Objetivos a cubrir **Código : MAT4-CDI.7**

- Polinomio de Taylor. Fórmula de Taylor. Cálculo de error.
- Series de Taylor. Series de Maclaurin.

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1. Encuentre el polinomio de Taylor de la función dada en el punto a indicado

1. $f(x) = x^3 - 1;\quad a = 1,\quad n = 3$	2. $f(x) = \operatorname{sen} x;\quad a = \frac{\pi}{6},\quad n = 3$
3. $f(x) = \cos x;\quad a = \frac{2\pi}{3},\quad n = 4$	4. $f(x) = \tan x;\quad a = \frac{\pi}{4},\quad n = 4$
5. $f(x) = e^{-x};\quad a = 1,\quad n = 4$	6. $f(x) = \cos x;\quad a = \frac{\pi}{4},\quad n = 3$
7. $f(x) = \sqrt{x};\quad a = 9,\quad n = 3$	8. $f(x) = \sqrt{x};\quad a = 100,\quad n = 3$
9. $f(x) = \frac{1}{(x-4)^2};\quad a = 5,\quad n = 5$	10. $f(x) = \tan x;\quad a = \frac{\pi}{4},\quad n = 5$
11. $f(x) = \cos x;\quad a = \pi,\quad n = 4$	12. $f(x) = \operatorname{sen} x;\quad a = \frac{\pi}{2},\quad n = 4$
13. $f(x) = x^{3/2};\quad a = 1,\quad n = 4$	14. $f(x) = \frac{1}{\sqrt{1-x}};\quad a = 0,\quad n = 4$
15. $f(x) = e^x \operatorname{sen} x;\quad a = 0,\quad n = 3$	16. $f(x) = e^x \cos 2x;\quad a = 0,\quad n = 3$
17. $f(x) = \frac{1}{\sqrt[3]{x}};\quad a = 8,\quad n = 3$	18. $f(x) = \ln \operatorname{sen} x;\quad a = \frac{\pi}{2},\quad n = 6$
19. $f(x) = \sec x;\quad a = \frac{\pi}{3},\quad n = 3$	20. $f(x) = \tan x;\quad a = 0,\quad n = 5$
21. $f(x) = \operatorname{sen}^2 x;\quad a = \pi,\quad n = 6$	

2. Encuentre la fórmula de Taylor para la función f dada en $a = 0$. Encuentre el polinomio de Taylor $P_n(x)$ del grado indicado n y el término del residuo $R_n(x)$

1. $f(x) = e^{-x};\quad n = 5$	2. $f(x) = \operatorname{sen} x;\quad n = 4$	3. $f(x) = \cos x;\quad n = 4$
4. $f(x) = \frac{1}{1-x};\quad n = 4$	5. $f(x) = \sqrt{1+x};\quad n = 3$	6. $f(x) = \ln(1+x);\quad n = 4$
7. $f(x) = \tan x;\quad n = 3$	8. $f(x) = \tan^{-1} x;\quad n = 2$	9. $f(x) = \operatorname{sen}^{-1} x;\quad n = 2$
10. $f(x) = x^3 - 3x^2 + 5x - 7;\quad n = 4$		

3. Determinar la serie de potencias en x para las funciones dadas

1. $f(x) = e^{-x}$	2. $f(x) = xe^{x^2}$	3. $f(x) = e^x + e^{-x}$	4. $f(x) = e^{2x} - 1 - 2x$

4. Escribiendo
$$\frac{1}{x} = \frac{1}{1-(1-x)}$$
y utilizando el desarrollo conocido de $\frac{1}{1-x}$, determine la serie de Taylor de $\frac{1}{x}$ en potencias de $x-1$.

