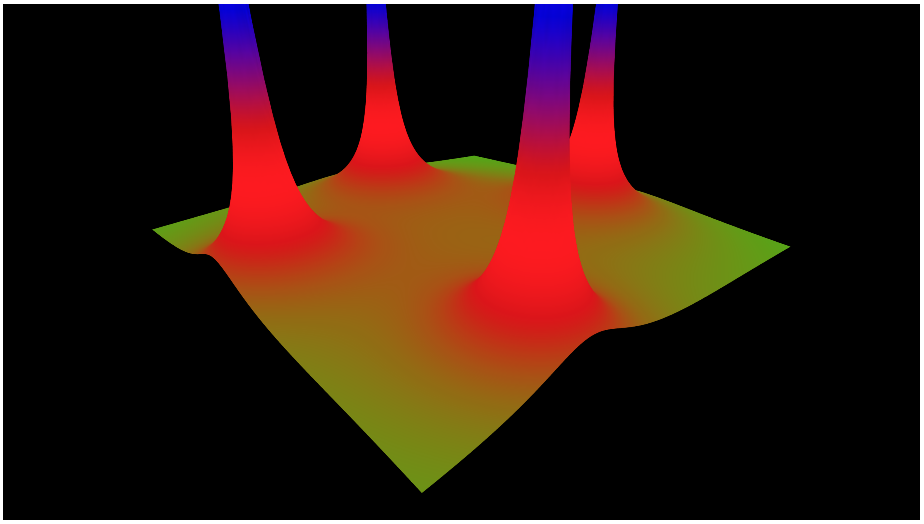

Figure 1: The electric potential visualisation without shading.

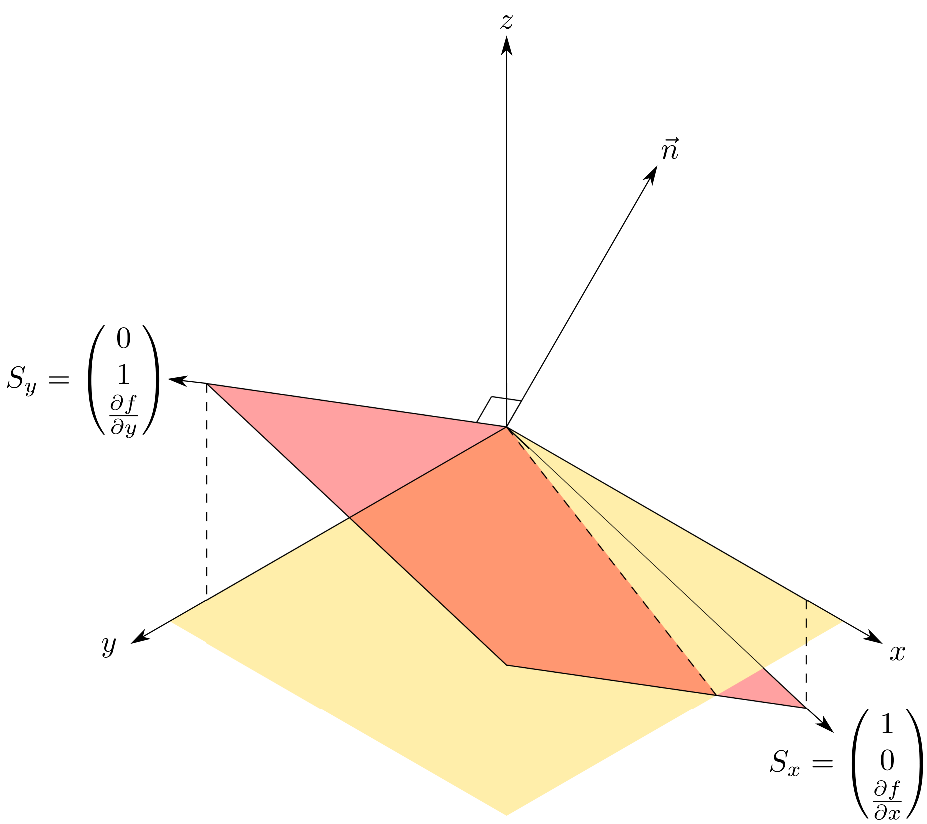

Figure 2: The electric potential visualisation with shading.

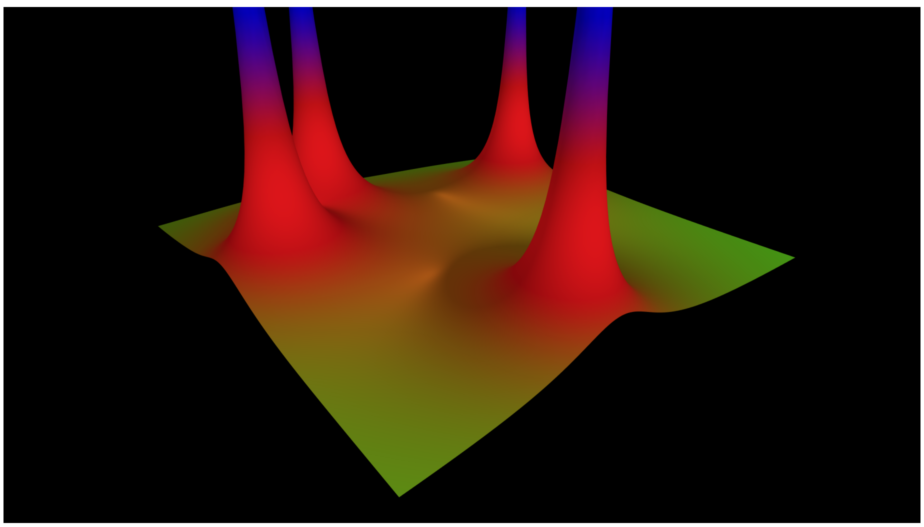

Figure 3: The electric potential visualisation with shading.