

Objetivos a cubrir**Código : MAT4-CDI.2**

- Series numéricas.
- Serie geométrica. Serie telescópica. Convergencia y divergencia de una serie numérica.

1. Escribir la fórmula del término general para las siguientes series

1. $1 + 2 + 3 + 4 + 5 + \dots$
2. $2 + 4 + 6 + 8 + \dots$
3. $1 + 3 + 5 + 7 + \dots$
4. $1 + 2 + 4 + 8 + 16 + \dots$
5. $\frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \dots$
6. $\frac{3}{7} + \frac{4}{8} + \frac{5}{9} + \frac{6}{10} + \dots$
7. $\frac{1}{7} + \frac{1}{9} + \frac{1}{11} + \frac{1}{13} + \dots$
8. $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \dots$
9. $2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \dots$
10. $3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \dots$
11. $2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \dots$
12. $x + x^2 + x^3 + x^4 + \dots$
13. $\sqrt{3} + \sqrt{4} + \sqrt{5} + \sqrt{6} + \dots$
14. $1 - x + x^2 - x^3 + \dots$
15. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$
16. $1 + e^{-1} + e^{-2} + e^{-3} + \dots$
17. $1 - 2 + 4 - 8 + \dots$
18. $\frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[4]{2}} + \dots$
19. $\frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots$
20. $1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots$
21. $4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots$
22. $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt[3]{3}} + \frac{1}{\sqrt[4]{4}} + \dots$
23. $1 + (1.01) + (1.01)^2 + (1.01)^3 + \dots$
24. $\frac{1}{2} + \frac{8}{3} + \frac{27}{4} + \dots$
25. $\frac{1}{2} + \frac{4}{4} + \frac{9}{8} + \frac{16}{16} + \frac{25}{32} + \dots$
26. $\frac{2}{3} + \frac{6}{8} + \frac{24}{15} + \frac{120}{24} + \dots$
27. $\frac{1}{2} + \frac{2}{5} + \frac{3}{10} + \frac{4}{17} + \dots$
28. $-1 + 0 + \frac{3}{2} + 4\frac{\sqrt{2}}{2} + 5\cos\frac{\pi}{5} + 6\frac{\sqrt{3}}{2} + \dots$

2. Verificar que la serie dada es convergente

1. $\sum_{i=1}^{\infty} \left(\frac{1}{i+1} - \frac{1}{i} \right)$
2. $\sum_{i=1}^{\infty} \left(\frac{1}{2i-1} - \frac{1}{2i+1} \right)$
3. $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$
4. $\sum_{n=1}^{\infty} \frac{1}{n(n+3)}$
5. $\sum_{i=7}^{\infty} \frac{1}{i^2+i}$
6. $\sum_{k=1}^{\infty} \frac{3}{k(k+3)}$
7. $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+5)}$
8. $\sum_{k=1}^{\infty} \frac{1}{(k+1)(k+2)}$
9. $\sum_{i=2}^{\infty} \frac{2}{i^2-1}$
10. $\sum_{i=3}^{\infty} \frac{1}{2i^2-6i+4}$
11. $\sum_{n=1}^{\infty} \frac{1}{4n^2-1}$
12. $\sum_{i=1}^{\infty} \frac{2}{i^2+4i+3}$
13. $\sum_{n=1}^{\infty} \frac{1}{9n^2-3n-2}$
14. $\sum_{m=1}^{\infty} \frac{1}{m^2+7m+12}$
15. $\sum_{n=1}^{\infty} \frac{2}{n(n+1)(n+2)}$
16. $\sum_{n=1}^{\infty} \frac{2n+1}{n^2(n+1)^2}$
17. $\sum_{n=1}^{\infty} \frac{1-4n^2-4n}{(2n-1)^2(2n+1)^2}$
18. $\sum_{n=3}^{\infty} \frac{6n}{n^4-5n^2+4}$
19. $\sum_{n=1}^{\infty} \frac{6}{n(n+1)(n+2)(n+3)}$
20. $\sum_{n=1}^{\infty} (\sqrt{n} - \sqrt{n-1})$