5. Encuentre la serie de Taylor de la función dada en el punto a indicado

1. $f(x) = e^x; \quad a = 1$ 2. $f(x) = e^{-x}; \quad a = -1$ 3. $f(x) = e^{2x}; \quad a = 0$

4. $f(x) = \dfrac{1}{x}; \quad a = 1$ 5. $f(x) = \dfrac{1}{x}; \quad a = -3$ 6. $f(x) = \dfrac{1}{1-x}; \quad a = 0$

7. $f(x) = \dfrac{1}{2x+3}; \quad a = 1$ 8. $f(x) = \dfrac{x+1}{x-1}; \quad a = -1$ 9. $f(x) = \dfrac{x+1}{x+2}; \quad a = 1$

10. $f(x) = \dfrac{1}{(1-x)^2}; \quad a = 0$ 11. $f(x) = \ln x; \quad a = 1$ 12. $f(x) = \ln(1+x); \quad a = 0$

13. $f(x) = \ln(3x-2); \quad a = 2$ 14. $f(x) = \ln(2x-1); \quad a = 1$ 15. $y = \operatorname{sen} x; \quad a = \dfrac{\pi}{2}$

16. $f(x) = \cos x; \quad a = \dfrac{\pi}{4}$ 17. $f(x) = \operatorname{sen} x; \quad a = \dfrac{\pi}{4}$ 18. $f(x) = \cos x; \quad a = \dfrac{\pi}{2}$

19. $f(x) = \operatorname{sen} x; \quad a = \dfrac{\pi}{6}$ 20. $f(x) = \cos x; \quad a = \dfrac{\pi}{3}$ 21. $f(x) = \tan x; \quad a = \dfrac{\pi}{4}$

22. $f(x) = \sqrt{1+x}; \quad a = 1$

6. Sea $f(x) = (1+x)^{1/2} + (1-x)^{1/2}$. Determine la serie de Maclaurin para f y utilicela para encontrar $f^{(4)}(0)$ y $f^{(51)}(0)$

7. Desarrollar las siguientes funciones en serie de Maclaurin e indicar los campos de convergencia de las series obtenidas

1. $f(x) = e^{-x}$ 2. $f(x) = e^{2x}$ 3. $f(x) = e^{-3x}$ 4. $f(x) = a^x; \ 0 < a \neq 1$

5. $f(x) = e^{x^3}$ 6. $f(x) = \operatorname{senh} x$ 7. $f(x) = \cosh x$ 8. $f(x) = \dfrac{1}{2+3x}$

9. $f(x) = \dfrac{x}{x-1}$ 10. $f(x) = \sqrt[3]{1+x^3}$ 11. $f(x) = \dfrac{x}{\sqrt[3]{8+x}}$ 12. $f(x) = \operatorname{sen} 2x$

13. $f(x) = \operatorname{sen}\left(\dfrac{x}{2}\right)$ 14. $f(x) = \operatorname{sen}\left(x^2\right)$ 15. $f(x) = \operatorname{sen}\left(x - \dfrac{\pi}{4}\right)$

16. $f(x) = \cos\left(2x - \dfrac{\pi}{4}\right)$ 17. $f(x) = \arctan x$ 18. $f(x) = \operatorname{sen}^2 x$

19. $f(x) = \ln\left(x + \sqrt{1+x^2}\right)$

8. Encuentre la serie de potencias que representa a la función dada f usando integración por términos

1. $f(x) = \displaystyle\int_0^x \operatorname{sen}\left(t^2\right) dt$ 2. $f(x) = \displaystyle\int_0^x \frac{\operatorname{sen} t}{t} dt$ 3. $f(x) = \displaystyle\int_0^x e^{-t^2} dt$

4. $f(x) = \displaystyle\int_0^x \frac{\arctan t}{t} dt$ 5. $f(x) = \displaystyle\int_0^x \frac{1-e^{-t^2}}{t^2} dt$ 6. $\tanh^{-1} x = \displaystyle\int_0^x \frac{1}{1-t^2} dt$

9. Use series de potencias en lugar de la regla de L'Hospital para evaluar los límites dados

1. $\displaystyle\lim_{x\to 0} \frac{1+x-e^x}{x^2}$ 2. $\displaystyle\lim_{x\to 0} \frac{x - \operatorname{sen} x}{x^3 \cos x}$ 3. $\displaystyle\lim_{x\to 0} \frac{1-\cos x}{x(e^x-1)}$ 4. $\displaystyle\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \arctan x}$

5. $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x} - \frac{1}{\operatorname{sen} x}\right)$ 6. $\displaystyle\lim_{x\to 1} \frac{\ln\left(x^2\right)}{x-1}$

Respuestas

1.1. $P_3(x) = 3(x-1) + 3(x-1)^2 + (x-1)^3$; 1.2. $P_3(x) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(x - \frac{\pi}{6}\right) - \frac{1}{4}\left(x - \frac{\pi}{6}\right)^2 - \frac{\sqrt{3}}{12}\left(x - \frac{2\pi}{3}\right)^3$;

