Homework 3

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Knights and Knaves:

Case	P	Q
1	Knight	Knight
2	Knave	Knight
3	Knight	Knave
4	Knave	Knave

- Knave: always tells lie
- Knight: always tells truth
- If P is a Knight then whatever he/she says is true. If P is a Knave, whatever he/she says is false. If P was a Knight, case 1 can't happen, but 3 can. if P was a knave, both case 2 and 4 couldn't have happen because no matter what the knave says, will be false. Row 3 is the only possible case.

Case	P	Q
1	Knight	Knight
2	Knave	Knight
3	Knight	Knave
4	Knave	Knave

- If P is a Knight then what he says would be true.. If P is a knave then what he says would be False. If P was a knight, case 1 and 3 couldn't have happened. If P was a knave, 4 couldn't have happened because whatever they say is a false. This leave case 2 being the only possible case.

Logical Identities:

- $-(p \to (q \to p))$
  - Implication Law: $p \to q \equiv \neg q \vee p$. so, $\neg(p \to (\neg q \vee p)$
  - Implication Law: $\neg(\neg p \vee (\neg q \vee p))$
  - De Morgans's Law: $\neg(a \vee b) \equiv \neg a \wedge \neg b$, $\neg p \equiv a \ and \ (\neg q \vee p) \equiv b$
    - $\neg(\neg p) \wedge \neg(\neg q \vee p)$
  - De Morgans's Law: $\neg(a \vee b) \equiv \neg a \wedge \neg b$,

        * $\neg q$ is a
        * p is b
        * $\neg(\neg p) \wedge \neg(\neg q) \wedge \neg p$
    - double negation law: $\neg(\neg p) \equiv p$
        * $p \wedge q \wedge \neg p$
        * $p \wedge \neg p \wedge q$
    - $p \wedge \neg p \equiv F$, thus
    - $FALSE \wedge q \equiv q$

- $-p((p \wedge q) \rightarrow (q \vee p))$
    - Let $(p \wedge q)$ be a, $(q \vee p)$ be b
    - Implication Law, $p \rightarrow q \equiv \neg p \vee q$
        * $\neg a \vee b$
    - Substituting the values of a and b,
        * $\neg(p \wedge q) \vee (q \vee p)$
    - De Morgan's Law: $\neg(p \wedge q) \equiv \neg p \vee \neg q$
        * $\neg p \vee \neg q \vee (q \vee p)$ ——— 1
    - associative law: $(p \vee q) \vee r \equiv p \vee (q \vee r)$
    - Rearranging 1,
        * $(\neg p \vee p) \vee (\neg q \vee q)$
    - negation laws, $p \vee \neg p \equiv T$
    - $TRUE \vee TRUE \equiv TRUE$

Logical Equivalences

- $p \rightarrow (q \rightarrow r)$ and $(p \wedge q) \rightarrow r$
    - Answer: $p \rightarrow (q \rightarrow r) \neq (p \wedge q) \rightarrow r$

- $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$
    - Answer: $p \rightarrow (q \rightarrow r) \neq (p \rightarrow q) \rightarrow r$

Logical Consequence

- 1.

  Jimmy is smart

  Smart People are rich

  ————————————-

Jimmy is Rich

Answer: Valid: Jimmy is rich
Jimmy is smart

Jimmy then smart

Jimmy $\rightarrow$ smart
Smart People are rich

Smart then rich

Smart $\rightarrow$ rich

Jimmy $\rightarrow$ rich $\equiv$ Jimmy is rich

- 2.
Islands are surrounded by water

Puerto Rico is surrounded by water

Puerto Rico is an island

Answer: Not Valid: We cannot say Puerto Rico is an island is always true.
Islands are surrounded by water

Islands $\rightarrow$ surrounded by water

Puerto Rico is surrounded by water

Puerto Rico $\rightarrow$ surrounded by water

Puerto Rico is not always an island