3. Verificar que la serie dada es convergente

$$\begin{array}{llll}
 1. & 2 - 1 + \frac{1}{2} - \frac{1}{4} + \frac{1}{8} + \dots & 2. & \sum_{n=0}^{\infty} (0.9)^n \\
 3. & \sum_{n=1}^{\infty} (-0.6)^n & 4. & \sum_{k=1}^{\infty} \frac{2^k - 1}{4^k} \\
 5. & 2 + \frac{3}{2} + \frac{9}{8} + \frac{27}{32} + \frac{81}{128} + \dots & 6. & \sum_{n=0}^{\infty} \frac{2^{n+1}}{3^n} \\
 7. & \sum_{k=1}^{\infty} \left[\left(\frac{1}{3} \right)^{k-1} + \left(\frac{1}{4} \right)^{k-1} \right]
 \end{array}$$

4. Verificar que la serie es divergente

$$\begin{array}{llll}
 1. & \sum_{n=1}^{\infty} \frac{3n}{n+1} & 2. & \sum_{n=1}^{\infty} \frac{n}{2n+3} \\
 3. & \sum_{n=1}^{\infty} \frac{n^2}{n^2+1} & 4. & \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \dots \\
 5. & \sum_{n=0}^{\infty} \left(\frac{4}{3} \right)^n & 6. & \sum_{n=0}^{\infty} 3 \left(\frac{3}{2} \right)^n \\
 7. & \sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}} & 8. & 3 - \frac{9}{2} + \frac{27}{4} - \frac{81}{8} + \frac{243}{16} - \dots \\
 9. & \sum_{n=1}^{\infty} \frac{n!}{2^n} & 10. & \sum_{n=1}^{\infty} \frac{2^n + 1}{2^{n+1}} \\
 11. & \sum_{n=0}^{\infty} 2(-1.03)^n & 12. & \sum_{n=0}^{\infty} 1000(1.055)^n
 \end{array}$$

5. Determine si la serie dada converge o diverge. Si converge encuentre su suma.

$$\begin{array}{llll}
 1. & 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots & 2. & 1 + e^{-1} + e^{-2} + e^{-3} + \dots \\
 3. & 1 + 3 + 5 + 7 + \dots & 4. & \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[4]{2}} + \dots \\
 5. & 1 - 2 + 4 - 8 + \dots & 6. & 1 - \frac{1}{4} + \frac{1}{16} - \frac{1}{64} + \dots \\
 7. & 4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots & 8. & \frac{1}{3} + \frac{2}{9} + \frac{4}{27} + \frac{8}{81} + \dots \\
 9. & 1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt[3]{2}} + \frac{1}{\sqrt[4]{2}} + \dots & 10. & 1 + (1.01) + (1.01)^2 + (1.01)^3 + \dots \\
 11. & \sum_{n=0}^{\infty} \frac{(-1)^n n}{n+1} & 12. & \sum_{n=1}^{\infty} \left(\frac{e}{10} \right)^n \\
 13. & \sum_{n=0}^{\infty} (-1)^n \left(\frac{3}{e} \right)^n & 14. & \sum_{n=0}^{\infty} \frac{3^n - 2^n}{4^n} \\
 15. & \sum_{n=1}^{\infty} (\sqrt{2})^{1-n} & 16. & \sum_{n=1}^{\infty} \left(\frac{2}{n} - \frac{1}{2^n} \right) \\
 17. & \sum_{n=1}^{\infty} \frac{n}{10n+17} & 18. & \sum_{n=1}^{\infty} \frac{\sqrt{n}}{\ln(n+1)} \\
 19. & \sum_{n=1}^{\infty} (5^{-n} - 7^{-n}) & 20. & \sum_{n=0}^{\infty} \frac{1}{1 + \left(\frac{9}{10} \right)^n} \\
 21. & \sum_{n=1}^{\infty} \left(\frac{e}{\pi} \right)^n & 22. & \sum_{n=1}^{\infty} \left(\frac{\pi}{e} \right)^n \\
 23. & \sum_{n=0}^{\infty} \left(\frac{100}{99} \right)^n & 24. & \sum_{n=0}^{\infty} \left(\frac{99}{100} \right)^n \\
 25. & \sum_{n=0}^{\infty} \frac{1 + 2^n + 3^n}{5^n} & 26. & \sum_{n=0}^{\infty} \frac{1 + 2^n + 5^n}{3^n} \\
 27. & \sum_{n=1}^{\infty} \sqrt[3]{2} & 28. & \sum_{n=0}^{\infty} \frac{7 \cdot 5^n + 3 \cdot 11^n}{13^n} \\
 29. & \sum_{n=1}^{\infty} \frac{n^2 - 1}{3n^2 + 1} & 30. & \sum_{n=1}^{\infty} \frac{2n}{\sqrt{4n^2 + 3}} \\
 31. & \sum_{n=1}^{\infty} \sin^n 1 & 32. & \sum_{n=1}^{\infty} \left[\left(\frac{7}{11} \right)^n - \left(\frac{3}{5} \right)^n \right] \\
 33. & \sum_{n=1}^{\infty} \tan^n 1 & 34. & \sum_{n=1}^{\infty} (\arcsen 1)^n \\
 35. & \sum_{n=1}^{\infty} (\arctan 1)^n & 36. & \sum_{n=1}^{\infty} \arctan n \\
 37. & \sum_{k=1}^{\infty} \ln \left(\frac{k}{k+1} \right) & 38. & \sum_{n=1}^{\infty} \frac{3^n + 2^n}{6^n} \\
 39. & \sum_{n=1}^{\infty} \frac{1}{n(n+1)(n+2)}
 \end{array}$$