1.3. $P_4(x) = -\frac{1}{2} - \frac{\sqrt{3}}{2}\left(x - \frac{2\pi}{3}\right) + \frac{1}{4}\left(x - \frac{2\pi}{3}\right)^2 + \frac{\sqrt{3}}{12}\left(x - \frac{2\pi}{3}\right)^3 - \frac{1}{48}\left(x - \frac{2\pi}{3}\right)^4$;

1.4. $P_4(x) = 1 + 2\left(x - \frac{\pi}{4}\right) + \frac{4}{2!}\left(x - \frac{\pi}{4}\right)^2 + \frac{16}{3!}\left(x - \frac{\pi}{4}\right)^3 + \frac{80}{4!}\left(x - \frac{\pi}{4}\right)^4$;

1.5. $P_4(x) = e^{-1} - e^{-1}(x-1) + \frac{e^{-1}}{2!}(x-1)^2 - \frac{e^{-1}}{3!}(x-1)^3 + \frac{e^{-1}}{4!}(x-1)^4$;

1.6. $P_3(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{4}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{12}\left(x - \frac{\pi}{4}\right)^3$; 1.7. $P_3(x) = 3 + \frac{1}{6}(x-9) - \frac{1}{108}\frac{(x-9)^2}{2!} + \frac{1}{648}\frac{(x-9)^3}{3!}$;

1.8. $P_3(x) = 10 + \frac{1}{20}(x-100) - \frac{1}{4000}\frac{(x-100)^2}{2!} + \frac{3}{800000}\frac{(x-100)^3}{3!}$;

1.9. $P_5(x) = 1 - 2(x-5) + \frac{6}{2!}(x-5)^2 - \frac{24}{3!}(x-5)^3 + \frac{120}{4!}(x-5)^4 - \frac{720}{5!}(x-5)^5$;

1.10. $P_5(x) = \frac{\sqrt{3}}{3} + \frac{4}{3}\left(x-\frac{\pi}{6}\right) + \frac{8\sqrt{3}}{9}\frac{1}{2!}\left(x-\frac{\pi}{6}\right)^2 + \frac{16}{3}\frac{1}{3!}\left(x-\frac{\pi}{6}\right)^3 + \frac{32\sqrt{3}}{3}\frac{1}{4!}\left(x-\frac{\pi}{6}\right)^4 + \frac{832}{9}\frac{1}{5!}\left(x-\frac{\pi}{6}\right)^5$;

1.11. $P_4(x) = -1 + \frac{1}{2!}(x-\pi)^2 - \frac{1}{3!}(x-\pi)^4$; 1.12. $P_4(x) = 1 - \frac{1}{2!}\left(x-\frac{\pi}{2}\right)^2 + \frac{1}{4!}\left(x-\frac{\pi}{2}\right)^4$;

1.13. $P_4(x) = 1 + \frac{3}{2}(x-1) + \frac{3}{4}\frac{(x-1)^2}{2!} - \frac{3}{8}\frac{(x-1)^3}{3!} + \frac{9}{16}\frac{(x-1)^4}{4!}$; 1.14. $P_4(x) = 1 + \frac{1}{2}x + \frac{3}{4}\frac{x^2}{2!} + \frac{15}{8}\frac{x^3}{3!} + \frac{105}{16}\frac{x^4}{4!}$;

1.15. $P_3(x) = x + 2\frac{x^2}{2!} + 2\frac{x^3}{3!}$; 1.16. $P_3(x) = 1 + x - 3\frac{x^2}{2!} - 11\frac{x^3}{3!}$; 1.17. $P_3(x) = \frac{1}{2} - \frac{(x-8)}{48} + \frac{1}{288}\frac{(x-8)^2}{2!} - \frac{7}{6912}\frac{(x-8)^3}{3!}$;

1.18. $P_6(x) = -\frac{1}{2!}\left(x-\frac{\pi}{2}\right)^2 - \frac{2}{4!}\left(x-\frac{\pi}{2}\right)^4 - \frac{16}{6!}\left(x-\frac{\pi}{2}\right)^6$; 1.19. $P_3(x) = 2 + 2\sqrt{3}\left(x-\frac{\pi}{3}\right) + \frac{14}{2!}\left(x-\frac{\pi}{3}\right)^2 + \frac{46\sqrt{3}}{3!}\left(x-\frac{\pi}{3}\right)^3$;

