$$\int \operatorname{erf}(z)\,dz = z\operatorname{erf}(z) + \frac{e^{-z^2}}{\sqrt{\pi}}$$

PROBLEM 4 → Convolution of Gaussian with slit

$$\underset{g}{\mathcal{L}} \circledast \underset{\Pi}{\mathcal{L}} = \int_{t-\frac{a}{2}}^{t+\frac{a}{2}} g(u)\,du$$

$$\Downarrow$$

$$f(t) = \int_{t-\frac{a}{2}}^{t+\frac{a}{2}} g(u)\,du = A_0 \int_{t-\frac{a}{2}}^{t+\frac{a}{2}} e^{-\frac{u^2}{2\sigma^2}}\,du \qquad s = \frac{u}{\sqrt{2}\sigma}$$

$$f(t) = A_0\sqrt{2}\sigma \int_{\sqrt{2}\sigma(t-\frac{a}{2})}^{\sqrt{2}\sigma(t+\frac{a}{2})} e^{-s^2}\,ds$$

$$f(t) = \frac{A_0\sqrt{2\pi}\sigma}{2}\left[\operatorname{erf}\left(\sqrt{2}\sigma\left(t+\tfrac{a}{2}\right)\right) - \operatorname{erf}\left(\sqrt{2}\sigma\left(t-\tfrac{a}{2}\right)\right)\right]$$

↓ Integrating over $t \in (-\infty, \infty)$ ⇓ through some algebra

$$\int_{-\infty}^{\infty} f(t)\,dt = \frac{A_0\sqrt{2\pi}\sigma}{2}\left(\sqrt{2}\sigma a\left(\operatorname{erf}\left(\sqrt{2}\sigma\frac{a}{2}\right) + 1\right)\right)$$

$$\| \qquad\qquad \| \qquad\qquad \| \qquad\qquad \| \qquad\qquad \|$$

$$A_f\sqrt{2\pi}\sigma = A_0\sqrt{\pi}\sqrt{2}\sigma^2 a\left(\operatorname{erf}\left(\frac{\sqrt{2}\sigma a}{2}\right) + 1\right)$$

$$A_f\sqrt{2}\sigma = A_0\sigma^2 a\left(\operatorname{erf}\left(\frac{\sqrt{2}\sigma a}{2}\right) + 1\right)$$

$$\frac{A_f \sqrt{2\pi}}{2A_0 \sigma} \frac{1}{a} = \operatorname{erf}\left(\frac{\sqrt{2}\sigma a}{2}\right) + 1$$

⇓

Equation that can be solved numerically or graphically:

$A_f = 0.29713\ldots$

$A_0 = 10.1000\ldots$

$\sigma = 2.7187654\ldots$

⇓ Solved graphically (see slitsolve.png)

$$a = 0.0132 \frac{mm}{\pi}$$

⇓

$$a = 0.0075 \text{ mm}$$ ← The slit width

$$\sigma = 2.7188 \text{ mm}$$ ← RMS size

↳ See python file with fit.