$$40. \sum_{n=1}^{\infty} \frac{6n^2 + 2n - 1}{n(n+1)(4n^2 - 1)} \quad 41. \sum_{n=1}^{\infty} \arctan\left(\frac{1}{2n^2}\right) \quad 42. \sum_{n=1}^{\infty} \left(\frac{1+n^2}{1-3n^2}\right)^2$$

6. Encuentre el conjunto de todos los valores de x para que la serie dada es una serie geométrica convergente.

$$1. \sum_{n=1}^{\infty} (2x)^n \quad 2. \sum_{n=1}^{\infty} \left(\frac{x}{3}\right)^n \quad 3. \sum_{k=1}^{\infty} \left(\frac{x}{2}\right)^{k-1} \quad 4. \sum_{k=1}^{\infty} (x+1)^k \quad 5. \sum_{n=1}^{\infty} (x-1)^n$$

$$6. \sum_{n=1}^{\infty} \left(\frac{x-2}{3}\right)^n \quad 7. \sum_{n=1}^{\infty} \left(\frac{x^2}{x^2+1}\right)^n \quad 8. \sum_{n=1}^{\infty} \left(\frac{5x^2}{x^2+16}\right)^n \quad 9. \sum_{n=1}^{\infty} \left(\frac{1}{x}\right)^{n-1}$$