1.20. $P_5(x) = x + \frac{2}{3!}x^3 + \frac{16}{5!}x^5$; 1.21. $P_6(x) = 2\frac{(x-\pi)^2}{2!} - 8\frac{(x-\pi)^4}{4!} + 32\frac{(x-\pi)^6}{6!}$;

2.1. $f(x) = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} + R_6(x)$; 2.2. $f(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + R_7(x)$; 2.3. $f(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + R_6(x)$;

2.4. $f(x) = 1 + x + x^2 + x^3 + x^4 + R_5(x)$; 2.5. $f(x) = 1 + \frac{1}{2}x - \frac{1}{4}\frac{x^2}{2!} + 45\frac{x^3}{6!} + R_4(x)$;

2.6. $f(x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + R_5(x)$; 2.7. $f(x) = x + \frac{x^3}{3} + R_5(x)$; 2.8. $f(x) = x - \frac{x^3}{3} + R_5(x)$;

2.9. $f(x) = x + \frac{x^3}{3!} + R_5(x)$; 2.10. $f(x) = x^3 - 3x^2 + 5x - 7$; 3.1. $e^{-x} = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$; 3.2. $xe^{x^2} = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{n!}$;

3.3. $e^x + e^{-x} = \sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$; 3.4. $e^{2x} - 1 - 2x = \sum_{n=2}^{\infty}\frac{2^n x^n}{n!}$; 4. $\frac{1}{x} = \sum_{n=0}^{\infty}(-1)^n(x-1)^n$;

5.1. $f(x) = \sum_{n=0}^{\infty}\frac{e}{n!}(x-1)^n$; 5.2. $f(x) = \sum_{n=0}^{\infty}(-1)^n\frac{e}{n!}(x+1)^n$; 5.3. $f(x) = \sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$; 5.4. $f(x) = \sum_{n=0}^{\infty}(-1)^n(x-1)^n$;

5.5. $f(x) = -\sum_{n=0}^{\infty}\frac{(x+3)^n}{3^{n+1}}$; 5.6. $f(x) = \sum_{n=0}^{\infty}x^n$; 5.7. $f(x) = \sum_{n=0}^{\infty}(-1)^n\frac{2^n}{5^{n+1}}(x-1)^n$; 5.8. $f(x) = -\sum_{n=1}^{\infty}\frac{(x+1)^n}{2^n}$;

5.9. $f(x) = \frac{2}{3} + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x+1)^n}{3^{n+1}}$; 5.10. $f(x) = \sum_{n=1}^{\infty}nx^{n-1}$; 5.11. $f(x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{(x-1)^n}{n}$;

5.12. $f(x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n}$; 5.13. $f(x) = \ln 4 + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{3^n}{4^n}\frac{(x-2)^n}{n}$; 5.14. $f(x) = \sum_{n=1}^{\infty}(-1)^{n+1}\frac{2^n}{n}(x-2)^n$;

5.15. $f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}\left(x-\frac{\pi}{2}\right)^{2n}$; 5.16. $f(x) = \sum_{n=0}^{\infty}(-1)^{\frac{n^2+n}{2}}\frac{\sqrt{2}}{2n!}\left(x-\frac{\pi}{4}\right)^n$; 5.17. $f(x) = \sum_{n=0}^{\infty}(-1)^{\frac{n^2-n}{2}}\frac{\sqrt{2}}{2n!}\left(x-\frac{\pi}{4}\right)^n$;

5.18. $f(x) = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}\left(x-\frac{\pi}{2}\right)^{2n+1}$; 5.19. $f(x) = \sum_{n=0}^{\infty}(-1)^{\frac{n^2-n}{2}}\frac{\sqrt{3}}{2n!}\left(x-\frac{\pi}{6}\right)^n$; 5.20. $f(x) = \sum_{n=0}^{\infty}(-1)^{\frac{n^2+n}{2}}\frac{\sqrt{3}}{2n!}\left(x-\frac{\pi}{3}\right)^n$;