7. Demuestre que $\sum a_n$ diverge y c es una constante diferente de cero, entonces $\sum ca_n$ diverge

8. Suponga que $\sum a_n$ converge y que $\sum b_n$ diverge. Demuestre que $\sum (a_n + b_n)$ diverge.

Respuestas

- 1.1. $\sum_{n=1}^{\infty} n$; 1.2. $\sum_{n=1}^{\infty} 2n$; 1.3. $\sum_{n=1}^{\infty} (2n-1)$; 1.4. $\sum_{n=0}^{\infty} 2^n$; 1.5. $\sum_{n=1}^{\infty} \frac{n}{n+1}$; 1.6. $\sum_{n=3}^{\infty} \frac{n}{n+4}$; 1.7. $\sum_{n=3}^{\infty} \frac{1}{2n+1}$;
1.8. $\sum_{n=1}^{\infty} \frac{1}{n^2}$; 1.9. $\sum_{n=0}^{\infty} \frac{3^n}{2^{2n-1}}$; 1.10. $\sum_{n=3}^{\infty} 3\left(-\frac{3}{2}\right)^n$; 1.11. $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{n-1}}$; 1.12. $\sum_{n=1}^{\infty} x^n$; 1.13. $\sum_{n=3}^{\infty} \sqrt{n}$;
1.14. $\sum_{n=0}^{\infty} (-1)^n x^n$; 1.15. $\sum_{n=0}^{\infty} \frac{1}{3^n}$; 1.16. $\sum_{n=0}^{\infty} e^{-n}$; 1.17. $\sum_{n=0}^{\infty} (-2)^n$; 1.18. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{2}}$; 1.19. $\sum_{n=0}^{\infty} \frac{2^n}{3^{n+1}}$;
1.20. $\sum_{n=0}^{\infty} \left(-\frac{1}{4}\right)^n$; 1.21. $\sum_{n=0}^{\infty} \frac{4}{3^n}$; 1.22. $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}}$; 1.23. $\sum_{n=0}^{\infty} (1.01)^n$; 1.24. $\sum_{n=1}^{\infty} \frac{n^3}{n+1}$; 1.25. $\sum_{n=1}^{\infty} \frac{n^2}{2^n}$;
1.26. $\sum_{n=1}^{\infty} \frac{(n+1)!}{n}$; 1.27. $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$; 1.28. $\sum_{n=1}^{\infty} n \cos\left(\frac{\pi}{n}\right)$; 2.1. Conv. a -1 ; 2.2. Conv. a 1 ; 2.3. Conv. a 1 ;
2.4. Conv. a $\frac{11}{18}$; 2.5. Conv. a $\frac{1}{7}$; 2.6. Conv. a $\frac{11}{6}$; 2.7. Conv. a $\frac{17}{60}$; 2.8. Conv. a $\frac{1}{2}$; 2.9. Conv. a 1 ;
2.10. Conv. a $\frac{1}{2}$; 2.11. Conv. a $\frac{1}{2}$; 2.12. Conv. a $\frac{5}{6}$; 2.13. Conv. a $\frac{1}{3}$; 2.14. Conv. a $\frac{1}{4}$; 2.15. Conv. a $\frac{1}{2}$;
2.16. Conv. a 1 ; 2.17. Conv. a -1 ; 2.18. Conv. a $\frac{3}{4}$; 2.19. Conv. a $\frac{1}{30}$; 2.20. Div.; 3.1. Conv. a $\frac{4}{3}$;
3.2. Conv. a 10 ; 3.3. Conv. a -0.375 ; 3.4. Conv. a $\frac{2}{3}$; 3.5. Conv. a 8 ; 3.6. Conv. a 6 ; 3.7. Conv. a $\frac{17}{6}$;
5.1. Conv. a $\frac{3}{2}$; 5.2. Conv. a $\frac{e}{e-1}$; 5.3. Div.; 5.4. Div.; 5.5. Div.; 5.6. Conv. a $\frac{4}{5}$; 5.7. Conv. a 6 ;
5.8. Conv. a 1 ; 5.9. Div.; 5.10. Div.; 5.11. Div.; 5.12. Conv. a $\frac{e}{10-e}$; 5.13. Div.; 5.14. Conv. a 2 ;
5.15. Conv. a $2 + \sqrt{2}$; 5.16. Div.; 5.17. Div.; 5.18. Div.; 5.19. Conv. a $\frac{1}{12}$; 5.20. Div.;
5.21. Conv. a $\frac{e}{\pi-e}$; 5.22. Div.; 5.23. Div.; 5.24. Conv. a 100 ; 5.25. Conv. a $\frac{65}{12}$; 5.26. Div.; 5.27. Div.;
5.28. Conv. a $\frac{247}{8}$; 5.29. Div.; 5.30. Div.; 5.31. Conv. a $\frac{\text{sen } 1}{1-\text{sen } 1}$; 5.32. Conv. a $\frac{1}{4}$; 5.33. Div.;
5.34. Div.; 5.35. Div.; 5.36. Div.; 5.37. Div.; 5.38. Conv. a $\frac{1}{2}$; 5.39. Conv. a $\frac{1}{4}$; 5.40. Conv. a 2 ;
5.41. Div.; 5.42. Div.; 6.1. $x \in \left(-\frac{1}{2}, \frac{1}{2}\right)$; 6.2. $x \in (-3, 3)$; 6.3. $x \in (-2, 2)$; 6.4. $x \in (-2, 0)$;
6.5. $x \in (0, 2)$; 6.6. $x \in (-1, 5)$; 6.7. $x \in \mathbb{R}$; 6.8. $x \in (-2, 2)$; 6.9. $x \in (-\infty, -1) \cup (1, \infty)$;

Bibliografía

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