5.21. $f(x) = 1 + 2\left(x-\frac{\pi}{4}\right) + \frac{4}{2!}\left(x-\frac{\pi}{4}\right)^2 + \frac{16}{3!}\left(x-\frac{\pi}{4}\right)^3 + \cdots$; 5.22. $f(x) = \sqrt{2} + \sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sqrt{2}}{4^n}\frac{(1\cdot3\cdot5\cdot7\cdot(2n-1))}{2n-1}\frac{(x-1)^n}{n!}$;

6. $f(x) = 2 - \sum_{n=1}^{\infty}((-1)^n+1)\frac{(2n-3)!}{2^n}\frac{x^n}{n!}$, $f^{(4)}(0) = -\frac{15}{8}$, $f^{(51)}(0) = 0$; 7.1. $e^{-x} = \sum_{n=0}^{\infty}(-1)^n\frac{x^n}{n!}$; 7.2. $e^{2x} = \sum_{n=0}^{\infty}\frac{2^n}{n!}x^n$;

7.3. $e^{-3x} = \sum_{n=0}^{\infty}(-1)^n\frac{3^n}{n!}x^n$; 7.4. $a^x = \sum_{n=0}^{\infty}\frac{\ln^n a}{n!}x^n$; 7.5. $e^{x^3} = \sum_{n=0}^{\infty}\frac{x^{3n}}{n!}$; 7.6. $\operatorname{sen} x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$;

7.7. $\cosh x = \sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$; 7.8. $\frac{1}{2+3x} = \sum_{n=0}^{\infty}(-1)^n\frac{3^n x^n}{2^{n+1}}$; 7.9. $\frac{x}{x-1} = -\sum_{n=0}^{\infty}x^n$;

7.10. $\sqrt[3]{1+x^3} = 1 + \frac{1}{3}x^3 - \frac{1}{9}x^6 + \frac{5}{81}x^9 - \frac{10}{243}x^{12} + \cdots$; 7.11. $\frac{x}{\sqrt[3]{8+x}} = 4\left(\frac{1}{8}x - \frac{1}{192}x^2 + \frac{1}{2304}x^3 - \frac{7}{165\,888}x^4 + \frac{35}{7962\,624}x^5 - \cdots\right)$;

7.12. $\operatorname{sen}(2x) = 2x - \frac{4}{3}x^3 + \frac{4}{15}x^5 - \frac{8}{315}x^7 + \cdots$; 7.13. $\operatorname{sen}\left(\frac{x}{2}\right) = \frac{1}{2}x - \frac{1}{48}x^3 + \frac{1}{3840}x^5 - \frac{1}{645\,120}x^7 + \cdots$;

7.14. $\operatorname{sen}\left(x^2\right) = \sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+2}}{(2n+1)!}$; 7.15. $\operatorname{sen}\left(x-\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\sum_{n=0}^{\infty}(-1)^{\frac{n^2-n}{2}}\frac{x^{n+1}}{(n+1)!}$;

7.16. $\cos\left(2x-\frac{\pi}{4}\right) = \sqrt{2}\sum_{n=0}^{\infty}(-1)^{\frac{n^2-n}{2}}\frac{2^{n-1}x^n}{n!}$; 7.17. $\arctan x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1}$;

7.18. $\operatorname{sen}^2 x = x^2 - \frac{1}{3}x^4 + \frac{2}{45}x^6 - \frac{1}{315}x^8 + \frac{2}{14\,175}x^{10} - \cdots$; 7.19. $\ln\left(x+\sqrt{1+x^2}\right) = x - \frac{1}{6}x^3 + \frac{3}{40}x^5 - \frac{5}{112}x^7 + \cdots$;

8.1. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{4n+3}}{(4n+3)(2n+1)!}$; 8.2. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+2}}{(2n+2)(2n+1)!}$; 8.3. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)n!}$; 8.4. $\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)^2}$;

8.5. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^{2n-1}}{(2n-1)n!}$; 8.6. $\sum_{n=1}^{\infty}\frac{x^{2n+1}}{2n+1}$; 9.1. $-\frac{1}{2}$; 9.2. 0; 9.3. $\frac{1}{2}$; 9.4. 1; 9.5. 0; 9.6. 2;

Bibliografía

1. **Purcell, E. - Varberg, D. - Rigdon, S.**: "Cálculo". Novena Edición. Pearson Prentice Hall.
2. **Stewart, J.**: "Cálculo". Grupo Editorial Iberoamericano.

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